Advances in Japanese Business and Economics 24

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International Governance and Risk Management

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Preface

This book develops a theory of risk management in an alliance with public goods. There has been an explosion of research in the last 10 years in a broad variety of areas in this literature, and the time seems right for a survey of this work. This book integrates likelihood of loss, magnitude of loss, and isolation from loss into a consolidated model. It extends existing concepts of individual risk management by a single person to decision theory for an entire country, managed by government bureaucracy and lodged in a universe of overlapping alliances. We also present the classic results to place the new developments in context.

This book is appropriate for advanced undergraduates, for graduate students who wish to learn the latest research in this area, and for practitioners who want to broaden their knowledge outside their own area of expertise. We present the background for each result and try to give the reader a feel for how a particular area of the literature developed. The technical results are provided and an intuitive explanation for them is also given. Each chapter is reasonably self-contained. A variety of models are studied in each chapter and are fully described, so the reader can open the book to any chapter and begin reading without missing any of the notation or technique.

This book was almost 3 years in the making. During the long gestation period of this book, we are indebted to a number of colleagues and students for helpful conversations along the way. In particular, we would like to thank Junichi Itaya, Keigo Kameda, Raymond Batina, Robin Boadway, Kai Konrad, Todd Sandler, and Richard Cornes. Professor Richard Cornes gave us his insightful comments on our research and sadly passed away in August 2015. We also wish to express deep gratitude to Prof. Ryuzo Sato, the Editor in Chief of the Advances in Japanese Business and Economic series. And last but not least, we wish to express gratitude to the editorial staff at Springer Japan for their help at various stages of the production of this book.

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Chapter 1
Introduction and Summary of the Book

1.1 Purpose of the Book

In the aftermath of World War II and height of the Cold War, the normative approach to the economics of alliances seemed paramount. Historically, an alliance formed when academic economics strived to assist the beleaguered civil and uniformed servants who were trying to formulate and execute policy. However, due in part to the Vietnam War and especially by the end of the Cold War, the gap between academic economics and policy implementation widened.

Academic focus has shifted and a large body of positive work—normatively neutral—on the political economy of international security has come into being; the number of journals and frequency of conferences has increased greatly, whereas the impact of economists on security policy decisions has declined. However, with the acceleration of globalization and a shift in the paradigm of international security toward trade and finance, the pendulum may swing back. This book will offer examples of both positive and normative analyses of alliances relevant to our era's burgeoning security problems against adversity.

In this book, we demonstrate how the economics of insurance, risk reduction, and damage control or limitation can be combined with concepts of collective choice and behavior to improve the analysis of the escalating threats and emergency costs faced by alliances throughout the world.

To do so, the book develops a theory of risk management with public goods, integrating the likelihood of loss, magnitude of loss, and isolation from loss into a consolidated model of alliances. It extends existing concepts of individual risk management by a single person to decision theory for an entire country, managed by a government bureaucracy and lodged in a universe of overlapping alliances.

Moreover, we uncover a tendency inherent in any bureaucracy for policy coordination in the realm of risk control to fail because of misunderstanding, misperception, disinterest, or perverse incentives. Understanding such incentives is essential to any sort of progress in the risk management of proliferating national and global threats.

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Olson and Zeckhauser (1966) developed an economic theory of alliances by characterizing deterrence as a pure public good. In particular, economists' models of voluntary public good (VPG) provision with many agents became very popular and widely used in the analysis of these topics. In this book, we first review the analytical results of the economics of alliances based on the conventional VPG models. However, although they are useful, they have not been well extended to understand the consequences of differences in risk aversion in this risk management context in which common defenses will reduce common hazards.

In their seminal contribution, Ehrlich and Becker (1972) introduced the terms "self-insurance" for effort that reduces the size of a loss and "self-protection" for effort that reduces the probability of a loss. These two measures are useful concepts in analyzing the risk management of alliances. We thus extend existing analyses of self-insurance and self-protection to the realm of public goods that countries may implement at a national level in pursuit of their security. That is, we provide an analysis of odds-improving self-protection and cost-alleviating self-insurance when they yield collective benefits to allied groups, such as the alliances of nations, for whom risks of loss are public bads and prevention of loss is a public good.

Self-insurance aims to reduce the cost of emergencies. This reduction may require the use, or threat or promise of the use, of defensive military weapons. Alternatively, self-protection aims to reduce the chances of loss. It may also require the use of offensive and defensive military weapons.

Regarding policy implications for Japan, the Constitution of Japan limits the use of defensive measures, although Japan and the US have formed a military alliance. This places Japan in the "political corner solution" of providing only self-insurance even when the optimal level of self-protection is positive. Recently, due to increasing threats in East Asia, the current Abe government intends to change the interpretation of the constitution so Japan can provide a full range of self-protection as well as self-insurance, which is a politically divisive issue. It is thus important to clarify the nature of "corner solutions" to evaluate this delicate political issue. This book investigates how corner solutions in self-protection and self-insurance can occur in alliances and explores the alliances' welfare implications. With the prospect of such constitutional change, this book thus becomes of special relevance to Japan's national security policy.

Finally, we believe the material contained in this book is especially useful for students who wish to learn some of the tools and techniques used by public finance economists and microeconomists interested in public economics, as well as some of the traditional literature on trade and risk management. We make use of standard techniques such as constraint maximization, differential incidence, and comparative statics analyses in general equilibrium models. We cover some traditional areas in public economics along with those currently receiving attention, including externalities, uncertain environments, and privately produced public goods, as well as several less traditional topics, such as security spending, emergency costs, alliances, and arms races. The derivations of the main results are included in the text to facilitate learning the technique.

1.2 Basic Concepts

1.2.1 Public Goods

In this book, the concept of public goods is important because security spending in an alliance has a spillover effect. We develop a variety of theoretical models of public goods. The classic case is of a pure public good, in which the marginal cost of providing another agent with the good is zero and no one can be excluded from enjoying its benefits. Examples include national defense, clean air and water, air travel safety, homeland security, peacekeeping operations, vaccinations that improve the general health of the population, and mosquito spraying to eradicate the West Nile virus (see, e.g., Batina and Ihori 2005).

Clearly, changes in technology can alter the nature of a public good. For example, a television signal in the 1950s can be interpreted as a pure public good because technology that can scramble signals did not exist at that time. The subsequent development of such technology created the possibility of exclusion. Once exclusion can be practiced, the private sector can provide the good, charge a price for it, and earn a normal rate of return on its investment. Moreover, it is possible for the private sector to provide a public good even in the absence of exclusion if another method of financing the good can be found. For example, advertising is used to pay for television programs in many countries, whereas the government provides the funding in others.

Some public goods are privately provided. Charity is a prime example and contributing to private education is another. In an alliance, member countries non-cooperatively contribute to an international public good within the alliance. Its benefit may be enjoyed by all allies but will not spillover beyond the alliance. In a non-cooperative situation, allied countries provide the good for the sake of their own interest. In this case, a framework of privately provided public goods becomes relevant to the analysis of the alliance.

There are numerous ways of modeling such behavior. Typically, one can assume that the individual cares about the total amount contributed. In this case, a framework of pure public goods under Nash equilibrium should be relevant. Alternatively, one can assume that the donor cares about her own contribution as well as the total, because she receives a "warm glow" from the act of donating. The difference between the two models is substantial. For example, in the former model, there is an externality across agents, whereas in the latter, this is absent if all donors only obtain a warm glow from donating. There are other possible ways of modeling privately provided public goods. In this book, however, we will mainly discuss the standard models of pure public goods.

In reality, many public goods are impure because they exhibit exclusion, congestion, or both. There are many examples of such goods at the local level, including city streets, sidewalks, and traffic lights. These goods have been labeled local public goods (LPGs) primarily because they are typically provided locally and the benefits tend to be local as well. In addition, there are other goods provided by the private sector that have public good characteristics. Examples include so-called "club" goods, such as tennis, golf, or swimming clubs. An alliance formation may therefore be regarded as a club good.

For many such goods, the quality of the facility is a pure public good, but there may also be congestion in the facility and this raises the issue of the size and characteristics of the group sharing it. Indeed, the notion of a sharing group is more general than this. In a sense, when one shops at the same grocery store as others, for example, one is in effect joining a "club". An alliance may therefore be regarded as a formation of clubs.

Local public goods and clubs share many features in common. However, the main difference between them in our view is that exclusion can be implemented with regard to clubs but not with respect to local public goods. This allows private "club firms" to charge those who join and use the club a fee. Establishing and maintaining alliances have similar features as club firms because allied countries only protect their member countries.

Let the representative country's preferences be represented by a utility function of the form u(x, G), where x is a vector of private goods and G is a public good, or possibly a vector of public goods. We will assume the utility function is typically quasi-concave, twice continuously differentiable, and monotone, increasing in each argument. The cost of the public good may depend on the number of agents enjoying the benefits of the public good when there is congestion.

1.2.2 Alliances

Historically or economically, some countries have common interests and/or face common threats. These countries may form an alliance to cope with common adversity from potential enemies or disasters. By creating an alliance, each member country may enjoy the benefit of protection, which comes from total security spending provided by all member countries. In this sense, security spending resembles a local public good with excludability and non-rivalness within the alliance.

In this book, we do not explicitly investigate how an alliance is formed. Rather, we investigate the economic consequences of an alliance coping with adversity.

As emphasized by Olson and Zeckhauser (1966), if defense is purely public among allies, the analytical results of VPG models can be applied. Among others, the following hypotheses should be noted.

First, defense burdens are expected to be shared unevenly among allies, which is called the "exploitation" hypothesis. Second, defense expenditures are predicted to be at inefficient or suboptimal levels in relation to a Pareto-optimal standard. This is the well-known property of a Nash equilibrium in the private provision of public goods. Third, there is no need to restrict alliance membership if the marginal cost of enlarging the size is zero. Fourth, an ally's optimal defense spending depends on relative prices, the ally's income, the level of its defense expenditures, and the perceived threat. We explain these results in the following chapters.

Moreover, we explicitly incorporate uncertainty or emergency risks into the theoretical framework and derive new results. This is a unique feature of our book.

1.2.3 Risk

Risk may be described by states and probabilities. The standard formulation is to specify a probability distribution function, which presents the relation between states and probabilities. In this book, we only consider two states: a good state and a bad state. This is a simple specification but we can generalize the model into a case of continuous states. A good state captures a state of peace and a bad state indicates a state of war. We assume that all allied countries know the true probability of each state. In Chap. 7, we also consider the possibility of misperceptions.

For many risks, people can buy insurance to protect against possible emergency losses. Moreover, they may take risk reducing actions. Although Ehrlich and Becker (1972) introduced self-insurance and self-protection in a private good setting of an individual's maximization problem, these activities may also benefit member countries of an alliance in a non-rival way. In fact, the security spending of member countries can be regarded as a self-protection and/or self-insurance act because it provides risk reduction benefits. Security spending in an alliance can be regarded as insurance against risks such as shipwrecks and floods.

The present book analyzes the insurance character of public goods when the risks targeted by public goods are correlated. The question whether risks are independent or correlated is crucial. For example, if an island experiences a significant earthquake, nearly all its inhabitants will be affected. In this example, risks are highly correlated. By contrast, we can consider cases in which the risks targeted by public goods are uncorrelated, as with lighthouses or isolated disasters. For instance, the risk of burglary affects all individuals in the population but is not correlated among individuals. Some military threats affect allied countries differently. In this book, we assume that a state of good or bad occurs with the same probability for all member countries within the same alliance and, hence, those risks are highly correlated. Security spending by any countries in the alliance can reduce the probability of a bad state and/or emergency costs in a bad state and, hence, it has the nature of a pure public good.

1.3 Outline of the Book

Here, we briefly summarize the contents of the book's chapters.

Chapter 2

Chapter 2 offers various examples of positive and normative analyses of national security economics. Positive analyses attempt to understand the economic origins of conflicts among nations and the economic foundations of success or failure in these conflicts. Normative economic analyses have a shorter pedigree, as they descend from the early days of the Cold War and the primacy of Soviet-American conflict.

When the pendulum moved toward a more positive focus, the reason was not that the Cold War normative issues had disappeared and been replaced. Rather, a new class of challenges emerged, layered over or parallel to earlier threats of great power conflict and competition. Meanwhile, because of technological developments and the globalization of the past quarter century, the number and range of security challenges have proliferated. Today, there are more threats and their possible resolution is substantially economic, as the present structural crisis in the world economy testifies.

To illustrate both positive and normative economics utilization, Chap. 2 develops two examples of analytic foundations of security studies and economic methods in the management of security challenges. The first illustrates the strivings of a political economy in general and abstract terms to describe the self-seeking behavior of nations—their competition for wealth, land, and power—and to derive the resultant equilibrium economic configuration of states in a nation-state system. The second provides an example of using economics to optimize policies against threats to energy or other resource security.

As a discipline, all economics contain both normative and positive elements. Applied to international security, economics seems to cycle between these two approaches with an emphasis on positive study dominating the past generation. Given the current crises in the world economy and financial system, however, one might reasonably expect the pendulum to swing back to an emphasis on normative analyses.

We show that rational calculation may exclude war for two distinct reasons. First, for the given offense-defense technologies, no country may have the sufficient resources to defeat or capture another. In this case, every country can mount a defense that physically or fiscally precludes its capture. We call this "Security by Making Conquest Unaffordable or Infeasible". Second, although affordable , conquest may be undesirable, in that it costs more than it is worth. Countries may mount sufficient defenses to make the costs to a victor exceed the benefits that capture will yield. We call this "Security by Denial of Benefits".

This chapter also shows how the possibilities for insurance against trade disruption (including "perfect" insurance) influences prescriptions as to the best mix of policies to manage the risk of international supply disruption in surprising and unambiguous ways. Specifically, in the course of describing this role of economics in security, we demonstrate that when the chance of trade disruption increases or the global risk aversion of a country declines, as instruments of risk management, a country should place greater reliance on stockpiling over artificial protection of domestic producers.

Chapter 2 thus presents examples of the substance and style of analysis appropriate in each of these arenas. For the positive analysis, we investigate how economic productivity and trade, military technology and strategy, and the political economy of governance can combine to determine a country's choice between peaceful trade and investment versus predation and conquest of others. For the normative analysis, we analyze the alternative means a country may employ to shelter itself from trade disruption.

In the appendix of Chap. 2, we expand the analysis of a country's preparation and response to an emergency disruption when it can purchase or sell insurance from a trusted alliance partner. That is, we select one of the alternative instruments examined in the previous sections of the chapter to focus on incentives among allies. Our results suggest that an increase in the emergency cost has different spillover effects, depending on where it occurs. If the penalty-income ratio rises in the demand country, it has a positive spillover effect on the supply country. On the other hand, if the penalty ratio rises in the supply country, it has a negative spillover effect on the demand/buying country. The reason is that an increase in the penalty ratio in either country will raise the cost of insurance. These theoretical results seem intuitively and practically plausible.

Chapter 3

An interesting application of non-cooperative private funding of public goods arises in international security and national defense. In their classic article on the economic theory of alliances, Olson and Zeckhauser (1966) explored why countries may allocate some fraction of national income to international (or regional) public goods to reduce regional and international tension and avoid random emergency costs.

One public good within an alliance may capture the benefits of risk sharing. Thus, by confederating in alliances each allied country can enjoy the benefits of its mutual insurance as an international public good as long as they belong to the alliance. Still, the non-cooperative levels of spending on national security by alliance members are sub-optimal from the perspective of the alliance as a whole.

In Chap. 3, we investigate how an international wealth or income transfer between allies influences the provision of public goods and the welfare of the allies. This is related to the realities of international defense alliances involving Japan and the United States. For example, in 1990, Japan offered $4 billion to the allied effort in the Gulf War and contributed an additional $9 billion in 1991. The $9 billion may be regarded as a transfer from Japan to the United States.

In Chap. 3, we present three models to examine this issue. A crucial assumption common to these models is that the public good is voluntarily and non-cooperatively provided by the allied countries' governments.

In a two-country model of international trade it is well known that international transfers would have the normal impacts: the donor loses and the recipient gains. We show that an international transfer between allies does not necessarily benefit the recipient or harm the donor in a three-country model. International transfers might have a paradoxical impact on the welfare of the donor and recipient. We refer to the situation in which both the donor and recipient of the transfer simultaneously benefit/lose from the transfer as a "weak paradoxical result". We also describe the surprising result in which a transfer of income increases the donor's welfare while decreasing the recipient's welfare as a "strong paradoxical result".

To apply this insight to the case of public goods, the assumption of an impure benefit of the public good is crucial. The response of the third country is also important. Spending on national defense is a good example of impure public goods in which one country's supply of international public goods may well be an imperfect substitute for another country's supply. It seems reasonable to assume that although countries have a common interest in providing the international public goods, their preferences over the public goods may not necessarily be the same. Alliances may consist of more than two countries and, hence, one allied country's security spending may not affect other countries in the same way.

The conventional neutrality result means that when agents voluntarily contribute to a public good, any income transfer between contributors affects neither the provision of the public good nor the welfare of the agents (provided all partners maintain some positive contribution both before and after the transfer). Because different countries' expenditures on these goods are imperfect substitutes, the neutrality result will not always apply here. Transfers affect both the equilibrium level of public good provision and the utility of the countries giving and receiving transfers, as well as the third country. We show that under certain conditions, such transfers have a strong paradoxical result: the reaction of the third country to an expenditure causes the giver's utility to rise and the receiver's to fall.

The appendix of Chap. 3 explicitly incorporates uncertainty into a model of an international public good to cope with some risks as an extension of the appendix of Chap. 2. With both a mutual insurance market and voluntary provision of an international public good, expected welfare is always equalized among allied partners, irrespective of differences in incomes, the penalty ratios, or the type of insurance as long as preferences are the same. Because welfare is equalized, we always have the weak version of the transfer paradox: both the receiving country and giving country lose when income is transferred to the country with the higher penalty ratio. We show that welfare implications of such transfers crucially depend on how protection against national emergencies is provided.

Chapter 4

This chapter investigates how the cooperation of members of an allied bloc influences their welfare when that bloc opposes another bloc. This is related to cooperative and non-cooperative defense spending in confronting blocs such as the North Atlantic Treaty Organization (NATO) and the Warsaw Treaty Organization (WTO) in the Cold War. By addressing this issue, we explore how the voluntary provision of an international public good by allied countries protects against threats from opposing alliances.

The security of an alliance is affected by the response of other alliances. The analytical model of this chapter is constructed to examine the interactions between two conflicting allied blocs. When one bloc increases its security expenditures, the other bloc may respond by increasing its spending. We call this effect an "arms race effect".

If cooperative behavior attains better outcomes than non-cooperative behavior, freeriding incentives may be an issue. However, in reality, we often observe the non-cooperative provision of security spending within an alliance. Thus, if non-cooperative behavior attains better outcomes than cooperative behavior within an alliance, freeriding incentives may not be a serious problem. We explore this issue by considering heterogeneity of preferences between conflicting blocs. We assume that blocs differ in their marginal valuations of security. We say that if one bloc's marginal valuation of security is higher than the other bloc's marginal valuation, the former agent has a "vital" interest in security and the latter agent has a "latent" interest. We refer to the former as the "vital bloc" and the latter as the "latent bloc".

Next, we assess the natural conjecture that a cooperative strategy is desirable for the countries in the vital bloc and a non-cooperative strategy is desirable for those in the latent bloc. Our analysis shows that the intuitive, natural conjecture can be mistaken. Assuming a significant arms race effect for the vital bloc but not for the latent bloc, the effect may dominate for the vital bloc. Otherwise, the welfare gains from cooperation within the alliance, which we refer to as a "cooperation effect," may dominate for the latent bloc. Accordingly, we may say that if NATO had a vital interest in the security issue during the Cold War, a non-cooperative outcome is desirable because the arms race effect can dominate the cooperation effects in the NATO bloc.

Chapter 5

Chapter 5 investigates the impact of self-protection on alliances and considers the impact of self-insurance as well as self-protection on security spending. We extend the existing analyses of self-insurance and self-protection in the case of private optimization to the case in which allied countries may implement them at a national level in pursuit of security. That is, we provide an analysis of odds-improving self-protection and cost-alleviating self-insurance when allied countries yield collective benefits to allied groups, such as alliances of nations, for whom risks of loss are public bads and prevention of loss is a public good.

As explained in Chap. 3, the economists' VPG models with many agents have not been well extended to understand the consequences of differences in risk aversion when common defenses will reduce common hazards. A short-coming of the conventional public goods model is that it assumes only constant returns in production. The provision of public good is given as a summation of contributions made by all agents. Diminishing marginal returns in the provision of the public good were, thereby, assumed away. We believe that scale considerations are appropriate for an entire country along an extensive margin as well as inclusion of other cooperating factors of production, as in Ehrlich and Becker (1972). This chapter emphasizes how diminishing returns in risk improvement can be folded into income effects.

Our simple solution to the problem of diminishing returns is to assume that the provision of public good is given by a non-linear transformation of the summation of contributions. This allows us to determine whether Olson's "exploitation of the great by the small" holds when the public good is risk reduction under diminishing returns and how differences in risk aversion influence equilibrium.

Furthermore, we show that the degree of common risk, relative wealth, and variability of risk aversion all interact in a novel and hitherto unrecognized fashion. In particular, we demonstrate that when risk aversion increases with income and risk is low (high), self-protection tends to be an inferior good for low (high) income agents. Conversely, if risk aversion decreases with income and risk is low (high), self-protection tends to be inferior for agents with high (low) income.

We consider that when agents form a group for public good provision, *ipso facto*, the full income of each agent increases (possibly dramatically). It follows, as we show, that these properties can have major and possibly quite unwelcome effects on the nature and stability of group behavior and thus of the Nash VPG solution.

In this Chap. 5, we first examine the income effect of self-protection. The income effect implies that whether the protection is inferior or normal depends on the risk aversion characteristics of underlying utility functions, the interaction between these, the level of risk, and marginal effectiveness of risk abatement. We demonstrate how public good inferiority is highly likely when the good is "group risk reduction" or self-protection. We also discover a natural or endogenous limit on the size of a group and the amount of risk-controlling outlay it will provide under Nash behavior. We call this limit an "inferior goods barrier" to voluntary risk reduction.

Because of interdependencies between risk aversion and degree of risk, the possibility of instability and multiple equilibria is magnified. Such interdependencies can also lead to shifting non-convexities in preference maps and, therefore, to shifting corner solutions and the implied instabilities.

In the second half of this chapter, we incorporate self-insurance into the model and investigate its income effect. That is, we extend the single public good model of self-protection to a two public good model of self-protection and self-insurance. By doing so, we discover a hitherto unrecognized tendency for misallocation between self-protection and self-insurance when both are available and considered together. Because of external effects running from self-protection to self-insurance, governments ruled by myopic bureaucracies that are trying to find the right balance can face incentives that encourage extreme, self-inflicted moral hazard to the detriment of self-protection. We demonstrate an inherent potential for "unstable conflict" in which centralized or decentralized specialization of the provision of these public goods occurs.

This chapter thus shows how, in a multi-country model, rather innocuous assumptions concerning countries' preferences lead to pervasive goods inferiority—at least for self-insurance. Evidently, many international problems in the world today resemble this common threat/loss management problem.

Chapter 6

Chapter 6 investigates how allies bear the burden of self-insurance and self-protection by examining the model of simultaneous voluntary provision of those public goods constructed in Chap. 5.

When self-insurance and self-protection are provided as pure public goods, all allies have freeriding incentives. Thus, public goods are not necessarily provided by all allied countries. In our two-country model, a single country may provide both public goods and the other ally may ride free or provide at most one public good. As we discuss in this chapter, the configuration of the contributors—who contributes to which public goods—is critically important in the sharing of the burden of risk management.

In the first half of this chapter, we theoretically show how countries share the burden of risk management. In the conventional model of voluntarily provided public goods developed by Olson and Zeckhauser (1966), the country endowed with the higher income contributes more than the country with the lower income assuming both public goods and private goods are normal goods and both countries have identical preferences. The difference in contribution is identical to the income difference, which implies that the high-income ally consumes the same amount of the private good and enjoys the same level of utility as the low-income ally.

In our framework, countries face the risk of a disastrous event and differ both in their income and loss in a bad state. Both these two differences may a source of exploitation. However, the conventional exploitation of the high-income agent by the low-income agent might not necessarily hold. We show how the nature of exploitation critically depends on the configuration of the equilibrium contributions. If the Nash equilibrium is interior so that each country contributes to both public goods, the difference in total security expenditures is precisely equal to the difference in national income, as in the conventional voluntary provision model. This situation is interpreted as an exploitation of a high-income country by a low-income country.

In contrast, in a self-insurance freeriding equilibrium, in which the high-income country contributes to both public goods and the low-income country freerides in self-insurance, the difference in total security expenditures is greater than the difference in the disposable income in the bad state. Thus, the exploitation of the high-income country by the low-income country is strengthened.

But, in a self-protection free-riding equilibrium, in which the high-income country contributes to both public goods and the low-income country freerides in self-protection, the exploitation of the high-income country by the low-income country is mitigated. The difference in total security expenditures is either lower than or as high as that of the national income. As a result, the high-income country consumes either more than or as much as the low-income country. The expected welfare of the high-income country is either higher than or as high as that of the low-income country. In this sense, the low-income country exploits the high-income country less than it does in the interior equilibrium.

Next, we investigate a decentralized specialization equilibrium, in which the high-income country contributes only to the self-protection public good and the low-income country contributes only to the self-insurance public good. We again observe the mitigated exploitation of the high-income country by the low-income country.

We also derive the neutrality result of income redistribution in our model. We show that when the self-protection public good is provided by both countries, any marginal income redistribution is canceled out by changes in the contributions to the self-insurance public good so that redistribution does not affect the private good consumption and the provision of the public good.

In the second half of this chapter, we utilize our model to conduct numerical simulations of burden sharing in NATO from 1970 to 2010. In this simulation, we show that whether the conventional exploitation hypothesis holds depends on the risk profile that NATO faces. Our calculated results closely simulate the actual development of the military spending ratio.

The appendix of Chap. 6 provides a theoretical analysis of the condition in which multiple players simultaneously contribute to multiple public goods. Cornes and Itaya (2010) considered an economy consists of two players, one private good, and two voluntarily provided public goods. They showed that if the players are different in preferences, the number of unknown variables is strictly lower than the number of equations in a Nash equilibrium in which both players simultaneously make positive contributions to both public goods. Thus, they claimed that "there 'almost surely' does not exist a Nash equilibrium in which both players simultaneously make positive contributions to both public goods" (Cornes and Itaya 2010, Proposition 2(i), p. 369).

In our model, players are different not only in their income but in their loss in a bad state. Thus, even if they are endowed with an identical income, they may have a different demand for public goods. We do not necessarily obtain any interior equilibrium in which all players contribute to all public goods. So, in this appendix, we show that if the number of resource constraints for each player is equal to the number of public goods, as in our model developed in this chapter, the claim proposed by Cornes and Itaya (2010) does *not* hold.

Chapter 7

In Chap. 7, we introduce misperceptions caused by interest groups that have special interests in security spending. When countries cope with the risk of disastrous events in a political economy, misperceptions by interest groups can occur. For example, international nongovernmental organizations (NGOs) that protect the global environment have an incentive to overestimate the cost of environmental emergencies. Similarly, arms-producing companies have an incentive to overestimate the cost of military emergencies or terrorist attacks. Such a political bias may affect a benevolent government in its choice of security spending. By explicitly incorporating misperceptions into a theory of security alliances, we investigate both the positive and normative impacts caused by interest groups and NGOs in a political economy framework.

Because the non-cooperative provision of public goods is sub-optimal, it might be desirable to consider some mechanisms to promote public good provision. The conventional mechanism is the use of a matching grant to reduce the effective cost of providing the public good. However, in an alliance, it is difficult to presuppose the existence of an upper-level organization that would design and manage such grants. Alternatively, in this book we consider the role of NGOs in promoting security spending. One possibility is the misperception of the threat.

In a one-country model, such misperceptions do not generate the optimal supply of security. However, in an alliance, the nature of a public good itself causes freeriding and the voluntary provision of a public good will be sub-optimal at a non-cooperative Nash solution. Therefore, if misperceptions stimulate the provision of the public good, they can improve the allies' welfare.

Chapter 7 consists of two main parts. In the first part, we examine the impact of misperceptions on burden sharing in an alliance. We consider an economy of two countries with identical preferences and concentrate on the interior equilibrium. If there is no misperception, both countries consume the same amount in both states of the world in the equilibrium. Unlike conventional models, we assume that countries decide their contributions without knowing the true loss of the bad event. We assume that their estimation of the loss is biased. Under the political bias consumption estimated by the government could differ from consumption actually realized. Next, we show that if both countries contribute to both public goods and if the absolute risk aversion of both countries sufficiently decreases with consumption, the estimated consumptions become identical, regardless of the difference in national income. However, realized consumption in a bad state will be higher than the estimated consumption due to the overestimation. The country that overestimates its loss in the bad state more than the other country (in the alliance) will consumes more than that other country in the bad state. This implies that the former country enjoys a higher true expected welfare than the latter.

In the second part of Chap. 7, we assume that self-insurance is not available and concentrate on the impact of misperception on self-protection and expected welfare. We show that a value of overestimation of the loss may exist, which can lead countries to voluntarily contribute a socially optimal amount to the self-protection public good. Furthermore, we extend our model to allow an NGO to endogenously determine the level of overestimation. Using numerical simulations, we present a case in which endogenous overestimation may improve the welfare of the allied countries.

1.4 Implication for Japan's Security Issue

We finish this chapter with a brief summary of results and conclusions about the research on Japan's national security and policy implications. We eschew strong policy conclusions because this is a thriving area of ongoing research activity. However, it is fair to say that we can rule out the extreme cases of spending at a high rate and the complete absence of spending on national security for a variety of reasons discussed throughout the book. The basic assumption of this book is that allied countries behave non-cooperatively in an alliance. Even if allied countries have the common goal to minimize risks of an emergency, each member country intends to maximize its own welfare, not the welfare of all others in the alliance. This analytical framework is relevant in Japan's case. Although Japan and the United States have an alliance, each country's main interest is its own welfare.

This section consists of three parts. First, we discuss the implication of the division of security into self-insurance and self-protection. Second, we derive policy implications from the exploitation hypothesis from Chap. 6. Finally, we discuss the impact of misperception about potential threats to Japan's security based on our analysis in Chap. 7. By investigating risk management roles of both self-insurance and self-protection in an alliance, this book explores the importance of the economic incentives of allies, including Japan and the US.

Although defense expenditures are not easily categorized into self-insurance and self-protection, we can classify military weapons into these two categories to some extent. For example, nuclear weapons and conventional forces may affect the probability of invasion and the loss from the invasion differently. Nuclear weapons are more effective in reducing the probability of invasion than conventional forces but are less effective in stopping invaders near the borders and minimizing the loss from an invasion. In this sense, nuclear weapons have a comparative advantage in self-protection, whereas conventional weapons have the advantage in self-insurance.

The Japan Self-Defense Forces has stocked its armory with weapons suited for self-insurance since its establishment in 1954. Article 9 of the Constitution of Japan states[1]:

> Aspiring sincerely to an international peace based on justice and order, the Japanese people forever renounce war as a sovereign right of the nation and the threat or use of force as means of settling international disputes.
>
> In order to accomplish the aim of the preceding paragraph, land, sea, and air forces, as well as other war potential, will never be maintained. The right of belligerency of the state will not be recognized.

The second paragraph has been interpreted not as prohibiting any military forces but as allowing maintenance of the Japan Self-Defense Forces. However, the article has been interpreted as still prohibiting the ownership of aggressive weapons such as ballistic missiles.

It is true that Japan's military forces, and the country's entire culture influence the risks that it refers to efforts that reduce such risks. At the same time it is also true that the self-protection of Japan mainly depends on the nuclear umbrella of the United States. Article V of the Treaty of Mutual Cooperation and Security between Japan and the United States pronounces, "Each Party recognizes that an armed attack against either Party in the territories under the administration of Japan would be dangerous to its own peace and safety and declares that it would act to meet the common danger in accordance with its constitutional provisions and processes".[2] Based on this treaty, Japan hosts US military bases and the United States will offer retaliation to armed attacks against Japan.

However, due to growing threats in East Asia from North Korea and China, there is a rising call for change in the Constitution of Japan. Prime Minister Shinzo Abe stated that his party should propose an amendment to Congress.[3] The Abe administration tends to pay more attention to aggressive weapons, such as aircraft carriers and long-range cruise missiles, and the overseas dispatch of military forces than the preceding administrations.

As explained in Chap. 6, the exploitation hypothesis proposes that a small country's (such as Japan) defense spending/GDP is lower than that of a large country

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[1]Cited from "Japanese Law Translation" by the Ministry of Justice, Japan (http://www.japaneselawtranslation.go.jp/law/detail/?id=174, accessed on October 9, 2018).

[2]Cited from the website of the Ministry of Foreign Affairs, Japan (https://www.mofa.go.jp/region/n-america/us/q&a/ref/1.html, accessed on October 9, 2018).

[3]Cited from the October 3, 2018, morning issue of *Asahi Shinbun* (p. 1).

(such as the US), which is consistent with the actual data. Similarly, the US accounts for higher military spending/GDP than other NATO countries. These outcomes are common and may well be explained in the standard framework of alliances. As for Japan, it has not maintained self-protection military measures and limited its military expenditures to less than 1% of its GDP.

We discuss the plausibility of a neutrality result. If this result holds in reality, the burden sharing issue loses its policy implication. For example, suppose Japan provides a subsidy to the United States. However, if Japan's disposable income declines, it may decrease its military spending to support national security in the Pacific Ocean and, therefore, the United States may raise its military spending. Thus, income transfers between Japan and the United States may have no real effects.

Japan's non-possession of offensive forces can have two explanations. First, because the optimal value of self-protection for Japan is negative, it does not choose to have self-protection as a corner solution. Second, although the optimal value is positive, it does not choose to have self-protection due to the political constraint of the Constitution of Japan. In the first case, we may justify the freeriding behavior of Japan with respect to self-protection as long as our theoretical framework is relevant. If the cost of a bad state rises in Japan, we may well have this situation. In this case, the call for a change in the Constitution of Japan does not have real effects. On the contrary, if the second case is relevant, it is beneficial to relax the political constraint to attain the optimal level of self-protection. Hence, whether Japan's optimal level of self-protection is negative is a crucial concern. Our analytical and numerical results suggest that multiple equilibria and corner solutions will likely occur if there are divergences among allied countries.

As for the total amount of security spending, Japan's 1% limitation of its military expenditures is difficult to justify with our model. As shown in Chap. 6, military expenditures vary with a country's security environment and economic growth. Japan has maintained its military expenditure target of less than 1% of GDP irrespective of the change in its security environment, including the end of the Cold War, and its economic stagnation in the last two decades. This suggests that Japan's national optimal military expenditure is likely to be higher than the 1% target of GDP. If so, the current political constraint might not be justified with sound economic reasoning.

As shown in Chap. 7, overestimation of the threat of the opponent may stimulate the allies' contribution to a public good and mitigate freeriding incentives. Because it seems difficult to estimate the true risks of North Korea's threat, misperceptions may have significant impact on Japan's security. For example, the reality of the nuclear armory of North Korea has not been grasped. If Japan overestimates the threat of North Korea, it may contribute more to self-insurance than it contributes in a Nash equilibrium without overestimation. If the United States overestimates the threat from North Korea, it may thereby also stimulate its provision of public goods. And such overestimation by the United States could indirectly benefit Japan. In summary, although overestimation can alleviate the under-provision of national security, its welfare implication can differ between allies.

In 2018, the United States and North Korea agreed to reduce military tensions. If denuclearization in the Korean Peninsula occurs, the international tension between the United States and North Korea might ease. In that case, the United States might reduce forces in the Far East. However, as long as the hostile relationship between Japan and North Korea is left unchanged, the indirect disadvantage from the reduction of the United States' forces will dominate. Thus, it becomes important to maintain the common interest against potential risks between allies—Japan and the United States.

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Chapter 2
Risk Management and International Security

2.1 Positive Versus Normative Economic Analyses

2.1.1 Introduction

Economics of national security or international security is so immense a topic as to be intimidating. As in most of economics it has both positive and normative components. The goal of this chapter is to offer examples of both. Positive analyses, attempt to understand the economic origins of conflicts among nations and the economic foundations of success or failure in these conflicts.[1] These are necessarily questions of political economy, and demand insights into many realms. Normative economic analysis of international security may have a shorter pedigree in so far as it descends from the early days of the Cold War and of the primacy of Soviet-American conflict. Analysts[2] associated with the RAND Corporation saw that ongoing expenditure for US national security goals could be subject to economic scrutiny and evaluation. Thus *cost effectiveness* analysis took hold in US defense circles with the goal of achieving desired security objectives without waste, at minimum cost. Beyond this, *cost-benefit* analysis proposed not merely to eliminate waste for achievement of given objectives, but more than that to quantify via economic calculation which objectives should be pursued, ideally those which produced security-benefits—benefits measured in the metric of dollars—greater than their costs.

In the aftermath of World War II and height of the Cold War the normative approach seemed paramount. Could economics assist in decisions as to which battles to pursue, how to arm at least cost, how to share missions and burdens among allies, and how to

[1] Such manifestations of the economic approach to conflict and peace have a considerable history, dating at least from Hobbes, Emmanuel Kant, Norman Angell the 1933 Nobel Peace Prize winner, and in recent days, J. Hirshleifer, H Grossman, E. Thompson and many others.

[2] Analysts such as T. Schelling, C. Hitch and R. McKean, A. Enthoven, and S. Enke, and H. Kahn, or political scientist like B. Brodie, A. Wholstetter and many others were focused largely on how to support defenses against feared Soviet strength.

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coordinate all the numerous instruments of state policy in the cause of security? An alliance formed where academics economics strived actually to assist the beleaguered civil and uniformed servants who were trying to formulate and execute policy.

However, due in part to the Vietnam War, and especially by the end of the Cold War the gap between academic economics and policy implementation has widened. Academic focus has shifted and a large body of positive work—normatively neutral—on the political economy of international security has come into being; the number of journals and frequency of conferences has increased greatly while the impact of economists on actual security policy decision has surely declined. However, with acceleration of globalization and a shift in the paradigm for international security toward trade and finance, the pendulum may swing back. This chapter will offer examples of both positive and normative analyses relevant to our era's burgeoning security problems.

2.1.2 Evolution of Security Issues

When the pendulum moved toward a more positive focus, the reason was not that the old Cold War normative issues have really disappeared and been replaced. Rather, a new class of challenges emerged, layered over or parallel to earlier threats of great power conflict and competition. Although many of the old geo-political rivalries persist fed by survival and expansionist needs of states (the "end of history[3]" will be delayed it seems) still some historical conflicts founded in territorial disputes and ambitions have dissolved.[4] Meanwhile because of developments in technology and globalization of the past quarter century the number and range of security challenges has proliferated.[5] Today more are threats and their possible resolution substantially economic as the present structural crisis in the world economy testifies.

To illustrate both positive and normative utilization of economics, this chapter will develop two illustrations of analytic foundations of security studies and economic method in the management of security challenge, based on McGuire (2010). *The first* illustrates employment of political economy in rather general and abstract terms to *describe* self-seeking behavior of nations, their competition for wealth, land and power, and to derive the resultant equilibrium economic configuration of states in a nation state system. The model is a partial synthesis of a much greater recent literature[6] of how formal political economy may provide insights into world

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[3]This refers to Fukuyama (1992).

[4]State boundaries in Europe and Central Asia for example.

[5]An important feature in this evolution has been the growing collective, public-good nature of most instances of security challenge. Accordingly, by a widely accepted corollary of the "logic of collective action" with increased globalization we should experience more unmanaged or mismanagement of international conflict, since no entity with effective enforcement power exists to act in the interest of the whole world of nations.

[6]Prominent in this literature is Alesina et al. (2000), Blainey (1991), Findlay (1996), Garfinkel and Skaperdas (2000), Grossman (1998), Grossman and Mendoza (2001), Hirshleifer (1988, 1991,

security challenges. *The second* example—drawing on another large literature[7] and illustrating the rising prominence of economic issues as opposed to military issues of security—will provide an example of using economics to optimize policy against threats to energy or other resource security.[8]

2.2 Old Fashioned Security: The Law of the Jungle Self-seeking Incentives and Conflict Among Countries[9]

2.2.1 Analytical Framework

Rational, self-serving behavior being the foundation of all economics, it is unsurprising that application of economic principles to analysis of the behavior of states and the resulting structure of the international system has a long history.[10] Countries can buy and sell and save and invest on the one hand, as well as invest, produce, and trade goods and services with each other; but on the other hand they can threaten war, attack, conquer, exploit and subjugate. How is one to understand the historical choices made between these alternatives, or the current incentives impinging on modern states?

In such a world national wealth will represent simultaneously (1) a required foundation for consumption (2) a necessary instrument for survival-by-self-protection and/or advancement-by-conquest and (3) an object coveted by other nations.[11] Especially to the extent that state-actors on the international scene can be idealized as individual utility, wealth, or power seeking entities an economic paradigm may provide understanding. Insight into how self-gain is driving the choices of states would constitute an extremely valuable legacy if economics could provide it. Policy makers from the highest strata of statecraft down to the details of trade or defense policy

2000), McGuire (1967), Powell (1999), Sandler (2000), Skaperdas (1992), Thompson (1974, 1979), Tullock (1974), Wittman (1991, 2000).

[7]References include Bergstrom et al. (1985), Bhagwati and Srinivasan (1976), Hilman and Ngo (1983), Hirshleifer (1953, 1987), McGuire (1990, 2000, 2006a), McGuire and Becker (2006), McGuire and Shibata (1985), Shibata (1986), Williams and Wright (1991).

[8]The role of alliances, or self-monitored international agreements among countries should be prominent in both these applications, and should be considered another central theme in evolution of security analyses of the past 50 years.

[9]This section of the chapter draws on McGuire (2006b). Professor Earl Thompson provided valuable criticisms of that paper.

[10]Depending on technology, and on wealth and its distribution, the competition for limited resources, limited and desirable geography, or access to foreign markets (for resource supplies an/or outlets for production), will cause tension and conflict among countries. Moreover, these sources of conflict will interact with the political organization of societies to determine how conflicts will be perceived and resolved—whether peacefully or by war, by production, trade, and investment, or war and conquest.

[11]These distinctions were, we believe, first emphasized by Thompson (1974).

execution crave insight into how friends and adversaries are motivated by various types of risks, opportunities, threats, rewards, and losses. Positive economics desires to offer better understanding of this system as an economic organism.[12]

All the many approaches to this problem seem to acknowledge similar elements in a description of the international system as an economic structure. The analytic issue then becomes how to fit these various elements together. These factors include:

a. Technologies of production, investment, transport, etc. including differences among countries.
b. Factor Endowments and their distribution across countries.
c. Costs of aggressive pursuit versus of defensive protection in conflict. This is sometimes collapsed into "a contest success function" (Tullock 1974; Hirshleifer 2000) to summarize how cost-effectiveness of offensive-conquest versus cost-effectiveness of defensive-protection influences the chances of success in armed conflict or division of spoils.
d. The internal system of governance in countries—ranging from despotic autocracy to utopian democracy, with many stages between these extremes.[13]
e. The decision framework for representing choices taken and interdependent outcomes, often represented as a mathematical game such as prisoner's dilemma, chicken etc.

The economist's approach to this problem follows from a belief that competition and survival in economic systems entail dissipation of surplus and rents.[14] All rents—pure surplus—are dissipated in equilibrium. This principle applies to equilibrium in an anarchic configuration of states, no less than to the equilibrium structure of firms and industries. A simple model of capture and defense among states, which utilizes this principle of surplus absorption, will show in the process how the aforementioned factors fit together. In the process of doing this we point out two generic causes for a state's being safe from capture by and absorption into another; both derive from the fact that defense expenditures raise the cost of conquering a country. This may make the country safe from capture because (1) potential victims defend themselves so as to become more expensive to capture than it is worth to a conqueror, or (2) potential victims become more expensive to capture than a conqueror who is limited by liquidity constraints can afford.

[12] An example of such benefit might be that one could answer the question "Why do countries fight wars with each other?" Wars are expensive. "Wouldn't it be wiser and much cheaper to bargain?" The loser of the war can afford a high payment and still be far better off than fighting and losing, while the winner could save the terrible expense of war by allowing itself to be "bought off." Does it follow that wars are all fought because of a mutual mistake? Or because of lack of information?

[13] As recognized as early as Immanuel Kant (1795) the system of governance influences how the costs and benefits of war versus peace are distributed among the citizenry of countries. It determines the costs of governance itself and thus the feasibility of conquest, suppression, exploitation etc. Kant and many authors since him have argued that autocratic organization favors war-conquer-exploit outcomes since an autocrat skims off the benefits from success without paying much in the way of the costs of failure.

[14] As the saying goes no "big bucks on the sidewalk."

2.2.2 Definitions and Notation

The first crucial concept to define is the required allocation to defense that a nation needs to secure its property, wealth, boundaries, and sovereignty, or conversely that an aggressor needs to effect conquest. Consider an initial distribution of property between two nations. This distribution will persist if each nation's benefit-cost analysis of the gains from conquest (very broadly and figuratively defined) and the costs of national survival sustain that distribution. We call such enduring distributions, "equilibria," noting that there may be many. Equilibrium is assumed to depend on those factors mentioned above including: the survival needs of both conquered and dominant societies, the resistance of subjugated societies to exploitation, and efficiency of rulers in extracting tribute from their colonies as measured by necessary allocations to ongoing subjugation.

For conciseness and brevity this chapter ignores possibilities for international borrowing and ignores the destructiveness of war itself, although the model easily can be extended in those directions.[15] We will not concern ourselves with benefit nor cost distributions within populations, instead only with aggregate wealth, consumption, survival needs, etc. of societies. Then to capture the most basic factors we limit our attention to just two countries, A, a potential attacker/conqueror and V a potential victim. As in Table 2.1, assumptions are extremely simple to allow a compact presentation.

2.2.3 Benefits and Costs of Conquest

Now to analyze equilibrium configurations of property aggregate and distribution we assume:

1. Confrontations whether actual wars or demonstrations that lead to capitulation or stand-off occur in a single period.
2. No international borrowing to finance attack or defense expenses is allowed to begin with, and past saving or stockpiling to build up a stock of military weapons is ignored also.[16] Therefore, one equilibrium condition is that during each period:

$$M_V \leq R_V - S_V; \tag{2.1}$$

and

$$M_A \leq R_A - S_A. \tag{2.2}$$

[15] For a more complete model see McGuire (2002b).

[16] We could incorporate past saving and present borrowing in the constraint as $M \leq R - S + S + \beta$ where β stands for borrowing and S stands for past military stockpiles accumulated at $t = 0$.

Table 2.1 Notation

T	Time/date index; $t = 0$ represents the present when conquest occurs or independence is maintained; $t = 1 \ldots \infty$, are future years when benefits from conquest or continued independence are enjoyed
R_V	Country V's current period ($t = 0$) resources; R_V is assumed to be constant throughout $t = 0 \ldots \infty$
R_A	Current period resources for A
S_V, S_A	Country V's current period ($t = 0$) population subsistence needs, S_V, also assumed to be constant throughout $t = 0 \ldots \infty$, A's population subsistence
$U_V = R_V - S_V$	Country V's surplus over survival needs. This surplus is available for military (offensive or defensive) allocations, or for ongoing resistance against occupation/subjugation, or for tribute to conquerors, or for consumption enjoyment. Earl Thompson's (1974) calls this "coveted capital stock"
$U_A = R_A - S_A$	Country A's surplus over survival needs
$B_A[R_V - S_V]$	Present value to A, the potential conqueror, of country V's net resources beginning from $t = 1 \ldots \infty$, net of V's survival needs, and net of A's ongoing costs of colonization. Function B_A incorporates economic advantages of merger between countries as well as redistributive transfers to A. This future surplus excludes M_V (defined below) since once conquered, country V's military forces are disbanded. Since B_A includes all discounted future flows of V's surplus, the graph of B_A versus U_V lies above 45° whenever the discount rate <100%. Higher discount rates will be to lower this curve but never below 45°. See Fig. 2.1
M_V, M_A	Country V's/A's military expenditure, serves simultaneously as defense against attack, and potentially for conquering other nations M_V and M_A must be paid out of current surplus. Multi-temporal treatment of savings and investment would allow M_V or M_A to be stockpiled from past savings, or enhanced by borrowing
$C_A = C_A(M_V)$	Military expenditure A required to conquer V; Cost C_A embodies comparative effectiveness of attack versus defense. Conquest occurs during $t = 0$. Included in this cost is the opportunity loss of foregone trade which a state may give up if it pursues conquest. Successful conquest of V brings to A the present value net benefit of $B_A[U_V]$ net of ongoing cost of rule implicit in $B_A[\,]$
$G_A = B_A[U_V] - C_A$	Net of C_A present value gain to A from conquering V. We ignore the destructiveness of war itself and its losses

$C_A(M_V)$ shows the amount M_A required by A to definitely prevail

Note that the costs of war or costs of conquest as given in the table above i.e. $C_A = C_A(M_V)$ may not satisfactorily reflect an *opportunity cost* factor of especially great importance for describing how equilibrium configurations change as countries grow richer. No doubt a major cost of conflict can be the opportunity cost caused by economic disruption at home, and loss of gains from specialization, investment, risk spreading, and trade with other countries. And the richer the prospective attacker, the greater should be this cost. Thus a more realistic characterization of costs of conquest might well be $C_A = C_A(R_A, M_V)$. This could easily imply a correlation between total world wealth and peace; and it could imply that distributions of smaller more numerous states coexist in equilibrium at greater levels of aggregate wealth. Including this feature in the analysis is entirely feasible but doing so we save this effect for a later exercise.

3. Full information with no uncertainty: as (2.1) or (2.2) indicate it can be rational for a country to spend as much as its entire current resource surplus over survival needs on *M*, if this allows it to conquer a rich enough country, or deters another from conquest. If, to settle disputes short of war, it is possible or necessary to leave some of the surplus "on the table" either for attacker or defender. Then M_V or M_A will be less than these amounts; i.e. this effect reduces the maximum extraction from a conquered population as well as the maximum possible effort (available from current resource flows) to be directed toward conquest.
4. Countries rationally calculate and compare the costs and benefits of aggression/conquest. If benefits exceed costs they allocate sufficient resources—provided they have the resources—to *M* to conquer/colonize; otherwise their military expenditures are restricted to defensive.[17]
5. If country V were conquered its future consumption stream would be forever reduced to its minimal survival needs, S_V. Therefore, V will be willing to devote its entire surplus, U_V, to defense if necessary to prevent conquest.
6. Country A accurately measures a present value benefit of $B_A[U_V]$ if it conquerors V. The cost to A of conquering V depends on V's defensive effort, but if A can overcome those defenses in the present period, it will enjoy the benefit forever, a durability that is captured in the function B_A. However, A may find that the benefit of conquest is not worth the cost.
7. Moreover, with no current borrowing and no inherited stockpiles, we also assume initially that the resources required by A to successfully conquer V must be available to A out of its current surplus. Similarly, resources required by V to maintain independence must be available from current surplus (Both of these stocks may be augmentable by previous stockpiling or by borrowing).

The Relative Advantage of Offense Versus Defense: Its Importance

In equilibrium country V either will persist as an independent state or it will be absorbed by a conqueror (If absorption occurs, V may eventually become an integrated equal partner with its conqueror, but that is not our present concern). Once all conquests/absorptions have been effected, equilibrium will exist in the entire international system since then no nation is both capable of and would benefit from conquest of another.

For example, suppose that a once conquered a country offers little resistance to exploitation; that is, assume that the ongoing costs to the conqueror of maintaining

[17] These decisions all occur in period 0. This assumption of rational calculation of benefits and costs ignores the entirely reasonable advantages a country may find in promising or committing to irrational action. The calculus of conquest and survival may be different when sequences of moves and commitments are allowed for the players. If conquest or submission is a gigantic game of international "chicken," then the capacity *credibly* to offer another the choice between annihilation and submission may not be adequately represented by force duel or conflict success function (CSF). The place of CSF's in making symmetric commitments may be a lot more complicated. But CSF's (as in our linear case) are still crucial to outcomes determined by "naively rational extortion," when not only "will power" but also a demonstrated capability to conqueror (not requiring actual application of force) may be necessary to evoke capitulation. For more see McGuire (2006a).

colonialism are low. Suppose in addition that the conquering power is patient, and therefore discounts future benefits rather little. Then quite possibly because a victim country's defense is limited to its surplus over survival needs in any single year while the temptation it offers to conquerors is high, it may not be able to secure its property/independence.

This conclusion however will depend crucially on the technology of defense versus offense, measured as the rate at which offensive and defensive expenditures neutralize each other—as defined by the function $C_A(M_V)$. More generally, we desire to explore how this outcome depends on the values of S, R, U, of M and C, and of the functions $B_A(\cdot)$ and $C_A(\cdot)$ and to construct a set of tools to facilitate such analysis. It may seem unnatural—or unrealistic—to assume that benefits from successful conquest are independent of the victim's defensive effort, i.e. of M_V—since the effects of defense on the conquered economy probably spill over into future productivity and available surplus.[18]

2.2.4 Two Categories of Autonomy-Preserving Allocation to Defense

First we identify the maximum defense effort required by V to secure property or preserve national independence. There are two distinct possibilities. First is the amount of M_V that makes conquest more expensive than it's worth to the conqueror. Second is the amount of M_V that makes conquest more expensive than the conqueror can afford.[19]

The value of M_V that raises the cost of conquest above its benefit to A is defined implicitly by:

$$B_A[R_V - S_V] - C_A(M_V) = 0 \quad \text{with} \quad C'_A > 0 \tag{2.3}$$

or

$$M_V = C_A^{-1}[B_A(U_V)] = g[U_V]: \quad \text{where} \quad g' > 0 \tag{2.4}$$

where C_A^{-1} represents the inverse function of $C_A(M_V)$. We denote this value of as M_V^{ND} ("ND" for "not desirable").

The value of M_V that raises the cost of conquest above A's resources is defined implicitly by:

[18]Thus a more complete formulation could represent $B_A(\cdot)$ by a present value of future tribute diminished each year by the effects of war i.e. by $B_A[PV_{t=1\ldots\infty}\{U_V^t(M_V^{t=0})\}]$. We will postpone including this improvement for later research.

[19]Borrowing and stockpiling will influence the relative importance of these two impediments to conquest.

$$R_A - S_A - C_A(M_V) = 0 \tag{2.5}$$

or

$$M_V = C_A^{-1}[U_A] = h[U_A] \tag{2.6}$$

We denote this value of M_V as M_V^{NF} ("NF" for "not feasible").

These two distinct ways in which one country can preclude being conquered by another have different effects on the nature of equilibrium. Because we assume here that countries cannot borrow internationally to finance defense outlays and have not stockpiled arms in the past, the amounts M_V and M_A cannot exceed the surplus over subsistence available in the respective home country.[20] The rule proposed here for V's equilibrium allocation to "defensive" or independence-sustaining military effort is:

$$z_V = \text{Min}[g(U_V), h(U_A)] = \text{Min}[M_V^{ND}, M_V^{NF}] \tag{2.7}$$

Because every country is potentially a "victim" country, V stands for every country in the international anarchic system. Then

$$z_V > U_V \rightarrow M_V^* = 0\text{: V loses its independence} \tag{2.8}$$

while

$$z_V \leq U_V \rightarrow M_V^* = z_V\text{: V maintains its independence} \tag{2.9}$$

This rule assumes that all countries know all others' military allocations/capabilities and all other aspects of full information. Each country calculates (a) the required defense that is just enough to make conquest more costly than it is

[20]So long as conquest is worth the cost and perfect international financial markets exist, an attacking country may be able to make up its resource shortfall by borrowing today and repaying out of the conquered surplus (if the attacker) in the future. Similarly, a defending country should be able to borrow the entire present value of its surplus over survival needs to defend itself against takeover and pay back loans from that surplus in the future. Total borrowings and past savings then would add to offense and defense capabilities.

Adding these factors first alters the resources constraints. We incorporate past saving and present borrowing in the constraint as $M_j \leq R_j - S_j + S_j + \beta_j$ and $\beta_j \leq \beta_j(R_j - S_j)$ where β_j stands for borrowing which is limited by some present value function of future surpluses, $\beta_j(R_j - S_j)$ which can be used to repay debts, and S stands for past military stockpiles accumulated at $t = 0$. These constraints apply to both attacker and defender, $j = (A, V)$.

Next the equilibrium conditions change: now for Eqs. (2.2) and (2.3) there is no change for M_V^{ND}, but for M_V^{NF}. Equation (2.4) now becomes $S_A + \beta_A + R_A - S_A - H_V^A(M_V) = 0$, with a corresponding adjustment in Eq. (2.4).

With perfect information in international markets, both attacker and defender could not simultaneously borrow these maxima, as the market might anticipate that only one of the loans would be repaid. This effect could be incorporated in the present value borrowing constraints $\beta_j(R_j - S_j)$.

worth, and (b) the required defense that is just enough to make the limited resources of any adversary insufficient to win a war of conquest. Every country then allocates the lesser of the above two amounts to defense in order to maintain its independence. Or if it cannot successfully defend itself, it allocates nothing to defense and is absorbed by a conquering state. Again note that Eqs. (2.9) and (2.10) imply that V will (if necessary and efficacious) allocate up to its entire surplus over survival needs to defense; this is because if conquered or obliged to capitulate (we assume) it will have to turn over that entire surplus to the conqueror. Country A, however, might allow V to keep some of this surplus in a negotiated merger. This provides V with a third choice, not analyzed here, between capitulate and fight.

To claim that one country, A, could "profit" from conquering another, V, but cannot "afford" to do so, may seem odd, since it implies that an economic surplus might be captured by reorganizing countries—a surplus which is somehow left unclaimed "on the sidewalk." And if A could borrow indefinitely in perfect markets, no such configuration would be possible. But with limits on resources available from present or past saved resources and on available borrowings against future re-payment the configuration is quite plausible. In other words, market or transaction "imperfections" may support or in a fashion even constitute defense, by limiting the amount of debt available.

2.2.5 Defense Allocations Illustrated

These rules for allocation to defense are pictured easily as in Fig. 2.1. There the origin of the resource endowment for A and V has been displaced by the amount of survival need, so that each axis represents $R - S = U$ for V and A. This allows us to use the x-axis simultaneously to depict the enticement-effect of V's resources to A as an object of capture, and the security they represent to V as a support for defense. Because A's and V's surpluses can be used for offensive or defensive forces, the x-axis can represent M_V and the y-axis can also represent C_A. Thus, given our simplifying assumptions and absent borrowing or stockpiling, for any value of $[R - S]$, M or C can be less than or equal that value but not exceed it. To further simplify the illustration, both benefits B_A and costs C_A are assumed to be linear functions of $[R_V - S_V]$ and of M_V respectively[21] (Thus, V's surplus on the x-axis has two different interpretations: (a) *first* regarded as the independent variable available to the conqueror, which extended into the future and discounted determines B_A, i.e. A's present value discounted today-benefits and (b) *second* regarded as V's present resources available for fighting which will induce costs C_A. Increases in costs

[21] It may seem unnatural to assume, as the diagram does, that the benefits of conquest are independent of M_V, since M_V (or maybe $M_V + M_A$) can be a good proxy for the destructiveness of war itself, and therefore reduce the present value of all future war spoils to the conqueror. This effect can be easily included by adding another quadrant to the diagram to show how the future surplus available to a conqueror ($U_V^{FUTURE} = R_V^{FUTURE} - S_V^{FUTURE}$) may depend on M_V and/or M_A.

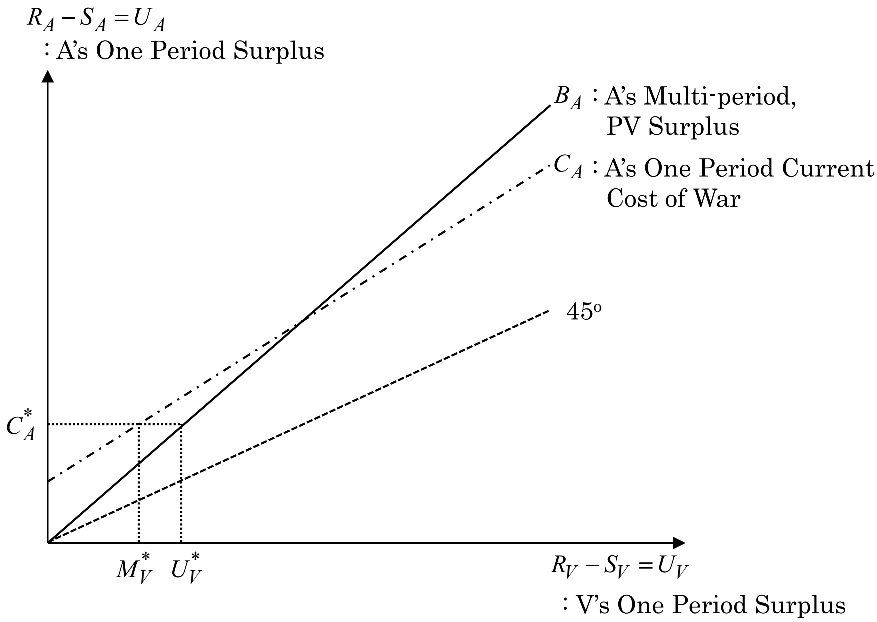

Fig. 2.1 The present value benefits and costs to A of conquering V: V's wealth as an object of theft and a support of defense. *Source* McGuire (2010)

of colonization or in the discount rate have the obvious implication to shift down the position of and/or reduce the slope of B_A (but not reduce it below 1:1).

Also the cost of conquest is assumed to require positive resources from A, even if V posts no defense at all. Accordingly, the C_A curve has a positive y-intercept. C_A also has a slope greater than 1 reflecting an assumption that defense has an advantage over offense at the margin $dM_A/dM_V > 1$. If this margin were increasing at greater scales of offense and defense then the slope of curve C_A would increase, i.e. $d^2M_A/dM_V{}^2 > 0$. If all this information is available to all sides in a conflict, then rather than actual war with its expense of M_A and M_V, plus the destructiveness of war (at $t = 0$) and future fallout from that destruction, a bargain might be struck where A simply buys out V, or V pays off A with a tribute. Just why such peaceful bribery is not the rule in international conflict remains a subject of ongoing puzzlement and research among economists.

The diagram is to be read as follows. Consider first values of U_V and (absent borrowing or stockpiling), therefore, necessarily of M_V that are less than that at the intersection of the B_A-curve with the C_A-curve—that is to the left of that intersection. In this range C_A is above B_A, so if V were to allocate its entire surplus to M_V it could make the cost of conquest to A (C_A) greater than its value (B_A). But V need not allocate its entire surplus to defense. To show V's required allocation to defense for any V-surplus, U_V^*

$$U_V^* = [R_V^* - S_V^*] \tag{2.10}$$

read up to B_A to obtain the cost which V must impose on A (this being the amount C_A^*) to make conquest uneconomic, i.e. worth less than it costs. A horizontal from that point will intersect curve C_A. From this intersection drop a vertical line to the x-axis to obtain the required level of defense M_V^* . This is the requirement for M_V if the attacker's surplus $U_A = [R_A - S_A]$ *equals or exceeds* C_A^*. If the attacker's net resources, say U_A^0, are less than C_A^*, then the required M_V is found at the intersection of $U_A^0 = [R_A - S_A]$ with curve C_A and then down to the x-axis.

Now consider values of U_V that exceed the value at which curves B_A and C_A intersect, that is to the right of the intersection; here B_A lies above C_A. Before, to the left that intersection M_V could be less than U_V, but to the right, absent borrowing and stockpiling, because of the constraint M_V cannot exceed U_V. Therefore, in this range, if for any value of M_V, A's endowment U_A exceeds $C_A(M_V)$ then the defender cannot impose sufficient costs on A to preclude conquest, and A absorbs V. However, if $U_A < C_A(M_V)$ then A *has inadequate resources* to conquer V. Thus to the right of the intersection and below the curve C_A, A would benefit from conquest of V but cannot afford the resources needed to win. Arguably in a rational world of perfect foresight, perfect capital markets and unrestricted borrowing, such a configuration might be unsustainable, as it implies an unexploited opportunity for gain. But that jumps to a conclusion possibly that could be illustrated or derived by introducing savings S and borrowing β.

Note that the diagram actually identifies the amount of M_V that V will spend to avoid capitulation to A if capitulation can be avoided, and the C_A curve shows the amount A must spend to induce V to capitulate. On this interpretation, anarchic property distribution results from a success or failure of a sort of "naively rational extortion." Analysis of hot war in which resources are destroyed on both sides is left for another exercise.

2.2.6 Conclusion

This section of the chapter built on the view of a country's wealth as providing an enticement to others to seize it through war or extortion, and simultaneously as a military resource for protecting its owner. Adding the technology of offense versus defense together with systematic evaluation of the benefits of conquest/theft then implies how, for some combinations of wealth, rational calculating countries should wage or threaten to wage war, while for other combinations rational calculation supports peace.

Our approach thus distills how these incentives vary with the shape and location of the pay-off function $B_A^J (J = 1, 2)$ and with the relative effectiveness of defense versus aggression $C_A = C_A(M_V)$. We show that rational calculation may exclude war for two distinct reasons. First for given offense-defense technologies no country may have sufficient resources to defeat and capture another. In this case every country can mount a defense that physically or fiscally precludes its capture. We call this "Security by Making Conquest Unaffordable or Infeasible." But second even though affordable, conquest may be undesirable, in that it costs more than it is worth. Countries may mount sufficient defense to make the costs to a victor exceed the benefits which capture will yield. We call this "Security by Denial of Benefits."

A principal empirical implication of this analysis concerns equilibrium distributions of property among States as would be illustrated with another diagram with both States depicted as both potential attackers and victims. First, the model defines a niche for small/poor countries with natural defenses. Second, the model implies that equal distributions may or may not represent peaceful equilibrium configurations, depending on the specifics of C and B curves, abilities to borrow, and to stockpile for wars, and specifics of surpluses available for theft or conquest. With this approach we can identify some unexpected interdependencies between resource distributions, technologies, and conflict, as well as confirm previously held intuitions.

For example, this set-up will identify cases in which a given distribution of income/wealth among countries tends to be unsustainable because one country is able to and would benefit from conquering another. Sometimes from such unstable distributions stability can be generated by transferring income "peacefully" among existing "competitor" states, without increasing the total (not an unexpected result). On the other hand cases in which the cost curves C_A^J overlap would indicate a state of distribution in which both countries would benefit from conquering the other and are capable of successful conquest. This could require striking asymmetry in the cost curves, or a pronounced comparative advantage for offensive forces, but it could indicate a "pocket of indeterminacy[22]" where objective incentives to fight (or capitulate) exist for both of two (or all) rivals. An unsuspected configuration such as this may be more likely to be a prelude to negotiated merger.

Extensions of this analysis also confirm and illustrate one's intuition that an ability to borrow and/or to stockpile weapons narrows down the set of distributions of world aggregate wealth for which war cannot be afforded. This shows in the diagrams as curves C_A and/or B_A shift downward and to the left. One's expectation should be that if any capture or predation is worthwhile in the sense that it is worth more than its cost, then prospective conquerors should be able to find the resources on world markets to conduct their wars and share (via debt repayments) their loot.

This setup also permits ready analysis of the effects of changes in war fighting technology—for example changes that favor offense or defense. Similarly the effects of changes in a society's benefit calculus including discount rate and costs of occupation are readily pictured as alterations in the position or shape of the benefit functions B_A^J. Note that both shifts in the C-curves and B-curves re-position the crossover point between B and C, and therefore the regions where security by non-affordability of conquest, versus non-desirability obtain.

Evidently, introducing the possibility that benefit and cost curves—B_A^J, and C_A^J—can be convex or concave may produce numerous other regions of stability or instability. Other extensions of the analysis should include better integration of borrowing, inclusion of population (in addition to wealth) as an independent variable in the benefit and cost functions, and a focus on the effects of conquest upon

[22]This terminology was suggested to me by Prof. Charles Anderton.

productivity of the conquered country. On the other hand, because the analysis is comparative static, it cannot do full justice to the inter-temporal multi-period nature of the phenomenon, nor to questions of dynamic consistency—disadvantages to be set off against the economy of expression of this approach. Several of these topics—but far from all of them—are treated in McGuire (2002a).

2.3 Optimal Preparations for Adversity

The second example that we present of the use of economics in international security will be more traditionally normative.[23] How should a country best prepare for risk of trade interruption? We choose this topic specifically for several reasons: because of its present relevance to resource insecurity particularly energy; because, therefore, of its special relevance to Japan; also because it represents a "security" problem that extends well beyond military security.

The ongoing globalization of the world economy allows each nation to enjoy the benefits of greater exploitation of scale economies and of specialization and exchange. But this greater dependence necessarily implies greater vulnerability to disruption of supply. How can and should nations prepare to reduce the effects of unhappy surprises, while retaining the benefits of interdependence? In the current international economic environment as potential for loss has increased there is a special need to understand better how the chances of crisis, the costs of various preparations, and the inherent willingness of countries to accept risk, all should come together to produce better policy and better anticipation of individual nation's behaviors. Accordingly, one observes a mounting need for countries to acquire risk management skills in an era of globalization.

This section of the chapter will show how possibilities for *insurance* against trade disruption (including "perfect" insurance) influences prescriptions as to the best mix of policies to manage risk of international supply disruption in surprising and unambiguous ways. Specifically, in the course of describing this role of economics in security, we will demonstrate that when the chance of trade disruption *increases* or the global risk aversion of a country *declines*, as instruments of risk management a country should place greater reliance on stockpiling over artificial protection of domestic producers.[24]

[23]This section draws on McGuire (2000, 730–751), and on McGuire and Becker (2006) We have benefited from earlier discussions and correspondence with Gary Becker, Christopher Clague, Jack Hirshleifer, Leonard Mirman, Mancur Olson, Hirofumi Shibata, Todd Sandler, George Tolley, and Murray Wolfson.

[24]Throughout, we employ the accepted concept of Kihlstrom and Mirman (1981) to define risk aversion (RA) in a multi-commodity world: locally first degree homogeneous utility functions are locally risk neutral; local decreasing returns to scale or diminishing marginal utility of income represents local risk aversion.

2.3.1 Alternative Policies to Combat Supply Disruption

The risks inherent in exploiting comparative advantage to the fullest and thus allowing one's country to become dependent on foreign markets have long served to justify (or rationalize) interference with free trade. The basic argument is that in an emergency disruption a country may not have time to reallocate internal resources to replace lost foreign supplies with domestic production. In the short run, if resources are immobile the country will suffer shortages unless it has planned in advance.

One advance action is to preserve higher cost domestic industry. Agricultural products, energy, strategic materials, and military equipment are foremost examples of commodities for which many countries strive to maintain greater independence than competitive free markets entail.[25] In addition to trade controls many other policy instruments may serve to manage risks of commercial disruption.[26] These might include incentives to consumers to change consumption patterns (e.g. auto gasoline efficiency), or diplomatic and politico-military efforts to lower risks of disruption (Ihori and McGuire 2007, 2008), or production base maintenance on a standby basis (Gansler 1980).

This present example of normative economic analysis for security presents a unified treatment of stockpiling versus protection including the effects of (1) probability of disruption, (2) degree/dispersion of comparative advantage in production, (3) intensity of a country's need for imports/exports, (4) costs of stockpiling, and (5) aversion to risk. The analysis examines how all these factors combine to determine the optimal level and mix of policies.

As for theoretical interest the technical literature includes varied analyses of risks of trade disruption and protection. Mayer (1977) provides a modern treatment of the classical argument for subsidy to domestic industry when probability of disruption is exogenous. Two studies in particular have investigated stockpiling versus protection, the dynamic analysis of Tolley and Wilman[27] (1977) and the two-period analysis of stockpiling as an investment by Shibata[28] (Shibata 1986).

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[25] Why governments should be more adept at predicting/managing supply disruption is of course a crucial issue. For a current summary of alternative positions see Williams and Wright (1991, 410–51).

[26] Such could include: (1) military to preparedness reduce the likelihood of trade disruption. (McGuire and Shibata 1985, 1988); (2) diversification of trading partners (McGuire and Becker 1994); (3) geographical dispersion of assets/factors-of-production across national borders (McGuire 1986); (4) insurance alliance formation to allow partners to exchange guaranties against disruptions or emergencies (Ihori and McGuire 2007, 2009).

[27] These authors conclude (p. 323) stockpiling should limit embargo price rise to unit cost of storage divided by embargo frequency. With low embargo frequency, the price rise based on the stockpiling criterion may be greater than with no stockpiling.

[28] Shibata (1986) concludes "that the smaller the storage costs and the degree of quality deterioration of the stockpile, the greater [the text reads "smaller" but the context make clear that "greater" is intended] the optimal amount of the stockpile. (p. 10) … And the smaller the domestic costs of (exports) X in terms of (imports) Y relative to international terms of trade, the greater the possibility of the stockpiling policy's superiority over the protective policy." (p. 14).

2.3.2 Insurance and Investment Components of Stockpiling

A useful way to frame comparisons between protection and stockpiling is to ask "if two policies yielded identical benefits, which would entail lower costs?" The question directs attention to crucial categories of cost associated with protection and stockpiling. Two of these cost categories are especially important. The first relates to the investment, multi-time-period, and inventory management nature of the problem and the corresponding "time dependent costs". The second category relates to hazard and insurance aspects of emergency management and the associated "contingency dependent costs."

Although both these elements and both cost components are always alloyed in any realistic problem or policy solution, this chapter focuses solely on inter-contingency costs. By assuming zero interest costs we investigate the anatomy of foreign dependence in its most basic form, identify conditions under which protection and stockpiling can yield the same benefit and show when the costs of one option outweigh the other or when both policy options should be pursued simultaneously. With a cost-minimizing analysis of protection and stockpiling so decided, the analysis can then proceed to the question of what overall level of benefits and costs maximizes national welfare.

2.3.3 Expected Welfare and Maximization: Assumptions and Notation

To reduce the analysis to bare essentials, we will impose further simplifying restrictions, summarized with related notation. The object studied is a small, price-taking country with a two-good Ricardian economy, constant average costs of production, and, therefore, a linear production possibility curve.

$x =$ export good;
$y =$ import good. The numeraire good is x.
$p_D = 1/q =$ unit domestic production cost of y in terms of good x.
$p_w = 1/t =$ world trading price of y in terms of good x.
$\bar{x} =$ maximum production of x at home.
$y_D =$ amount of y produced at home: $y_D = qx_D$;
or $x_D = p_D y_D =$ amount of home production of x foregone to produce y_D.
$y_M =$ amount of y imported: $y_M = tx_E$;
or $x_E = p_W y_M =$ amount of x exported to purchase y_M on world markets.
$y_M + y_D =$ home consumption of y when exports and imports are permitted.
$\bar{x} - x_D - x_E =$ home consumption of x when exports and imports are permitted.

The country is a unitary monolith. It's expected utility depends only on its consumption of x and y. It's time horizon extends over two periods. The first is the present—denoted by superscript k—just about to begin and during which no trade disruption can occur. The second time period is the future when the country faces two distinct contingencies; exports and imports permitted with probability π and forbidden with probability $\omega = (1 - \pi)$, contingencies represented by superscripts π (for "peace") and ω (for "war") respectively.

If exports/imports are cut off, all home production must be consumed at home; no resource reallocations between sectors can be made.[29] The private competitive sector is assumed not to anticipate or make provision for trade interruption because (a) private producers have less adequate information about trade disruption than do governments and/or (b) domestic or foreign policy derived externalities of potential trade loss are not internalized by private markets.[30] In anticipation of embargo risks, therefore, the authorities will attempt to control (y_M, y_D, x_D, x_E). Also they may set aside some exports during the present (normal times) to buy imports for future emergencies, which are then stockpiled rather than consumed.

$x_S =$ the amount of exports used to purchase a stockpile of good y.
$rx_S = y_S =$ the amount of imports so obtained and stockpiled.
$p_S = 1/r =$ unit cost of y-stockpiles in terms of good x, including spoilage and transaction cost of delivering to and retrieving stockpiles.

The price of acquiring, maintaining, and dispensing stockpiles, i.e. p_S may depend on the risks of disruption. It may, therefore, involve further "out-of-pocket" or transaction costs (although parts of p_S may be fixed independent of π). We will assume in all cases that $p_S = p_S(\pi)$ remains a parameter since π is not chosen. These distinctions may have import for the relative desirability of protection and stockpiling, to be addressed presently.

Now the two-time-period objective function of a small country can be written.

$$\begin{aligned} W = {} & U^k[(\bar{x} - p_D y_D - p_W y_M^k - p_S y_S), (y_D + y_M^k)] \\ & + \pi U^\pi[(\bar{x} - p_D y_D - p_W y_M^\pi), (y_D + y_M^\pi + y_S)] \\ & + (1-\pi) U^\omega[(\bar{x} - p_D y_D), (y_D + y_S)] \\ \text{s.t.} \quad & y_D \geq 0, y_M^\pi \geq 0, y_M^k \geq 0, y_S \geq 0 \end{aligned} \tag{2.11}$$

Equation (2.11) indicates that stockpiles not used in an emergency are available in ordinary times, since today's stockpile $y_S = rx_S$ is available under both of the next period contingencies. This two period welfare function as in Eq. (2.11) is chosen for its simplicity. Typically, a realistic analysis with two time periods viz. "the present" known with certainty and "the future" subject to known risk $[\pi, (1 - \pi)]$ would require that future benefits and costs be discounted. This chapter ignores that issue, assuming zero discount. Equation (2.11) thus represents the complete two-period,

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[29] As the duration of the emergency trade disruption lengthens, this assumption of complete factor immobility becomes less and less realistic. In fact the duration of an emergency which requires advance preparation is just that over which factors of production are immobile!

[30] For further discussion of this assumption, see Tolley and Wilman (1977) and Nichols and Zeckhauser (1977).

two- contingency welfare function when both protection and stockpiling are permitted. To investigate the structure of the welfare maximum we will explore various restricted versions of Eq. (2.11) piecemeal. Then we will assemble the sub-analyses. We will begin with analysis of "protection only," followed by "stockpiling only," followed by protection plus stockpiling. For heuristic purposes, we collapse the two-period model to only a single period with two uncertain contingencies. We could re-extend it later to two periods, but with zero time discount and zero investment loss from stockpiling, as we demonstrate in Sect. 2.4, the difference in structure between a the two-period and single-period heuristic assumption is minimal.

2.3.4 Optimum Protection in the Absence of Stockpiling

The standard classical argument for trade intervention in the form of subsidy for importables is especially easy to illustrate if we collapse the expected utility function to one period with two-contingencies. Equation (2.12) shows the objective function for this special case; x_S and y_S do not appear since only protection, no stockpiling is allowed. The two-contingency objective function of a small country can be written

$$\begin{aligned} W = E(U) = \pi U^{\pi}\big[(\bar{x} - p_D y_D - p_W y_M^{\pi}), (y_D + y_M^{\pi})\big] \\ + (1-\pi)U^{\omega}[(\bar{x} - p_D y_D), (y_D)] \\ \text{s.t.}: \quad y_D \geq 0, \quad y_M^{\pi} \geq 0 \end{aligned} \tag{2.12}$$

The superscripts on U (i.e. U^{ω} and U^{π}) serving to distinguish the two contingencies, indicate different realized values of the same utility function in adversity (ω being "war" for short) and regular times (π being "peace") respectively.[31] The ex post immobility of productive resources is captured by the assumption that y_D and $x_D = p_D y_D$ must have the same values under both outcomes, whereas freedom to trade in peacetime and the loss of this option in war follows from the entry of y_M and $x_E = p_W y_M^{\pi}$ in peace but not war. Necessary conditions for a maximum of Eq. (2.12) with respect to (y_D, y_M^{π}) are given as (where $U_x^{\pi} = \partial U^{\pi}/\partial x$ etc.):

$$U_x^{\pi}/U_y^{\pi} = 1/p_W = t, \tag{2.13}$$

$$\frac{\pi U_x^{\pi} + (1-\pi)U_x^{\omega}}{\pi U_y^{\pi} + (1-\pi)U_y^{\omega}} \geq \frac{1}{p_D} = q. \tag{2.14}$$

In peace, trade is optimally pursued, and MRS, the left hand side of (2.13) is equal to the world price. But, comparative advantage is not fully exploited; rather domestic

[31] More realistically the utility function might differ between war and peace; some goods are simply more valued in war. To incorporate this idea calls for introducing state-dependent utility where the marginal rate of substitution (MRS)—i.e. relative value—between x and y shifts systematically, but elaboration along these lines is left for later.

production of y is subsidized (so that $x_D < \bar{x}$, or $y_D > 0$) so as to sustain an internal domestic marginal cost equal to a weighted composite of wartime and peacetime marginal utilities as in (2.14). Evidently if war is certain $\pi = 0$, and $MRS^{\omega} = q$; if war is impossible $\pi = 1$, $MRS^{\pi} = t > q$, and at a cut-off $\pi = \pi^0 < 1$, a strict equality obtains in Eq. (2.14) (see Shibata 1986).[32] As π (probability of peace) declines to zero, x_D increases from zero to x_D^n.

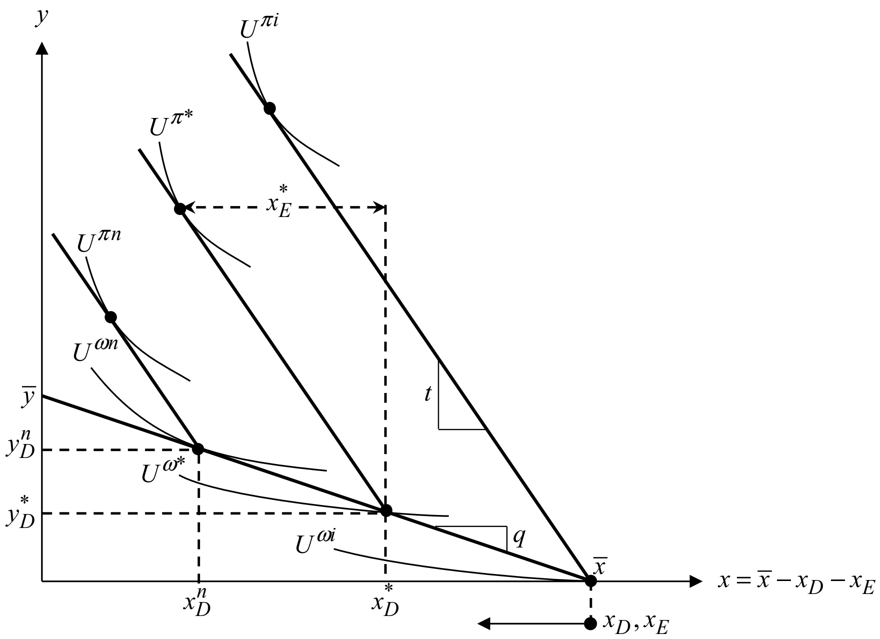
These outcomes are illustrated in Fig. 2.2, with x_D measured to the left from $\bar{x}$ and optimal values indicated by "*" . The diagram shows a Ricardian production possibility curve, linear between $\bar{x}$ and $\bar{y}$. A peacetime terms-of-trade line through $\bar{x}$ (the point of complete specialization in x) is drawn with slope $1/p_W = t$. If no plans were made for the possibility of trade disruption or war, the country would specialize at $\bar{x}$, and if war is actually avoided it would export some of x in return for imports of y, and consume where utility is maximized at the tangency with $U^{\pi i}$. But if war happens then with no preparations the country could only consume what it produces at $\bar{x}$, which would result in $U^{\omega i}$.

If the country made maximal preparation for war or trade disruption, it would give up x_D^n to produce y_D^n at home. Then if war did actually happen the country would produce and consume at $U^{\omega n}$ along the home Production Possibility Curve, while in the event of peace it would produce at that same point but be able to trade to a higher indifference curve, $U^{\pi n}$. We assume that this country recognizes that a wartime utility of $U^{\omega} > U^{\omega i}$ can be achieved only if peacetime utility is reduced to $U^{\pi} < U^{\pi i}$. The combination, $U^{\pi *}$, $U^{\omega *}$—assumed for illustration to be an interior expected utility optimum—is reached by subsidizing domestic production of y giving up x_D^* to produce y_D^* at home thereby fixing production at those levels for both contingencies. This subsidy package assures $U^{\omega *}$ in war and it allows trade in peacetime up to $U^{\pi *}$. Note that when protection is the sole instrument of national policy, at an expected utility maximum, the outcome in peace, U^{π}, is necessarily superior to that in war, U^{ω}, $U^{\pi *} > U^{\omega *}$.

If instead of assuming only a single time period we changed the model to include two time periods then, as shown (later in Sect. 2.4 of this chapter where details are provided) by Eq. (2.16), the qualitative implications of the one and two period models are basically identical.[33] Therefore, our analysis will continue with a single period, two contingency model. (Introduction of a positive utility discount for future periods would simply alter the weights between time-periods further in favor of free trade).

[32] For $\pi < \pi^0$ the negative protection, i.e. $y_M < 0$ could be desired, but this is eliminated by assumption.

[33] Some may find the assumption of only one period (with outcome uncertain as between two contingencies) too unrealistic. See Sect. 2.4 below.



$\overline{y}$ - Maximum possible domestic production of y.
$\overline{x}$ - Maximum domesitic production of x.
$1/q$ - Unit cost of y produced at home.
$1/t$ - Unit cost of y bought on world markets.
$U^{\omega i}$ - Wartime utility without protection.
$U^{\pi i}$ - Peacetime utility without protection.
$U^{\omega *}$ - Wartime utility with protection.
$U^{\pi *}$ - Peacetime utility with protection.
U^n, x_D^n - Choice which maximizes wartime utility.
* - Example of optimal choice when $p < 1$.

Fig. 2.2 Optimal protection without stockpiling: one time period—two contingencies. *Source* McGuire (2010)

2.3.5 Stockpiling and Standby Production

We can now consider the option of stockpiling—first as an alternative to and then in conjunction with protection. To compare the benefits of protection and of stockpiling, one first must understand that in the absence of offsetting anticipatory policies, war or embargo causes a consumption deficit of good y during wartime (i.e. a deficit in y^{ω}, or equivalently a surplus of y^{π}), and a surplus of good x in wartime (i.e. a surplus in x^{ω}, or equivalently a deficit of x^{π}).

An ideal first best solution would be to transfer y from peace to war, and to transfer x from war to peace. If one could transfer x- and y-consumption independently between contingencies—at insurance prices say v_x, for x, and v_y for y—the wartime surpluses and deficits would be curtailed individually. An insurance premium of v_y is then paid for each unit of y transferred from peace to war, while a premium of v_x is received for each unit of x transferred from war to peace. And if v_x, and v_y, reflected actuarially fair exchange rates $v_x = v_y = \pi/(1-\pi)$, then piecemeal optimization will equalize marginal utilities $U_x^\omega = U_x^\pi$ and $U_y^\omega = U_y^\pi$, and this would actually equalize consumption across contingencies and therefore equalize utility also (see Ehrlich and Becker 1972; or McGuire and Becker 2006).

Protection alone is partially effective in achieving such an ideal goal, and stockpiling alone is partially effective, but neither policy—neither alone nor in combination—actually achieves a first best optimum. Thus stockpiling partially substitutes for insurance, allowing some of the intercontingency deficit/surplus to be corrected but not all of it. In particular, stockpiling effects a transfer from x^π to y^ω. In other words, stockpiling relieves the deficit in wartime consumption of good y but creates or exacerbates the deficit in peacetime consumption of x (see Sect. 2.4 below and Eq. (2.19) for further discussion). As shown by Eq. (2.19), a crucial parameter in evaluating stockpiling as protection is the unit price of stockpiling, p_S, which determines relative and absolute benefits of acquiring stockpiles (versus other alternatives). A satisfactory model of this cost would allow for:

(1) a pure insurance or actuarial element in the price, where the amount of stockpile available in an emergency depends on the chance of actually needing the stockpile, or the relative proportion of time spent depleting the stockpile to the time spent accumulating it $(1-\pi)/\pi$.
(2) a proportionate spoilage or storage cost, α, and
(3) a constant unit cost independent of frequency or risk of emergency, β.

Putting these together gives Eq. (2.15)—numbered out of sequence.

$$p_S = [\{\alpha(1-\pi)/\pi\} + \beta]p_W. \tag{2.15}$$

One might also include costs of stockpile extraction during the emergency, war, embargo etc. but this effect will be neglected. Now we use a two-contingency one-period model to see how optimal $x_S = p_S y_S$ depends on the parameters in Eq. (2.15).

2.3.6 *The Optimal Stockpile*

In order to limit space we assume[34]:

(1) stockpile cost is unrelated to π, i.e. $\alpha = 0$ in Eq. (2.15)[35];

[34] More varied, and realistic alternatives are analyses in McGuire (2000, 2005, 2006).

[35] Note that Eq. (2.15) is out of sequence.

(2) no cost of stockpiling (no spoilage, storage, or retrieval cost) other than the inherent resource cost of commodity purchase;
(3) the stockpile is purchased at world price during peace for consumption in war so that $\beta = 1$, and $p_S = p_W$ in Eq. (2.15) or equivalently $r = t$ in the notation set up; and
(4) that all stockpiles become useless with zero utilization or salvage value if the bad event war is avoided.

These simplifying assumptions will serve to establish a baseline for analysis of optimum storage.[36]

A depiction of pure stockpiling decisions following Shibata (1986) is given in Fig. 2.3. Stockpiling essentially shifts Production Possibilities leftward by the amount, say, x_S^* during peace so that peacetime consumption is shown by point "a" along curve t^*, with optimum utility $U^{\pi *}$. Curve t^* originates at $\bar{x} - x_S^*$ since free trade and complete specialization in producing good x is maintained. The return for reducing peacetime utility to $U^{\pi *}$ from initial value $U^{\pi i}$ is that wartime utility rises from $U^{\omega i}$ to $U^{\omega *}$. The best (U^{ω}, U^{π}) combination follows from Eq. (2.21). Figure 2.3 shows this maximum (m) as $(U^{\pi m}, U^{\omega m})$. As π declines, greater peacetime sacrifices are warranted, and conceivably with high enough likelihood of war, the optimum can require so much stockpiling that $U^{\omega *} > U^{\pi *}$, an effect due partly to the assumption that costs of stockpiling are unrelated to risk, π.[37]

This result should be compared with the optimum under "pure protection"; rational trade control alone can never proceed so far that utility in peacetime is pushed below utility in war. But with stockpiling as an instrument, if $\alpha = 1$ and $\beta = 0$ then a rationally planning nation could conceivably prepare for an emergency so strenuously that it is worse off if the emergency fails to happen. This means that the prima facie policy case for truly serious stockpiling preparations could be much stronger than casual observation might suggest. Even though fair, zero-loading, insurance prices are unrealistic and, therefore, the reversal possibility (i.e. that $U^{\omega *} > U^{\pi *}$) is suspect, the argument strongly supports further serious analysis of stockpiling or its equivalent as preparation for supply disruption.

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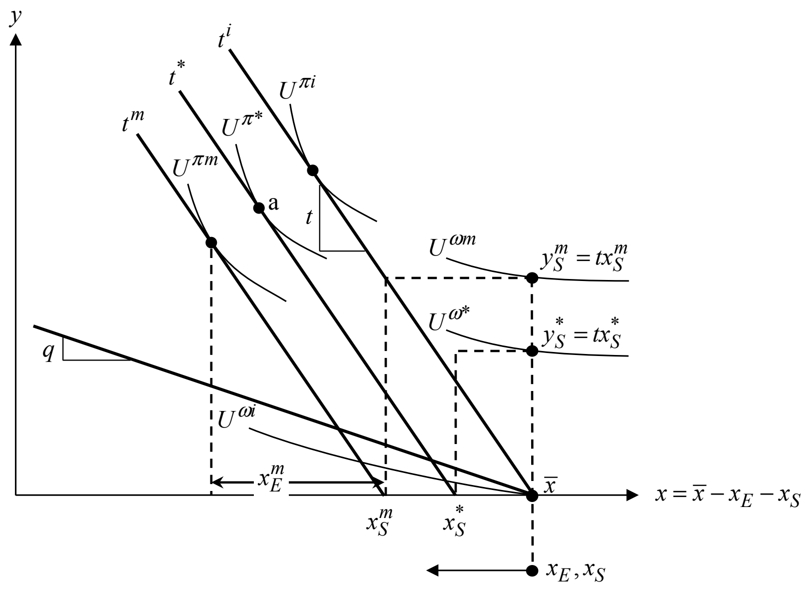
[36] A good alternative to this assumption might be that stockpiles can be provided at an actuarially fair price. In this case we would write $p_S = [(1 - \pi)/\pi]p_W$.

[37] In fact, if U is homothetic and stockpiling can be purchased at a "fair price" as in $p_S = [(1 - \pi)/\pi]p_W$, then as shown by McGuire and Becker (2006) for even the smallest chance of war/emergency, optimal stockpiling for adversity will always entail so much preparation that a reversal of utility positions results. That is, the optimal stockpile will be so great that utility is actually higher in the "bad" event, the emergency. The utility reversal result follows from the fact that fair insurance requires equalization of marginal utility of good y across contingencies; and homothetic utility, therefore, implies the stated reversal (McGuire 1991; McGuire and Becker 1994, 2006). The geometric depiction of the optimal stockpile with perfect insurance is similar to Fig. 2.3. In this case however, the price line for transforming x^{ω} into y^{π} i.e. $p_S = p_W(1 - \pi)/\pi$ is very steep at low risks of adversity indicating how very cheap it is to stockpile for a very unlikely event.



$1/q$	- Unit cost of domestic production of y.
$1/t$	- Unit cost of y bought on world markets, assumed the same as unit cost of stockpiling/ retreival.
$U^{\omega i}, U^{\pi i}$	- Wartime/peacetime utilities with no stockpile $x_S = 0$.
x_S^m, x_S^*	- Alternative amounts of x set aside to buy y – stockpiles.
y_S^m, y_S^*	- Corresponding amounts of y purchased and available during wartime.
$U^{\omega m}, U^{\omega *}$	- Wartime utilities corresponding to x_S^m, x_S^*.
$U^{\pi m}, U^{\pi *}$	- Peacetime utilities corresponding to x_S^m, x_S^*.

Fig. 2.3 Optimal stockpile without protection: one period—two contingencies. *Source* McGuire (2010)

2.3.7 The Optimal Mix of Stockpiling and Trade

Next we allow protection and stockpiling at the same time, i.e. $y_D > 0$ and $y_S > 0$ in Eq. (2.19) below. This generates another condition relating marginal benefits of protection (MBP) to marginal costs (MCP) in addition to Eqs. (2.20a), (2.20b) and (2.21). As Sect. 2.4 demonstrates, it also yields surprising discovery as to how the best mix of preparation depends on chances of adversity, and on the country's aversion to risk embodied in its utility function. One new condition (Eq. 2.22a below) from maximization when $y_D > 0$ simply and unexpectedly says that $MBP = MCP$ at an optimum. Then we can use the requirement from optimum peacetime trade—Eq. (2.3) above. As shown in Eq. (2.22c), this implies that the expected benefit from an increment of domestic y-production in "war" must equal its expected marginal cost in war plus the expected extra out of pocket domestic cost of home production over imports during peace. The important conclusion is that this approach allows one to compare costs of stockpiling alone, stockpiling in combination with protection, or protection alone at the same marginal benefit in war i.e. the same $(1-\pi)U_y^{\omega}$.

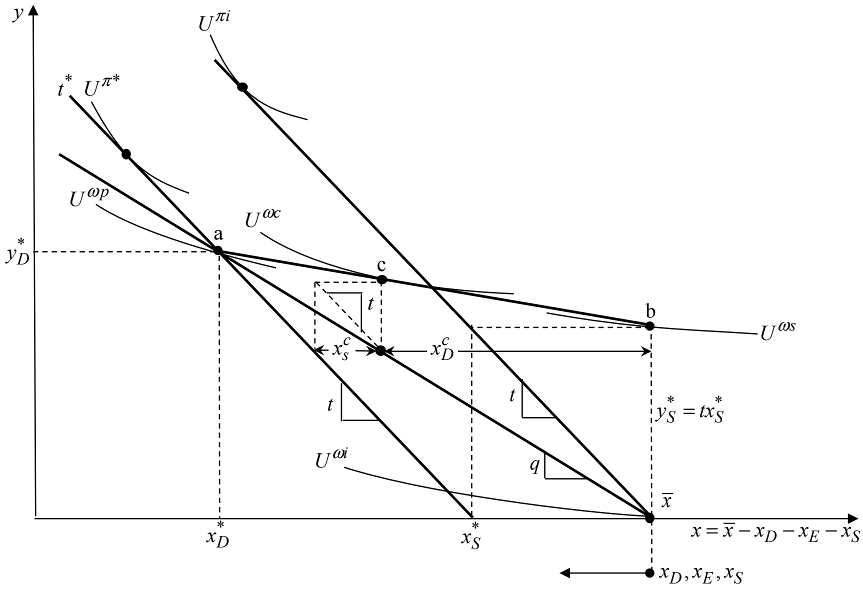
Equations (2.22c) and (2.20a) make this comparison and show that stockpiling alone, or stockpiling in combination with protection, or protection alone is the optimal provision for adversity, according as MCS $\lesseqgtr$ MCP. Equation (2.22d) gives this condition. If at an optimum both stockpiling and protection should be used simultaneously, then combining all three necessary conditions Eqs. (2.20) and (2.21) allows us to summarize in one formula—Eq. (2.23)—whether and to what degree to combine both y_D and y_S i.e. both protection and stockpiling.

Note first if all the conditions in (2.23) obtain, the optimum is "interior." Accordingly when p_S is independent of π, (as we assume here to simplify the argument) the solution marginal rate of substitution (MRS) does not depend on chances of war or peace. Figure 2.4 will be helpful for understanding this heuristic result, which is valid for any ordinary diminishing MRS utility function. It shows q—internal cost of good x—as moderately less than t, the world price of x. That is p_D, the domestic cost of y, is only somewhat greater than p_W, the world price of y. As before, with neither stockpiling nor protection the country obtains $U^{\pi i}$ and $U^{\omega i}$ as initial outcomes in peace and adversity respectively.

Now suppose the country is prepared to accept a lesser welfare, say $U^{\pi *}$, in peace. What is the maximum utility under adversity attainable as constrained by $U^{\pi *}$, domestic resources, technology, and world prices? (Remember we assume no spoilage or other loss of the stockpile). The diagram pictures the answer to this question. To attain $U^{\pi *}$ in peace a country may produce y_D^* internally foregoing x_D^* to do so. This option would give $U^{\omega p}$ in war and $U^{\pi *}$ in peace. On the other hand, the country might stockpile y_S^*. To do this it should pay x_S^* as exports. So using a pure stockpile-only option, its peacetime consumption opportunities via trade are shown by the line t^*, with the same slope as t. Line t^* also allows $U^{\pi *}$ in peace, but it yields $U^{\omega s}$ in war. Thus, either $U^{\omega p}$ or $U^{\omega s}$ are achievable in war at the same peacetime sacrifice of $(U^{\pi i} - U^{\pi *})$. But by a suitable linear combination of x_S^* and x_D^*, a wartime consumption at any point on the line "ab" connecting these two "pure" outcomes can be reached, while maintaining peacetime consumption and utility $U^{\pi *}$.

For example, combination x_S^c and x_D^c gives point c on line ab. As shown in the diagram, the optimum mix between x_D and x_S—i.e., between protection and stockpiling—occurs where the indifference curve is just tangent to opportunity line ab. To derive this optimal allocation no restriction on the utility function (other than diminishing MRS) is required.

Now we can use this construction to illuminate the interdependence between (1) chance of adversity as represented by π, and (2) the optimal mix between protection and stockpiling. To do this Fig. 2.5 then shows that as the optimal peacetime



$1/q$	- Unit cost of domestic production of y.
$1/t$	- Unit price of y bought on world markets, assumed to be same as the cost of storing and retreiving one unit of y.
$U^{\pi i}, U^{\omega i}$	- Peacetime/wartime utilities with no stockpiling and no protection: $x_D = 0, x_S = 0$.
$U^{\omega p}$	- Wartime utility with protection only and no stockpiling: $x_D = x_D^*, x_S = 0$.
$U^{\omega s}$	- Wartime utility with stockpiling only and no protection: $x_S = x_S^*, x_D = 0$.
$U^{\pi *}$	- Peacetime utility resulting from either pure stockpiling or pure protection: $x_D = x_D^*$ OR $x_S = x_S^*$.
ab	- Set of wartime consumption possibilities consistent with peacetime utility $U^{\pi *}$, derived from weighted combinations of pure stockpiling and pure protection.
$U^{\omega c}$	- Wartime optimum constrained by consumption possibilities ab.
x_S^c, x_D^c	- Optimum stockpile and domestic production to acheive $U^{\omega c}$.

Fig. 2.4 The optimal mix between stockpiling and protection: one period—two contingencies. *Source* McGuire (2010)

utility $U^{\pi *}$ declines, *ceteris paribus* the optimal wartime utility $U^{\omega *}$ increases.[38] Moreover, the figure shows how the mix between stockpiling and protection varies as acceptable peacetime welfare $U^{\pi *}$ changes. As $U^{\pi *}$ takes on values $U^{\pi a}$, $U^{\pi b}$, and $U^{\pi c}$, these allow corresponding $U^{\omega a}$, $U^{\omega b}$, and $U^{\omega c}$, and with this progression the relative importance of stockpiling increases. At $U^{\omega a}$ no reliance is placed on stockpiling, whereas at $U^{\omega c}$ no reliance is placed on protection.

[38]This assertion can be proven provided only that the utility function $U(x, y)$ is not concave from above, i.e. that the country is risk neutral or risk averse, not risk preferring. See McGuire and Becker (2006).

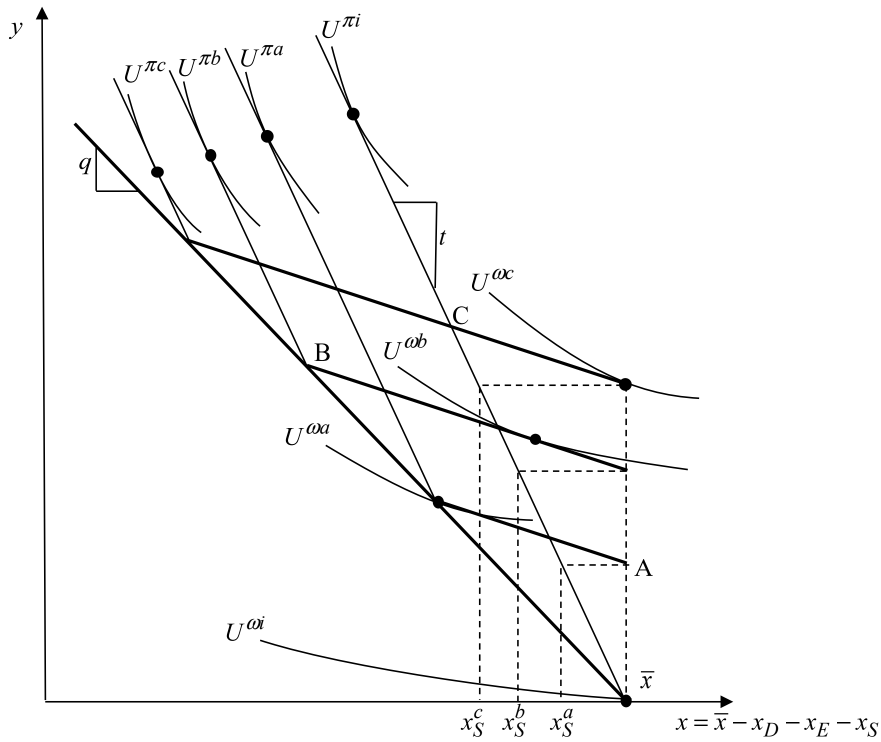


Fig. 2.5 The shift toward stockpiling in the optimum mix between protection and stockpiling as the risk of war increases: one time period—two contingencies. A, B, C = Different wartime consumption opportunity sets (S) available at greater sacrifice of peacetime utility. Each S is derived from linear combinations of protection and stockpiling. As risk of war increases, lower peacetime utility *U* acceptable and higher S-curve is available. As S shifts out, the optimum moves rightward

2.3.8 Effects of Risk Aversion

Now we can ask: "how does this progression (from points "a" to "b" to "c") relate to π, and to a country's aversion to risk (RA)? Intuitively one might think that a big risk should have qualitatively similar effects on decisions as would strong risk aversion, but as we will see that is not the case.

First suppose RA is given at some positive value and let π increase: then with higher risk of war, the optimal peacetime $U^{\pi *}$ declines, $U^{\omega *}$ increases, and more reliance is place on stockpiling. Next, conversely suppose π is given and let RA (i.e. the declining returns to scale in the utility function $U[x, y]$) decline: as risk aversion vanishes, and the utility function approaches linear homogeneity, all utility-costs of protection against risk are borne in peacetime (McGuire 2005, 2006a). Consequently as RA declines, so does the best choice of $U^{\pi *}$ decline, and the best mix shifts toward stockpiling, just as when π increases.

Conversely, the more risk averse a utility function is (i.e. one with more curvature in $U[x, y]$ to changes in scale) the greater relatively is the utility loss from the reduction in income during peace (income which is transferred to war) compared to the utility gain from the increase in income during war which results from that transfer. In response the optimizing country shifts resources losses away from peace time and toward war—i.e. reduces the size of the resource transfer from peace to war. Therefore, greater RA ceteris paribus raises optimal $U^{\pi *}$ and as in Fig. 2.5 shifts the best mix of policy toward protection. Again the same unexpected conclusion: higher RA produces the same result[39] at higher π!

2.3.9 Remarks

This section of the chapter has had a heuristic purpose to present an example of normative economics applied to a significant security issue. Using a deliberately frugal set of assumptions, we have integrated the classic security argument for protection of domestic industries against international competition with one prominent generic alternative—stockpiling and/or standby production capacity maintenance. The mathematical and diagrammatic formulations developed allow a clear picture of the relative benefits of the two instruments to emerge in comparison with their costs.

First, we show how the likelihood of interruption, has an income effect which favors stockpiling as a policy response, and a substitution effect which favors protection as a policy response. The substitution effect follows whenever costs of stockpiling bear some positive relation to chances of disruption. Expectedly, the optimum relative mix of protection and stockpiling will depend on all of the three cost parameters considered in the analysis (i.e., internal production costs, world terms of trade, and stockpiling costs). But unexpectedly, even when stockpiles can be accumulated at world prices and stored at zero interest with no loss or deterioration, some degree of protection may still be justified. Moreover, the risk aversion of the subject country has been shown systematically to influence the optimal mix; lower risk aversion ceteris paribus entails stronger preparation and increased importance for stockpiling.

The analysis has numerous gaps to be filled in before it becomes practically applicable to economic and security policy. However, we have constructed a method capable of shedding light onto an important area of interdependence between economic policy and international security, an area deserving more attention as globalization accelerates.

[39] This conclusion depends critically on independence of stockpile costs from chance of war.

2.4 Technical Discussion

Here we record certain technical details to support the argument of that portion of the main text that concerns protection against trade disruption.

2.4.1 *Equivalence Between One Period-Two Contingency and Two Period Analysis*

Even in a context where protection is the sole instrument for defense against risk of embargo some may find the assumption of only one period (with outcome uncertain as between two contingencies) too unrealistic. After all the present situation we assume is known; it is the future that is uncertain; and arguably to account for this fact at least two periods are essential—at a minimum the present known with certainty and the next period comprised of two contingencies.

To capture this environment—including immobility of resources among contingencies—we require one single decision as to protection to hold both in the certain present (designated by superscript k) and in the two future contingencies ("peace" designated by π, and "war" by ω). Again ignoring inter-temporal discounting and interest, this two-period context with stockpiling ruled out (i.e. $x_S = 0$) changes the objective function from Eq. (2.11) to one in which present utility and future peacetime utility are identical i.e. ($y_M^k = y_M^\pi = y_M; U^k = U^\pi$) which gives Eq. (2.16).

$$\begin{aligned} W &= U^k[(\bar{x} - p_D y_D - p_W y_M), (y_D + y_M)] \\ &\quad + \pi U^\pi[(\bar{x} - p_D y_D - p_W y_M), (y_D + y_M)] \\ &\quad + (1-\pi)U^\omega[(\bar{x} - p_D y_D), (y_D)] \\ \text{s.t. :} &\quad y_D \geq 0, y_M \geq 0 \end{aligned} \tag{2.16}$$

If instead of assuming only a single time period we changed the model to include two time periods[40] then, as in Eq. (2.16) above, for a maximum, necessary conditions derived from (2.16) become:

[40]As an alternative to (2.11), a repeated, "rolling", two-period decision context might be used to analyze stockpiling and protection. In such a model the same question would arise of how to handle the effect—on the decision to stockpile itself—of goods stocked in the present for consumption in the future when those stores become available even if no emergency actually materializes in the future. One approach to this question is to assume that inherited stockpiles add to initial wealth, with the stockpile repeatedly chosen period after period in steady state equilibrium. This approach is followed by Tolley and Wilman (1977). But an assumption of inheritance of (possibly depreciated) stockpiles would only influence the solu by an income effect in the steady state. Therefore, it will not be explicitly modeled here. We could represent this assumption in Eq. (2.11)—with other terms in W unchanged—by chang U^k to:
$U^k\big[(\bar{x} - p_D y_D - p_W y_M^k - p_S y_S), (\bar{y}_S + y_D + y_M^k)\big]$.

$$U_x^\pi / U_y^\pi = 1/p_W = t, \tag{2.17}$$

and

$$\frac{(1+\pi)U_x^\pi + (1-\pi)U_x^\omega}{(1+\pi)U_y^\pi + (1-\pi)U_y^\omega} \geq q = \frac{1}{p_D} \tag{2.18}$$

Comparing (2.14) from earlier in the text and (2.18), shows that if protection is the sole policy, the more realistic two-period assumption makes no qualitative difference in efficiency conditions; it merely changes the weights placed on marginal utilities from $[\pi, (1-\pi)]$ to $[(1+\pi), (1-\pi)]$ and therefore raises the critical value of risk of war $(1-\pi)$, identifying that value only above which some non-zero degree of protection becomes desirable. This shows that the qualitative implications of the one and two period models are identical. For this reason, the analysis in the main body of the text continues with a single period, two contingency model. (Introduction of a positive utility discount for future periods would simply alter the Eq. (2.18) weights further in favor of free trade).

2.4.2 Inter-contingency Gains and Losses from Stockpiling

To focus on how stockpiling and protection transfer goods across contingencies consider protection alone first: it reduces the x-surplus in war (reduces x^ω) and alleviates the y-deficit in wartime (increases y^ω) reallocating consumption in the same directions as would perfect insurance. But because protection supports inefficient production, it exacerbates the misallocations in peacetime—reallocating consumption just the opposite of perfect insurance. This suggests that the function of stockpiling is to substitute (imperfectly) for the absence of ideal insurance, because in fact x- and y-consumption cannot be independently transferred and the ideally desired first best outcomes cannot be realized.

Equation (2.19) shows how stockpiling partially substitutes for insurance, allowing some of the intercontingency deficit/surplus to be corrected but not all of it. In particular stockpiling effects a transfer from x^π to y^ω. In other words, stockpiling relieves the deficit in wartime consumption of good y but exacerbats the deficit in peacetime consumption of x.

$$\begin{aligned} W = \; & \pi U^\pi[(\bar{x} - p_D y_D - p_W y_M - p_S y_S), (y_D + y_M)] \\ & + (1-\pi)U^\omega[(\bar{x} - p_D y_D), (y_D + y_S)] \\ \text{s.t.:} \quad & y_D \geq 0, \quad y_M \geq 0, \quad y_S \geq 0 \end{aligned} \tag{2.19}$$

Here $\bar{y}_S$ represents the inherited stockpiles from the previous rolling decision. Steady state consistency requires that $y_S^* = \bar{y}_S^*$, i.e. requires that stockpiles chosen period after period be the same as inherited stockpiles.

First, if stockpiling, y_S, is the only allowed policy—complete free trade, $y_M > 0$, being assumed in peace and protection prohibited $y_D = 0$—then necessary conditions for an optimum (using $p_S = p_W$) from maximizing Eq. (2.8) become:

$$-\pi p_W U_x^\pi + (1-\pi)U_y^\omega = 0 \tag{2.20a}$$

$$-p_W U_x^\pi + U_y^\pi = 0. \tag{2.20b}$$

The first term in (2.20a) represents the marginal cost of stockpiling (MCS) and the second term represents the marginal benefit (MBS); so Eq. (2.20a) just requires MCS = MBS, while (2.20b) gives the optimal free trading condition during peace. Whence at an optimum combining (2.20a) and (2.20b) gives

$$1/p_W = t = \frac{U_x^\pi}{U_y^\pi} = \frac{\pi U_x^\pi}{(1-\pi)U_y^\omega}. \tag{2.21}$$

Equation (2.21) is to be compared with Eq. (2.3) or (2.6) to see the effect of stockpiling on the necessary conditions for optimality.

Next we allow protection and stockpiling at the same time, i.e. $y_D > 0$ and $y_S > 0$ in Eq. (2.19). This generates another condition relating marginal benefits of protection (MBP) to marginal costs (MCP) in addition to Eqs. (2.20a), (2.20b) and (2.21). As we shall see, this is what produces the surprising discovery as to how the best mix of preparation depends on chances of adversity, and on the country's aversion to risk embodied in its utility function. Equation (2.22a) is the new condition from maximizing (2.8) when $y_D > 0$.

$$\pi U_y^\pi + (1-\pi)U_y^\omega - \pi p_D U_x^\pi - (1-\pi)p_D U_x^\omega = 0. \tag{2.22a}$$

Equation (2.22a) simply says that MBP = MCP at an optimum. Rearranging and using $p_W U_x^\pi = U_y^\pi$ (from optimum peacetime trade) yields:

$$\pi p_W U_x^\pi + (1-\pi)U_y^\omega - \pi p_D U_x^\pi - (1-\pi)p_D U_x^\omega = 0. \tag{2.22b}$$

whence, as shown in Eq. (2.22c), for an optimum the expected benefit from an increment of domestic y-production in "war" must equal its expected marginal cost in war plus the expected extra out of pocket domestic cost of home production over imports during peace.

$$(1-\pi)U_y^\omega = \pi(p_D - p_W)U_x^\pi + (1-\pi)p_D U_x^\omega. \tag{2.22c}$$

Comparing Eqs. (2.22c) and (2.20a) at the same marginal benefit in war $(1-\pi)U_y^\omega$ shows that stockpiling alone, or stockpiling in combination with protection, or protection alone is the optimal provision for adversity, according as MCS $\lesseqgtr$ MCP. We write this as a formula in Eq. (2.22d).

$$\pi p_W U_x^\pi \lesseqgtr \pi(p_D - p_W)U_x^\pi + (1-\pi)p_D U_x^\omega \tag{2.22d}$$

If both stockpiling and protection are used simultaneously, then combining all three necessary conditions Eqs. (2.20a) and (2.21) gives (2.23a) to explain whether and to what degree to combine y_D and y_S i.e. protection and stockpiling.

$$\frac{U_x^\pi}{U_y^\pi} = t = \frac{1}{p_W} \tag{2.23a}$$

$$\frac{U_x^\pi}{U_y^\omega} = t\frac{1-\pi}{\pi} = \frac{1-\pi}{\pi p_W} \tag{2.23b}$$

$$\frac{U_x^\omega}{U_y^\omega} = 2q - t = \frac{2p_W - p_D}{p_W p_D} \tag{2.23c}$$

Note first if all these conditions obtain, the optimum is "interior" and thus when p_S is independent of π, Eq. (2.23a) shows that the solution marginal rate of substitution (MRS) does not depend on chances of war or peace. Second, wartime MRS—assuming it to be positive—can only equal $(2q - t)$ provided $(2q - t) > 0$, i.e. $q > t/2$ (or $p_W > p_D/2$). For $q < t/2$ (i.e. a very severe comparative disadvantage in home production of good y compared to importation and stockpiling) only stockpiling, and no trade protection should be chosen. For $q > t/2$ a mix of stockpiling and protection is optimal. These conclusions are highly dependent on the special simple assumed formula for p_S so they should be taken merely as illustrative of the potential application of such modeling as ours to trade and security policies.

The main text argues that by a suitable linear combination of x_S^* and x_D^*, a wartime consumption at any point on the line "ab" connecting the two "pure" outcomes (stockpiling only and protection only) can be reached, while maintaining peacetime consumption and utility $U^{\pi *}$. For example combination x_S^c and x_D^c gives point c on line ab. And the optimum mix between protection and stockpiling occurs where the indifference curve is just tangent to the opportunity line connecting the two "pure" outcomes. In our illustration, the slope of line ab is $(2q - t)$. Again note that this is a special case.[41]

2.5 Conclusion

As a discipline all economics contains both normative and positive elements. Applied to international security economics seems to cycle between these two approaches with emphasis on positive study having dominated the past generation. Given the

[41] The explanation of this slope is as follows: when one unit less of x is "stockpiled" $\Delta x_S = -1$, and two more units of x are allocated to internal production $\Delta x_D = +2$, the net effect is one less unit of x-consumption available during war; this generates in turn $+2q$ units of y from the internal reallocation, but $-t$ units of y from the stockpile; thus the slope $\Delta y/\Delta x$ becomes $(2q - t)$.

current world crises in the world economy and financial system, however, one might reasonably expect the pendulum to swing back to an emphasis on normative analysis.

This chapter presents examples of the substance and style of analysis appropriate in each of these arenas. For positive analysis, we have chosen the question of how economic productivity and trade, military technology and strategy, and political economy of governance can combine to determine a country's choice between peaceful trade and investment versus predation and conquest of others. For normative analysis, we have chosen analysis of alternative means a country could employ to shelter itself from trade disruption.

While positive advance of the foundations of security will always be valued, perhaps the pendulum is now swinging back to how governments might better manage international perils.

Appendix: Mutual Insurance Model

Analytical Framework

Here we will expand the analysis of a country's preparation and response to an emergency disruption when it can purchase or sell insurance from a trusted alliance partner. That is, we select one of the alternative instruments examined in the previous sections of the chapter, to focus on incentives among allies. The results are neither obvious nor expected.

There are two partner countries in the alliances, country 1 and country 2. They are identical in preferences but may be heterogeneous in income and emergency costs. They do not provide an international public good. There is an intra-alliance mutual insurance market.

Assuming additively separable utility functions, country i's expected utility is given by

$$W_i = (1-\alpha)V(c_i^A) + \alpha V(c_i^B), \tag{2.24}$$

where W_i is expected welfare of country i, c_i is private consumption of country i $(i = 1, 2)$. c_i is subject to uncertainty. In state A, which occurs at the probability of $1-\alpha$, country i enjoys c_i^A. In state B, which occurs at the probability of α, country i cannot enjoy c_i^A but can enjoy c_i^B. α indicates the probability of an economically disruptive production emergency or a 'war'. Note that subscript i refers to the country. Country 1 may buy insurance from or sell insurance to 2.

Country i's budget constraint in each state is given by

$$c_i^A = Y_i - ps_i \tag{2.25a}$$

$$c_i^B = (1-\pi_i)Y_i - ps_i + s_i$$

$$= c_i^A - \pi Y_i + s_i \tag{2.25b}$$

where Y_i is exogenously given (identical) national income of country i. $\pi_i(>0)$ indicates the net fraction of resources lost for each dollar of private production during the period of contingency when a war or natural disaster actually occurs—resources lost by reason of being diverted to a war effort or being cut off because of disruptions in production activities, thus leaving $(1 - \pi_i)Y_i$ of production under contingency B. p is the price of insurance, and π is called the penalty ratio.

s is the insurance return in event of emergency with p the price of insurance. In other words, ps indicates a demanding or buying country's premium paid to a supplying or selling country during state A. If $s > 0$ (< 0), s represents the demand (supply) country's return in event of state B with p the price of insurance, that is, the premium per dollar of insurance coverage. Country 1's return s_1 may be positive or negative with country 2's return, s_2, necessarily opposite in sign. It is also assumed that uncertainty is restricted to production so that the insurance premium paid by a buyer country to a seller country in state B is risk free and is not subject to the penalty ratio.

We assume that each government (or consumer) determines its insurance demand or supply, treating exogenous parameters α, π and insurance price p as given.

From Eqs. (2.25a) and (2.25b) we have

$$pc_i^B + \rho c_i^A = (1 - p\pi_i)Y_i \tag{2.26}$$

where $\rho = 1 - p$. We assume $p < 1$ $(\rho > 0)$ to investigate a meaningful realistic problem. The price of insurance, p, also means the price of consumption in state B, while $1 - p$ means the price of consumption in state A. Effective income in the left hand side of Eq. (2.26) evaluates emergency costs $\pi_i Y_i$ using p, the price of consumption in state B.

Each country maximizes its expected welfare (2.24) subject to its budget constraint (2.26). The first order condition for each country's optimization is given by

$$\alpha\rho V_c^B = p(1 - \alpha)V_c^A \tag{2.27}$$

where $V_c^A \equiv dV/dc^A$, $V_c^B \equiv dV/dc^B$. At the optimum, the marginal utility gain from allocation of an extra dollar to s (LHS of 2.27) just equals the expected marginal utility cost of that last dollar (RHS of 2.27). For simplicity subscript i in Eq. (2.27), referring to country i, is omitted and c_i is rewritten as c.

For example, consider the log-linear utility function

$$W = (1 - \alpha)\log c_i^A + \alpha \log c_i^B \tag{2.28}$$

Then the ordinary demand functions for c_i^A, c_i^B, and s_i are respectively given by

$$c_i^A = \frac{1 - \alpha}{1 - p}(1 - p\pi_i)Y \tag{2.29a}$$

$$c_i^B = \frac{\alpha}{p}(1 - p\pi_i)Y \tag{2.29b}$$

$$s_i = \frac{1 - p - (1 - \alpha)(1 - p\pi_i)}{p(1 - p)}Y \tag{2.29c}$$

Eq. (2.29c) implies that if

$$\frac{p(1 - \pi_2)}{1 - p\pi_2} > \alpha > \frac{p(1 - \pi_1)}{1 - p\pi_1} \tag{2.30}$$

then, $s_1 > 0 > s_2$; which is compatible with or consistent with mutual insurance.

From Eqs. (2.29a) and (2.29b), we also have

$$c_i^B - c_i^A = \frac{(1 - p\pi_i)(\alpha - p)}{p(1 - p)}Y \tag{2.31}$$

which is negative if $p > \alpha$.

Compensated Demand Function

Let us define the following expenditure function:

$$\text{Min} E_i \equiv pc_i^B + \rho c_i^A \quad \text{subject to } W_i \geq \overline{W}_i$$

Since preferences are identical between countries, the functional form is also the same between them. Considering Eq. (2.26), we have as the expenditure function

$$E(W_i, \alpha, 1 - p, p) = \tilde{E}(W_i, \alpha, \rho, p) = (1 - p\pi_i)Y_i \tag{2.32}$$

From Eq. (2.32) expenditure function E will determine W_i as a function of income Y_i, the probability of "war" α, the penalty ratio π_i, and the price of insurance p.

By a variant of Shephard's Lemma we know

$$\tilde{E}_{ip} \equiv \partial \tilde{E}(W_i, \alpha, \rho, p)/\partial p = c^B(W_i, \alpha, \rho, p) \tag{2.33}$$

$$\tilde{E}_{i\rho} \equiv \partial \tilde{E}(W_i, \alpha, \rho, p)/\partial \rho = c^A(W_i, \alpha, \rho, p) \tag{2.34}$$

$$E_{ip} \equiv \partial E(W_i, \alpha, 1 - p, p)/\partial p = c_i^B - c_i^A = s(W_i, \alpha, 1 - p, p, \pi, Y_i) - \pi_i Y_i \tag{2.35}$$

where $s(\ )$ is the compensated demand (or supply) function for insurance and $c^j(\ )$ is the compensated demand function for private consumption in state j $(j = A, B)$. It follows from Eq. (2.35) that $s_\pi \equiv \partial s/\partial \pi Y = 1$.

Alternatively, from Eqs. (2.24) and (2.26), we solve for s, c^A and c^B respectively as functions of W and p, which give the compensated demand (or supply) functions for s, c^A and c^B; Eqs. (2.35), (2.33) and (2.34), respectively.

Totally differentiating (2.24) and (2.26), yields

$$\begin{bmatrix} \alpha\rho V_c^B/p, & \alpha V_c^B \\ -p(1-\alpha)V_{cc}^A, & \rho\alpha V_{cc}^B \end{bmatrix} \begin{bmatrix} dc^A \\ dc^B \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} dW + \begin{bmatrix} 0 \\ \alpha V_c^B/p \end{bmatrix} dp \tag{2.36}$$

where $V_{cc}^A \equiv d^2V^A/dc^Adc^A$, $V_{cc}^B \equiv d^2V^B/dc^Bdc^B$, $V^A \equiv V(c^A)$, $V^B \equiv V(c^B)$.

Hence we have

$$c_W^A \equiv \partial c^A/\partial W = \rho\alpha V_{cc}^B/\gamma > 0 \tag{2.37}$$

$$c_W^B \equiv \partial c^B/\partial W = p(1-\alpha)V_{cc}^A/\gamma > 0 \tag{2.38}$$

$$s_W \equiv \partial s/\partial W = c_W^B - c_W^A = \left[p(1-\alpha)V_{cc}^A - \alpha\rho V_{cc}^B\right]/\gamma \tag{2.39}$$

$$\begin{aligned} E_W &\equiv \partial E/\partial W = p\partial c^B/\partial W + \rho\,\partial c^A/\partial W \\ &= \left[p^2(1-\alpha)V_{cc}^A + \rho^2\alpha V_{cc}^B\right]/\gamma = \frac{p}{\alpha V_c^B} > 0 \end{aligned} \tag{2.40}$$

$$c_p^A \equiv \partial c^A/\partial p = -\alpha^2 V_c^B V_c^B/p\gamma > 0 \tag{2.41}$$

$$c_p^B \equiv \partial c^B/\partial p = \alpha^2 V_c^B V_c^B \rho/p^2\gamma < 0 \tag{2.42}$$

$$s_p \equiv \partial s/\partial p = c_p^B - c_p^A = \alpha^2 V_c^B V_c^B/p^2\gamma < 0 \tag{2.43}$$

where $\gamma \equiv \alpha V_c^B\left[p^2(1-\alpha)V_{cc}^A + \rho^2\alpha V_{cc}^B\right]/p < 0$.

Greater values of W require higher values of c^A, c^B, and E. An increase in p raises c^A but reduces c^B and s—results that are qualitatively plausible. However, the sign of Eq. (2.43) is ambiguous. To see this, suppose relative risk aversion is constant $\left(\frac{cV_{cc}}{V_c} = -\lambda\right)$. Then considering the first-order condition (2.26), (2.43) may be rewritten as

$$s_W = \alpha\rho\lambda V_c^B(c^A - c^B)/c^Ac^B\gamma \tag{2.43$'$}$$

Thus, if $c^A > c^B$, as we will assume then it follows that $s_W < 0$ and $E_p \equiv \partial E/\partial p = c^B - c^A < 0$. In summary, in so far as $\pi_1 Y_1 \geq \pi_2 Y_2$ and $Y_1 \leq Y_2$ (and strict inequality obtains for one or both), then $W_1 < W_2$ and $s_1 > s_2$.

Two Country Model

The two country model will then be summarized by

$$E(W_1, p) = (1 - p\pi_1)Y_1 \tag{2.44}$$

$$E(W_2, p) = (1 - p\pi_2)Y_2 \tag{2.45}$$

$$s(W_1, p, \pi_1 Y_1) + s(W_2, p, \pi_2 Y_2) = 0 \tag{2.46}$$

Equation (2.44) gives expected welfare of country 1 as a function of insurance price, p, and its own effective income, $(1 - p\pi_1)Y_1$. Similarly, Eq. (2.45) shows expected welfare of country 2. Equation (2.46) gives the equilibrium condition for the private mutual insurance market and shows the equilibrium value of insurance price, p. Since there are no externalities, this equilibrium is Pareto efficient.

The benefits of insurance-alliance formation depend crucially on the nature of relative risks of emergency. Equation (2.46) implies that s_1 and s_2 must have opposite signs. Without loss of generality, assume $s_1 > 0$ and $s_2 = -s_1 < 0$. Then, country 1 is the buyer, while country 2 is the seller or writer of insurance. If $Y_1 = Y_2$ and $\pi_1 = \pi_2$, both countries are identical whence $W_1 = W_2$ and $s_1 = s_2 = 0$. Accordingly, heterogeneous income and/or penalty ratios are essential for the mutual insurance. We now briefly explore how such a situation would occur.

Suppose that Y is the same between two countries. Nevertheless, if π is high in country 1 and low 2, it is still possible to have mutual insurance. With $s_\pi = 1$, s_i is necessarily increasing with π_i. Therefore, when $\pi_1 > \pi_2$, it is possible for $s_1 = -s_2 > 0$. From Eqs. (2.44) and (2.45) we then would know $W_1 < W_2$ when $\pi_1 > \pi_2$.

Comparative Statics

Next we investigate some comparative statics results in this insurance model. Totally differentiating Eqs. (2.44), (2.45) and (2.46), gives

$$\begin{bmatrix} E_{1W} & 0 & s_1 \\ 0 & E_{2W} & s_2 \\ s_{1W} & s_{2W} & s_{1p} + s_{2p} \end{bmatrix} \begin{bmatrix} dW_1 \\ dW_2 \\ dp \end{bmatrix} = \begin{bmatrix} -pY_1 \\ 0 \\ -Y_1 \end{bmatrix} d\pi_1 \tag{2.47}$$

Hence, we have

$$\frac{dW_1}{d\pi_1} = -\frac{Y_1}{\Delta}\left\{p\left[E_{2W}(s_{1p} + s_{2p}) - s_2 s_{2W}\right] - s_1 E_{2W}\right\} \tag{2.48}$$

$$\frac{dW_2}{d\pi_1} = -\frac{Y_1}{\Delta} s_2 (ps_{1W} - E_{1W}) \tag{2.49}$$

$$\frac{dp}{d\pi_1} = \frac{Y}{\Delta} E_{2W} (ps_{1W} - E_{1W}) \tag{2.50}$$

where $\Delta \equiv E_{1W}E_{2W}(s_{1p} + s_{2p}) - s_1 E_{2W} s_{1W} - s_2 E_{1W} s_{2W}$ and the first subscript on the partial derivatives refers to the country. As shown in Eqs. (2.40) and (2.43), $E_W > 0$, $s_p < 0$. Moreover, Δ is negative from the stability condition.

Therefore, it is easy to see that Eq. (2.50) is positive. To see this note that an increase in π_1 raises the price of insurance, p; an increase in the penalty ratio will raise the price of insurance. Since $s_2 < 0$, Eq. (2.49) is positive. An increase in p, of course, is desirable for the seller country; and an increase in π_1 has a positive welfare spillover into country 2—this being the ordinary price effect. Since $s_1 > 0$, the sign of Eq. (2.48) is negative. An increase in π_1, therefore, directly reduces the expected income of country 1—this being the ordinary the income effect. As country 1 is a buyer, an increase in p implies an unfavorable price effect, which will strengthen the unfavorable income effect of the increase in the penalty ratio. In sum then, an increase in π_1 hurts country 1, while it benefits country 2.

An increase in π_2 may be analyzed in the similar way. That is, it also increases the price of insurance, hurting country 1 due to the price effect. On the other hand, while greater π_2 directly reduces country 2's own effective income, it indirectly raises 2's welfare due to the price effect. Since the latter price effect offsets the former income effect, the overall welfare effect on country 2 is ambiguous.

These results suggest that an increase in the emergency cost has different spillover effects, depending on where it occurs. If the penalty ratio rises in the demand country, it has a positive spillover effect on the supply country. On the other hand, if the penalty ratio rises in the supply country, it has a negative spillover effect on the demand/buying country. The reason here is that an increase in the penalty ratio in either country will raise the price of insurance. All these theoretical results seem intuitively and practically plausible.

Note particularly that a decrease in π_i has the same qualitative effect as an increase in Y_i, national income of country i, by reducing the price of insurance. Although the penalty ratio does not directly affect the price of insurance changes in that ratio will affect the economy by changing emergency costs. Therefore, an increase in Y_1 (income of the insurance buyer) will hurt country 2, while an increase in Y_2 (income of the seller of insurance) benefits country 1. Each country directly gains from its own economic growth due to the income effect but country 2 gets smaller benefits of its own economic growth than country 1 due to a reduction in the price of the insurance that it sells. A surprising and underappreciated result that follows: World-wide economic growth is more beneficial to the country that buys insurance than it may be to the country that provides or sells it.

This structure of benefits and costs next leads us to analyze the effect of transferring income between the countries within our alliance. An income transfer like this is equivalent to a change in the penalty ratios: where the penalty ratio rises in one country but it decreases in another country. To capture this idea let the constraint of $dY_1 + dY_2 = 0$. Then from Eqs. (2.44), (2.45) and (2.46) we have

$$\frac{dW_1}{dY_1} = \frac{1}{\Delta}\left[(1 - p\pi_1)E_{2W}(s_{1p} + s_{2p}) + (\pi_2 - \pi_1)s_1(ps_{2W} - E_{2W})\right] \tag{2.51}$$

$$\frac{dW_2}{dY_2} = -\frac{1}{\Delta}\left[(1 - p\pi_2)E_{1W}(s_{1p} + s_{2p}) - (\pi_2 - \pi_1)s_2(ps_{1W} - E_{1W})\right] \tag{2.52}$$

$$\frac{dp}{dY_1} = \frac{1}{\Delta}[-(1 - p\pi_1)s_{1W}E_{2W} + (1 - p\pi_2)s_{2W}E_{1W} + (\pi_2 - \pi_1)E_{1W}E_{2W}] \tag{2.53}$$

These equations lay bare the relationships among differences between countries in loss from emergency, π, value of p the price of insurance, and welfare consequences of income/wealth transfer. Equation (2.53) implies that as $\pi_2 > \pi_1$, then p must be of a lower value and vice versa. That is, if the penalty ratio is higher in country 1, a transfer from country 2 to country 1 will raise the price of insurance, hurting the country that buys and benefiting the supplier country. This impact represents the price effect. As for income effect from greater π, the buyer country gains and the supplying country loses. Thus, Eq. (2.51) is positive if $\pi_2 > \pi_1$. In this case both income and price effects benefit buyer country 1. However, if $\pi_1 > \pi_2$, the price effect of higher "p" hurts country 1, while the income effect benefits that country. Equation (2.51) becomes positive when the income effect dominates the price effect.

Equation (A29) is negative if $\pi_2 > \pi_1$. In that case both the income and price effects hurt country 2 the seller or writer of insurance. However, if $\pi_1 > \pi_2$, the sign of Eq. (A29) becomes ambiguous since the income and price effects offset each other. To sum up, when $\pi_2 > \pi_1$, the country which receives a transfer becomes better off while the country which pays out the transfer becomes worse off. However, if $\pi_2 < \pi_1$, we could have a transfer paradox: namely that a giving country gains and a receiving country loses.

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Chapter 3
One-Dimensional Public Goods in Models of Alliance

3.1 Introduction

An interesting application of non-cooperative private funding of public goods arises in international security and national defense. In their classical article on the economic theory of alliances, Olson and Zeckhauser (1966) demonstrated that countries may allocate some fraction of national income to international (or regional) public goods (for example, special trading agreements, formation of international organizations, military preparedness, active international diplomacy, foreign aid) to reduce regional and international tension and to avoid random emergency costs. For example, deterrence as provided by the nuclear triad is non-rival among allies especially if the retaliation is automatic.

To summarize Olson and Zeckhauser (1966): the interdependence of economic activities among allies collectivizes the security of those allies. The public good within an alliance may capture the benefits of risk sharing. And thus by confederating economically with each other, countries create alliances. Each allied country can enjoy the benefits of its mutual insurance as an international public good. Still, the non-cooperative levels of spending on national security by alliance members are sub-optimal from the cooperative perspective of the alliance as a whole.

In this chapter, we develop models of alliance with one-dimensional public goods and investigate how an international wealth transfer between allies influences the provision of international public goods and the welfare of the allies. This question is related to the realities in international defense alliances involving Japan and the United States. For example, in 1990, Japan offered $4 billion to the allied effort in the Gulf War, and contributed an additional $9 billion in 1991. The $9 billion may be regarded as a transfer from Japan to the United States. How would it hurt Japan and benefit the United States?

We consider one-dimensional international public good models. A crucial assumption common to these models is that the public good of an alliance is voluntarily and non-cooperatively provided by the allied countries' governments.

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In this chapter, we will present two models to examine the above-mentioned question. The first model is the basic model of this chapter, which comes from the standard framework of private provision of public goods. See Bergstrom et al. (1986) among others. We incorporate publicness of defense among allies into the utility function assuming that the public good here is a pure public good. We then analyze the effects of international transfers on welfare within the alliance.

We will show that an international transfer between allies does not necessarily benefit the recipient and harms the donor. First, we review the so-called "neutrality" result. It was pointed out by Shibata (1971) and Warr (1983) and applied to alliance defense spending by Kemp (1984). They showed that when agents voluntarily contribute to a public good, any income transfer between contributors affects neither the provision of the public good nor the welfare of the agents (provided all partners maintain some positive contribution both before and after the transfer).

Furthermore, international transfers might have other paradoxical impacts on the welfares of the donor and the recipient. In a different framework of international trade, the welfare effects of a unilateral transfer have been extensively studied by trade theorists [see Bhagwati et al. (1983) and Yano (1983), among others], and conditions for paradoxical phenomena of immiserizing transfers from abroad and enriching transfer payments to occur have been obtained. We will refer to the situation in which both the donor and the recipient of the transfer simultaneously benefit/lose from the transfer as a "weak paradoxical result." We also describe the surprising result in which a transfer of income makes the donor better off, while it makes the recipient worse off as a "strong paradoxical result."

A three-country framework is essential to obtain these paradoxical results. Gale (1974) constructed a three-agent example in which both the transfer giver and the receiver gain after a transfer, harming the third agent. Using the standard trade model with three countries, Bhagwati et al. (1983) and Yano (1983) demonstrated that a country may gain by giving a transfer, that the receiver may lose, and that these two phenomena may appear at the same time.

To apply this insight to the case of public goods, we then develop the second model, which is the three-country model where an assumption of impure benefit of the public good is crucial. Spending on national defense is a good example of impure public goods where one country's supply of international public goods may well be an imperfect substitute for another country's supply. It seems reasonable to assume that although countries have a common interest in providing the international public goods, their preferences over the public goods may not necessarily be the same.

The second model of this chapter is based on Ihori (1992a, b), and explains the possibility of paradoxical results by extending the first model. Accordingly, we extend the conventional model of non-cooperative private funding of pure public goods to the case of impure public goods using a three-country framework. Because different countries' expenditures on these goods are imperfect substitutes, the neutrality result will not always apply. Transfers affect both the equilibrium level of public good provision and the utility of the countries giving and receiving transfers. We will show that under certain conditions such transfers have a strong paradoxical result:

the reaction of the third country to expenditure causes the giver's utility to rise and the receiver's to fall.

The Appendix of this chapter explicitly introduces economic protection against random emergency cost by mutual insurance into the two-country framework of the first model, which is an extension of the Appendix of Chap. 2. To mitigate the effects of emergencies, each country creates a private mutual insurance market in addition to providing voluntarily an international public good. We will explore how protection through voluntary provision of insurance as an international public good together with mutual insurance affects welfare. The existence of both mutual insurance and an international public good is crucial to obtain welfare equalization and a weak paradox of international transfer.

To sum up, the organization of this chapter is as follows. First, Sect. 3.2 presents the basic and conventional model of pure public goods in an allied economy. Section 3.2 also describes Nash equilibria in which independent national governments contribute voluntarily to an international public good. Section 3.3 explains the neutrality result of income transfer and discuss the plausibility of this theorem. Section 3.4 extends the basic model to a three-country world with impure public goods, and investigates the welfare effects of international transfers, and discusses possibilities of weak and strong paradoxical results. Section 3.5 investigates effects of economic growth on welfare among allies. Finally, Sect. 3.6 concludes the chapter. Furthermore, Appendix develops a model of economic protection against random emergency costs with mutual insurance and provision of public goods in a two-country framework.

3.2 The Theory of Public Good Provision: The Nash Equilibrium Approach

3.2.1 Private Provision of Public Goods

It is plausible to assume that each allied country non-cooperatively determines the provision of security spending in an alliance. For example, we may say that members of the NATO alliance behave non-cooperatively with respect to the provision of international public goods in the alliance. See McGuire (1990), Sandler (1977), and Sandler and Hartley (1995) among others. Thus, this chapter provides a positive theoretical analysis of an alliance with the private provision of public goods.

When the allied countries cannot provide public goods optimally at the cooperative solution, how does each allied country behave? They may have an incentive to provide the public goods privately. The standard approach with the private provision of public goods is called the Nash equilibrium approach, whereby each agent optimizes the provision of public goods at a given level of public goods provided by other agents. See Bergstrom and Varian (1985) and Bergstrom et al. (1986) among others.

If we consider the non-cooperative provision of an international public good in an alliance, each country may be regarded as a private contributor since it determines its provision of the international public good based solely on its own interest. Thus, this framework is well applicable for variety cases beyond defense spending. For example, the total emissions of CO_2 could be reduced by the abatement activities of any countries in the world; thus, the provision of abatement spending by each country may be regarded as a private provision of public goods. Then, the Nash equilibrium approach becomes relevant since each country behaves non-cooperatively to maximize its own welfare.

3.2.2 A Two-Country Model

Consider a two-country alliance of countries 1 and 2. In this context, we formulate country 1's optimization problem. By replacing index 1 with 2, we also formulate the problem of country 2.

Country 1's utility function is given as

$$U_1 = u_1(c_1, G), \tag{3.1}$$

where U_1 is its utility, $u_1(.)$ is its utility function, c_1 its consumption of private goods, and G its consumption of pure public goods. We assume that utility function $u_1(.)$ is twice continuously differentiable, increasing in all arguments, and strictly concave. We also assume that both goods are normal goods, which means that country 1's demands for both goods increase with its income. We do not necessarily assume that preferences are identical between countries.

Country 1's budget constraint is given as

$$c_1 + g_1 = Y_1, \tag{3.2}$$

where Y_1 is its income, and g_1 denotes the private (or voluntary) provision of public goods by country 1. For simplicity, the marginal cost of public goods is fixed and normalized to be unity. Thus, production technologies are Ricardian and identical across countries, and units are chosen such that the constant marginal rate of transformation (MRT) between the private good and the public good is unity for all countries. We assume that the amount of public good enjoyed by both countries comprises the sum of the private provisions. Then, we have

$$\sum_{i=1}^{2} g_i = G. \tag{3.3}$$

From Eqs. (3.2) and (3.3), the budget constraint of country 1 is rewritten as[1]

[1]Similarly, the budget constraint of country 2 is written as $c_2 + G = Y_2 + g_1$.

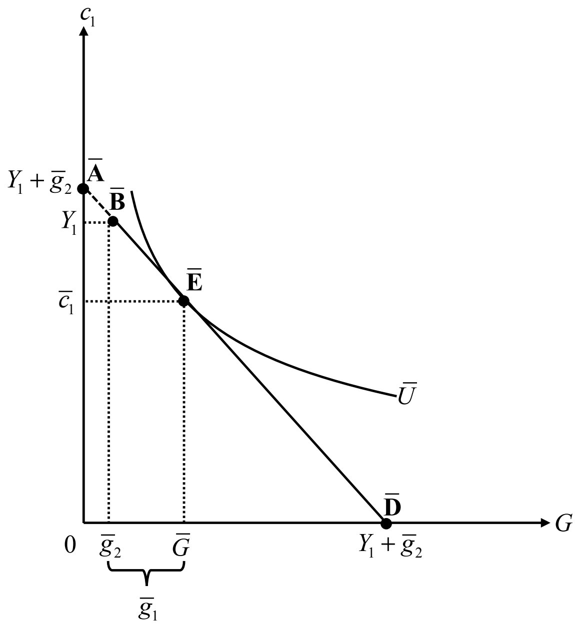


Fig. 3.1 Indifference curve and budget line. *Source* Authors

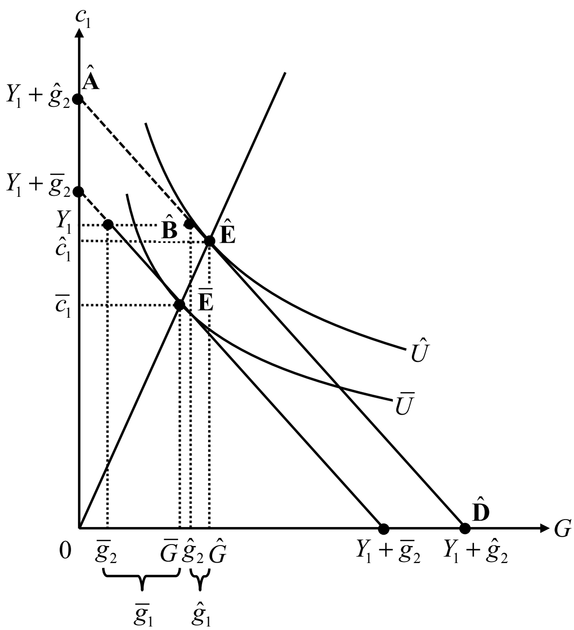
$$c_1 + G = Y_1 + g_2 \tag{3.4}$$

The right-hand side of Eq. (3.4) is called the effective income of country 1, which includes the benefit of the spillover from public goods provided by the other country, country 2. Country 1, therefore, effectively chooses c_1 and G as it maximizes welfare, Eq. (3.1), subject to budget constraint, Eq. (3.2), at given levels of Y_1 and g_2. Remembering both private and public goods are assumed normal goods, we note that country i's demands for both goods increase with its effective income.

The foregoing formulation assumes that country 1 regards others' provisions of public goods g_2 as fixed when it chooses values for its own decision variables, c_1 and G. We also assume that country 2 regards the provision of public good by country 1 as given. This is known the Nash equilibrium approach, as it exemplifies the definition of non-cooperative Nash equilibrium.

In Fig. 3.1, the vertical axis represents private goods, c_1, and the horizontal axis is public goods, G. Line $\bar{\mathbf{A}}\bar{\mathbf{D}}$ corresponds to the budget constraint, Eq. (3.4) where g_2 is assumed to be fixed at $\bar{g}_2$. Note that country 1 cannot choose points on $\bar{\mathbf{A}}\bar{\mathbf{B}}$, where its private provision of the public good becomes negative. At point $\bar{\mathbf{B}}$, country 1 allocates its entire endowed income to consumption, c_1, and does not provide any public good. At point $\bar{\mathbf{D}}$, country 1 allocates its entire income to the public good and does not consume any private good. The slope of the budget line represents the MRT between private good and public good, which is assumed to be unity. Thus, country 1 may choose any point on line-segment $\bar{\mathbf{B}}\bar{\mathbf{D}}$.

Fig. 3.2 Expansion path. *Source* Authors



Curve $\overline{U}$ draws 1's indifference curve, showing combinations of private goods and public goods needed to maintain country 1's utility at a specific level or value. Although we draw one indifference curve in Fig. 3.1, it is only one of indifference curves each of a different utility level. Since we assume that the utility function is concave, indifference curves are concave from above toward the origin. From now on we assume an interior solution.

The optimal point, the highest utility point, on $\overline{\mathbf{B}}\,\overline{\mathbf{D}}$ is given by point $\overline{\mathbf{E}}$, where line $\overline{\mathbf{B}}\,\overline{\mathbf{D}}$ is tangent to indifference curve $\overline{U}$. Country 1 consumes private good of $\bar{c}_1$ and public goods of $\overline{G}$. Thus, at the given level of country 2's public good provision, $\bar{g}_2$, country 1's optimal provision is given as $\bar{g}_1$.

Now, we investigate how country 1 responds to a change in country 2's provision. We draw a new budget line and an indifference curve on Fig. 3.1 to obtain Fig. 3.2. Line-segment $\widehat{\mathbf{B}}\,\widehat{\mathbf{D}}$ is the new budget line of country 1, showing its budget constraint when country 2's contribution, g_2, is fixed at $\widehat{g}_2$. The new optimal choice is given at point $\widehat{\mathbf{E}}$ where line $\widehat{\mathbf{B}}\widehat{\mathbf{D}}$ is tangent to indifference curve $\hat{U}$. The line connecting the optimal points of country 1, $\overline{\mathbf{E}}$ and $\widehat{\mathbf{E}}$, is called the expansion path of country 1.[2] We note that when country 2 increases its provision of public good to $\hat{g}_2$, country 1 also increases its optimal amount of public good to $\widehat{G}$, but reduces its contribution to $\hat{g}_1$.

[2]In Figs. 3.1 and 3.2, the expansion path is drawn as a ray from the origin. When country 1 has a homothetic preference, its expansion path becomes a ray from the origin as in the figure. If it were not so, its expansion path would not become a ray. Please refer to the textbooks of microeconomics (e.g. Mas-Collel et al. 1995) for the details of expansion path.

Based on the above formulation, the optimal level of g_1 for country 1 is a function of g_2. This relationship gives the Nash reaction function of country 1, $N_1(.)$. Thus, we have

$$g_1 = N_1(g_2). \tag{3.5}$$

If country 2 provides more in the way of public goods, country 1 will reduce its own contribution and raise its consumption of private goods ($dN_1/dg_2 < 0$); however, country 1 will not reduce its allocation to public goods so much as to cause a reduction in the total supply ($-1 < dN_1/dg_2$). The reason for it is that when g_2 increases, country 1's effective income $Y_1 + g_2$ rises. Then because of positive income effects, country 1 raises its consumption of both private goods *and* public goods. Thus, its consumption of private goods increases, and g_1 declines but nevertheless consumption of public goods, G, increases.

Country 2's optimizing behavior is similarly formulated. Its Nash response function is then given as

$$g_2 = N_2(g_1). \tag{3.6}$$

The combination of g_1 and g_2, which satisfies Eqs. (3.5) and (3.6) at the same time, gives the Nash equilibrium.

3.2.3 The Inefficiency of Nash Equilibrium

Let us compare a Nash equilibrium with a Pareto optimum allocation, that is, compare a non-cooperative solution and a cooperative solution. As we will see below, the provision of public goods at a Nash equilibrium is smaller than at a Pareto optimum. This follows because under non-cooperative Nash behavior, each country chooses its provision by considering only its own welfare.

At a Nash equilibrium point, the marginal benefit enjoyed by country, (given by its marginal rate of substitution, or MRS), is equal to the marginal cost of public goods (given by its marginal rate of transformation, or MRT). Thus at such an equilibrium, we obtain

$$MRS_1^N = MRS_2^N = MRT, \tag{3.7}$$

where MRS_i^N $(i = 1, 2)$ is the MRS of country i at the Nash equilibrium. If the good considered were not a public good but a private good, with benefit limited solely to the country that provides it, then the allocations characterized by (3.7) would be socially efficient and—subject to distributive justice—"optimal." Private markets without public goods in general yield Pareto efficient/optimal allocations, as follows from the first fundamental theorem of welfare economics.[3]

However, for public goods in, say, a multiple country alliance, the marginal social benefit of providing the public good is larger than any single country's marginal benefit. To attain a first best allocation, it is necessary that each partner or ally include in its allocative decision calculus the spillover effect on other allies. According to Samuelson (1954), to achieve an optimum the sum of all country's marginal rates of substitution gives the marginal *social* benefit of public goods, and this sum must be set equal to the marginal rate of transformation, which constitutes the true marginal cost of the public goods. Thus, we obtain

$$MRS_1^P + MRS_2^P = MRT, \tag{3.8}$$

where MRS_i^P $(i = 1, 2)$ is the MRS of country i at the social optimum. Equation (3.8) is well-known as the Samuelson rule. Equation (3.8) clearly entails

$$MRS_i^P < MRS_1^P + MRS_2^P = MRT \quad (i = 1, 2).$$

Since the MRS decreases with the total quantity of public good, G, the amount of the public good at a Nash equilibrium must be smaller than that at the social optimum. In other words, the public good is undersupplied at the Nash equilibrium.

As is well known in the public economics literature [see Cornes and Sandler (1996) and Hindriks and Myles (2006) among others)], the Nash equilibrium may be displayed by using a diagram with the Nash reaction functions and indifference curves in the plane $g_1 - g_2$. In that plane, Fig. 2.1 shows a set of indifference curves for country 1. These are extracted or derived from Fig. 3.1 by travelling along an indifference curve there, and for that constant utility recording the combinations of g_1 and g_2 required to maintain said utility, then entering them in Fig. 3.3.

Now suppose that country 2 provides $\overline{g}_2$ of the public good as in Fig. 1.1. Then, the choices open to country 1 lie along the horizontal line drawn at $g_2 = \overline{g}_2$ in Fig. 2.1. Curve $\overline{U}$ is an indifference curve of country 1, which shows combinations of g_1 and g_2 needed to maintain that specific level of utility. Any indifference curve drawn above (below) $\overline{U}$ shows combinations of g_1 and g_2 achieving a higher (lower) utility level in country 1 than curve $\overline{U}$. The choice that maximizes 1's utility occurs at the tangency of the indifference curve and the horizontal line, shown as point $\overline{\mathbf{E}}$. Country 1 chooses $\bar{g}_1$ at that point. Varying the fixed level of g_2 will lead to another best reaction for country 1. For example, when g_2 is fixed at $\hat{g}_2$, country 1's optimal choice is given at point $\widehat{\mathbf{E}}$. Doing this for all possible g_2 traces out country 1's optimal choices. The line labeled with $g_1 = N_1(g_2)$ represents the locus known as the Nash reaction curve, which depicts the value of g_1 that will be chosen in response to values of g_2.

[3]The first fundamental theorem states that every market equilibrium in perfectly competitive markets of private goods is Pareto optimal. Please refer to the textbooks of microeconomics (e.g. Mas-Collel et al. 1995) for the details of the fundamental theorem of welfare economics.

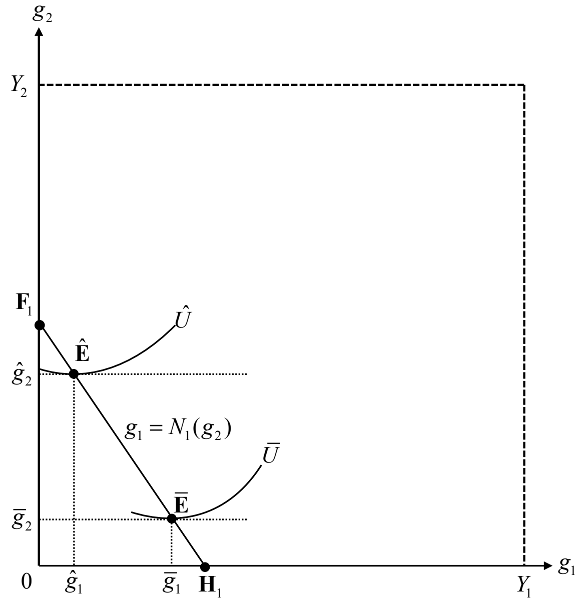


Fig. 3.3 Indifference curve of country 1 in (g_1, g_2)-plane. *Source* Authors

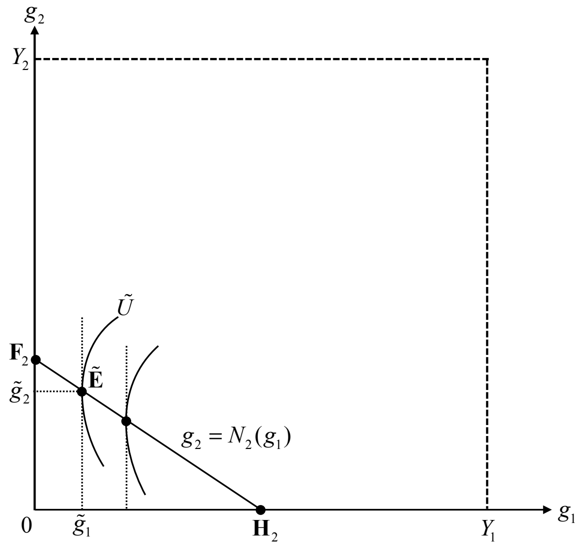


Fig. 3.4 Indifference curve of country 2. *Source* Authors

A similar construction can be repeated for country 2 and leads to Fig. 3.4. There, point $\tilde{\mathbf{E}}$ represents country 2's optimal choice when 1's provision of public good is given as $\tilde{g}_1$. In Fig. 3.4, any indifference curve drawn to the right (left) of curve $\widetilde{U}$ represents a locus of (g_1, g_2) required to maintain a higher (lower) utility level than $\widetilde{U}$. The line labelled $g_2 = N_2(g_1)$ then gives the best response curve of country 2.

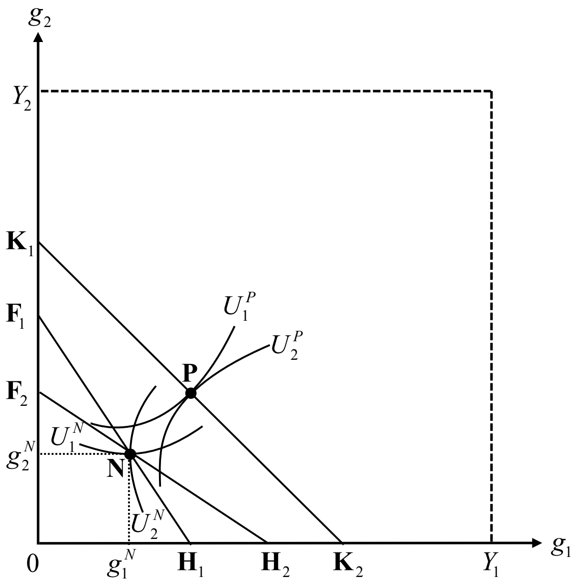


Fig. 3.5 Nash equilibrium and pareto efficient allocation. *Source* Authors

Next, Fig. 3.5 illustrates a Nash equilibrium and its relation to Pareto efficiency. As before, the vertical axis is country 1's provision, g_1, and the horizontal axis country 2's provision, g_2. Line $N_1(g_2)$ shows country 1's reaction curve and $N_2(g_1)$ shows 2's reaction curve. Since $-1 < dN_1/dg_2 < 0$ and $-1 < dN_2/dg_1 < 0$, the slope of curve $N_1(g_2)$ is negative and steeper than $N_2(g_1)$. The intersection of reaction curves gives the Nash equilibrium point **N**. At the Nash equilibrium, country 1 voluntarily provides the public goods of g_1^N, while country 2 provides g_2^N.

In Fig. 3.5, we identify one particular Pareto efficient point **P**, where the indifference curves of both countries are tangent to each other. Curves U_1^P and U_1^N give indifference curves of country 1, while U_2^P and U_2^N show indifference curves of country 2. Both U_1^N and U_2^N pass through point **N**. Comparing point **P** to point **N**, where the indifference curves of both countries intersect, gives confirmation that point **N** is not Pareto efficient, since from **N** the utility of both parties can be increased by moving to **P**. Line $\mathbf{K}_1\mathbf{K}_2$ shows the locus of tangent points of the indifferent curves of countries 1 and 2. Note that indifference curve U_1^P is tangent to a horizontal line at the point where it intersects $N_1(g_2)$. Similarly, indifference curve U_2^P is tangent to a vertical line at its intersection with curve $N_2(g_1)$.

3.3 Effects of Income Transfer

3.3.1 Neutrality Theorem

Shibata (1971) and Warr (1983) showed that in the formulation of the Nash provision of public goods, redistribution policy cannot have any real effects. This policy is perfectly offset by the private reaction to the provision of public goods. This is called the neutrality theorem of public goods. The outcome is consistent irrespective of preferences and the distribution of income. See also Bergstrom and Varian (1985).

From the optimizing behavior of each country, we have as the compensated demand functions for c_i and G, respectively

$$c_i = c_i(U^i) \tag{3.9.1}$$

$$G = G_i(U^i) \quad (i = 1, 2). \tag{3.9.2}$$

These two Eqs. (3.9.1) and (3.9.2) give the optimal combination of private consumption and public good as a function of each country's utility, respectively. Note that consumer price is fixed in the conventional model of public goods. Namely, optimal levels of private good consumption and public good are an increasing function of each country's utility, respectively. These compensated demand functions may be derived by the expenditure function. See Sect. 3.4.

Then, from the above equations, we have as the overall feasibility condition

$$Y_1 + Y_2 = c_1(U^1) + c_2(U^2) + G^1(U^1) \tag{3.10}$$

From the definition of pure public goods, we also have

$$G_1(U^1) = G_2(U^2). \tag{3.11}$$

These two Eqs. (3.10) and (3.11), determine the equilibrium values of U^1 and U^2. Since the total allied income, $Y_1 + Y_2$, appears in the first equation, the total allied income in the economy determines welfare and other economic variables. So long as the distribution of income between allies does not affect the total income $Y_1 + Y_2$, redistribution does not matter either. In other words, income redistribution does not affect real economic variables at interior solutions. This is the neutrality theorem of public goods.

Another formulation of the neutrality result is to incorporate publicly provided public good X in addition to privately provided public good G. Suppose the allied global government or international organization exists and it can provide the same public goods X by levying taxes on allies. Since the non-cooperative level of G is smaller than the optimal cooperative level, the allied global government may have an incentive to provide security spending X by imposing an extra burden on allied countries.

Then, the utility function, Eq. (3.1), is rewritten as

$$U^i = U_i(c_i, G + X), \tag{3.1'}$$

where X and G are perfect substitutes. We assume that the allied global government can levy a lump sum tax T_i in order to provide the public good X. Then, each allied country's budget constraint, Eq. (3.2), is rewritten as

$$Y_i = c_i + g_i + T_i. \tag{3.2'}$$

The allied central government budget constraint is given as

$$T_1 + T_2 = X.$$

In this formulation, the feasibility condition, Eq. (3.10), may be rewritten as

$$Y_1 + Y_2 = c_1\left(U^1\right) + c_2\left(U^2\right) + Z_1\left(U^1\right), \tag{3.10'}$$

where $Z = X + G$ denotes the overall supply of allied public good, which is an increasing function of each allied country's utility. $Z_i(U^i)$ is the compensated demand function for the overall public good Z by country i. Then, in place of Eq. (3.11), we have

$$Z_1\left(U^1\right) = Z_2\left(U^2\right). \tag{3.12}$$

Equations (3.10′) and (3.12) determine U^1, U^2 as a function of the total income $Y_1 + Y_2$. It is easy to see that an increase in X by raising T_i does not affect the equilibrium level of Z at interior solutions. It would crowd out private provision of public good, G, by one to one. The allied organization cannot affect the equilibrium level of Z by changing X. This means the perfect crowding out of public goods.

Thus, we have another version of the neutrality result here. If we consider the allied global government's public provision as well as the private provision of public goods, public provision financed by taxes can crowd out private provision by 100%.

3.3.2 *Plausibility of the Neutrality Theorem*

In relation to the Nash equilibrium approach, each country regards others' provision of public goods as fixed. This is a one-shot game. Analytically, we can consider other formulations of the voluntary provision of public goods. Namely, suppose that an allied country becomes the leader and chooses its optimal provision by incorporating others' reaction functions. Others follow the leader's provision. This is a two-shot game, called the Stackelberg equilibrium approach. Even in this leader–follower approach, the neutrality theorem can be maintained.

The neutrality theorem applies to various instances of the private provision of public goods. For example, national governments provide international public goods and some international organizations redistribute income among countries. Alternatively, local governments may provide nationwide public goods voluntarily and central government may conduct regional redistribution policies. In such instances, redistribution may not have any real effect on the Nash solution.

In a realistic situation, however, this proposition may not well hold. First, the assumption of fixed cost of providing the public good seems rather restrictive. Second, the neutrality theorem is not preserved in relation to non-negative constraint. The private provision of public goods by each agent cannot be negative. If income is low and/or the evaluation of public goods is low, the optimal level of public goods may well be smaller than the sum of others' provision. In this regard, the optimal level of each country's provision can be negative. However, because of the non-negativity constraint, it spends its income on private consumption in accordance with the corner solution. Then, public redistribution has a real effect. This is because the agent cannot adjust to the redistribution policy in accordance with the corner solution. It has been shown that the larger the size of agent and the larger the inequality of income, so the likelihood of this situation occurring increases.

The third possibility is the case of impure public goods. If the public good is not pure, the neutrality result does not always hold. We consider this case in a three-country model of Sect. 3.4. Finally, Andreoni (1989), Bernheim (1986), and Bernheim and Bagwell (1988) also criticized the plausibility of neutrality result by questioning the conventional formulation of utility function. Namely, if the agent has a warm glow motive, the neutrality result does not hold.

3.4 Transfer Paradoxes in Three-Country Models with Impure Public Good

3.4.1 Analytical Framework

Assume that there are three countries in the world, 1, 2, and 3. As before, country 1's utility function is given by Eq. (3.1). Country 2's (or 3's) utility function may be given in the same way.

Public good G_i is now given by

$$G_i = g_i + \sum_{i \neq j} \varepsilon_{ij} g_j, \quad i \neq j \tag{3.3'}$$

where ε_{ij} is the degree of externalities of the public good provided by country j to country i. ($i, j = 1, 2, 3$) If $0 < \varepsilon_{ij} < 1$, the public good is called impure. If $\varepsilon_{ij} = 1$ for all i, j, then the good is a pure public good. If $\varepsilon_{ij} = 0$ for all i, j, then the good is a private good. In the context of international defense alliances ε_{ij} can be more than 1. If $\varepsilon_{ij} > 1$ for some i, j, then country i can be better defended from some other country j. If we consider the adversarial relationship, ε_{ij} can be negative. In such a case some countries are regarded as enemies: g_j is then regarded as a public bad by country i. In particular if $\varepsilon_{ij} = -1$ between enemies, there exists an extreme case of "arms race" relationship where an equal increase in national defense by a country and its enemy leaves the national security of both unchanged.

As before, country i's budget constraint is given by Eq. (3.2). Substituting Eq. (3.3′) into country i's budget constraint (2), we have

$$c_i + G_i = Y_i + \sum_{i \neq j} \varepsilon_{ij} g_j, \tag{3.4′}$$

As in Sect. 3.2, we assume that each allied government determines its public good provision non-cooperatively, treating the other's public spending as given. It is useful to draw on duality theory in terms of compensated demand functions. Define the expenditure function as

Minimize $E^i = c_i + G_i$ subject to $U^i \geq \overline{U}^i$.

Then, the following equation determines U^i as a function of real income, $Y_i + \sum_{i \neq j} \varepsilon_{ij} g_j$, which contains actual income and the externalities from the other countries' provision of public goods.

$$E^i(U^i) = Y_i + \sum_{i \neq j} \varepsilon_{ij} g_j. \tag{3.13}$$

Since the consumer price vector is fixed here, we omit the consumer price vector from the expenditure function for simplicity. From the well-known property of the compensated demand function we have Eq. (3.9.2) as in Sect. 3.1.

$$G_i = G^i(U^i), \tag{3.9.2}$$

where $G^i = \partial E^i / \partial p$ and p is the price of providing the public good, which is assumed to be unity. By definition we have

$$g_j = Y_j - c_j = Y_j + G_j - E^j, \tag{3.14}$$

From Eqs. (3.4′), (3.8), (3.13) and (3.14) the three-country model is summarized by the following three equations.

$$E^1(U^1) = Y_1 + \sum_{1 \neq j} \varepsilon_{1j} \left[Y_j - E^j(U^j) + G^j(U^j) \right] \tag{3.15.1}$$

$$E^2(U^2) = Y_2 + \sum_{2\neq j} \varepsilon_{2j}\big[Y_j - E^j(U^j) + G^j(U^j)\big] \tag{3.15.2}$$

$$E^3(U^3) = Y_3 + \sum_{3\neq j} \varepsilon_{3j}\big[Y_j - E^j(U^j) + G^j(U^j)\big] \quad (j = 1, 2, 3). \tag{3.15.3}$$

These three Eqs. (3.15.1) determine U^1, U^2, and U^3 as a function of Y_i and ε_{ij}. We assume that the existence of an interior Nash equilibrium with $g_i > 0$.

As discussed in Sect. 3.2 [also see Bergstrom et al. (1986) and Andreoni (1988)], a non-negativity constraint on providing g, may well be binding as a solution if the number of countries becomes large. However, in order to present the results in the simplest way and in their strongest form, we assume therefore that non-negativity constraints on providing public goods are non-binding in an interior equilibrium of this section. As in the context of allied world, the number of allied countries is relatively small in the context of providing international public goods and hence it may plausible to assume an interior solution. Nevertheless, we may not obtain the neutrality result.

3.4.2 Comparative Statics

If the public good is an impure public good, the neutrality result does not necessarily hold and hence transfers between countries in general have real effects. Let us investigate the welfare effects of transfers in such situations. Without loss of generality we consider the effects of a transfer from country 2 to country 1 ($dY_1 + dY_2 = 0$).

Totally differentiating three Eqs. (3.15.1), (3.15.2) and (3.15.3), we have

$$\begin{bmatrix} E^1_U, \varepsilon_{12}\widehat{E}^2_U, \varepsilon_{13}\widehat{E}^3_U \\ \varepsilon_{21}\widehat{E}^1_U, E^2_U, \varepsilon_{23}\widehat{E}^3_U \\ \varepsilon_{31}\widehat{E}^1_U, \varepsilon_{32}\widehat{E}^2_U, E^3_U \end{bmatrix} \begin{bmatrix} dU^1 \\ dU^2 \\ dU^3 \end{bmatrix} = \begin{bmatrix} 1 \\ \varepsilon_{12} \\ \varepsilon_{31} \end{bmatrix} dY_1 + \begin{bmatrix} \varepsilon_{12} \\ 1 \\ \varepsilon_{32} \end{bmatrix} dY_2 + \begin{bmatrix} \varepsilon_{13} \\ \varepsilon_{23} \\ 1 \end{bmatrix} dY_3 \tag{3.16}$$

where $\widehat{E}^i_U \equiv E^i_U - G^i_U$ and $E^i_U \equiv \partial E^i/\partial U^i$, $G^i_U \equiv \partial G^i/\partial U^i$ (i = 1, 2, 3).

Considering the definition of $1 - e^i (= \widehat{E}^i_U / E^i_U)$ and the transfer constraint $dY_1 + dY_2 = 0$, we have

$$\begin{aligned} \frac{dU^1}{dY_1} = \Delta E^2_U E^3_U \big[&(1 - \varepsilon_{12})\big\{1 - \varepsilon_{32}\varepsilon_{23}(1 - e^3)(1 - e^2)\big\} \\ &+ (1 - \varepsilon_{21})\big\{\varepsilon_{12}(1 - e^2) - \varepsilon_{32}\varepsilon_{13}(1 - e^3)(1 - e^2)\big\} \\ &+ (\varepsilon_{31} - \varepsilon_{32})\big\{\varepsilon_{12}\varepsilon_{23}(1 - e^3)(1 - e^2) - \varepsilon_{13}(1 - e^3)\big\}\big] \end{aligned} \tag{3.17.1}$$

$$\begin{aligned}\frac{dU^2}{dY_1} &= -\Delta E_U^1 E_U^3\big[(1-\varepsilon_{21})\big\{1-\varepsilon_{31}\varepsilon_{13}(1-e^3)(1-e^1)\big\}\\ &\quad + (1-\varepsilon_{12})\big\{\varepsilon_{21}(1-e^1)-\varepsilon_{31}\varepsilon_{23}(1-e^3)(1-e^1)\big\}\\ &\quad + (\varepsilon_{32}-\varepsilon_{31})\big\{\varepsilon_{21}\varepsilon_{13}(1-e^3)(1-e^1)-\varepsilon_{23}(1-e^3)\big\}\big]\end{aligned} \tag{3.17.2}$$

where $1/\Delta$ is the determinant of the matrix of LHD of Eq. (3.16). e^i is the marginal propensity to consume the international public good of country i. Δ is positive from the stability condition.

3.4.3 Neutrality Result Revisited

If the public good is a pure public good ($\varepsilon_{ij} = 1$ for all i, j), then U^i is given by a function of the total income $Y_1 + Y_2 + Y_3$. So long as $Y_1 + Y_2 + Y_3$ is fixed during transfers, any transfers among three countries ($dY_1 + dY_2 + dY_3 = 0$) do not affect the real equilibrium. We have the well-known neutrality result, as explained in Sect. 3.3.

Furthermore, the neutrality result can hold in some restricted cases even if public goods are impure and some ε_{ij} are not equal to one. Suppose $\varepsilon_{12} = \varepsilon_{21} = 1$ and $\varepsilon_{31} = \varepsilon_{32} < 1$. Then it is easy to see from Eq. (3.15.1) that U^i is a function of $Y_1 + Y_2$ and Y_3: any transfers between country 1 and country 2 will not affect U^i. Even if $\varepsilon_{31} \neq \varepsilon_{13} \neq \varepsilon_{23} \neq \varepsilon_{32} \neq 1$ and hence the public good is not a pure public good in a three-country economy, the neutrality result may still hold with respect to transfers between country 1 and country 2. An intuitive explanation is as follows: For given g_3, country 1 and country 2 have the same preference over the locally pure public good within countries 1 and 2, $g_1 + g_2$. Thus, for given g_3, a transfer between the two countries does not affect the equilibrium. The neutrality result holds for the transfer between country 1 and country 2 so long as g_3 is fixed.

Furthermore, country 3 does not have an incentive to change g_3. Since $\varepsilon_{31} = \varepsilon_{32}$, country 3 is indifferent to the allocation of $g_1 + g_2$, and is only concerned with the total amount of G. In other words, if the public good is locally a pure public good within the sub-world of countries 1 and 2 and in addition if country 3 receives the same strength of the externalities from country 1 and country 2, then any transfers between countries 1 and 2 will not affect the real equilibrium. The neutrality result can hold in some restricted cases of an impure public good. This is an extended neutrality result.

It is also easy to see that if the third country 3 does not receive any externalities from 1 or 2 ($\varepsilon_{32} = \varepsilon_{31} = 0$) and $\varepsilon_{12}, \varepsilon_{21} < 1$, then we do not obtain either neutral or paradoxical results.

In a two-country model we always have $\varepsilon_{32} = \varepsilon_{31} = 0$. In such a case if the strength of the externalities is less than one, the receiving country always gains, while the giving country always loses. We have the normal result. When $\varepsilon_{12}, \varepsilon_{21}$ are negative, we still have the normal result. This suggests that when $\varepsilon_{ij} < 1$, paradoxical results could happen only in the three (or more)-country framework.

Finally, if $\varepsilon_{12}, \varepsilon_{21} > 1$, the receiving country 1 loses and the giving country 2 gains. Since the strength of the externalities is now more than unity, this result is intuitively plausible and may not be regarded as a paradoxical result.

3.4.4 Redistribution Among Allied Countries

Paradoxical results may occur due to the third term of Eqs. (3.17.1) and (3.17.2), the indirect externality effect through the reaction of the third country. In order to explore this possibility, it is useful to assume $\varepsilon_{12} = \varepsilon_{21} = 1$ for simplicity. We also assume the symmetry case of $\varepsilon_{ij} = \varepsilon_{ji}$ except Sect. 3.4.2. The public good is locally a pure public good within the sub-world of countries 1 and 2. In this case the first two terms of Eqs. (3.17.1) and (3.17.2) become zero, so that the third term dominates the sign of Eqs. (3.17.1) and (3.17.2). Suppose without loss of generality $\varepsilon_{31} > \varepsilon_{32}$. Then from Eq. (3.17.1) $dU^1/dY_1 \geq 0$ if and only if $\varepsilon_{13} \leq (1 - e^2)\varepsilon_{23}$. Note that $(1 - e^2)\varepsilon_{23}$ actually corresponds to the marginal response of g_2 when g_3 changes. This is because $(1 - e^2)$ is by definition the marginal propensity to consume the private good. When g_3 decreases, country 2's real income decreases by the amount of ε_{23}, reducing G_2 by the amount of $e^2\varepsilon_{23}$. Thus, g_2 increases by the amount of $(1 - e^2)\varepsilon_{23}$ so as to reduce G_2 by the amount of $e^2\varepsilon_{23}$. This effect benefits country 1 by the amount of $(1 - e^2)\varepsilon_{23}$, while a reduction of g_3 directly hurts country 1 by the amount of ε_{13}. If the former positive effect outweighs the latter negative effect, country 1 gains. Similarly, from Eq. (3.17.2) $dU^2/dY_2 \leq 0$ if $\varepsilon_{13}(1 - e^1) \leq \varepsilon_{23}$.

Case 1 Japan, United States, and Germany

Consider the case of $\varepsilon_{13} = \varepsilon_{31} = 1$, $\varepsilon_{23} = \varepsilon_{32} = 0$, $\varepsilon_{12} = \varepsilon_{21} = 1$. Since ε_{12} and ε_{13} are very high, country 1 is concerned with the total amount of the international public good. On the contrary, since ε_{23} and ε_{32} are very low, country 2 and country 3 are concerned with the partial amount of the public good. This case may reflect some realities in international defense alliances of the Western World. As shown in Fig. 3.6, suppose country 1 is the United States, country 2 is Japan, and country 3 is Germany. It seems reasonable to assume that the United States is concerned with the total amount of national defense in the Western World. However, Japan's main concern is the amount of national defense in Asia, while Germany's main concern is the amount of national defense in Europe.

Suppose g_3 is fixed. A transfer from Japan to the United States does not affect either U^1 or U^2, because both countries have the same interests in the public good $(g_1 + g_2)$ if g_3 is fixed. Then, the neutrality result holds and we have $dg_1 = dY_1 > 0$ and $dg_2 = dY_2 < 0$. On the contrary, Germany is concerned with the allocation of $g_1 + g_2$. Since $\varepsilon_{31} > \varepsilon_{32}$, by an increase in g_1, $(dg_1 > 0)$, Germany gains, and hence it reacts to decrease its supply of the public good, $dg_3 < 0$. Since ε_{13} is high, this is now harmful to the United States. The United States loses, and hence it increases its supply of security spending further by $\varepsilon_{13}(1 - e^1)$ which benefits Japan as well as Germany. Japan gains because the negative welfare effect of the decrease in g_3 is

Fig. 3.6 USA-Japan-Germany model. *Source* Authors

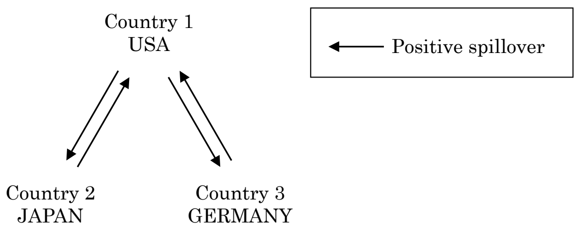
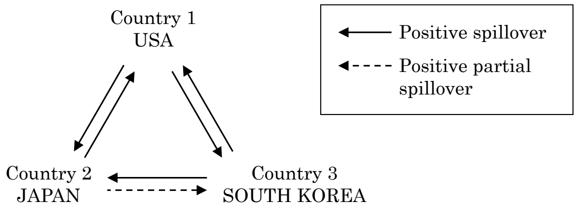


Fig. 3.7 USA-Japan-South Korea model. *Source* Authors



dominated by the positive welfare effect of the increase in g_1, $(\varepsilon_{13}(1 - e^1) > \varepsilon_{23})$ when ε_{23} and e^1 are low.

This example explores the importance of a third country's response on the welfare of two allied countries. Japan may free ride on an extra provision of the United States caused by a reduction of Germany's provision. If Japan anticipates this outcome, it might have an incentive to make a transfer to the United States.

Case 2 Japan, United States, and South Korea

Consider next the case of $\varepsilon_{13} = \varepsilon_{31} = 1, \varepsilon_{23} = 1, 1 > \varepsilon_{32} \geq 0$, and $\varepsilon_{12} = \varepsilon_{21} = 1$. Namely, both country 1 and country 2 are concerned with the total amount of the international public good ($\varepsilon_{13} = \varepsilon_{31} = 1$). However, country 3 is concerned with the partial level of the international public good ($\varepsilon_{23} = 1, 1 > \varepsilon_{32} \geq 0$). Although 3's spending has a large externality to 2, 2's spending has a smaller externality to 3 than 1's spending.

As shown in Fig. 3.7, this case also may reflect some realities in an international defense alliance in East Asia. We may assume that country 1 is the United States, country 2 is Japan, and country 3 is South Korea. Suppose both the United States and Japan are concerned with the total amount of national defense in East Asia. However, South Korea may not be very happy with an increase in national defense of Japan because of past bad experiences before WWII.

As in Case 1, by the transfer we have $dg_1 = dY_1 > 0$ and $dg_2 = dY_2 < 0$ for given g_3. Since $dg_1 > 0$, $g_1 + g_3$ again increases and hence South Korea responds to decrease its supply of the public good, $dg_3 < 0$. This is harmful to both the United States and Japan. When ε_{23} is high and e^1 is also high, Japan now loses because the negative welfare effect of the decrease in g_3 dominates the positive welfare effect of the increase in g_1, $(\varepsilon_{13}(1 - e^1) < \varepsilon_{23})$, which is caused by a reduction of g_3.

In this case Japan might not have an incentive to make a transfer to the United States. Rather, Japan would like to welcome a transfer from the United States to

Japan, which also benefits the United States since it stimulates security spending of South Korea. On the contrary, South Korea would welcome a transfer from Japan to the United States.

Case 3 Japan, United States, and North Korea

We briefly consider a three-country world with adversarial relationships. As before we assume $\varepsilon_{12} = \varepsilon_{21} = 1$; countries 1 and 2 are perfect allies. Now, allies 1 and 2 both treat country 3 as an enemy while country 3 treats both 1 and 2 as enemies Thus, we investigate the case where ε_{i3} and ε_{3i} are negative for i = 1 and 2. As shown in Fig. 3.8, suppose country 1 is the United States, country 2 is Japan, and country 3 is now North Korea. Then we know that if $\varepsilon_{31} = \varepsilon_{32}$, the transfer between Japan and the United States is neutral. If ε_{31} is not equal to ε_{32}, the transfer is not neutral.

If the strength of the negative externalities received from North Korea is similar for the United States and Japan, or if marginal propensities to consume the public good are relatively high in both the United States and Japan, we have the weak paradox. Both countries could lose or gain. Furthermore, if North Korea regards the receiving country (the United States) less (more) threatening than the giving country (Japan) does, North Korea responds to reduce (increase) its security spending, benefiting (hurting) the United States. The United States then responds to reduce (increase) its security spending, hurting (benefiting) Japan. Hence, we have the normal result (strong paradox).

Thus, if North Korea receives more threat from the United States than Japan, Japan might have an incentive to make a transfer to the United States.

Case 4 United States, Pakistan, and Iran

Finally, let us investigate the case where ε_{3i} and ε_{i3} are negative for $i = 1$ or 2. One of the allies has an adversarial relationship with country 3, but the other country has a friendly relationship with country 3. For example, suppose $\varepsilon_{13}, \varepsilon_{31} > 0$ and $\varepsilon_{23}, \varepsilon_{32} < 0$. As shown in Fig. 3.9, suppose country 2 is the United States, country 1 is Pakistan, and country 3 is Iran. Pakistan has a good relationship with the United States as well as Iran. However, Iran has a hostile relation with the United States. Then, a transfer from the United States to Pakistan raises g_1 and hence benefits Iran, U^3 and reduces its security spending g_3, which benefits the United States and hurts Pakistan. We always have the strong paradox in such a case. The giving country (the United States) gains and the receiving country (Pakistan) loses.

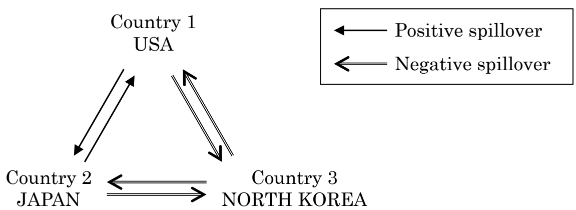


Fig. 3.8 USA-Japan-North Korea model. *Source* Authors

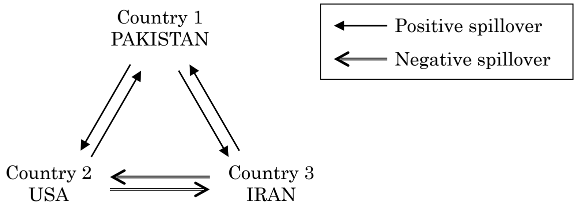


Fig. 3.9 Pakistan-USA-Iran model. *Source* Authors

3.4.5 Stackelberg Equilibrium

Within the non-cooperative framework, we can also consider the Stackelberg leader–follower case where one country, for whatever reason, can make a credible first move. This asymmetry may arise because the country is dominant in the alliance in some sense or has a less flexible economy so that the level of defense spending it chooses is credibly maintained.

Suppose country 1 acts as a Stackelberg leader. Denote by

$$G_i = e^i\left(Y_i + \sum_{j\neq 1}\varepsilon_{ij}g_j\right) \quad (i = 2, 3, \quad 0 < e^i < 1), \tag{3.18}$$

the reaction function (ordinary demand function for the impure public good) of country $i\,(i = 2, 3)$. From Eqs. (3.2), (3.3) and (3.18), country 1's budget constraint (3.3) is rewritten as

$$c_1 + \left[\widehat{S}^2(1 - e^2)\varepsilon_{12}\varepsilon_{21} + \widehat{S}^3(1 - e^3)\varepsilon_{13}\varepsilon_{31} + 1\right]G_1$$
$$= Y_1 + \widehat{S}^2 e^2\varepsilon_{12}Y_2 + \widehat{S}^3 e^3\varepsilon_{13}Y \tag{3.19}$$

where $\widehat{S}^i \equiv 1/\left[1 - (1 - e^i)\varepsilon_{i1}\varepsilon_{1i}\right]$ $(i = 2, 3)$. Then, the following equation determines U^1 in the Stackelberg case as a function of effective income, $Y_1 + S^2e^2\varepsilon_{12}Y_2 + S^3e^3\varepsilon_{13}Y$, where $S^i \equiv \widehat{S}^i e^i$.

$$E^1(U^1) = Y_1 + S^2e^2\varepsilon_{12}Y_2 + S^3e^3\varepsilon_{13}Y. \tag{3.20}$$

Since the consumer prices in Eq. (3.19), $\widehat{S}^2(1 - e^2)\varepsilon_{12}\varepsilon_{21} + \widehat{S}^3(1 - e^3)\varepsilon_{13}\varepsilon_{31} + 1$, is fixed, we omit it from Eq. (3.20) for simplicity. Since Eq. (3.4′) holds for the followers, countries 2 and 3, the three-country model in the Stackelberg case is summarized by Eqs. (3.20) and (3.13) where $i = 2, 3$ and $j = 1, 2, 3$.

In the two-country world where the public good is a pure public good, Bruce (1990) shows that the Stackelberg equilibrium also exhibits neutrality. When $\varepsilon_{12} = \varepsilon_{21} = 1$, $S^i = 1$. Thus, it is easy to see from Eqs. (3.13) and (3.20) that the neutrality result holds if $\varepsilon_{31} = \varepsilon_{32}$ as in the one-shot Nash case. This is a generalized neutrality result in the Stackelberg case. For example, suppose country 1 and country 2 are perfect allies ($\varepsilon_{12} = \varepsilon_{21} = 1$) and the allies 1 and 2 and country 3 are perfect adversaries ($\varepsilon_{31} = \varepsilon_{32} = -1$). Then a redistribution of resources between the allies is still neutral.

If the public good is an impure good, international transfers in the Stackelberg case would have real effects as in the Nash case. As shown in Ihori (1992b), in the Stackelberg case we do not have the possibility of strong paradoxical results. We may have the weak paradox due to the indirect externality effect. But we do not have the weak paradox due to the direct externality effect. The leader always gains by receiving a transfer.

3.5 Economic Growth and Spillover Effects

We can investigate welfare implications of an exogenous increase in national income in one country using the similar framework. In the case of pure public goods, only total contributions by all allies enter the utility function. In such a case economic growth of any country benefits all the allies. Ihori (1994) relaxed this assumption and considered the case of impure public goods.

He showed that the growing country always gains if the spillover is identical among countries. This holds also in the pure public good case as well as the private good case. Paradoxical results may occur only when the good is impure and its externalities are heterogeneous across allies. For example, consider the three-country case of Japan, the United States, and Germany in Sect. 4.4, where the United States is the central and growing country. The growing country may lose due to a negative direct-spillover effect when the marginal propensities to spend the security good of other allies are low and the good produces a high degree of externalities between the central country and other allies.

Intuition is as follows. When Y_1 of the growing country increases, the effective incomes of allied countries 2 and 3 increase. If the marginal propensities to spend the security are low, the desired levels of impure public good of other allies, G_2 and G_3, do not increase much. Thus, g_2 and g_3 decline to a great extent so as to offset an increase in g_1. This is called the negative direct-spillover effect. In a two-country model, the negative direct-spillover effect cannot dominate the overall effect since growth itself always benefits the growing country. However, if the number of allies is large, this negative effect becomes large and could dominate the overall effect. In other words, when the number of allies increases, the negative effect becomes large and hence the growing country will be more likely to lose.

3.6 Concluding Remarks

As is stated in the introduction of the present chapter, Olson and Zeckhauser (1966) pointed out that countries may allocate some fraction of national income to international (or regional) public goods (for example, special trading agreements, formation of international organizations, military preparedness, active international diplomacy, foreign aid) to reduce regional and international tension and to avoid random emergency cost. In Sect. 3.2 we have first summarized conventional analytical results of economic outcomes of the standard one-dimensional international public goods in allied countries.

The non-cooperative provision of public good at a Nash equilibrium is smaller than at a Pareto optimum. In the formulation of Nash provision of public good, redistribution policy cannot have any real effect. This policy is perfectly offset by private reactions to the provision of public good. This is called the neutrality theorem of public good. The outcome obtains irrespective of preferences and the distribution of income. We have also explained another version of the neutrality result. Namely, if we consider public provision as well as private provision of public good, public provision financed by taxes can crowd out private provision by 100%.

Next, Sect. 3.4 has explored possibilities such that if the non-cooperatively provided public good is not a pure public good for all the countries, transfers may lead to paradoxical results in a three-country economy. The giver may benefit, or the receiver may lose, or both. Spending on national defense is a good example of impure public good where one country's supply of international public good need not be a perfect substitute for another country's supply. This feature may reflect realities in international defense alliances.

Section 3.4 has shown that the reaction of the third country to expenditure changes brought about by a transfer between the other two countries may be important when the privately provided public good is an impure public good in a three-country model. In a hypothetical example of Japan, the United States, and Germany case, Japan may free ride on an extra provision of the United States caused by a reduction of Germany's provision. If Japan anticipates this outcome, it might have an incentive to make a transfer to the United States. In a three-country case of Japan, the United States, and South Korea, Japan might not have an incentive to make a transfer to the United States. On the contrary, Japan might welcome a transfer from the United States.

We have also investigated some interesting results involving adversarial relationships by incorporating enemy countries in case of Japan-the United States-North Korea and the United States-Pakistan-Iran models. Our hypothetical examples of international defense alliances suggest that if the impure public good is provided non-cooperatively, paradoxical results may occur because of the direct and indirect externality effects of defense.

We have finally investigated welfare implications of an exogenous increase in national income in one country using the similar framework. Normally, growth itself always benefits the growing country. However, when the number of allies increases, the negative direct-spillover effect becomes large and hence the growing country would be more likely to lose.

Appendix: Insurance and International Public Goods

Analytical Framework

We now explicitly incorporate uncertainty into a model of an international public good, G to cope with some risks, as an extension of the Appendix of Chap. 2. Consider a two-country model. As to income of country $i\,(i = 1, 2)$, we have Y_i in good state A, but $(1 - \pi_i)Y_i$ in bad state B. π_i is the penalty ratio in country i in bad state B. c_i is also subject to uncertainty. In good state A, which occurs at the probability of $1 - \alpha$, country i enjoys c_i^A. In bad state B, which occurs at the probability of α, country i cannot enjoy c_i^A but can enjoy c_i^B. α indicates the probability of an economically disruptive production emergency or a "war". Then, the utility function (3.1) may be rewritten as:

$$W_i = (1 - \alpha)V(c_i^A) + \alpha V(c_i^B) + U(G) \tag{3.21}$$

where W_i is the expected welfare of country i. In this Appendix we assume that preferences are the same between two allies.

It might be plausible to assume that an international public good is more beneficial when emergency occurs than when it does not. For simplicity we consider the case where the benefit of an international public good is independent of the state of nature. The qualitative results are almost the same even if G is assumed to be more beneficial when emergency occurs.

In this Appendix G is assumed to be a pure public good. Thus, as in Sect. 3.2, G is given by

$$G = g_i + \sum_{j \neq i} g_j \tag{3.3}$$

where g_i is an international public good voluntarily provided by country i.

Country i's budget constraint in each state is now given by

$$c_i^A = Y_i - g_i \tag{3.22.1}$$

$$c_i^B = (1 - \pi_i)Y_i - g_i = c_i^A - \pi_i Y_i \tag{3.22.2}$$

where the relative price of public goods in terms of private consumption is assumed to be unity. Substituting Eq. (3.22.2) into Eq. (3.21), we have

$$W_i = (1-\alpha)V(c_i^A) + \alpha V(c_i^A - \pi_i Y_i) + U(G) \tag{3.21$'$}$$

We assume that each government in a non-cooperative setting determines its public good provision, treating the other country's public spending as given. Note that $\pi_i Y_i$ appears in the expected welfare function. Considering Eqs. (3.3) and (3.22.1) may be rewritten as

$$c_i^A + G = Y_i + \sum_{j \neq i} g_j \tag{3.23}$$

Let us now define the following expenditure function:

$$\text{Minimize } E^i \equiv c_i^A + G \text{ subject to } W_i \geq \overline{W}_i$$

Then in place of Eq. (3.23) we have

$$E(W_i, \pi_i Y_i) = Y_i + \sum_{j \neq i} g_j \tag{3.24}$$

The two-country model with voluntary provision of the public good is summarized by

$$E(W_1, \pi_1 Y_1) + E(W_2, \pi_2 Y_2) = Y_1 + Y_2 + G(W_1, \pi_1 Y_1) \tag{3.25}$$

$$G(W_1, \pi_1 Y_1) = G(W_2, \pi_2 Y_2) \tag{3.26}$$

where $G(\ )$ is the compensated demand function for the public good. Equation (3.25) comes from Eqs. (3.23) and (3.24). Equation (3.26) means that each country demands the same amount of the pure public good in equilibrium.

Since preferences are assumed to be the same between countries, from Eq. (3.26) if emergency costs are equal between countries, that is,

$$\pi_1 Y_1 = \pi_2 Y_2$$

then, expected welfare is also equalized.

$$W_1 = W_2$$

Even if income is different between countries, expected welfare is equalized when emergency costs are the same between countries. This is because the voluntary provision of pure public good can effectively offset the difference with respect to income before uncertainty. Similarly, if the emergency cost in country 1 is greater than in country $2(\pi_1 Y_1 > \pi_2 Y_2)$, then welfare in country 1 is less than in country

$2(W_1 < W_2)$. This result comes from the property that the compensated demand for G increases with the emergency cost; $G_\pi \equiv \partial G/\partial \pi Y > 0$.

The first order condition with respect to the public good provision is written as

$$(1-\alpha)V_c^A + \alpha V_c^B = U_G \tag{3.27}$$

where $U_G \equiv dU/dG$. Let us investigate properties of expenditure and compensated demand functions.

Totally differentiating Eqs. (3.21) and (3.27), we have

$$\begin{bmatrix} (1-\alpha)V_{cc}^A + \alpha V_{cc}^B & -U_{GG} \\ (1-\alpha)V_c^A + \alpha V_c^B & U_G \end{bmatrix} \begin{bmatrix} dc^A \\ dG \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} dW + \begin{bmatrix} \alpha V_{cc}^B \\ \alpha V_c^B \end{bmatrix} \pi_i Y_i \tag{3.28}$$

Hence we have

$$c_W^A = \frac{1}{\bar{\gamma}} U_{GG} > 0 \tag{3.29.1}$$

$$G_W \equiv \frac{\partial G}{\partial W} = \frac{1}{\bar{\gamma}}\left[(1-\alpha)V_{cc}^A + \alpha V_{cc}^B\right] > 0 \tag{3.29.2}$$

$$c_\pi^A \equiv \frac{\partial c^A}{\partial \pi Y} = \frac{1}{\bar{\gamma}}\left[\alpha V_{cc}^B U_G + \alpha V_c^B U_{GG}\right] > 0 \tag{3.29.3}$$

$$G_\pi \equiv \frac{\partial G}{\partial \pi Y} = \frac{\alpha}{\bar{\gamma}}\left\{\left[(1-\alpha)V_{cc}^A + \alpha V_{cc}^B\right]V_c^B - \left[(1-\alpha)V_c^A + \alpha V_c^B\right]V_{cc}^B\right\} \tag{3.29.4}$$

$$E_\pi \equiv \frac{\partial E}{\partial \pi Y} = \frac{\alpha}{\bar{\gamma}}\{V_c^B U_{GG} + [(1-\alpha)V_{cc}^A + \alpha V_{cc}^B]V_c^B\} > 0 \tag{3.29.5}$$

$$E_W = c_W^A + G_W > 0 \tag{3.29.6}$$

where $\bar{\gamma} \equiv \left[(1-\alpha)V_{cc}^A + \alpha V_{cc}^B\right]U_G + \left[(1-\alpha)V_c^A + \alpha V_c^B\right]U_{GG} < 0$. Suppose the relative risk aversion is constant $(cV_{cc}/V_c = -\lambda)$. Then (A9.4) may be rewritten as

$$\frac{\partial G}{\partial \pi Y} = \frac{\alpha \lambda V_c^A V_c^B \alpha(1-\alpha)}{c^A c^B}(c^A - c^B) > 0 \tag{3.29.4'}$$

Comparative Statics

We now investigate some comparative statics results. Totally differentiating Eqs. (3.24) and (3.25), we have

$$\begin{bmatrix} E_{1W} - G_{1W} & E_{2W} \\ G_{1W} & -G_{2W} \end{bmatrix} \begin{bmatrix} dW_1 \\ dW_2 \end{bmatrix} = \begin{bmatrix} -Y_1 c_{1\pi}^A \\ -Y_1 G_{1\pi} \end{bmatrix} d\pi_1 + \begin{bmatrix} -\pi_1 E_{1\pi} + \pi_2 E_{2\pi} + G_{1\pi}\pi_1 \\ -\pi_1 G_{1\pi} - \pi_2 G_{2\pi} \end{bmatrix} dY_1$$

We assume $dY_1 = -dY_2$ when we consider the income transfer from country 2 to country 1.

Or, we have

$$\frac{dW_1}{d\pi_1} = \frac{1}{\bar{\Delta}}\left[Y_1 c_{1\pi}^A G_{2W} + Y_1 G_{1\pi} E_{2W}\right] \tag{3.30.1}$$

$$\frac{dW_2}{d\pi_1} = \frac{1}{\bar{\Delta}}\left[(E_{1W} - G_{1W})(-Y_1 G_{1\pi}) + G_{1W} Y_1 c_{1\pi}^A\right] \tag{3.30.2}$$

$$\frac{dW_1}{dY_1} = \frac{1}{\bar{\Delta}}\left[(-\pi_1 c_{1\pi}^A + \pi_2 E_{2\pi})(-G_{2W}) + (\pi_1 G_{1\pi} + \pi_2 G_{2\pi})E_{2W}\right] \tag{3.30.3}$$

$$\frac{dW_2}{dY_1} = \frac{1}{\bar{\Delta}}\left[(-\pi_1 c_{1\pi}^A + \pi_2 E_{2\pi})(-G_{1W}) - (\pi_1 G_{1\pi} + \pi_2 G_{2\pi})(E_{1w} - G_{1W})\right] \tag{3.30.4}$$

where $\bar{\Delta} \equiv (E_{1W} - G_{1W})(-G_{2W}) - E_{2W}G_{1W} < 0$, $c_\pi^A \equiv \partial c^A / \partial \pi Y > 0$, and $E_\pi \equiv \partial E / \partial \pi Y > 0$.

First of all, let us examine the welfare effect of an increase in π_1. Equation (3.30.1) is negative but the sign of Eq. (3.30.2) is ambiguous. An increase in π_1 reduces the total effective income, hurting both countries. This is called the total income effect. An increase in π_1 raises the demand for the public good in country 1, inducing more provision of g_1 and less provision of g_2. This response hurts country 1 but benefits country 2, which is called the relative burden effect. Thus, country 1 loses but the welfare effect on country 2 is ambiguous. Country 2 loses when the total income effect dominates the relative burden effect.

Let us then consider the welfare effect of the income transfer policy. Suppose country 2 gives a transfer to country 1. Y_1 increases, while Y_2 decreases. This policy affects the emergency costs. The sign of the first term in Eqs. (3.30.3) and (3.30.4) is ambiguous. This is because an increase in emergency costs $\pi_1 Y_1$ in country 1 hurts both countries, while a decrease in emergency cost $\pi_2 Y_2$ in country 2 benefits both countries. The second term is positive in Eq. (3.30.3), while it is negative in Eq. (3.30.4). An increase in the emergency cost in country 1 induces more provision of g_1, while a decrease in the emergency cost in country 2 induces less provision of g_2. This is the relative burden effect. Thus, Eq. (3.30.3) could be positive, while Eq. (3.30.4) could be negative. We could have the strong paradox if the relative burden effect dominates the sign of (3.30.3) and (3.30.4), respectively.

Insurance Market and International Public Good

This section considers both the insurance market and the international public good. Namely, we now incorporate mutual insurance of the Appendix of Chap. 2 into the above model. With mutual insurance country i's budget constraint is now rewritten as

$$c_i^A = Y_i - ps_i - g_i \tag{3.22.1$'$}$$

$$c_i^B = (1 - \pi_i)Y_i - ps_i + s_i - g_i \tag{3.22.2$'$}$$

where s_i is insurance for country i and p is the price of insurance. Hence, we have

$$pc_i^B + \rho c_i^A + g_i = (1 - p\pi_i)Y_i \tag{3.31}$$

where $\rho \equiv 1 - p$.

The game is a single-shot one-stage game. Before the game the equilibrium price of insurance p is announced and binding commitments regarding insurance are determined in the private insurance market. Then, two countries simultaneously choose the provision of the public good g_i and insurance s_i, and have Nash conjectures about the choice of the other's public good provision. Finally, nature decides which state (A or B) occurs.

Considering Eq. (3.24), Eq. (3.31) may be rewritten as

$$pc_i^B + \rho c_i^A + G = (1 - p\pi_i)Y_i + \sum_{j \neq i} g_j \tag{3.32}$$

Let us now define the following expenditure function:

$$\text{Min } E_i \equiv pc_i^B + \rho c_i^A + G \quad \text{subject to } W_i \geq \overline{W}_i$$

Then in place of Eq. (3.24) we have

$$E(W_i, \alpha, 1 - p, p) = (1 - p\pi_i)Y_i + \sum_{j \neq i} g_j \tag{3.33}$$

Let us investigate properties of compensated demand and supply functions of private consumption, public good, and insurance. With both the public good and the insurance market the first order condition is written as

$$\alpha\rho V_c^B = p(1 - \alpha)V_c^A = p\rho U_G \tag{3.34}$$

Totally differentiating Eqs. (3.21) and (3.34), we have

$$\begin{bmatrix} \alpha\rho V_c^B/p, & \alpha V_c^B, & \alpha V_c^B/p \\ -p(1-\alpha)V_{cc}^A, & \alpha\rho V_{cc}^B, & 0 \\ 0 & \alpha V_{cc}^B, & -pU_{GG} \end{bmatrix} \begin{bmatrix} dc^A \\ dc^B \\ dG \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} dW + \begin{bmatrix} 0 \\ \alpha V_c^B/p \\ \alpha V_c^B/p \end{bmatrix} dp \tag{3.35}$$

Hence we have

$$c_W^A = \frac{1}{\gamma^*}\left[-\alpha\rho p V_{cc}^B U_{GG}\right] > 0 \tag{3.36.1}$$

$$c_W^B = \frac{1}{\gamma^*}\left[-p^2(1-\alpha)V_{cc}^A U_{GG}\right] > 0 \tag{3.36.2}$$

$$G_W = \frac{1}{\gamma^*}\left[-p(1-\alpha)\alpha V_{cc}^A V_{cc}^B\right] > 0 \tag{3.36.3}$$

$$c_p^A = \frac{1}{\gamma^*}\left[\alpha^2 (V_c^B)^2 \left(U_{GG} + \frac{\alpha}{p}V_{cc}^B\right)\right] > 0 \tag{3.36.4}$$

$$c_p^B = \frac{1}{\gamma^*}\left[-\alpha^2\rho(V_c^B)^2 U_{GG}/p - (1-\alpha)\alpha^2 V_{cc}^A V_c^B V_{cc}^B\right] < 0 \tag{3.36.5}$$

$$G_p = \frac{1}{\gamma^*}\frac{\alpha^2}{p}(V_c^B)^2\left[-\alpha\rho V_{cc}^B + p(1-\alpha)V_{cc}^A\right] \tag{3.36.6}$$

where $\gamma^* \equiv -V_c^B\left[p^2\alpha(1-\alpha)V_{cc}^A U_{GG} + \rho^2\alpha^2 V_{cc}^B U_{GG} + (1-\alpha)\alpha^2 V_{cc}^A V_{cc}^B\right] < 0.$

Since $s = c^B - c^A + \pi Y$, we have

$$s_W = \frac{1}{\gamma^*}(-p)U_{GG}\left[p(1-\alpha)V_{cc}^A - \alpha\rho V_{cc}^B\right] \tag{3.36.7}$$

$$s_p = c_p^B - c_p^A < 0 \tag{3.36.8}$$

Finally, we have

$$E_W = pc_W^B + \rho c_W^A + G_W > 0 \tag{3.37.1}$$

$$E_p = c^B - c^A \tag{3.37.2}$$

The properties of c^A, c^B, E, and s functions are qualitatively the same as in the Appendix of Chap. 2. We also have $G(W, p)$; the compensated demand function for the public good as in Sect. 4.1 of this chapter. It is intuitively plausible to see that $G_W \equiv \partial G/\partial W$ is positive. From Eqs. (3.36.6) and (3.36.7) $G_p \equiv \partial G/\partial p$ is negative since we assume $s_W < 0$. G is substitutable with c^A and complement able with c^B.

By definition we have

$$s(W_i, p, \pi_i Y_i) \equiv c^B(W_i, p) - c^A(W_i, p) + \pi_i Y_i \tag{3.38}$$

Outcome of Two-Country Model

The two-country model with mutual insurance motives and public goods is then summarized as follows:

$$E(W_1, p) + E(W_2, p) = Y_1 + Y_2 - p(Y_1\pi_1 + Y_2\pi_2) + G(W_1, p) \tag{3.39}$$

$$G(W_1, p) = G(W_2, p) \tag{3.40}$$

$$c^B(W_1, p) - c^A(W_1, p) + \pi_1 Y_1 + c^B(W_2, p) - c^A(W_2, p) + \pi_2 Y_2 = 0 \tag{3.41}$$

Equations (3.39) and (3.40) correspond to Eqs. (3.25) and (3.26), respectively. Equation (3.41) is the equilibrium condition for the insurance market, $s_1 + s_2 = 0$, and comes from Eq. (3.38).

From (3.40) when preferences are identical between countries we always have

$$W_1 = W_2$$

We also have

$$s_1 = s_2 = 0$$

if emergency costs are equal between countries:

$$\pi_1 Y_1 = \pi_2 Y_2$$

$W_1 = W_2$ means that $c_1^A = c_2^A$, $c_1^B = c_2^B$. Hence, from Eq. (3.41) we know

$$s_1 > 0 > s_2 \text{ if and only if } \pi_1 Y_1 > \pi_2 Y_2$$

Suppose condition $\pi_1 Y_1 = \pi_2 Y_2$ holds but incomes are different between countries ($Y_1 \neq Y_2$). Then condition $s_1 = s_2 = 0$ holds when we allow for an international public good as well as the mutual insurance market. In such a case insurance is not necessary.

These results show how the interdependence between allies is affected by the existence of insurance motives and an international public good. It should be stressed that welfare equality $W_1 = W_2$ does not hold without incorporating international public goods in addition to the mutual insurance market. Once we incorporate both the mutual insurance market and the international public good, expected welfare is equalized at the Nash solution, irrespective of differences in income, the penalty ratio, or the type of insurance. The symmetry preference is the only requirement. When the insurance market works and each country voluntarily provides an international public good, the divergence with respect to the effective income ($(1 - p\pi_i)Y_i$) does not matter. On the contrary, the total effective income $Y_1 + Y_2 - p(\pi_1 Y_1 + \pi_2 Y_2)$ and total emergency cost $\pi_1 Y_1 + \pi_2 Y_2$ do matter. Each country's expected welfare is affected in the same way. This is because both countries face the same price of insurance and hence different emergency costs have the income effect only.

Remember that in the insurance market model of the Appendix of Chap. 2, welfare is equalized if the expected disposable income $(1 - p\pi_i)Y_i$ is equal. In the public good provision model in Sect. 4.1, welfare is equalized if emergency cost $\pi_i Y_i$ is equal. Here in a two-ally model with both insurance and the public good expected welfare is always equalized so long as the preference is the same between countries.

If preferences are the same between two allies, the three-equation model of Eqs. (3.39), (3.40) and (3.41) reduces to:

$$2E(W, p) = (1 - p\pi_1)Y_1 + (1 - p\pi_2)Y_2 + G(W, p) \tag{3.42}$$

$$s(W, p, \pi_1 Y_1) + s(W, p, \pi_2 Y_2) = 0 \tag{3.43}$$

Hence, assuming that an income transfer from country 2 to country 1 occurs before the true state (A or B) is known and hence considering $dY_1 = -dY_2$, we have

$$\begin{bmatrix} 2E_W - G_W & -G_p \\ s_{1W} + s_{2W} & s_{1p} + s_{2p} \end{bmatrix} \begin{bmatrix} dW \\ dp \end{bmatrix} = \begin{bmatrix} -pY_1 \\ -Y_1 \end{bmatrix} d\pi_1 + \begin{bmatrix} -p(\pi_1 - \pi_2) \\ -\pi_1 + \pi_2 \end{bmatrix} dY_1 \tag{3.44}$$

or

$$\frac{dW}{d\pi_1} = \frac{-Y_1\left[p(s_{1p} + s_{2p}) + G_p\right]}{(2E_W - G_W)(s_{1p} + s_{2p}) + (s_{1W} + s_{2W})G_p} \tag{3.45.1}$$

$$\frac{dW}{dY_1} = \frac{-(\pi_1 - \pi_2)\left[p(s_{1p} + s_{2p}) + G_p\right]}{(2E_W - G_W)(s_{1p} + s_{2p}) + (s_{1W} + s_{2W})G_p} \tag{3.45.2}$$

We know $2E_W - G_W > 0$, $s_p < 0$, $G_p < 0$. The denominator is negative from the stability condition.

An increase in π_1 reduces the total expected income, hurting both countries. This is the total income effect. An increase in π_1 raises the price of insurance, inducing less supply of the pure public good. This price effect also hurts both countries. Hence, Eq. (3.45.1) is always negative.

Let us then investigate the welfare effect of an international transfer. The conventional conjecture is that the transfer policy usually has normal welfare effects; a receiving country becomes better and a giving country becomes worse when the direct income effect dominates the price effect. Since expected welfare is equalized, both countries may either gain or lose from the international income transfer. We have the weak paradox here. We now investigate how both countries are affected by the transfer policy.

The numerator in Eq. (3.45.2) is positive if and only if $\pi_2 > \pi_1$. Thus, if $\pi_2 < \pi_1$, Eq. (3.45.2) is negative; a transfer from country 2 to country 1 will reduce welfare of each country. Both a receiving country and a giving country lose when income is transferred to the country that has a larger penalty ratio.

Intuition is as follows. $dY_1 = -dY_2$ means that in state A country 1 gains 1\$ and country 2 loses this dollar. But in state B country 1 gains $(1 - \pi_1)$\$ while country 2 loses $(1 - \pi_2)$\$. If $\pi_1 > \pi_2$, it is clear that aggregate income in this state decreases. The existence of an insurance market and a public good means welfare equalization, which then implies that both countries' expected welfare decreases. In this sense, the existence of both the insurance market and the international public good is crucial to obtain the weak paradoxical result of international transfer.

Summary

In this Appendix we have investigated welfare implications of the interdependence between allied countries through insurance motives and voluntary provision of international public good toward country risk. We have considered effects of redistribution in the case where two countries are identical in preferences but they may be heterogeneous with respect to income and the penalty ratio on production.

With the international public good alone, total income effects and relative burdens are shown to be relevant. A country's welfare with larger emergency cost becomes worse than a country's welfare with smaller emergency cost. Each country loses from an increase in its own penalty ratio due to the total income effect, but it creates a beneficial spillover on the other country due to relative burden effects. Welfare equalization occurs only when emergency costs are the same between countries.

With both a mutual insurance market and voluntary provision of international public good, expected welfare is always equalized among allied partners, irrespective of differences in incomes, the penalty ratios, or the type of insurance so long as preferences are the same. In such a case any divergence between effective incomes does not matter since total effective incomes and total emergency costs affect each country's welfare in the same way. Furthermore, as to redistribution between countries we could have the strong paradox of transfers; a giving country gains and a receiving country loses when the relative burden effect dominates the total income effect. Since welfare is equalized, we always have the weak version of the transfer paradox; both a receiving country and a giving country lose when income is transferred to the country with the larger penalty ratio. International transfers between allies may well occur in the real world. We have shown that welfare implications of such transfers crucially depend on how protection against national emergency is provided.

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Chapter 4
Defense Expenditures and Allied Cooperation in Conflicting Alliances

4.1 Introduction

This chapter investigates how the cooperation of members of an allied bloc influences their welfare when that bloc opposes another bloc. This question is related to cooperative and non-cooperative defense spending in the confronting blocs such as the NATO and the WTO in the Cold War. By addressing this question, we can explore how voluntary provision of an international public good within allied countries protects against national emergency from the adversarial alliance.

Since Olson and Zeckhauser (1966), it has been emphasized that cooperation in contribution by the agents in a bloc is required for an efficient outcome. Needless to say, the security of an alliance is affected by the response of other alliances. In this chapter, we construct a model to examine the interactions between two conflicting allied blocs. Once we consider the interactions of two conflicting blocs, the conventional conjecture does not necessarily hold. When one bloc increases its security expenditure, the other bloc may respond to it by increasing its spending. We call this effect an "arms race effect." Bruce (1990) showed that in a three-country world with two allies and an adversary, all countries may be worse off when the allies cooperate than when they do not. Ihori (2000) showed that by incorporating multiple countries into the two conflicting blocs respectively, when the number of countries is large, the cooperative behavior may be a Nash equilibrium even if a negative spillover effect from the adversarial bloc is high. Ihori also showed that within a two-stage game, cooperative behavior may obtain as a sub-game perfect solution.

If cooperative behavior attains better outcomes than non-cooperative behavior, we have to worry about the free riding incentives. However, in reality we often observe non-cooperative provision of security spending. Thus if non-cooperative behavior could attain better outcomes than the cooperative behavior within an alliance, then free riding incentives may not be serious problems. In this chapter we explore this possibility by considering heterogeneity of preferences between conflicting blocs. We may say that if one agent's marginal valuation of the security issue is higher

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than the other agent's marginal valuation, the former agent has a "vital" interest in the issue and the latter agent has a "latent" interest. We assume that one bloc has relatively a vital interest in a security issue and that the other bloc has relatively a latent interest in the issue, and refer to the former as the "vital bloc" and the latter as the "latent bloc".

Then, we assess the natural conjecture that a cooperative strategy is desirable for the countries in the vital bloc and a non-cooperative strategy is desirable for those in the latent bloc. Our analysis shows that the above, intuitive, natural conjecture can be mistaken. Assuming that an arms race effect is large for the vital bloc but not for the latent bloc, then that arms race effect may dominate for the vital bloc. Otherwise, the welfare gain from cooperation within alliance, which we refer to as a "cooperation effect," may dominate for the latent bloc. Accordingly, we may say that if the NATO had a vital interest in the security issue during the Cold War, a non-cooperative outcome would be desirable because the arms race effect could dominate the cooperation effects in the NATO bloc.

This chapter thus investigates the implications of non-cooperative and cooperative spending on defense of allied countries of conflicting blocs using two-stage game models.

4.2 Cooperation Effect and Arms Race Effect

It is widely recognized that each member of an allied bloc has an incentive to free ride on spending on the security activities of other allied members although cooperation should benefit all. In their classical paper, Olson and Zeckhauser (1966) applied the theory of private provision of public goods to allied countries and concluded that all allied countries gain when they determine the level of spending on security cooperatively within an alliance. They highlighted the importance of allied cooperation in setting the spending on security activities. When the size of the alliance is large, the cooperation effect, or gains from cooperation would also be large. See also McGuire (1974, 1990), Sandler (1977), and Kemp (1984) among others.

If the cooperation effect is large, the free riding problem becomes serious. However, in a framework of conflicting blocs, the cooperation benefit might not be strong if we consider the arms race effect. Conflicting allied blocs often compete on the security issue. When two conflicting allied blocs engage in security activities, we may observe an arms race effect: an adversary's spending would rise in response to an increase in spending by the alliance. This arms race effect would apply to various areas of contentious public goods such as security activities, environmental issues, and defense spending.

In fact, Bruce (1990) pointed out that cooperation among allies in setting their defense spending is not necessarily welfare improving because of the arms race effect. He considered a three-country model with two allied countries and an adversary and showed that all countries may be worse off when the allies cooperate on defense spending than when they do not. This is because defense spending by the adversary rises in response to a cooperative increase in defense spending by the alliance. If this arms race effect is strong, that cooperation among allies in setting defense spending is not necessarily welfare-improving for them. Even if allied countries can cooperate, allied corporation will not become an equilibrium. This is an interesting result. "The whole notion of suboptimality of defense provision must be reconsidered when adversaries' reactions are included." (Sandler and Hartley 1995, p. 42).

However, his analysis was rather restrictive in that one bloc has two countries and another bloc has only one country. It should be stressed that an allied bloc usually has multiple countries. When the number of countries within the same bloc is large, gains from cooperation would also be large. Thus, if each bloc has a large number of allied countries, we would expect that cooperative behavior produces large benefits. By developing a simple multi-country model of arms races between two blocs, Ihori (2000) investigated to what extent such a conjecture would be plausible. When the number of countries in one bloc is larger than that in another bloc, countries in the larger bloc might be better off by cooperating than by not cooperating even if there is a negative spillover from the adversarial smaller bloc. Thus, in a two-stage game cooperative behavior becomes a subgame perfect solution although the free riding behavior within each group was not allowed in the cooperative case. These results suggest that the cooperation outcome may well be plausible even if an adversarial response of the opposing bloc is explicitly incorporated into the model of the two conflicting blocs. In this sense, bloc size divergence does matter in cooperation–non-cooperation issues.

Nevertheless, the arms race effect could outweigh the cooperation effect if preferences are divergent between the conflicting blocs but the bloc size is almost the same. In this chapter, we would like to focus on heterogeneous preferences between conflicting blocs by assuming that the bloc size is the same between conflicting blocs. In reality, a latent bloc and a vital bloc would have different preferences. We assume that a latent bloc has more interests in national security than a vital bloc. Put it another way, if one bloc's marginal valuation of national security is larger than that of another bloc, the former bloc is called vital while the latter is called latent.

Consider, for example, the performance of NATO (North Atlantic Treaty Organization) and WTO (Warsaw Treaty Organization) during the Cold War. Suppose for the NATO bloc, the security issue was vital since many member countries were seriously concerned with the potential threat from the USSR, whereas for the WTO bloc the security issue was not vital since most member countries were mainly concerned with economic standards or resources for consumption. If this is true, the natural conjecture is that the vital NATO countries behaved cooperatively, while the latent WTO countries behaved non-cooperatively during the Cold War. However, during the Cold War the NATO countries did not often organize a strong political body to seek more security whereas the WTO countries usually organized a strong political body to support security benefits. We may say that NATO's decision making was actually non-cooperative, while WTO's decision making was cooperative within allies. Even if it is true that the NATO had the vital interest in the security issue, it seems that they did not have an incentive to organize a strong political party, the counterpart of WTO.

Two explanations could hold for this seemingly paradoxical outcome. First, a plausible political explanation is that the degree of democracy is important to make joint decisions. From this viewpoint, it may not be easy for democratic countries to internalize the free riding incentives. Thus, democratic NATO members preferred non-cooperative decision making, while non-democratic WTO members conducted cooperative decision making under the strong leadership of the USSR.

In this chapter, we would like to offer another explanation based on standard economics, using a simple game between two conflicting blocs. Namely, if the NATO bloc had vital interests in the security issue, a non-cooperative outcome would be desirable, anticipating the reaction by WTO. On the other hand, if the WTO bloc had vital interests in the non-security issue such as private consumption, a cooperative outcome would be desirable, anticipating the adversary reaction by NATO.

It is useful to investigate the effects of divergent sizes and different preferences separately. In the present chapter we assume that the two conflicting blocs have different preferences over the security issue but the group size is the same. Thus, we do not consider differences in group size in this section. We examine the plausibility of the conventional conjecture that a vital group cooperates whereas a latent group does not. By developing a simple multi-member model of two conflicting groups with the same size, we explore an interesting counterexample against the conventional wisdom by showing that the latent group cooperates but the vital group does not cooperate at the Nash solution.

Another example, which might be relevant to this issue, would be the recent situation in the Eurozone. Zimmermann (2015) categorized 10 Euro area countries into two blocs: the northern one of Finland, Luxemburg, Germany, Austria, and the Netherlands, and the southern one of Spain, Portugal, France, Italy, and Greece. It seems that the northern countries are less vital than southern countries with respect to the sustainability of the Eurozone. They behave cooperatively, while the southern countries behave non-cooperatively. The numbers of the two bloc are almost the same. Since funding from the member countries to stabilizing facilities such as ESFS (European Financial Stability Facility) and ESM (European Stability Mechanism) is a kind of public good provision, our analytical framework can be applied.

4.3 Analytical Framework

Consider a simple competition model in which two blocs compete for security. Assume that there are $n + n$ countries and two opposing allies, α and β, in the world. Each allied bloc consists of n allied countries. Country i's utility function is given by Eq. (4.1).

$$U_i = u_i(c_i, G), \tag{4.1}$$

G is regarded as an international public good for bloc α and as a public bad for bloc β. c_i is private consumption of country i. An increase in G benefits bloc α but hurts bloc β. In order to win the Cold War game, both blocs may spend resources to either raise G or reduce G. Thus, one bloc's public good is another's public bad, and each bloc can take action to shift the total security influence toward its own preferred level.

Country i's budget constraint can be given by

$$c_i + g_i = Y_i, \tag{4.2}$$

For simplicity we assume that $Y_\alpha = Y_\beta = Y$.

We formulate that the actual level of G is determined by

$$G = G\left(\sum_{i \in \alpha} g_i, \sum_{i \in \beta} g_i\right) = \sum_{i \in \alpha} g_i - \sum_{i \in \beta} g_i \tag{4.3}$$

where g_i is the amount of security spending provided by country i. Following the seminal studies of Tullock (1980) and Becker and Mulligan (1998), the outcome of political conflict/contest between blocs α and β is summarized by a modified version of contest success function, Eq. (4.3). The conflict/contest involved is presumably complicated, but a key factor used to determine the "output" of the conflict/contest is the "input" expended by the players. Function (4.3) is a reduced-form end result of what may be a very complicated process of a Cold War game. In this reduced-form end result, the size of G directly depends on the amount spent by both blocs to gain security influence.

We formulate that the outcome of the Cold War conflict/contest is a function of the difference between the security expenditures of blocs. More pressure by bloc α increases the size of G, whereas more pressure by bloc β decreases it. In order to simplify our analysis, we assume that it is the net of the pressures applied by the blocs that determines the actual security influence, G. Equation (4.3) exhibits the property of homogeneity of degree one such that the same proportional increase or decrease in g_α and g_β raises the conflict/contest outcome by the same magnitude. Cornes and Rubbelke (2012) use a formulation similar to G as Eq. (4.3) and investigate the contentious public characteristics. They present conditions under which the existence of a unique non-cooperative equilibrium is retained and analyze its normative and comparative static properties.

For simplicity, we assume that the utility function is specified in a Cobb–Douglas form for each bloc.

$$U^{\alpha i} = c_{\alpha i}^{1-\theta}(A + G)^{\theta} \tag{4.4.1}$$

$$U^{\beta i} = c_{\beta i}^{1-\mu}(A - G)^{\mu} \tag{4.4.2}$$

where superscript (subscript) α or β denotes the bloc that a member of allies belongs to. We denote by A the initial level of vested security for countries. $2A$ means the total amount of "pie." An increase in g_α at the given g_β results in an increased distribution of pie, $2A$, in favor of bloc α but against bloc β, and vice versa. Put in another way, a given amount of $2A$ is allocated according to the net pressure G. If $G > 0$, bloc α can get more than half of the total amount of "pie", and vice versa.

Variables θ and μ are parameters representing the preferences of the allies on national their security. Variable θ is assumed to be relatively small for bloc α, whereas μ is assumed to be relatively large for bloc β. Hence, we have $1 > \mu > \theta > 0$. Since θ is small, the marginal valuation of G by bloc α is small, which means that G is not vital but latent for bloc α. On the contrary, since μ is large, the marginal valuation of security by bloc β is large, which implies that G is not latent but vital for bloc β. In this sense, we call bloc α the latent bloc and bloc β the vital bloc. Condition $A > 0$ is incorporated into Eq. (4.4.2) so that $A - G > 0$. It is true that the Cobb–Douglas functional form is very restrictive. However, in order to obtain concrete results and provide a counterexample against the conventional conjecture, this formulation is useful as a first step of this research.

In the example of the Cold War game, we may regard bloc α as the WTO bloc and β as the NATO bloc. G may capture the size of security benefits of WTO. In order to exclude the effects of differences in bloc size, we assume that both blocs have the same number of members and that each bloc consists of identical members. By doing so, we focus on the implications of differences in preference.

The structure of the game is as follows:

Stage I: *A country of each bloc determines whether to cooperate or not within the bloc.*
Stage II: *The country determines its spending on security activities, and a Nash equilibrium is obtained.*

Since all the countries in each bloc are identical, they behave in the same way. We do not consider the free riding behavior of some members within each bloc in the cooperative case. Therefore, each country may commit itself to the cooperation decision in the cooperative case. Thus, in stage I the allied countries uniformly decide whether to cooperate or not within the bloc. When cooperation is chosen, each bloc determines a representative who will then decide the per-capita contribution from inside the bloc in stage II. In case of noncooperation, each member determines its own contribution non-cooperatively in stage II. We do not consider cooperation between the conflicting two blocs.

4.4 Second Stage

4.4.1 Noncooperative Case

G is determined as a Nash equilibrium of a "game" between two blocs. First, we investigate the non-cooperative case where each country determines its own spending on security in stage II, treating the rest of the allied countries' spending on security as given. In other words, allied countries within the same bloc do not make any cooperative decisions with respect to spending on allied security activities but behave at Nash conjectures. As in Chap. 3, in order to present the results in the simplest way and in their strongest form, we consider only the case where the non-negativity constraints are non-binding in equilibrium. See Bergstrom et al. (1986) among others.

We now derive a reaction function of country i of bloc α. Country i maximizes Eq. (4.4.1) subject to its budget constraint

$$c_{\alpha i} + G_\alpha + A = Y_\alpha + (n-1)g_{\alpha j} - ng_\beta + A \tag{4.5}$$

taking security spending of other countries $g_{\alpha j}, g_\beta$ as given. Here, $g_{\alpha j}$ denotes spending on security by country $j(\neq i)$ of bloc α, and g_β denotes spending on security by any identical country of bloc β.

From the first-order condition, we have

$$g_{\alpha i} + (n-1)g_{\alpha j} - ng_\beta + A = \theta[Y_\alpha + (n-1)g_{\alpha j} - ng_\beta + A].$$

Since all the countries of bloc α are identical, we have $g_{\alpha i} = g_{\alpha j} = g_\alpha$ at any interior solution. Substituting $g_{\alpha i} = g_{\alpha j} = g_\alpha$ into the above equation, we finally obtain

$$g_\alpha = \frac{1}{n-(n-1)\theta}[\theta Y_\alpha - (1-\theta)A + (1-\theta)ng_\beta], \tag{4.6}$$

which is a reduced reaction function of each country belonging to bloc α in the non-cooperative case.

Variable g_α is an increasing function of the "real income" of the country, which is defined as a weighted sum of its own income and spending on security by the rival bloc, $\theta Y_\alpha - (1-\theta)A + (1-\theta)ng_\beta$. An increase in θ raises g_α, which is intuitively plausible. Equation (4.6) also indicates the arms race response of g_α to g_β,

$$\frac{dg_\alpha}{dg_\beta} = \frac{(1-\theta)n}{n-(n-1)\theta},$$

which is positive and decreases with θ from 1 at $\theta = 0$ to 0 at $\theta = 1$. In other words, if θ is small, an increase in g_β induces a large increase in g_α. The intuition is as follows. When θ is small, a change in the real income of bloc α would not affect the demand for G to a great extent, and hence, a decrease in real income due to an increase in g_β induces little decrease in the demand for G. On the contrary, it reduces private consumption to a great extent, resulting in a large increase in g_α.

Similarly, the non-cooperative reaction function of country of bloc β is given as

$$g_\beta = \frac{1}{n-(n-1)\mu}[\mu Y_\beta - (1-\mu)A + (1-\mu)ng_\alpha] \tag{4.7}$$

An increase in μ raises g_β, which is intuitively plausible.

Henceforth, we call country α (or β) the representative country of bloc α (or β). In Fig. 4.1, curve α represents country α's reaction curve, and curve β represents country β's reaction curve. Both curves are upward sloping. Spending on security is a strategic complement reflecting the arms race between rival blocs. An intersection of both curves, N, represents the non-cooperative Nash equilibrium point.

From Eqs. (4.6) and (4.7), the Nash equilibrium security spending for both countries can be respectively given as

$$g_\alpha = \frac{[n-(n-1)\mu][\theta Y_\alpha - (1-\theta)A] + (1-\theta)n[\mu Y_\beta - (1-\mu)A]}{[n-(n-1)\theta][n-(n-1)\mu] - (1-\theta)(1-\mu)n^2} \tag{4.8.1}$$

$$g_\beta = \frac{[n-(n-1)\theta]\left[\mu Y_\beta - (1-\mu)A\right] + (1-\mu)n[\theta Y_\alpha - (1-\theta)A]}{[n-(n-1)\theta][n-(n-1)\mu] - (1-\theta)(1-\mu)n^2} \tag{4.8.2}$$

When the marginal valuation of the security issue for the rival bloc μ increases, the spending on security g_α also increases. Since an increase in the marginal valuation of the rival bloc β raises its political spending, g_β, this raises threats to the countries of the other bloc α, giving rise to the arms race effect. Thus, both g_α and g_β increase with θ and μ. G also increases with θ but decreases with μ.

Thus, we have

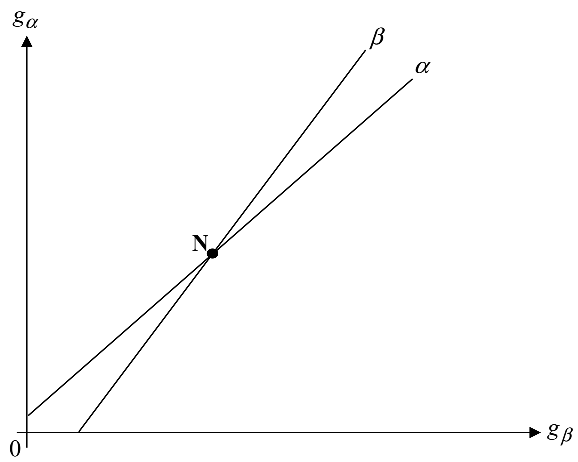


Fig. 4.1 Non-cooperative case on the second-stage game. *Source* Authors

$$G + A = \frac{\theta}{n(\theta + \mu - 2\theta\mu) + \mu\theta}\left\{[2n(1-\mu) + \mu]A + \mu n[Y_\alpha - Y_\beta]\right\}, \quad (4.9.1)$$

$$C_\alpha = \frac{1-\theta}{n(\theta + \mu - 2\theta\mu) + \mu\theta}\left\{[2n(1-\mu) + \mu]A + \mu n[Y_\alpha - Y_\beta]\right\}, \quad (4.9.2)$$

$$U^\alpha = \frac{(1-\theta)^{1-\theta}\theta^\theta}{n(\theta + \mu - 2\theta\mu) + \mu\theta}\left([2n(1-\mu) + \mu]A + \mu n[Y_\alpha - Y_\beta]\right), \quad (4.9.3)$$

$$U^\beta = \frac{(1-\mu)^{1-\mu}\mu^\mu}{n(\theta + \mu - 2\theta\mu) + \mu\theta}\left([2n(1-\theta) + \theta]A + \theta n\left[Y_\alpha - Y_\beta\right]\right) \quad (4.9.4)$$

As shown in Eqs. (4.9.1) and (4.9.2), the marginal rate of substitution of $G + A$ with respect to c_α equals 1, which is the marginal cost of providing security spending. This is the well-known result of non-cooperative solutions on public goods within a bloc. The same applies to country β. Equations (4.9.3) and (4.9.4) also suggest that the welfare of each country decreases with a rival's marginal valuation of the issue, whereas it may well increase with the country's own marginal valuation of the issue. This is intuitively plausible as well.

The actual level of security, G, increases with the number of allied countries of bloc α; this is consistent with McGuire (1974). It is also interesting to note that the real variables including G, c, and U are independent of Y if $Y_\alpha = Y_\beta = Y$. More precisely, the net income, $n(Y_\alpha - Y_\beta)$, matters in the provision of contentious public goods. As shown in Cornes and Rubbelke (2012), a net increase in the resources available to the economy $dY_\alpha = dY_\beta > 0$ may have no real consequences in the provision of contentious public goods. This is the so-called super-neutrality result, which holds in a general functional form of the utility function. It should also be noted that a redistribution between two conflicting blocs does have a real impact. A giving bloc loses, whereas a receiving bloc gains. On the contrary, a redistribution within a bloc does not have real effects. The conventional neutrality result, which was first pointed by Shibata (1971) and Warr (1983), holds within each bloc.

4.4.2 Intra-alliance Cooperative Case

We now consider a cooperative case where the allied countries cooperate within their bloc. Note that there is still no cooperation (or negotiation) between the two conflicting blocs.

Consider the joint optimization problem of representative country α of bloc α. Adding Eq. (4.2) up to n and considering $g_{\alpha i} = g_{\alpha j} = g_\alpha$, country α's consolidated budget constraint may be written as

$$nc_\alpha + G_\alpha + A = nY_\alpha - ng_\beta + A \quad (4.10)$$

Thus, country α jointly maximizes Eq. (4.4.1) subject to the above consolidated budget constraint Eq. (4.10), taking g_β as given. From the first-order condition, we have

$$ng_\alpha - ng_\beta + A = \theta(nY_\alpha - ng_\beta + A)$$

Thus, the cooperative reaction function of country α can be given as

$$g_\alpha = \theta Y_\alpha - \frac{1-\theta}{n}A + (1-\theta)g_\beta \tag{4.11}$$

Equation (4.11) implies that $\frac{\partial g_\alpha}{\partial g_\beta} = 1 - \theta > 0$, which decreases with θ. This property is qualitatively the same as in the non-cooperative case. When θ is low and the latent bloc α does not recognize the benefit of G to a great extent, it is desirable for bloc α not to change $A + G$ to a great extent. In order to reduce $A + G$ to a small extent, bloc α raises its spending on security activities to a great extent when bloc β increases its spending on security activities.

Note that if $n = 1$, Eq. (4.11) reduces to Eq. (4.6). When $n > 1$, dg_α/dg_β, the slope of the reaction function, $(1-\theta)$, is less than $(1-\theta)n/\{n-(n-1)\theta\}$ in the non-cooperative case. Thus, when the adversarial bloc β raises the level of security pressure, rival bloc α reacts by spending less in the cooperative case than in the non-cooperative case. This is because in the cooperative case, a member of bloc α incorporates the positive reaction of other allied members when g_β increases, resulting in a smaller increase in g_α than in the non-cooperative case, where it does not consider the positive reaction of other allied members. We also need to note that g_α is higher in the cooperative case than in the non-cooperative case at the same level of g_β, since a cooperative behavior can internalize the free riding motive. If $\theta = 1$, Eq. (4.11) again reduces to Eq. (4.6). In other words, if θ is large, the gap between cooperative g_α and non-cooperative g_α becomes small.

Similarly, the cooperative reaction function of country β can be given as

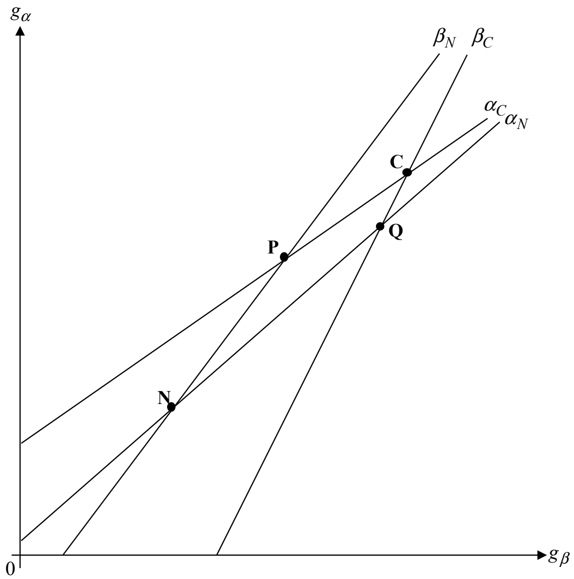
$$g_\beta = \mu Y_\beta - \frac{1-\mu}{n}A + (1-\mu)g_\alpha \tag{4.12}$$

Again, if $n = 1$, Eq. (4.12) reduces to Eq. (4.7). If μ is large, the gap between cooperative g_β and non-cooperative g_β is small.

4.4.3 Three Cooperative Cases

There are three cases where cooperation is chosen in at least one bloc. First, let us investigate the cooperative case where all the countries of bloc α as well as bloc β cooperate. In Fig. 4.2, curve α_N represents the non-cooperative reaction curve of country α when no countries of bloc α coop-

Fig. 4.2 Four Nash equilibria. *Source* Authors



erate, curve β_N represents the non-cooperative reaction curve of country β when no countries of bloc β cooperate, curve α_C represents the cooperative reaction curve of country α when all the countries of bloc α cooperate, and curve β_C represents the cooperative reaction curve of country β when all the countries of bloc β cooperate. The intersection of curves α_C and β_C, denoted by point C, corresponds to the full cooperative case where both blocs α and β cooperate respectively. The cooperative equilibrium levels of g_α, g_β are respectively given as

$$g_\alpha = \frac{\theta Y_\alpha + \mu(1-\theta)Y_\beta - (1-\theta)A(2-\mu)/n}{\mu + \theta - \mu\theta} \tag{4.13.1}$$

$$g_\beta = \frac{\mu Y_\beta + \theta(1-\mu)Y_\alpha - (1-\mu)A(2-\theta)/n}{\mu + \theta - \mu\theta} \tag{4.13.2}$$

An increase in θ or μ raises the security activities of both blocs, g_α, g_β, and G increases with θ but decreases with μ. An increase in μ lowers the welfare of bloc α by reducing G, and an increase in θ lowers the welfare of bloc β by raising G. Thus, we have

$$G + A = \frac{\theta}{\mu + \theta - \mu\theta}\left\{[2-\mu]A + \mu n[Y_\alpha - Y_\beta]\right\}, \tag{4.14.1}$$

$$c_\alpha = \frac{1-\theta}{(\mu + \theta - \mu\theta)n}\left\{[2-\mu]A + \mu n[Y_\alpha - Y_\beta]\right\}, \tag{4.14.2}$$

$$U^\alpha = \frac{(1-\theta)^{1-\theta}\theta^\theta}{\mu + \theta - \mu\theta}\frac{1}{n^{1-\theta}}\left\{[2-\mu]A + \mu n[Y_\alpha - Y_\beta]\right\}, \tag{4.14.3}$$

$$U^{\beta} = \frac{(1-\mu)^{1-\mu}\mu^{\mu}}{\mu+\theta-\mu\theta}\frac{1}{n^{1-\mu}}\big\{[2-\theta]A + \theta n[Y_{\beta} - Y_{\alpha}]\big\}. \tag{4.14.4}$$

As shown in Eqs. (4.14.1) and (4.14.2), the total marginal rate of substitution of $G + A$ with respect to c_{α} equals 1, which is the marginal cost of providing political pressure. This condition is nothing but the Samuelson rule on public goods within the bloc, which is the well-known result of cooperative solution. Although the Samuelson rule holds for each bloc, it does not apply to the overall world. Since there is no cooperation between two conflicting blocs, the cooperative solution within the bloc here cannot attain the first-best result. The second-best theory suggests that the second-best utility is not necessarily higher at the cooperative solution than at the non-cooperative solution.

We may also consider the partial cooperative case where the countries of bloc α do not cooperate whereas the countries of bloc β cooperate. In this case, country α's reaction curve can be given as Eq. (4.6) while country β's reaction curve can be given as Eq. (4.12). We may also consider the partial cooperative case where the countries of bloc α cooperate while the countries of bloc β do not cooperate. We need to note that the net income $n(Y_{\alpha} - Y_{\beta})$ matters in the provision of contentious public goods. Thus, the neutrality and super-neutrality results hold in the three cooperative cases as well.

4.5 First Stage

Table 4.1 indicates the hypothetical payoffs of four cases in the second stage of the game: (1) either bloc α or bloc β does not cooperate at point N, (2) bloc α cooperates while bloc β does not cooperate at point P, (3) bloc α does not cooperate while bloc β cooperates at point Q, and (4) both blocs α and β cooperate at point C.

$$\Delta \equiv \frac{n(\theta+\mu-2\theta\mu)+\theta\mu}{(1-\theta)^{1-\theta}\theta^{\theta}A}, \quad \Delta^{*} \equiv \frac{n(\theta+\mu-2\theta\mu)+\theta\mu}{(1-\mu)^{1-\mu}\mu^{\mu}A},$$
$$\Phi \equiv \frac{\theta+\mu n-n\theta\mu}{(1-\theta)^{1-\theta}\theta^{\theta}A}, \quad \Phi^{*} \equiv \frac{\theta+\mu n-n\theta\mu}{(1-\mu)^{1-\mu}\mu^{\mu}A},$$

Table 4.1 Hypothetical payoffs of four cases in the second stage of the game

		Bloc β	
		N	C
Bloc α	N	$\frac{2n(1-\mu)+\mu}{\Delta}, \frac{2n(1-\theta)+\theta}{\Delta^{*}}$	$\frac{2-\mu}{\Phi}, \frac{2n(1-\theta)+\theta}{n^{1-\theta}\Phi^{*}}$
	C	$\frac{2n(1-\mu)+\mu}{n^{1-\theta}\Lambda}, \frac{2-\theta}{\Lambda^{*}}$	$\frac{2-\mu}{n^{1-\theta}\Gamma}, \frac{2-\theta}{n^{1-\mu}\Gamma^{*}}$

Note N noncooperation, *C* cooperation

$$\Lambda \equiv \frac{n\theta + \mu - n\theta\mu}{(1-\theta)^{1-\theta}\theta^{\theta}A}, \quad \Lambda^* \equiv \frac{n\theta + \mu - n\theta\mu}{(1-\mu)^{1-\mu}\mu^{\mu}A},$$
$$\Gamma \equiv \frac{\theta + \mu - \theta\mu}{(1-\theta)^{1-\theta}\theta^{\theta}A}, \quad \Gamma^* \equiv \frac{\theta + \mu - \theta\mu}{(1-\mu)^{1-\mu}\mu^{\mu}A}.$$

Now, we can investigate the Nash equilibrium by comparing four possible payoffs at the second stage: the non-cooperative payoffs where no countries cooperate and the three cooperative payoffs where at least some allied cooperation occurs in blocs α and/or β.

In order to internalize the positive spillover effect between members within the same bloc, the countries of the same bloc, say α, should choose a representative or have an agreement to determine security activities cooperatively. By doing so, the bloc's spending on security is stimulated and it benefits all the countries of the bloc. This is the cooperation effect. However, in such a case the members of the rival bloc β react by raising their security activities, which would hurt the countries of bloc α. We call this negative spillover the arms race effect. If the negative spillover due to the arms race effect outweighs the positive spillover due to the cooperation effect, such cooperation hurts bloc α. This possibility was first pointed out by Bruce (1990) in the study of national defense. Ihori (2000) showed that the cooperation effect might well dominate the arms race effect when the number of allied members is larger than the number of rival members. However, if preferences are divergent and the group size is the same, the arms race effect may well dominate the cooperation effect.

By excluding the differences in bloc size, the present chapter focuses only on the differences in preferences between the two blocs; θ is smaller than μ. Let us first investigate the optimal strategy of bloc α in stage I. Suppose bloc β cooperates. As shown in Table 4.1, it is desirable for bloc α to cooperate if and only if

$$\begin{aligned} D &= n^{1-\theta}(\mu + \theta) - \mu\theta n^{1-\theta} - (\theta + \mu n - \theta n\mu) \\ &= \mu n(n^{-\theta} - 1)(1-\theta) + \theta(n^{1-\theta} - 1) < 0 \end{aligned}$$

Since $n^{-1} < n^{-\theta} < 1$, this sign could be negative when μ is relatively large. Suppose now that bloc β does not cooperate. Then, it is desirable for bloc α to cooperate if and only if

$$\begin{aligned} E &= \mu n^{1-\theta} + \theta n^{2-\theta} - \theta n^{2-\theta}\mu - n(\theta + \mu - 2\theta\mu) - \mu\theta \\ &= \mu n(n^{-\theta} - 1) + \theta n(n^{1-\theta} - 1) + \theta\mu(2n - 1 - n^{2-\theta}) < 0 \end{aligned}$$

This sign could be negative when μ is relatively large, since $2n < n^{2-\theta} + 1$ for a small θ.

Tables 4.2, 4.3 and 4.4 shows the values of D and E calculated using their definitions in order to evaluate utilities of bloc α. Table 4.2 suggests that D becomes negative if $\mu > 0.5$ for $n = 3$. In other words, if μ is relatively large, bloc α gains by cooperating within the bloc. Table 4.3 shows that E becomes negative if $\mu \geq 0.9$ for n = 3. Table 4.4 further shows that E becomes negative if $\mu \geq 0.8$ for $n = 6$.

Table 4.2 Value of D for n = 3: bloc α

		θ					
		0.8	0.7	0.6	0.5	0.4	0.3
μ	0.9	−0.12	−0.16	−0.19	−0.20	−0.20	−0.18
	0.8		−0.11	−0.13	−0.14	−0.14	−0.12
	0.7			−0.07	−0.08	−0.07	−0.07
	0.6				−0.01	−0.01	−0.01
	0.5					0.05	0.05
	0.4						0.11

Table 4.3 Value of E for n = 3: bloc α

		θ					
		0.8	0.7	0.6	0.5	0.4	0.3
μ	0.9	−0.08	−0.11	−0.12	−0.13	−0.13	−0.11
	0.8		−0.00	0.00	0.01	0.01	0.01
	0.7			0.12	0.14	0.15	0.14

Table 4.4 Value of E for n = 6: bloc α

		θ					
		0.8	0.7	0.6	0.5	0.4	0.3
μ	0.9	−0.31	−0.40	−0.48	−0.51	−0.50	−0.44
	0.8		−0.03	−0.01	0.03	0.07	0.11
	0.7			0.46	0.57	0.64	0.66

In other words, if μ is relatively large, it is always desirable for bloc α to cooperate within the bloc. Hence, cooperation is the dominant strategy for bloc α when μ is relatively large.

Tables 4.5, 4.6 and 4.7 compares the payoffs of bloc β in the cooperative case and non-cooperative case. Values of D and E here are calculated using their definitions where θ is replaced with μ, vice versa, in order to evaluate utilities of bloc β. Table 4.5 shows that D is positive if $\theta \leq 0.5$ for $n = 3$. In other words, if θ is relatively small, bloc β gains by not cooperating within the bloc. Table 4.6 shows that E is positive if $\theta \leq 0.7$ for $n = 3$. Table 4.7 further shows that E becomes positive if $\theta \leq 0.7$ for $n = 6$ as well. In other words, if θ is relatively small, it is desirable for bloc β not to cooperate within the bloc. Therefore, non-cooperation is the dominant strategy for bloc β when θ is relatively small. Hence, a Nash outcome is likely that the latent bloc cooperates and the vital bloc does not cooperate.

The intuition is as follows. When μ is relatively large, the effect of an increase in g_α on g_β is small. In such a case, an increase in the security activities of bloc α due to cooperation within the bloc would not stimulate security spending in the rival bloc β to a great extent. This is because the gap between cooperative g_β and non-cooperative g_β becomes small when μ is relatively large. Hence, the arms race effect is small and bloc α gains much by cooperating. Furthermore, when θ is relatively

Table 4.5 Value of D for n = 3: bloc β

		θ					
		0.8	0.7	0.6	0.5	0.4	0.3
μ	0.9	−0.05	−0.03	−0.01	0.01	0.03	0.05
	0.8		−0.05	−0.01	0.02	0.06	0.09
	0.7			−0.02	0.03	0.08	0.13
	0.6				0.04	0.10	0.16

Table 4.6 Value of E for n = 3: bloc β

		θ					
		0.8	0.7	0.6	0.5	0.4	0.3
μ	0.9	−0.00	0.04	0.08	0.11	0.15	0.19
	0.8		0.07	0.14	0.22	0.29	0.37
	0.7			0.20	0.31	0.41	0.51
	0.6				0.37	0.50	0.62
	0.5					0.55	0.69
	0.4						0.70

Table 4.7 Value of E for n = 6: bloc β

		θ					
		0.8	0.7	0.6	0.5	0.4	0.3
μ	0.9	−0.03	0.11	0.24	0.38	0.51	0.65
	0.8		0.22	0.49	0.75	1.01	1.28
	0.7			0.72	1.10	1.48	1.86
	0.6				1.41	1.88	2.35

small, the negative spillover from an increase in the security spending of bloc β does not hurt bloc α to a great extent. In such a case, bloc α does not lose to a great extent from an arms race reaction by rival bloc β. In other words, if θ is relatively small, the cooperation effect dominates the arms race effect for bloc α. Qualitatively, the opposite mechanism applies to bloc β if μ is relatively large. That is, the arms race effect may dominate the cooperation effect for bloc β.

The above mechanism is affected by the rate of substitution of the two goods. Tables 4.8 and 4.9 shows the remainder of utility levels of bloc α in the non-cooperative case after deducting that in the cooperative case when bloc β cooperates, using the CES utility function $U^{\alpha i} = \left[(A+G)^{\sigma/(1-\sigma)} + (1-a)c_{\alpha i}^{\sigma/(1-\sigma)}\right]^{(1-\sigma)/\sigma}$, which should be compared with Table 4.2. As shown, our results strengthen when the rate of substitution $\sigma > 1$ and weaken when $\sigma < 1$ because an increase or a decrease in σ works like that in θ in the Cobb–Douglas utility case when $\theta > 0.5$. The same mechanism operates for as well, thus our results that a Nash equilibrium of the latent bloc and the cooperative equilibrium in the vital bloc is more plausible when $\sigma > 1$.

Table 4.8 $U^{\alpha i}(N, C) - U^{\alpha i}(C, C)$ for $\sigma = 1.5$

		θ					
		0.8	0.7	0.6	0.5	0.4	0.3
μ	0.9	−0.53	−0.65	−0.63	−0.54	−0.41	−0.29
	0.8		−0.62	−0.62	−0.54	−0.42	−0.30
	0.7			−0.57	−0.51	−0.41	−0.29
	0.6				−0.39	−0.32	−0.24
	0.5					−0.04	−0.02
	0.4						0.62

Table 4.9 $U^{\alpha i}(N, C) - U^{\alpha i}(C, C)$ for $\sigma = 0.5$

		θ					
		0.8	0.7	0.6	0.5	0.4	0.3
μ	0.9	0.36	0.32	0.30	0.27	0.26	0.24
	0.8		0.51	0.47	0.44	0.42	0.39
	0.7			0.65	0.61	0.58	0.55
	0.6				0.79	0.76	0.73
	0.5					0.97	0.93
	0.4						1.19

In the real economy, it is often observed that a small number of countries form a powerful and well-organized allied bloc to cooperate within the bloc, whereas a large number of countries do not always build a powerful allied bloc. It is true that bloc size divergence has an important role since it affects the size of the cooperation effect. In addition to this, a natural conjecture is that if the former bloc has a vital interest in the issue, a cooperative strategy would be desirable although it stimulates security activities of the rival bloc, and it would be desirable for the latter bloc not to cooperate since it has a latent interest in the issue. One could argue that if the issue is vital, cooperation becomes desirable.

Our analytical result suggests that the above conjecture is not necessarily valid if the bloc size effect is controlled for. We have shown that the arms race effect is large for the vital bloc but not for the latent bloc. Hence, the arms race effect could dominate the cooperation effect for the vital bloc whereas the cooperation effect could dominate the arms race effect for the latent bloc. The cooperative behavior of the vital bloc might stimulate the offsetting security spending of the latent rival bloc to a great extent, hurting the vital bloc very much. The opposite mechanism applies to the latent bloc. Hence, a Nash outcome may well be that the latent bloc cooperates and the vital bloc does not cooperate. In reality, conflicting blocs in the Cold War might have such features.

4.6 Conclusion

This chapter has investigated the cooperation and arms race effects of non-cooperative and cooperative spending on the security activities of the allied members of two rival blocs with the same bloc size. We have shown that if the two rival blocs' preferences on the security issue are different, a Nash outcome is likely that the latent bloc cooperates and the vital bloc does not cooperate. As for the vital bloc, the non-cooperative supply of security activities is close to the cooperative level so that gains from cooperative behavior may not be large, and hence, the non-cooperative (free riding) behavior of each member does not hurt the vital bloc to a great extent compared with the cooperative choice. Moreover, it could benefit the other allied members by depressing the magnitude of the arms race effect. On the contrary, as for the latent bloc, the non-cooperative (free riding) behavior hurts each member to a great extent since the gap between the cooperative and non-cooperative levels of security activities are large. The vital bloc may lose much by cooperation since it would induce the considerable magnitude of the arms race effect from the latent countries. We have shown that the arms race effect may be dominant for the vital bloc and that the cooperative effect may be dominant for the latent bloc.

In the case of the Cold War game, it was often observed that the NATO countries did not organize a strong political body to seek more security benefits whereas the WTO countries organized a powerful political body under the strong leadership of the USSR. We have offered a plausible explanation of why such outcome occurred during the Cold War.

Another example is the bailout plan for the recent European financial crisis. The bailout was strongly needed by the southern countries such as Greece. However, they could not form an alliance against the conditions contended by the northern member coalition. On the contrary, northern members united in demanding the fiscal reform of the southern members although their benefits were relatively weak. Although the Cobb–Douglas formulation is rather restrictive, our simple model may explain this outcome. It is true that this model does not endogenize political bloc membership. This is a topic of interest for future research.

This chapter has investigated implications of non-cooperative and cooperative spending on defense of allied countries within the two rival blocs using two-stage game models of arms races. It is well known that in the three-country world with two allies and a common adversary all countries may be better off when the allies do not cooperate than when they do. By incorporating multiple countries into two opposing blocs, cooperative behavior may well produce a Nash equilibrium although negative spillover effects from rival blocs are high. Furthermore, in a two-stage game cooperative behavior will obtain as a sub-game perfect solution. This cooperation result may well be valid in a world of endogenous threat among adversarial countries.

However, if preferences are divergent, cooperative behavior may not result in higher welfare than non-cooperative behavior due to a strong arms race effect for a vital bloc. We have shown that if the two rival blocs' preferences on an issue are different, a Nash outcome is likely in which the latent bloc cooperates and the vital bloc does not. As for the vital bloc, non-cooperative provision of security is close to the cooperative level so that the gains from cooperative behavior may not be large, and hence, non-cooperative (free riding) behavior of each member would not hurt the vital bloc to a great extent (compared to cooperative choice). On the other hand, as for the latent bloc, non-cooperative (free riding) behavior would hurt each member to a great extent since the gap between cooperative and non-cooperative levels of security activities is large. We have also shown that an arms race effect may be dominant for the vital bloc and that the cooperative effect may be dominant for the latent bloc.

It has been assumed that all countries in the same bloc behave in the same way. The analysis could be generalized to allow for diverse game-theoretic behavior among allied countries. It will be also useful to investigate the impact of allied cooperation on arms races in a dynamic setting.

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Chapter 5
Self-protection and Self-insurance Against Adversity

5.1 Introduction

5.1.1 Object of This Chapter

Based on Ihori and McGuire (2007), this chapter first investigates the impact of self-protection on alliances and then, based on Ihori and McGuire (2010), considers the impact of self-insurance as well as self-protection on security spending. We extend existing analyses of self-insurance and self-protection that countries may implement at a national level in pursuit of their security. That is, we provide an analysis of odds-improving self-protection and cost-reducing self-insurance when they yield collective benefits to allied groups, such as alliances of nations, for whom risks of loss are public bads and prevention of loss is a public good.

The classic of economic analysis for how an individual expected-utility maximizer should deal with threat of loss is based on Ehrlich and Becker, (1972). Their rational agents can choose to allocate resources among risk reducing self-protection, preparations to curtail losses if they happen, or insurance contracts with others to compensate for losses that occur. Their model also should apply to policy choices of groups of countries considered as monolithic rational decision units facing common international risks, a topical subject nowadays as in global warming, pollution, security, or finance.

There is of course a giant literature on collective provision of insurance, wherein the risks of loss are assumed to be fixed (e.g. Genicot and Ray 2003, Eeckoudt and Gollier 2000, Schlesinger 2000). But application of the Ehrlich and Becker model to collective probability improving, "self-protection" and/or collective damage reducing, "self-insurance" is sparse, except for some work on terrorism such as Lapan and Sandler (1988) or more recently Sandler (1992, 1997, 2005), McGuire (2000, 2002, 2006), McGuire and Becker (2006), McGuire et al. (1991), and Arce et al. (2005).

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In particular, as explained in Chap. 3, economist's models of voluntary public good provision with many agents (VPG) have not been extended to understand the consequences of differences in risk aversion in this risk management context where common defense will reduce common hazard. As explained in Chap. 3, the VPG model (Olson 1965; Olson and Zeckhauser 1966) has come to have a standard stylized structure and format that has yielded especially striking properties with respect to Cournot-Nash equilibria among members of a public good consuming group (Bergstrom et al. 1986; Andreoni, 1988, 1989; Andreoni and McGuire (1993), McGuire and Shrestha 2003; Cornes and Sandler 1984, 1996; Warr 1983).

This chapter emphasizes how diminishing returns in risk improvement despite its "non-summation" consumption technology can be folded into income effects. In Sect. 5.2 we first examine the income effect of self-protection. This income effect then implies that whether the protection is inferior or normal depends on the risk aversion characteristics of underlying utility functions, and on the interaction between these, the level of risk, and marginal effectiveness of risk abatement. We demonstrate how public good inferiority is highly likely when the good is "group risk reduction" or self-protection. In fact, we discover a natural or endogenous limit on the size of a group and of the amount of risk controlling outlay it will provide under Nash behavior. We call this limit an "Inferior Goods Barrier" to voluntary risk reduction.

Because of interdependencies between risk aversion and degree of risk, possibilities of instability and multiple equilibria are magnified. Such interdependencies also can lead to shifting non-convexities in preference maps, and therefore to shifting corner solutions and the implied instabilities. Our analysis thus raises serious doubts overall about the reliability of the Nash-VPG model in group protection problems explained in Chap. 3, and it suggests a troubling source of instability for nations attempting to cooperate for provision of collective security.

In the second half of this chapter Sect. 5.3 incorporates self-insurance into the model and then investigates the income effect of such self-insurance. By doing so, we discover a hitherto unrecognized tendency for misallocation between self-protection and self-insurance when both are available and considered together. Because of external effects running from self-protection to self-insurance, governments ruled by myopic bureaucracies and trying to find the right balance can face incentives that encourage extreme, self-inflicted moral hazard, to the detriment of self-protection. This chapter thus shows how in a multi-country model rather innocuous assumptions concerning countries' preferences lead to pervasive goods inferiority for at least self-insurance.

To sum up, the object of this chapter is to make that extension, with the VPG model applied to groups of risk adverse agents who share common chances of loss and common benefit when those risks are curtailed. These agents can protect or defend themselves with self-protection and self-insurance, but such measures, we assume, necessarily spillover to the benefit of other agents in the group or coalition—equally improving their odds as well. Thus, all have incentives to free ride, and their individual actions are influenced by the reciprocal free spill-in benefits they receive from others. Evidently, many international problems in the world today resemble this common threat/loss management problem.

5.1.2 Summation Aggregator

A limit of the conventional model was that it assumed a linear or summation aggregator for the consumption technology ("summation in consumption"), and also a summation aggregator for group contributions or finance ("summation in finance"). Diminishing marginal returns in provision of the public good were, therefore, assumed away; only constant cost linear production was addressed. Later beginning with the "weakest-link/best-shot" contributions of Hirshleifer (1983, 1985), subsequent literature including Cornes (1993), Vicary (1990), Sandler and Vicary (2002), recognized a variety of ways that individual consumption or enjoyment of the public good depended on individual own and other contributions. These variations in the "aggregation of consumption technology," have become standard in such treatises as Cornes and Sandler (1996) or Mueller (2003).

In our analysis diminishing returns are essential in risk reduction. With risk of loss defined as $1 - p(M)$, $(0 \leq p \leq 1)$ and M as total expenditures on deterrence or defense to reduce the chance of loss, then assumedly $p = p(M)$ shows diminishing returns, $p'(M) > 0$, $p''(M) < 0$. Also, self-insurance differs from standard market insurance in that self-insurance function $\mathbf{L}$ should show diminishing returns or increasing costs. $\mathbf{L}' > 0, \mathbf{L}'' < 0$. Ehrlich and Becker (1972) make this assumption also, and refer glancingly to the role of human capital in providing for self-insurance as a source of diminishing returns. We believe that scale considerations are appropriate for an entire country along an extensive margin as well as other cooperating factors of production, as in Ehrlich and Becker (1972). We also argue that "self-insurance" has declining marginal productivity as shown earlier.

Thus, to handle collective probability improvement we must cope with "non-summation consumption aggregation" in public good provision. Although this difference may represent only the simplest beginning departure from the standard VPG model and although it is implicit in more advanced studies where individual contributions are not perfect substitutes such as Cornes (1993), nevertheless it has not been explicitly treated elsewhere. (Actually, the fact that our "innovation" here is so minimal makes the perversity of our results, all the more notable!)

Our simple solution to the problem of diminishing returns and distribution of infra marginal costs/gains will be to assume a "summation *finance* aggregator," $M = \sum m$, in the provision of public good p, even though $p(M)$ and $\mathbf{L}(M)$ represent a "non-summation *consumption* aggregator". Then, importing an idea from contest theory we take primitive preferences as being over *contributions* to risk reduction, rather than risk reduction itself. This allows the differences caused by diminishing returns in $p(M)$ and $\mathbf{L}(M)$ and the effects of positive and/or variable differences in risk aversion to be folded into VPG income effects. It will confirm that a "summation aggregator in consumption" is not necessary and that "summation finance" alone is sufficient to maintain the Warr neutrality properties of the original model (as implicit in the treatments of Cornes and Sandler,1996, and Mueller 2003) And we will learn whether Olson's "exploitation of the great by the small," property obtains when the

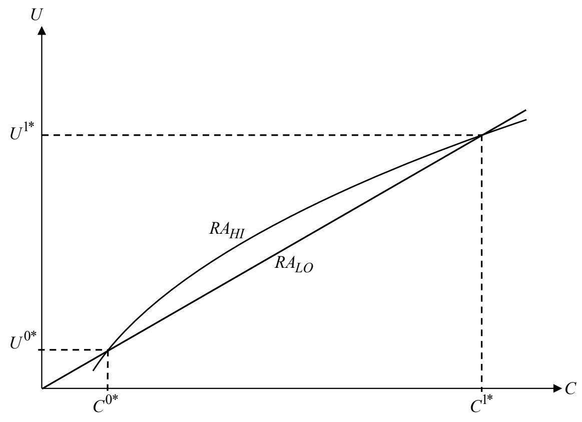


Fig. 5.1 Risk/risk aversion interaction-effect. *Source* Authors

public good is risk reduction under "summation finance", and how differences as to risk aversion influence equilibrium.

Most especially, we will learn that the degree of common risk, relative wealth, and variability of risk aversion all interact in a novel and hitherto unrecognized fashion. In particular, we demonstrate that when risk aversion increases with income, and risk is low (high) then self-protection (expenditure thereon) tends to be an inferior good for low (high) income agents. On the other hand, if risk aversion decreases with income and risk is low (high) then self-protection tends to be inferior for agents with high (low) income. Now consider the fact that when agents form a group for public good provision then *ipso facto* the full income of each agent increases (possibly dramatically). It follows, as we will show, that these properties can have major, and possibly quite unwelcome effects on the nature and stability of group behavior and thus of the Nash VPG solution.

As an intuitive preview to motivate interest, consider this problem. Suppose that an agent has allocated resources to probability improvement as per Ehrlich an Becker (1972) to an optimum—designated p^*, $E^*[U]$—where expected marginal benefits of probability improvement equal expected marginal costs. Suppose his risk aversion is nil, and let his optimum be illustrated in Fig. 5.1 by the straight linear utility function.

Now let this agent's resource allocation be fixed (giving C^{0*}, C^{1*}, U^{0*}, U^{1*} in Fig. 5.1) but let his utility function become more risk averse; and just for illustration assume that it coincides with the first utility function at both outcomes. With this new configuration will the old allocation continue to be optimal? There is no change in expected marginal benefit, MB_M. However, expected marginal costs do change—with MC_M at the bad contingency becoming greater, and MC_M at the good contingency becoming smaller due to the higher risk aversion. Expected MC will increase or decrease depending on the weights p^*, $(1-p^*)$. Thus, whether greater risk aversion induces more (less) outlay on self-protection depends on whether expected MC decreases (or increases) and therefore on status quo probabilities.

As we will demonstrate in Sect. 5.2, this risk/risk-aversion interaction-effect profoundly influences the behavior of self-protecting groups especially when group size/wealth changes. It can cause risk improvement to become inferior with income growth, can endogenously limit the degree of protection a group will provide for itself, and it introduces an endogenous limit on voluntary group size. Moreover, when we consider mutual provision of public goods by nations, we find another difficulty, namely, inferiority. That is, if countries are risk averse, we could have the situation where both types of public goods are inferior, leading to Nash equilibrium would at corner solutions.

In Sect. 5.3, by extending the single public good model of self-protection to a two public good world of self-protection and self-insurance, we investigate how provisions of self-insurance and self-protection make public good provision more complicated, and demonstrate an inherent potential for "unstable conflict" where centralized or decentralized specialization of provision of these public goods will occur.

5.2 Self-protection

5.2.1 Analytical Framework

Let the world consist of two countries; country 1 and country 2, and have two states, a good state "1" and a bad state, "0". Ignoring all insurance and compensation possibilities, expected utility for a single country i $(i = 1, 2)$ is given as:

$$W_i = pU^1(C_i) + (1 - p)U^0(C_i - L_i) \tag{5.1}$$

or

$$W_i = W_i(C_i, p) \tag{5.1'}$$

where W_i is expected utility for country i, C_i is i's private consumption, L_i is i's loss in the bad state, and p is the chance of a good state. Our analysis in this section will focus on the first Ehrlich-Becker modality of defense—raising p and reducing $(1 - p)$—Ehrlich-Becker's "self-protection." Here we take L_i to be fixed—eliminating Ehrlich-Becker's "self-insurance". Later in Sect. 5.3 we include self-insurance into the model. The variable "p" might be risk of war, shared indivisibly by two coalition members. Utility function $U(.)$ is assumed the same whether luck is good or bad. U^1 denotes realized utility if the good event happens, and U^0 if the bad event happens. We assume that $U(.)$ is increasing and strictly concave.

The individual country's budget constraint is given as

$$Y_i = C_i + m_i \tag{5.2}$$

where Y_i is a fixed national income and m_i denotes allocations to risk reduction; that is, m_i gives the voluntary input to the public good by country i. Here "p" has the nature of a public good (as conventionally defined) for countries 1 and 2, since we assume that an increase in "p" benefits both countries in a non-rival non-excluded fashion. That is, both m_i spent on self-protection reduce the chance of a bad event i.e. a decrease in what we later call "baseline risk of $[1 - p(0)]$," increasing "p," the probability of a good event, for both parties. So we can account for the collective summation technology quality of the inputs to p by writing

$$p = p(M) \tag{5.3}$$

with summation finance

$$M = m_1 + m_2. \tag{5.4}$$

Protective expenditures by countries 1 and 2 are equally effective in reducing the common risks. Specifically, m_1 and m_2 are perfect substitutes for each other. M, therefore, is the aggregate voluntary expenditure on the public good, giving the uncoordinated group's total amount of resources devoted to reducing the probability of the bad state or the risk of loss. A priori, many risk reduction or self-protection functions are plausible: quadratic, exponential, logistic etc. One expects $p' > 0$ throughout for all of these; while p'' may vary, we assume here that $p'' < 0$ throughout.

5.2.2 *Individual Optimizations*

For improvement effected through changes in p, expected utility (5.1) is maximized with respect to m_i subject to constraints (5.2), (5.3), and (5.4). This gives Eqs. (5.5) and (5.6) as first and the second order conditions:

$$\text{FOC} \quad p'(U^1 - U^0) - [pU_Y^1 + (1-p)U_Y^0] = 0 \tag{5.5}$$

$$\text{SOC} \quad p''(U^1 - U^0) - 2p'(U_Y^1 - U_Y^0) + [pU_{YY}^1 + (1-p)U_{YY}^0] < 0 \tag{5.6}$$

where $U_Y \equiv \partial U/\partial Y_i, U_{YY} \equiv \partial U^2/\partial Y_i^2$.

We will introduce simplifying notation $W_Y, W_{YY}, \Delta, \Delta_Y, MC_M$, and MB_M to interpret Eqs. (5.5) and (5.6) (Tables 5.1 and 5.2).

Table 5.1 Notations

$W_Y = pU_Y^1 + (1-p)U_Y^0 > 0$	$W_{YY} = pU_{YY}^1 + (1-p)U_{YY}^0 < 0$
$\Delta = (U^1 - U^0) > 0$	$\Delta_Y = (U_Y^1 - U_Y^0) < 0$

Table 5.2 Marginal cost and marginal benefit of self-protection

$pU_Y^1 + (1-p)U_Y^0 = W_Y = MC_M =$ Marginal Cost
$p'(U^1 - U^0) = p'\Delta = MB_M =$ Marginal Benefit

Table 5.3 Second derivatives of marginal benefit and marginal cost

$p''(U^1 - U^0) = p''\Delta = MB_{MM} < 0$	$[pU_{YY}^1 + (1-p)U_{YY}^0] = W_{YY} = MC_{MY} < 0$	$p'(U_Y^1 - U_Y^0) = p'\Delta_Y = MB_{MY} < 0$

Thus, the term $p'(U^1 - U^0) = p'\Delta$ in Eq. (5.5) gives the marginal benefit—MB_M—of expending m_1 in country 1, (or m_2 by its partner country), that is the marginal gain in expected utility from increasing public good M and, therefore, p. The term $pU_Y^1 + (1-p)U_Y^0 = W_Y$ in Eq. (5.5) gives marginal expected cost of providing the public good—MC_M—i.e. the marginal loss of expected utility from reducing private consumption by m_1.

Apparently, the second order condition (5.6) requires some combination of large absolute value of $p''(U^1 - U^0) < 0$ or of $[pU_{YY}^1 + (1-p)U_{YY}^0] < 0$, and/or small absolute value of $p'(U_Y^1 - U_Y^0) < 0$. If this SOC obtains, then FOC (5.5) will represent a maximum rather than a minimum. Table 5.3 extends the compact notation of Tables 5.1 and 5.2, with the second subscripts indicating second derivatives of MB_M and of MC_M.

5.2.3 *Interior Nash Equilibria*

5.2.3.1 Treatment of Cost-Input as Public Good

Now, to exploit the advantages of "summation financing" instead of the conventional expression for expected welfare, i.e. $W_i(C_i, p)$, we propose—as described elsewhere in the literature (e.g. Cornes and Hartley 2005)—to work with "induced" preferences over C_i and M and adjust terminology slightly so that M is called a "public good". An increase in M at given C_i changes the welfare of both parties (in the relevant range raises welfare) even though M is not itself directly an object of consumption per se; nor is it an argument in the conventional direct expected utility function $W_i(C_i, p)$. But providing this "public good" does instrumentally raise expected utility of consuming the private good for both parties in a non-rival non-excluded fashion. Moreover, so long as the Nash equilibrium is interior so that both parties make positive contributions, M is effectively chosen by and agreed on by both as noted first by Becker (1974).

We want to introduce this innovation, extending the term "public good" to the indirect productive input, M, because doing so permits us to use conventional geometric properties of the VPG model to derive and illustrate the unconventional insights of this article. These relate to connections among (a) risk aversion in the utility function, (b) status quo risk, (c) normality/inferiority of the public good M, (d) stability and therefore attainability of Nash solutions, and (e) effects of group size on equilibrium. Thus, in place of Eq. (5.1′)—i.e. $W_i(C_i, p)$—we will write a country's expected welfare objective function as:

$$W_i = W_i(C_i, L_i, M) \tag{5.7}$$

First of all, just for completeness we show that Cournot-Nash solutions and their known properties continue to obtain for the induced or primitive utility functions $W(C, L, M)$.

5.2.3.2 Nash Equilibria When the Public Good Is Measured by Summation of Finance

Equation (5.7) gives expected utility W_i for country i for given L_i. From expressions (5.2) and (5.4) we obtain the effective full income budget constraint.

$$C_i + M = Y_i + m_j, \tag{5.8a}$$

where the right-hand side defines full income of country i, $Y_i^* \equiv Y_i + m_j$. Therefore, utility maximizing behavior may be expressed in terms of the expenditure function, $E_i(\ )$

$$E_1(W_1, L_1) = Y_1 + m_2 \tag{5.8b}$$

$$E_2(W_2, L_2) = Y_2 + m_1 \tag{5.8c}$$

The expenditure function on the left-hand side depends on expected utility and loss in the bad state (taken as a parameter). Adding these two equations gives as the world-wide feasibility condition

$$E_1(W_1, L_1) + E_2(W_2, L_2) = Y_1 + Y_2 + M_1(W_1, L_1) \tag{5.8d}$$

where $M_1(\ )$ denotes the compensated demand function for M in country 1. If the equilibrium solution to E_1 and E_2 is interior then both countries must have chosen the same value of M. Thus in equilibrium

$$M_1(W_1, L_1) = M_2(W_2, L_2) \tag{5.8e}$$

Equations (5.8d) and (5.8e) then determine expected welfare of each country, (W_1, W_2) as a function of incomes Y_1, Y_2 and losses in the bad state, (L_1, L_2). A diagram in the space of (W_1, W_2) would show loci for the two conditions (5.8d) and (5.8e) intersecting where and if Nash equilibria exist.

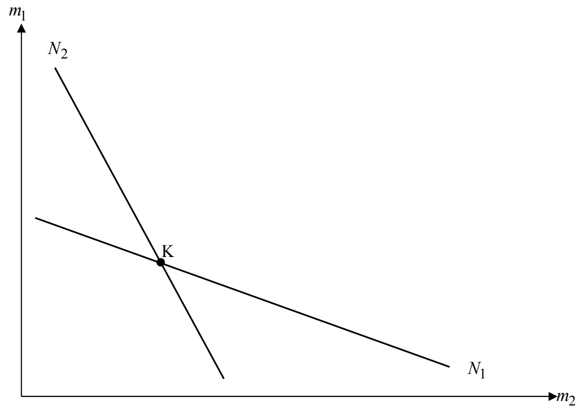


Fig. 5.2 Stable Nash equilibrium. *Source* Ihori and McGuire (2007)

5.2.3.3 Nash Reaction Functions

More important is to show the Nash equilibrium with reaction functions in space (m_1, m_2). Figure 5.2 has curve N_1 for country 1 and N_2 for 2. As with any good if M is normal $(dM/dY_1^* > 0)$, the absolute value of the slope of N_1 with respect to the m_2-axis is less than 1 ($-1 < dm_1/dm_2 < 0$, as in Cornes and Sandler 1986, 1996). Here, as shown in Fig. 5.2, the equilibrium point K is stable. If the public good M is inferior $(dM/dY_1^* < 0)$, then $dm_1/dm_2 < -1$. The (absolute) slope of country 1's reaction curve (with respect to the m_2-axis) is greater than 1, and the Nash equilibrium point would be unstable (not drawn).

5.2.4 *Income Effects: Normal and Inferior*

5.2.4.1 Sign of Income Effect

In the standard VPG model all interactions among participants are propagated by income effects. Thus our adaptation of Ehrlich and Becker must explore the effects of income change or differences between individuals on how public good provision is shared, and how free rider incentives operate. Doing this will reveal a surprising insight into the influence of risk aversion on these income effects, which are crucial to the stability and dynamic attainability of Cournot-Nash equilibria. Here we will see how taking M as the public good (rather than p) has facilitated this task.

For comparative static results it is the sign of the income effect we must investigate. Specifically, we want to determine whether M is an inferior or a normal "good". Taking total differentiation of FOC (5.5) gives:

$$\frac{dM}{dY^*} = -\frac{p'(U_Y^1 - U_Y^0) - [pU_{YY}^1 + (1-p)U_{YY}^0]}{p''(U^1 - U^0) - 2p'(U_Y^1 - U_Y^0) + [pU_{YY}^1 + (1-p)U_{YY}^0]}$$
$$= -\frac{p'\Delta_Y - W_{YY}}{p''\Delta - 2p'\Delta_Y + W_{YY}} \tag{5.9}$$

where Y^* is the effective individual "full" income that obtains at an interior solution. Assuming identical preferences and wealth to illustrate would give: $Y^* \equiv Y_1 + m_2 (\equiv Y_2 + m_1)$.

Condition (5.7), the SOC, determines the sign of the denominator in Eq. (5.9) as negative at an optimum, if the second order condition actually obtains, but the sign of numerator is ambiguous, and the normality or inferiority of M depends on this numerator.

5.2.4.2 Interior and Corner Solutions

We define a family of indifference curves $W(C, M)$; these are constant expected utility contours in the space (C, M) for a given value of L. We write these as $M = M(C, \ W)$ shown in Fig. 5.3. For a given expected utility and given loss, L, as in Eq. (5.1), we write the absolute value of the slope of an indifference curve:

$$MRS = \left.\frac{dM}{dC}\right|_{W\,const} = -M'(C) = \frac{pU_Y^1 + (1-p)U_Y^0}{p'(M)(U^1 - U^0)} = \frac{W_Y}{p'\Delta} = \frac{MC_M}{MB_M} > 0 \tag{5.10}$$

with W_Y, Δ, MB_M, and MC_M as defined above.

For an interior solution to the problem of risk reduction, the SOC's of Eq. (5.7) require

$$p''/p' \; - 2\Delta_Y \big/ \Delta \; + W_{YY} \big/ W_Y < 0$$

But at a tangency point, where FOC's obtain, the curvature of indifference curve $M = M(C, W)$ is given as

$$d^2M \big/ dC^2 = -W_{YY} \big/ W_Y \quad + 2\Delta_Y \big/ \Delta \quad - p''/p'$$

This means that for an interior maximum the curvature of the indifference curve must be positive

$$d^2M \big/ dC^2 > 0$$

confirming our geometric intuition of tangency between budget line (with slope −1) and indifference curve $M = M(C, W)$. If the SOC is not satisfied and hence $d^2M \big/ dC^2$ is negative, then the curvature of the indifference curve is "wrong" and

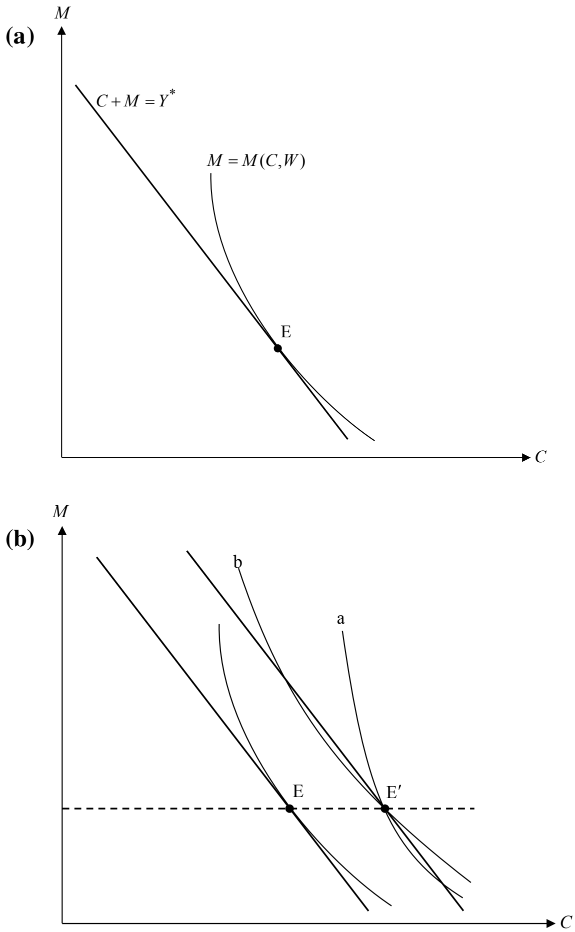


Fig. 5.3 **a** Interior solution and indifference curve. *Source* Authors, **b** Income effect on expenditure on public good. *Source* Ihori and McGuire (2007)

no interior solution is possible. In this case one member of the group will provide all the public good (M or p) and others will free ride. Obviously the first and last terms of $d^2M\big/dC^2$ are positive, tending to give indifference curves of the "right" curvature. However, if these terms are sufficiently small, then the middle negative term can dominate and the curvature of indifference curves be negative, in which case corner solutions become likely.

5.2.4.3 Inferior Goods

Hence, if the numerator is negative, given the SOC, the sign of Eq. (5.9) is negative, and M becomes an inferior good. *Intuitively when M is inferior, an increase in income reduces marginal benefit more than it reduces marginal cost*.

To see this, consider the numerator of Eq. (5.9). The term $p'\Delta_Y$ gives the change in marginal benefit or MB_{MY} (of providing the public good, M) that would be caused by an increase in Y; this is negative. When Y^* increases, U^0 rises more than U^1 and hence Δ_Y, the difference between U_Y^0 and U_Y^1 declines. Thus, an increase in Y^* reduces the marginal benefit of enjoying M measured from any initial optimal level of that public good. Next consider the second term of (5.9). W_{YY} measures the change (due to an increase in Y) in the expected marginal utility cost of providing M, or MC_{MY}. This also is negative when Y^* increases since U_Y declines with an increase in Y^*. That is, when Y^* increases and a country becomes richer, U_Y^0 and U_Y^1 (the utility cost components of providing the public good) both decline. Thus, an increase in Y^* reduces the marginal cost of providing public good, M. So to summarize from Eq. (5.9), if marginal benefit of providing M declines in absolute amount more than marginal cost,—i.e. if $p'(U_Y^1 - U_Y^0) < [pU_{YY}^1 + (1-p)U_{YY}^0]$ —then the numerator of Eq. (5.9) becomes negative, so that $dM\big/dY^* = -(-/-) < 0$. Here an increase in Y depresses the demand for M, meaning that M is inferior.

Now consider an increase in Y^* shown by an outward shift in the budget line of Fig. 5.3'. For given level of M, let C increase from E to E'. If MRS increases in absolute value at E' compared with E (curve a in Fig. 5.3') the new equilibrium will be to south-east of point E' and desired M will decline in value. This is the case of a negative income effect on M. But if MRS declines in absolute value at E' compared with E (curve b in Fig. 5.3'), the new equilibrium point is to north-west of E' and desired M rises, showing a positive income effect on the public good.

To evaluate how MRS varies at an optimum when income changes in Eq. (5.10), partial differentiation of MRS with respect to C and insertion of FOC (5.5) yields:

$$\begin{aligned} MRS_C = \frac{\partial MRS}{\partial C} &= \frac{\{[pU_{YY}^1 + (1-p)U_{YY}^0] - p'(M)(U_Y^1 - U_Y^0)\}}{p'(M)(U^1 - U^0)} \\ &= \frac{W_{YY} - p'\Delta_y}{p'\Delta} = \frac{W_{YY}}{p'\Delta} - \frac{\Delta_Y}{\Delta} \\ &\overset{sign}{=} \frac{W_{YY}}{p'} - \Delta_Y \end{aligned} \tag{5.11}$$

Evaluation of $MRS_C = \partial MRS/\partial C$ is important because it will establish a formula, Eq. (5.11) for seeing if M is inferior or normal, depending on the sign of the formula. We will see presently that its sign depends in turn on risk plus the risk aversion properties of U. Thus it establishes the connection between inferiority and risk-plus-risk-aversion. Since all interaction among group members are propagated as income effects, $MRS_C = \partial MRS/\partial C$ captures or displays these interactions.

Note that $dM\big/dY^* < 0$ in Eq. (5.9) and M is inferior if and only if $MRS_C = \partial MRS/\partial C > 0$ in Eq. (5.11). Then, given that $p' > 0$, from Eq. (5.11) if $\partial(W_Y - p'\Delta)/\partial Y = 0$ i.e. $\partial(MC_M - MB_M)/\partial Y = 0$ then MRS_C is zero, so that M then is borderline normal. Using our compact notation, we can summarize succinctly by writing:

- The first order condition requires $MB_M = MC_M$,

- The second order condition requires $MB_{MM} < MC_{MM}$,
- Non-inferiority requires $MB_{MY} > MC_{MY}$,

The sign of MRS_C and thus normality or inferiority of M is seen from Eq. (5.11) to depend on three factors: p', U_{YY}, and p. Taken together, however, in the aggregate, they imply the hitherto unrecognized connection to risk aversion that we have mentioned above, and to which we now turn.

5.2.5 Properties of Interior Nash Equilibria When the Public Good Is Defined by Summation Financing

5.2.5.1 Neutrality of Wealth Redistributions

For public good as a common improvement in probabilities our transformation of the problem to take primitive preferences over cost inputs has produced surprising new results. Even though the consumption technology for public good p is "non-summation" the feature of "summation finance" leads to rather striking results. Because this technique is so advantageous for managing the problematic nature of Nash equilibrium it is of all the more interest to inquire whether this success is only achieved at the cost of giving up the other desirable properties of Nash equilibrium in the standard "goods summation" of the VPG model.

Reassuringly, our unconventional treatment of M as the public good in $W(C, M)$, is compatible with many established properties of interior Cournot Nash solutions including the neutrality of wealth redistributions within a group. This equivalence result—which we explained in Chap. 3—is of importance more widely to any VPG problem where costs of public good provision are increasing rather than constant and where the public good depends only on the summation of input contributions by group members.

5.2.5.2 Effects of Productivity Differentials in Nash Equilibrium

Now consider differences in technology of providing the pubic good between countries. In place of Eq. (5.4), (with ε_i denoting the relative productivity of providing M for country i) we have

$$M = \varepsilon_1 m_1 + \varepsilon_2 m_2 \tag{5.12}$$

From Eqs. (5.2) and (5.8a), the effective budget constraint ($c_i = i$'s consumption) for country 1 becomes:

$$\varepsilon_1 c_1 + M = \varepsilon_1 Y_1 + \varepsilon_2 m_2 \tag{5.13}$$

Then, Eqs. (5.8b) and (5.8c) may be rewritten as

$$E_1(W_1, L_1, \varepsilon_1) = \varepsilon_1 Y_1 + \varepsilon_2 m_2 \tag{5.14a}$$

$$E_2(W_2, L_2, \varepsilon_2) = \varepsilon_2 Y_2 + \varepsilon_1 m_1 \tag{5.14b}$$

And Eq. (5.8d) will be rewritten as

$$E_1(W_1, L_1, \varepsilon_1) + E_2(W_2, L_2, \varepsilon_2) = \varepsilon_1 Y_1 + \varepsilon_2 Y_2 + M_1(W_1, L_1, \varepsilon_1) \tag{5.15}$$

Finally, Eq. (5.8e) becomes

$$M_1(W_1, L_1, \varepsilon_1) = M_2(W_2, L_2, \varepsilon_2) \tag{5.16}$$

In the effective budget constraint ε_i may be regarded as the relative price of private consumption in terms of the public good for country i. That is, $\partial M_i/\partial \varepsilon_i$ denotes the substitution effect of an increase in the relative price of private consumption on the public good, and is positive. The effect of an increase in ε_i on relative welfare is qualitatively the same as of a decrease in L_i so long as M is normal ($M_W > 0$). And with M normal, in equilibrium $\varepsilon_1 > \varepsilon_2$ entails $W_1 < W_2$; a country of higher productivity benefits less from group formation than a country of low productivity (see Jack, 1991; and Ihori, 1996). Here both countries can gain by transferring income from country 2 to 1. However, if M is inferior, the inequality $\varepsilon_1 > \varepsilon_2$ implies $W_2 < W_1$; low productivity is worse for a country, and high productivity is better since $m_1 < m_2$. Since effective aggregate income is always given by $\varepsilon_1 Y_1 + \varepsilon_2 Y_2$, it remains true that both countries gain by transferring income from country 2 to country 1 even when M is inferior.

5.2.6 Normal and Inferior Income Effects: Details

5.2.6.1 Effect of p': Marginal Productivity of Security Expenditures

Peculiarities of common risk control as we have identified them derive from interactions between risk aversion, status quo risk, and wealth. But underlying these are certain prior factors that lead to further surprising insight into the structure of Nash equilibrium. We explore these briefly. Qualitatively, the sign of Eq. (5.11)—which we repeat below—and therefore normality or inferiority of M can be seen to depend on three factors, p', U_{YY}, which influences both W_{YY} and Δ_Y, and p, $(1-p)$, which influences W_{YY} as well.

$$MRS_C = \frac{\partial MRS}{\partial C} = \frac{\{[pU^1_{YY} + (1-p)U^0_{YY}] - p'(M)(U^1_Y - U^0_Y)\}}{p'(M)(U^1 - U^0)}$$

$$= \frac{W_{YY} - p'\Delta_y}{p'\Delta} = \frac{W_{YY}}{p'\Delta} - \frac{\Delta_Y}{\Delta}$$

$$\overset{sign}{=} \frac{W_{YY}}{p'} - \Delta_Y \tag{5.11}$$

Note first that the overall sign of Eq. (5.11) depends on the relative importance of (W_{YY}/p')—which is negative—and $-\Delta_Y$—which is positive. If the first term is negligible then with the remainder $-\Delta_Y > 0$ Eq. (5.11) will tend to be positive and M therefore inferior. Now with diminishing marginal returns to risk reduction, $p'' < 0$, and p' will be greatest when p is least. *Accordingly, there is an inherent tendency for expenditures on risk improvement to be inferior when risk* $(1 - p)$ *is great, and for these expenditures to be normal or superior when risk is small.* In other words and most paradoxically, the more is security needed the more likely, *ceteris paribus*, is greater wealth a counter-indicator of provision.

5.2.6.2 Effect of $(1 - p)$: Baseline Risk

Should $U_{YY} = 0$ throughout, then Eq. (5.11) is identically zero and M is borderline normal. But if $U_{YYY} = 0$, then for any given value of p' the sign of $[(W_{YY}/p') - \Delta_Y]$ in (5.11) will **vary** systematically with p, *independently of p'*, since p is a component of W_{YY}. We know $\partial W_{YY} / \partial p = U^1_{YY} - U^0_{YY}$ is positive if and only if $U_{YYY} \equiv \partial^3 U/\partial Y_i^3 > 0$. Hence, $[(W_{YY}/p') - \Delta_Y]$ is increasing with p if $U_{YYY} > 0$, which is the standard case where U_{YY} is smoothly declining. Hence, for low p *ceteris paribus* the expression $[(W_{YY}/p') - \Delta_Y]$ will be negative with the first term dominating, while for high values of p with the second term dominating, the overall expression tends to be positive. For any given value of p' then, this analysis implies the existence of a cross-over value of p' where $MRS_C = 0$ and M switches from normal to inferior.

5.2.6.3 Effects of U_{YY} and of U_{YYY}: Diminishing Marginal Utility

Quantitatively, all these tendencies become less important as $U_{YY} \to 0$ for then both parts of Eq. (5.11) vanish and risk reduction expenditures will approach borderline normal. Note the indeterminacy here of Nash equilibrium when M is borderline normal. Reaction functions of identical countries have 45° slope all overlap throughout.

For a wide class of utility functions, lower values for the term $|U_{YY}|$ correlate with the agent being rich or close to wealth satiation (given $U_{YYY} > 0$). So in this case Eq. (5.11) implies that the sensitivity top of expenditure on M declines as wealth increases. Conversely, as $|U_{YY}|$ and the differential Δ_Y increase—corresponding to small value of L and/or lower wealth—the sign of Eq. (5.11) and therefore the inferiority or normality of M becomes more sensitive to the factors $[p, (1 - p)]$ and p' as analyzed above. Such paradoxical effects will be buried, but always to some degree operational, in the other elements which go to make up inferiority/normality of the public good M.

5.2.7 Normality, Inferiority and Risk Aversion: The Inferior Goods Barrier to Public Good Provision

5.2.7.1 Risk Aversion and Inferior Good

It was shown above that for commonly beneficial expenditures allocated to risk-reduction, the sign of income effects depended on the sign of the numerator in Eq. (5.11). But, as we now demonstrate, this numerator depends crucially and systematically on the risk aversion properties of the underlying utility function and on the interaction of these with p. Absolute risk aversion (R) is defined as

$$R = -U_{YY}\big/U_Y \quad \text{or} \quad -U_{YY} = R \cdot U_Y \tag{5.17}$$

Then the numerator of Eq. (5.9)—with C_0 and C_1 as consumption in each contingency and R_0 and R_1 as the associated absolute risk aversion—can be written as

$$H = \underset{(-)}{p'(U_Y^1 - U_Y^0)} + \underset{(+)}{[pR_1U_Y^1 + (1-p)R_0U_Y^0]} \tag{5.18}$$

So normality or inferiority of M now depends on the sign of Eq. (5.18): for $H < 0$, M is inferior. But with the sign of each part of Eq. (5.18) indicated in parentheses, the overall sign is ambiguous depending on magnitude and properties of R interacting with $(p, [1-p])$. To see this, multiply FOC (5.5) by R_1 to obtain

$$R_1p'(U^1 - U^0) = [pR_1U_Y^1 + (1-p)R_1U_Y^0] \tag{5.19}$$

Then re-write Eq. (5.18) as:

$$H = p'(U_Y^1 - U_Y^0) + [pR_1U_Y^1 + (1-p)R_1U_Y^0] + (1-p)R_0U_Y^0 - (1-p)R_1U_Y^0 \tag{5.20}$$

Substituting Eq. (5.19) into Eq. (5.20) gives:

$$H = \underbrace{p'[(U_Y^1 + R_1U^1) - (U_Y^0 + R_0U^0)]}_{defined\ as = Q} + \underbrace{[(1-p)U_Y^0 + p'U^0][R_0 - R_1]}_{defined\ as = T} \tag{5.21a}$$

i.e.

$$H = Q + T \tag{5.21b}$$

As it is demonstrable that $d[U_Y + R(Y)U]\big/dY = R'U$ it follows that:

$$R' > 0 \rightarrow \{Q > 0; T < 0\},\ R' < 0 \rightarrow \{Q < 0; T > 0\} \quad \therefore (Q + T) \gtreqless 0 \quad (5.22)$$

And

$$R' = 0 \rightarrow \{Q = 0; T = 0\}, \text{ in which case } N = Q + T = 0 \quad (5.23)$$

5.2.7.2 Risk Aversion Along Indifference Curves $M = M(C, W)$

Constant Risk Aversion:
We can now dissect the relation shown by Eq. (5.18) if we begin with the knife-edge case of constant risk aversion. When $R = R^*$ a constant or $R' = 0$, expression (5.18) conclusively implies that $H = 0$, that good M, therefore, is on the borderline between normality and inferiority and the income effect is zero. This case represents a break-even case of no interdependence between risk aversion and the magnitude of risk as determinants of optimal allocations to defense/deterrence. The negative exponential utility function $U = 1 - e^{-R*Y}$ generates this case. The reason we call it 'breakeven' is that for all other preference functions other than that of constant R, there is a variable, fluctuating relation between p and desired M, with potential for wobbling between normality and inferiority. We examine this next.

Increasing Risk Aversion:
If $R_0 < R_1$ then Eq. (5.18) shows that low risk and high risk aversion interact. When risk is low, and $(1 - p)$ small, high risk aversion combines with high p to weight expression (5.18) positively—toward normality. Here chance favors the outcome where risk aversion is greater. When R increases with income rational agents will insure against low probability events, low $(1 - p)$, and the richer they are the more will they so insure. Thus, if risk is high so that $(1 - p)$ is great, the positive part of Eq. (5.18) weighs less because R_0 is small; therefore, Eq. (5.18) tends to be negative so that M is an inferior good. Here chance gives the less risk averse outcome more weight. Expenditure on M resembles a gamble, not insurance, and rational agents will gamble by wagering on improvement to p by expending on M. But the richer they are the less will they so gamble.

Decreasing Risk Aversion:
On the other hand, the opposite risk profile causes (5.18) to be negative and M to tend toward inferiority. If risk aversion is decreasing, $R_0 > R_1$, then high risk and low risk aversion reinforce each other. Now high risk aversion, i.e. large R_0 which obtains at lower wealth, interacts with high risk $(1 - p)$ to weight expression (5.18) positive and the indifference curve toward normality. Here the rational, expected utility maximizing agent will gamble on improving the less likely event and the richer he is the more will he gamble. But the opposite combination of lower risk (meaning high p) and a greater weight, therefore, on lesser risk aversion R_1 leads to inferiority. That is, when p is big, low risk aversion correlates with the higher wealth outcome, and due to the higher weight on small R_1 Eq. (5.18) tends to be negative and provision of M inferior. So here the rational agent will insure, but the richer he

is the less will he do so. Figure 5.1 and the argument it illustrates contain the same idea that risk averse agents insure against unlikely events while risk tolerant agents gamble on them is covered in detail in McGuire, Pratt, and Zeckhauser (1991).

5.2.7.3 Critical Risk p*

We can give a heuristic summary of these forces by introducing the idea of a critical crossover value of p in Eq. (5.18). That is, for given R_0 and R_1 and assuming $R_0 < R_1$ there is a critical value of p^* (with $0 \leq p^* \leq 1$) such that for $p > p^*$ good M is normal, and for $p < p^*$ the good is inferior. Thus, when $R_0 < R_1$ if $p^* = 1$, M is necessarily inferior and for $p^* = 0$, good M is necessarily normal. Correspondingly, if $R_0 > R_1$ there is a critical risk p^* such that for $p > p^*$ the good is inferior, while for $p < p^*$ M is normal. Here if $p^* = 1$, $p \leq p^*$ and M is necessarily normal, while if $p^* = 0$, $p \geq p^*$ necessarily and M is inferior.

Generally, we expect absolute risk aversion to decrease with wealth and the taste for risk correspondingly to rise, so that the amount of <u>insurance</u> <u>purchase declines</u>, *ceteris paribus*, as the rational agent grows richer and his <u>propensity to gamble increases</u>. But in our analysis, expenditure on odds-improving self-protection resembles <u>insurance</u> when the chance of a bad outcome is low. So, decreasing absolute risk aversion is congruent with <u>inferiority</u> of M—a negative income effect—when risk is low leading to <u>lower</u> "insurance" purchase as income increases.

But, if risk is high, and thus the weight on high R_0 is great, the decision to improve $p(M)$ by expending M resembles a <u>gamble</u> rather than insurance, and a propensity to gamble correlates with low risk aversion. Here when risk is high with $R_1 < R_0$ risk aversion is also high, so that an increase in income increases the propensity to gamble, and thus here expenditure on M is a normal good. Still, we cannot exclude the possibility that risk aversion will increase with income in which case we would anticipate a reversal of the negative/positive income effect described above. So, to summarize the whole situation, including the effects of ΔY on ΔM we must include the increasing risk aversion case. We present this summary in Table 5.4a and Table 5.4b derived from 5.4a. [Corresponding cells are labeled A, B, C, and D].

5.2.8 *The Cornes-Hartley "Replacement"*

5.2.8.1 Solutions for Nash Equilibria

Cornes and Hartley (2000, 2005) have suggested an elegant construction that allows a direct visualization of which members of a public good group will make positive contributions in Nash equilibrium, how much will be supplied, and how this outcome changes with group size and composition (See also Andreoni and McGuire, 1993). We can use this method to demonstrate the consequences of public good inferiority on

Table 5.4 Summary of inferiority/normality of M and effect of increasing full income

a Relation between risk, risk aversion, and Inferiority/normality of M[a]

In Eq. (5.18)	Additional M raises $p(M)$	Effect on optimal choice of M when Income, Y, Increases	
		Case of increasing R: $R_1 > R_0$	Case of decreasing R: $R_1 < R_0$
Low risk p is high: $(1 - p)$ is low R_1 dominates	Insures against unlikely event $(1 - p)$	**A** Agent wants to insure and can do so R_1 is big. Equation (5.18) > 0: normal Therefore, greater Y makes R_1 and R_0 bigger and leads to more insurance **M increases**	**C** Agent wants to gamble but must insure R_1 is small. Equation (5.18) < 0: inferior. Therefore, greater Y makes R_1 and R_0 smaller and leads to less insurance **M declines**
High Risk	Gambles on unlikely event p	**B** Agent wants to gamble and can do so R_0 is small. Equation (5.18) < 0: inferior Therefore, greater Y makes R_1 and R_0 bigger and leads to less gambling **M declines**	**D** Agent wants to insure but must gamble R_0 is big. Equation (5.18) > 0: normal Therefore, greater Y makes R_1 and R_0 smaller and leads to more gambling **M increases**

[a] This assumes that variables in Eq. (5.18) are such that H will change sign as p varies from 0 to 1. And just to repeat, low risk aversion correlates with a propensity to gamble but not insure; high risk version correlates with a propensity to insure but not gamble

Source Ihori and McGuire (2007)

b Effect of increasing full income of members of a group by adding a new participants

		Increasing risk aversion: $R_0 < R_1$	Decreasing risk aversion: $R_0 > R_1$
Initial level of self-protection	p_{HIGH}	Normal **A**	Inferior **C**
	p_{LOW}	Inferior **B**	Normal **D**

Source Authors

group provision and its stability. Cornes and Hartley define a “replacement function” (or more generally replacement correspondence) as follows.

Let the Nash reaction function be given as

$$m_i = N^i(Y_i, M_{-i}) = N^i(Y_i, M - m_i); \qquad (5.24)$$

where M_{-i} indicates public good provided by all agents except agent i, $M_{-i} = \Sigma_{j \neq i} m_j$. Then the replacement function is derived as:

$$m_i = r^i(Y_i, M) \tag{5.25a}$$

and hence

$$r_M^i \equiv \partial r^i \big/ \partial M = N_{M-i}^i \big/ (1 + N_{M-i}^i), \tag{5.25b}$$

where $N_{M-i}^i \equiv \partial m_i \big/ \partial M_{-i}$. Let us define the aggregate of individual replacement functions by $R \equiv \sum_i r^i(Y_i, M)$ (different from the R^0, R^1 of risk aversion). Then, this immediately yields the Cournot-Nash equilibrium where the aggregate of individual replacement function crosses the 45° through the origin, i.e. at

$$M = \sum\nolimits_i [r^i(Y_i, M)] \equiv R \tag{5.26}$$

To use r^i and R, it will be helpful first to relate their properties to the underlying Nash reaction functions. Thus, if M is a normal good, then $-1 < N_{M-i}^i < 0$, and $r_M^i < 0$. Then the individual replacement function is decreasing with M and we designate the function as "Normal." On the other hand, if M is an inferior good, then $1 + N_{M-i}^i < 0$, whence $N_{M-i}^i < -1$ and $r_M^i > 1 > 0$. Now the individual replacement function is increasing with M with its slope greater than 1, and i we call this function "Inferior".

The replacement function for individual i, of course, incorporates the effects of i's income level Y_i on i's contribution to the group provision of the public good M. To see these effects consider the individual replacement function $m_i = r^i(Y_i, M)$. Note that an increase in income Y_i will raise m_i whether M is normal or inferior. That is, regardless of income effects

$$\frac{\partial m_i}{\partial Y_i} \equiv r_Y = \frac{\partial N^i \big/ \partial Y_i}{1 + \partial N^i \big/ \partial M_{-i}} \equiv \frac{N_Y^i}{1 + N_{M-i}^i} \geq 0 \tag{5.27}$$

where $N_Y \equiv \partial N^i \big/ \partial Y_i$ gives the incremental effect on m_i (given M_{-i}) of an increase in Y_i along agent i's Nash reaction function; this is negative for M an inferior good, positive if M is normal. The denominator is also negative when M is inferior, and positive if normal. Thus, if M is inferior, $\partial m_i \big/ \partial Y \equiv r_Y$ is positive—the same as in the normal good case where both numerator and denominator are positive.

To proceed to analysis of the equilibrium of the group as in Eq. (5.26) we focus on the case when all agents are identical, and therefore omit individual i specific notation. In this symmetric case the equilibrium condition is

$$nr(Y, M) = nm = M \tag{5.28}$$

So that

$$\frac{dM}{dY} = \frac{nr_Y}{1 - nr_M(M)} \tag{5.29}$$

Here dM/dY indicates the incremental change in aggregate equilibrium provision of M when the income of each and every individual increases incrementally. If the public good is normal, both numerator and denominator of Eq. (5.29) are positive. An increase in income raises the provision of public good. On the other hand, if the public good is inferior, the numerator is positive, while the denominator is negative. Hence in this case the sign of Eq. (5.29) is negative. Thus, the direction of change in the aggregate equilibrium provision of M correlates with negative or positive income effects. However, in the case of inferiority the new equilibrium is unstable, and thus most probably unattainable.

Further relations between $r^i(Y_i, M)$ and $N^i(Y_i, M_{-i})$ as given by

$$r^i_Y = N^i_Y/(1 + N^i_{M-i}) \tag{5.30a}$$

$$r^i_M = N^i_{M-i}/(1 + N^i_{M-i}). \tag{5.30b}$$

allow us to express (5.29) in terms of the reaction functions:

$$\frac{dM}{dY} = \frac{nr_Y}{1 - nr_M} = \frac{n[N_Y/(1 + N_{M-i})]}{1 - n(N_{M-i}/(1 + N_{M-i}))}; \tag{5.30c}$$

that is

$$\frac{dM}{dY} = \frac{nN_Y}{1 + N_{M-i} - nN_{M-i}} \tag{5.30d}$$

Note that when income effects are negative and M inferior, even though as per Eq. (5.27) each individual agent would contribute more when only his income increases, nevertheless when every agent's income increases and M is an inferior good the aggregate equilibrium provision declines. The reason for the difference lies in the interaction between income effects. The denominator of Eq. (5.30c) or Eq. (5.30d) indicates how in a group interaction the negative effects of reciprocal reductions in M_{-i} when M is inferior outweigh the positive individual effects of Eq. (5.27).

5.2.8.2 Multiple Equilibria and Stability

The collective provision of risk reduction as a public good seems beset by effects of changing risk aversion interacting with risk itself. Table 5.4a in Sect. 5.7.3 shows this, and so suggests the likelihood of multiple equilibria, instabilities, and corner

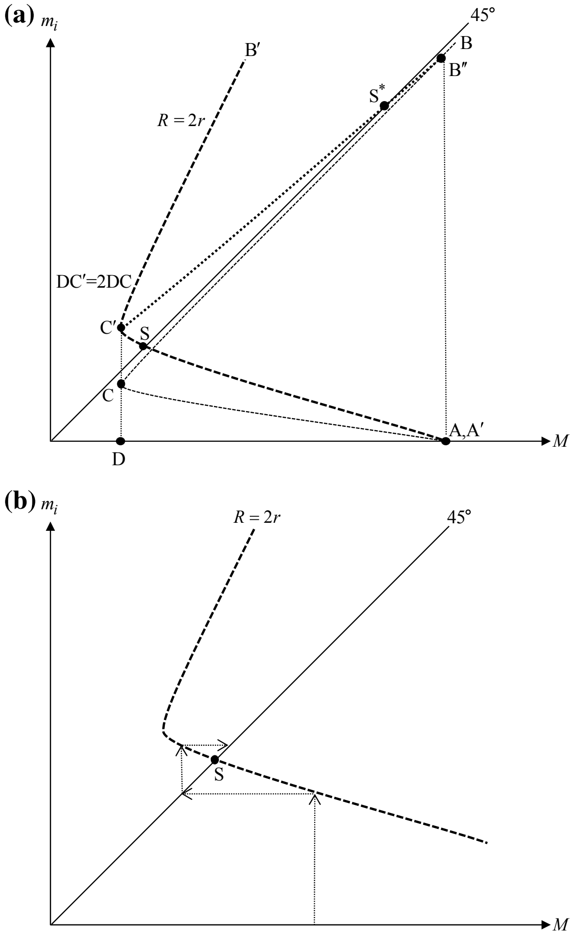


Fig. 5.4 **a** Multiple Nash equilibrium when public good is normal/inferior, *Note* At point *S* where the aggregate replacement function (A′C′B″) intersects 45° line, *M* is normal. At point *S** where the aggregate replacement function (A′C′B″) intersects 45° line, *M* is normal for one agent but inferior for the other. **b** Stable Nash equilibrium, *Note* intersection of aggregate replacement function *R* and 45° line gives a stable Nash equilibrium at point S. *Source* Ihori and McGuire (2007)

solutions. Fortunately, the replacement function construct is ideally suited to analysis of such effects. To illustrate we confine our attention to two identical countries, country 1 and country 2, assuming for both that risk aversion and status quo p are such that M is inferior for low incomes but normal at high incomes.

The individual replacement function for either country is then shown in Fig. 5.4a by curve AB; section AC applies when M is normal and BC when it is inferior. For each section we have the replacement correspondences m^A and m^B:

$$\{m^A, m^B\} = r(M) \tag{5.31}$$

or

$$m^A = r^A(M) \tag{5.32}$$

and

$$m^B = r^B(M) \tag{5.33}$$

where $m^A < m^B$ and m^A belongs to curve AC, while m^B belongs to curve BC. Thus, for M normal in region AC, m is decreasing with M, and for M inferior in region BC m is increasing with M. Again, the Nash equilibrium condition is: $m_1 + m_2 = M$.

This condition (5.31) gives multiple and/or unstable equilibria depending on the mix of replacement functions—normal and inferior—among agents. Figure 5.4a also shows the vertical sum of two ACB curves, one for country 1 and one for country 2. The vertical summation is denoted A'C'B'. The drawing shows a case where A'C'B' intersects the 45° line at S, so that equilibrium occurs when each country is on the M-normal or AC section of its individual replacement function. When countries are homogeneous and their equilibrium positions identical, we have

$$R = 2r(M) = M \tag{5.34}$$

and each provides the same m; therefore, $m_1 = m_2$. This is the standard case: with r_1 and r_2 normal equilibrium is stable as indicated by the direction of the arrows in Fig. 5.4b. However, if replacement functions are such that the inferior section (BC) obtains symmetrically for both 1 and 2 then while still $m_1 = m_2$ equilibrium is unstable. This is shown in Fig. 5.5a, with arrows pointing to the instability drawn in Fig. 5.5b.

Note, however, that even if countries are homogeneous with identical individual replacement functions, the equilibria may not be symmetric if it occurs on the normal section of one's replacement function but simultaneously on the inferior section of the other's. Of course, if countries 1 and 2 are not homogeneous there can be a multitude of other, asymmetric equilibria. To illustrate, go back to Fig. 5.4a and construct the aggregate replacement function from dissimilar sections of ACB (i.e. from AC + BC). Suppose equilibrium occurs where M is a normal good for country 1 but an inferior good for country 2. Then for the individual replacement functions we write

$$m_1 = m^A, \quad m_2 = m^B \tag{5.35}$$

with equilibrium condition

$$R \equiv m^A + m^B = M \tag{5.36}$$

We illustrate this equilibrium as S^* in Fig. 5.4a, where the vertical sum of AC curve and CB curve intersects the 45° line. But even for such limited asymmetry (population still homogeneous), Nash equilibrium may be stable or unstable and depending on the exact shape and positioning of individual replacement functions ACB, and numerous stable/unstable sequences are entirely possible.

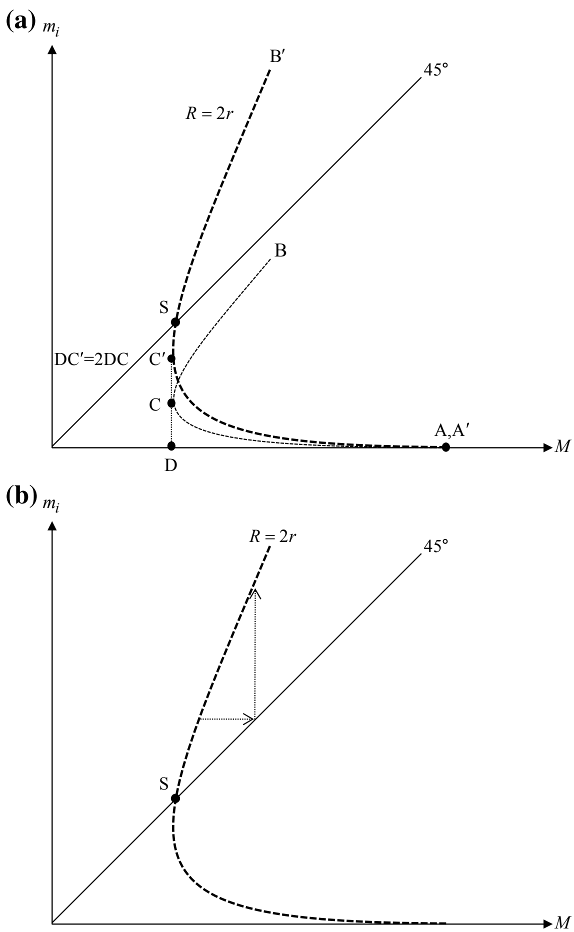


Fig. 5.5 **a** Symmetric unstable equilibrium, *Note* At Point *S* where the aggregate replacement function (A′C′B′) intersects 45° line, *M* is inferior. At intersection with 45° line, slope of aggregate replacement function is greater than one. **b** Unstable Nash equilibrium, *Note* Intersection of the aggregate replacement function from below 45° line gives an unstable Nash equilibrium. *Source* Ihori and McGuire (2007)

For example, in Fig. 5.5a even though the sum of identical inferior sections of individual replacement functions gives an equilibrium at "S," we could aggregate AC for one country plus CB for the other. This would give another aggregate replacement function (A'C'B" not shown in Fig. 5.5a) with another Nash equilibrium, S^*, this time stable. The figures therefore illustrate that at a symmetric equilibrium, if the aggregate replacement curve is upward-sloping (downward-sloping), *M* is inferior (normal) and the equilibrium is unstable (stable), as shown in Fig. 5.5b. But if the equilibrium is asymmetric, and the slope of aggregate replacement curve is greater (smaller) than 1, then it is unstable (stable). Of course, if countries are not homogeneous there can be a multitude of other, asymmetric equilibria.

5.2.9 *Effects of Change in Group Membership: Size and the Inferior-Good Barrier to Public Good Supply*

5.2.9.1 Greater Income with Larger Group Size

As discussed in Sect. 5.7.3, effects of Y on M are of special interest, and emphasized in Table 5.4a because of the association of greater income with larger group size that we know obtains under Nash-Cournot behavior. Specifically, we know from Becker (1974) Atkinson and Stiglitz (1980) and Bergstrom et al. (1986) that formation of a group for public good provision—assuming interior solutions—always increases "full income" for each and every member of the group, and the larger the group of positive contributors, *ceteris paribus*, the larger is this full income. Therefore, group formation or increase in group size by augmenting each agent's "full income" would have a tendency to change the choice of M for each agent depending on the cell in the table.

Thus, Table 5.4b implies certain rather surprising effects when hitherto disconnected countries form a group and react (in a Nash-Cournot manner) to the income effects they confer upon one another. It suggests we might compare static interior Nash equilibria before and after a new entry when all members of a group (new and old) are identical. Since the table implies that many Nash "equilibria" would be unstable we say little about the dynamics nor likely end result of enlarging group size. However, we do know that strong incentives exist for corner solutions to arise when interior Nash outcomes are unstable. In light of this fact generalizations to be derived from our analysis are arresting. To see these effects, consider the case where at an interior equilibrium (stable or unstable) all countries make positive contributions to the public good.

First, it is clear that the degree of status quo protection, i.e. the initial value of p, and the direction of increase in risk aversion, basically will determine the qualitative result of enlarging a Nash group. For example, if an agent's risk aversion declines with increases in wealth (as expected) and if a group is basically unsafe and p therefore is low, [Cell D in Table 5.4a, b] then adding a new member will increase M (normal good, stable equilibrium). However, if more members continue to be added so that more and more is spent on M, the value of p will increase and the situation will migrate toward Cell C, where the public good is inferior and equilibrium is unstable, as depicted in the previous diagrams. This suggests a natural or endogenous limit on the size of a group and of the amount of M and therefore of $p(M)$—a new conclusion markedly different from the standard VPG model. Once a region where M is inferior is reached and the Nash equilibrium becomes unstable new agents will probably induce chaotic adjustments leading to a corner solution. And even if the new unstable equilibrium were somehow reached, since M becomes inferior on transition from Cell to C to D, the total voluntary provision of M as we prove below will decline, notwithstanding that incentives reward all parties for enlarging the group.

On the other hand, returning to Table 5.4a, b if the initial status quo is very hazardous (p is low) but risk aversion is increasing for all agents [cell B] then adding new members actually *reduces* the (comparative static) equilibrium provision of M and therefore of p. So if risk aversion is increasing—absent some global agreement to collaborate in the universal provision of p—addition of members who behave by Nash-Cournot rules, can NEVER achieve a high level of protection, crossing the critical level of risk that separates Cell A from Cell B. This phenomenon also has never before been identified. It calls for a more organized rigorous definition of the connections between critical risk and group membership that separates Cells A/B or Cells C/D. The replacement function gives us a tool to do this.

5.2.9.2 The Replacement Function and Effects of Change in Size of Group

To pursue the analytics of an increase in the number of countries, start with an interior Nash equilibrium in an identical agent model with n members. Using the replacement function the Nash equilibrium for a homogeneous identical membership is given as

$$R = nr(M) = M \tag{5.37}$$

With comparative statics

$$dM \big/ dn = r(M) \big/ [1 - nr_M(M)] \tag{5.38}$$

It follows if M is normal, $r_M < 0$ and hence the sign of Eq. (5.38) is positive: an increase in the size of group n raises the total level of public goods M. However, when M increases, p rises and hence sooner or later M becomes inferior. But once M becomes inferior, $r_M > 1$ and hence the sign is negative: an increase in the size of group n reduces the total level of public goods M. Thus we observe an "endogenous barrier" to public good provision.

Proposition 5.1 Inferiority limits the ability of a group to increase public good provision by means of membership expansion.

Also we derive the effect of an increase in group size on the welfare of the initial membership. We show this for comparative static Nash equilibria when agents are homogeneous and identical irrespective of the stability or instability of such equilibria. To see this, the worldwide feasibility condition (5.8d) (where "F" designates full income) gives:

$$E(W) = Y + [(n-1)/n]M(W) = F \tag{5.39}$$

Y^* represents full income throughout this chapter. Here we introduce F as notation for full income to emphasize its functional dependence on size of group. Differentiating gives:

$$\frac{dW}{dn} = \frac{M}{n^2\left[E_W - \frac{n-1}{n}M_W\right]} > 0 \tag{5.40}$$

Thus, an increase in n always raises welfare, independently of the sign of M_W. Since welfare is increasing with full income F, Eq. (5.40) shows that $F(n+1) > F(n)$. If M is inferior, $M_W < 0$. In such a case, we also have $M(n+1) < M(n)$. Thus, the combination of $M(n+1) > M(n)$ and $F(n+1) < F(n)$—or decreasing full income combined with increasing public good provision—is excluded. Accordingly, adding new members is always beneficial but it eventually makes the public good inferior and hence it cannot indefinitely raise the total provision of the public good.

To sum up, when at the initial state the public good is normal, adding new members would tend to change the nature of public good to inferior, and hence cause the total provision of public good to decline. On the other hand, when at the initial state the public good is inferior, adding new members could not make the nature of public good normal, and hence the total provision of public good will still decline. Thus in a sense, the effect of risk aversion on the desire to insure and/or gamble creates "an inferior good barrier" that obstructs the ordinary consequences of adding new members to an alliance of states or other relevant group.

5.2.9.3 The Effects of Economic Growth

Based on Eqs. (5.8d) and (5.8e) we see that the equilibrium conditions are independent of redistribution of income. Equilibrium is dependent on total income, $Y_1 + Y_2$ but not on Y_1 or Y_2 separately. Thus, the neutrality result holds as in the conventional VPG model. See Cornes and Sandler (1984, 1996), Warr (1983), Bergstrom et al. (1986), and Jack (1991). Provided redistribution of income between members does not change the set of positive contributors, it will not affect the real equilibrium.

Now suppose both countries are identical except for income; preferences and loss in the bad state are the same: $E_1() = E_2(),\ L_1 = L_2$. This implies $W_1 = W_2$. Then from (5.8b) and (5.8c) it follows that in equilibrium $Y_1 + m_2 = Y_2 + m_1 = Y^*$, i.e. individual full income is identical for all interior Nash equilibria. If $Y_1 > Y_2$, then $m_1 > m_2$ and a higher income implies greater relative contribution to public good M. This outcome, with good M assumed to be normal is shown in Fig. 5.6, where an increase in Y_1 shifts curve N_1 upwards. With greater Y_1 at the new equilibrium point m_1 rises and m_2 declines. However, if M is inferior, an increase in Y_i need not raise i's equilibrium expenditure on m_i. In Fig. 5.7 with the "public good" inferior an increase in Y_1 shifts curve N_1 downwards to N_1'. At the new Nash point, m_1 has risen and m_2 declined due to the negative income effect of Y^*; but since these equilibria are unstable, rather than N', changes in Y would most probably result in corner solutions as shown.

Thus "normality" of good M and summation finance are consistent with "exploitation of the great by the small" (Olson 1965) since the richer country has a disproportionately stronger incentive to provide for security. But if M is inferior, Olson's idea fails since the richer country now has a weaker incentive to provide for

Fig. 5.6 Effects of an increase in wealth on expenditures to reduce risk when risk improvement is normal. *Source* Authors

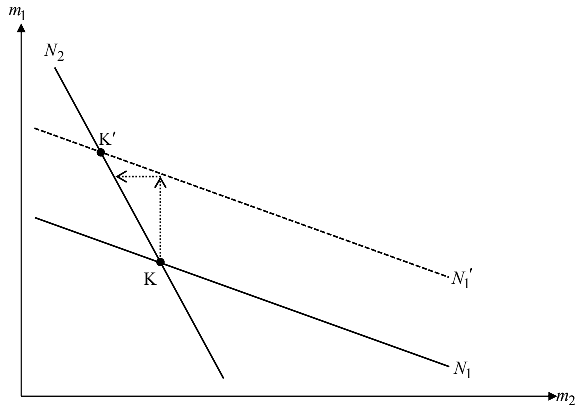
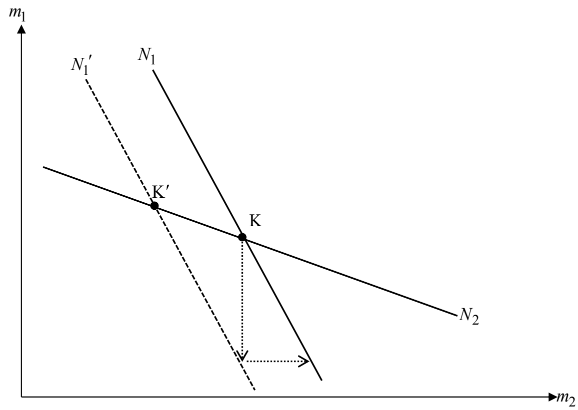


Fig. 5.7 Effects of an increase in wealth on expenditures to reduce risk when risk improvement is inferior. *Source* Authors



security (aside from instability and corner solution problems associated with inferiority). From Eq. (5.39) it is easy to see that an increase in Y (economic growth) has qualitatively the same effect as an increase in n (bigger group size). Thus, economic growth may give rise to an "inferior good barrier" to public good provision just as an expansion of group size can.

5.3 Self-insurance and Self-protection

5.3.1 Inclusion of Self-insurance

From now on we consider self-insurance as well as self-protection. Economists' VPG models have not been well extended to understand incentives and behaviors of sovereign agents desiring to manage risks along multiple channels analogous to

Ehrlich and Becker's (1972) self-protection and self-insurance. Analysis of multiple instruments of collective risk management must extend the standard market insurance paradigm along several dimensions.

First, sovereign agents will face increasing costs for self-insurance coverage in contrast to the competitive prices relevant to individual market insurance. Second, when instruments both for self-insurance and for self-protection are available to a single decision maker, reducing risks through protection changes the price of self-insurance and thus the two allocations will interact in an unfamiliar manner. Third, if both self-insurance and self-protection are available to all members of a group, we must deal with problems with incentives which can preclude determinate Nash equilibria. Lastly, as explained in Sect. 5.2, since inter-agent incentives will be transmitted by income effects in the VPG model the issue of normality vs inferiority of protection and insurance is of special importance. Therefore, we must integrate known results concerning inferiority of insurance (Mossin 1968, Hoy and Robson 1981) and of protection in Sect. 5.2 (See also Ihori and McGuire, 2007).

If the agent provides security spending at an insurance market, then the standard result states that with fair linear pricing, complete coverage (net of premiums) is purchased from resources available in the good contingency. We use this benchmark result to compare with *self-insurance* by an entire country, and also in examination of the effect of insurance on the choice of protection. It is shown that if the nation provides the public goods in terms of self-insurance, complete coverage (net of premiums) may not be purchased from resources available in the good contingency due to decreasing productivity of self-insurance spending.

On the one hand, if the small agent tries to provide both self-insurance and self-protection, she cannot provide both at the same time under the linear technology with a price taker. On the other, the provision by nations could solve this difficulty since self-insurance is well described by the decreasing technology and they incorporate the actuarially fair condition at their optimization as a price maker.

However, when we consider mutual provision of the public goods by nations, we have another difficulty of inferiority. Namely, if countries are risk averse, we could have the situation that both types of public goods are inferior, and hence the Nash equilibrium would be corner solutions. In such a case we observe specialization of provision of the public goods. In the case of centralized specialization the free-riding country always gains, while in the case of decentralized specialization both countries free-ride each other and the welfare comparison becomes ambiguous.

5.3.2 Analytical Framework for a Single Country

5.3.2.1 An Example of Flooding

Let us begin with a narrative of the concept that our models might try to capture, or summarize. Imagine there is an island nation that is subject to flooding. See also Cornes (1993). Whenever a flood happens, there is a big loss. No matter how big the flood, the loss is the same, L.

To protect itself against this loss the country can reduce the frequency of flooding by building flood-barriers, dikes, channels etc. If it builds no dikes a flood happens every other year. If it builds dikes that are 10 feet tall, the country will be flooded every 6 years. If this country builds dikes 15 feet tall it will be flooded every 11 years. The frequency of flooding depends on the height of its flood barriers. The relationship between cost of dikes and frequency of flood will be known with certainty.

For simplicity, ignore time discounting and assume to start that if a flood happens every other year, each year without a flood the country can set aside x pounds of goods for the next year when there is a flood and have available in that year x pound of goods. If a flood happens every 6 years then to have x pounds available during the flood, the country only needs to give up $x/5$ lbs during each of the 5 dry years. If a flood happens every 11 years, the country can provide x lbs during the flood by giving up only $x/10$ lbs in each of the dry years. In other words, as an initial assumption we may suppose the country can self-insure at an actuarially fair price, $(1 - p)/p$.

5.3.2.2 Risk Profiles and Emergency Cost

Congruent with the foregoing story we consider a single agent i with two states, a good state "1" and a bad state, "0". Ignoring all insurance and compensation possibilities (that is, taking L as a fixed parameter) expected utility for this agent is given as Eq. (5.1) and Eq. (5.2) as in Sect. 5.2. This section now includes self-insurance to focus on two types of Ehrlich-Becker defense; (i) Ehrlich-Becker's "self-protection;" which raises p and reduces $(1 - p)$, (ii) Ehrlich-Becker's "self-insurance" which reduces L. Aside from our flood narrative, the variable "p" might be risk of trade interruption, disease outbreak, environmental calamity, or war. Later in Sect. 5.3.6 we will assume these are shared indivisibly by the two coalition members.

From now on we use new notations with respect to self-protection m_1 and self-insurance m_2. m_k $(k = 1, 2)$ is now allocated to risk reduction, $p(m_1)$, as self-protection, m_1, and or loss reduction, $-L(m_2)$, as self-insurance, m_2. m_1^i and m_2^i mean self-protection and self-insurance provided by country i $(i = 1, 2)$. When agents provide collective or mutual self-insurance to reduce losses, for example, "summation finance" implies Eq. (5.41).

$$L = L(M_2) \tag{5.41}$$

where M_2 is the total supply of self-insurance by two countries, $M_2 = m_2^1 + m_2^2$ (with only one agent in the picture $M_k = m_k^i$). Thus our simple solution to the problem of diminishing returns and distribution of infra marginal costs/gains will be to assume a "summation *finance* aggregator," $M_k = \sum_i m_k^i$, in the provision of public good L, even though $L(M_2)$, later $p(M_1)$, represents a "non-summation *consumption* aggregator" (e.g. $p(M_1) \neq \sum_i p_i(m^i)$) Then, importing an idea from contest theory we take primitive preferences as being over *contributions* to insurance or to risk reduction, rather than insurance coverage or risk reduction itself.

But before turning to public good provision, first we establish the incentives for a single agent. Therefore, if our concern was with insurance only—with p taken to be a parameter—Eq. (5.1) can be written

$$\tilde{W} = \tilde{W}(C, m_2) \tag{5.42}$$

where now, for a single agent Eq. (5.42) shows how it is natural and helpful to consider m or M rather than L to be the public good.

5.3.2.3 Market Insurance: Standard Result

As we will argue presently, the structure and context of self-insurance for a large entity such as an entire nation is inherently quite different from market insurance. So it is for later comparative purposes useful to set out in brief summary the standard market insurance model, where again m_2 represents expenditures in good times to insure against bad times.

A basic feature of the standard market insurance model (which distinguishes it from self-insurance) is that the payoff to insurance coverage in bad times is linear. Suppressing agent-i indices, for a linear insurance recovery function instead of $-L(M)$ we write with numeraire income being consumption in good times.

$$C^1 = Y - m_2 \tag{5.43}$$

and

$$C^0 = Y - \left(\bar{L} - (m_2/\pi)\right), \tag{5.44}$$

where m_2 gives the expenditure on market insurance in good times (measured in units of C^1), and m_2/π represents in units of L or C^0 the amount of insurance coverage purchased at price π. Then welfare becomes

$$W = pU^1[Y - m_2] + (1 - p)U^0\left[Y - \left(\bar{L} - \frac{m_2}{\pi}\right)\right] \tag{5.45a}$$

If instead we took m_2 to mean the amount of insurance coverage rather than contingency- expenditures, we would write

$$W = pU^1[Y - \pi m_2] + (1 - p)U^0\left[Y - \left(\bar{L} - m_2\right)\right] \tag{5.45b}$$

Equations (5.45a) and (5.45b) are equivalent in the standard linear case, and we will use Eq. (5.45a).

Then for optimal insurance, maximizing Eq. (5.45a) with respect to m_2 yields necessary condition (5.46).

$$-p\pi U_Y^1 + (1-p)U_Y^0 = 0 \tag{5.46}$$

If the price of insurance happens to be actuarially fair, then we have

$$\pi = (1-p)\big/ p \tag{5.47}$$

And now the FOC entails $U_Y^1 = U_Y^0$ whence $U^1 = U^0$ and, therefore, $C^1 = C^0$ or

$$Y - m_2 = Y - \left(\bar{L} - \frac{m_2}{\pi}\right) \tag{5.48}$$

And from this it follows that at the optimum, insurance coverage purchased is $m_2/\pi = p\bar{L}$ so that the total cost of the optimum of such fairly priced insurance becomes

$$m_2 = (1-p)\bar{L} \tag{5.49}$$

This standard result states that with fair linear pricing, complete coverage (net of premiums) is purchased from resources available in the good contingency. We will use this summary as the benchmark later to compare with *self-insurance* by an entire country, and also in examination of the effect of insurance on the choice of protection.

5.3.3 *Self-insurance*

Self-insurance provided to itself by a large entity such as a nation differs in two important respects from standard market insurance. To show this, we extend notation slightly. Rather than $-L(m)$ where $L(0)$ was a threatened loss if nothing is spent on insurance, we write

$$-L(m) = -[\overline{L} - \mathbf{L}(m)] \tag{5.50}$$

Now the entire, total, insurance benefit is shown by $\mathbf{L}$, and $L(0)$ is given by $\bar{L}$.

5.3.3.1 Diminishing Returns

First of all, self-insurance differs from standard market insurance in that self-insurance function $\mathbf{L}$ should show diminishing returns or increasing costs.

$\mathbf{L}' > 0, \mathbf{L}'' < 0$. In our story of flood protection and insurance, as greater quantities of consumables are set aside during dry years, their costs of preservation and delivery during good years might increase more than proportionately. For example, if $p = 1/2$ setting aside $m_2 = 1$ provides 1 unit in bad times, but saving $m_2 = 8$ yields only 4 units in bad times, etc. Ehrlich and Becker make this assumption also, and refer glancingly to the role of human capital in providing for self-insurance as a source of diminishing returns. Scale considerations should be appropriate for an entire country along an extensive margin. Hence, "self-insurance" has declining marginal productivity as shown earlier. National self-insurance may often involve actions like stockpiling or standby production maintenance and these surely will show diminishing returns.

Declining "productivity" of "m_2" thus is the first source of a distinction between sovereign self-insurance versus lesser scale decentralized market insurance. Diminishing returns also will introduce issues in the formulation of inter-contingency pricing of self-insurance that are absent from market insurance—a fact not recognized in the literature as far as we can determine. First, definitions and formulation of "actuarial fairness" become ambiguous. Among formulations of pricing when insurance is non-linear, two seem salient both being consistent with an inter-contingency price π as shown in Eq. (5.51):

$$W = pU^1[Y - \pi m_2] + (1-p)U^0\big[Y - \big\{\overline{L} - \mathbf{L}(m_2)\big\}\big] : \ m_1 \ \textit{not shown} \quad (5.51)$$

where π shows the actuarial price per unit in good times necessary to yield m_2 units of resources in adversity.

Thus the following equation—consistent with Eq. (5.51)—gives one variant of expected utility, where m_2 now indicates the total premium in good times and $1/\pi$ indicates the unit yield of those premiums in adversity per unit of m_2. This amount is entered into the self-insurance function, $\mathbf{L}$ to give the net amount of insurance benefit.

$$W = pU^1[Y - m_2] + (1-p)U^0\left[Y - \left\{\overline{L} - \mathbf{L}\left(\frac{m_2}{\pi}\right)\right\}\right] : \ m_1 \ \textit{not shown} \quad (5.52a)$$

An alternative to Eq. (5.52a) and also consistent with Eq. (5.51) is Eq. (5.52b).

$$W = pU^1[Y - m_2] + (1-p)U^0\left[Y - \left\{\overline{L} - \frac{1}{\pi}\mathbf{L}(m_2)\right\}\right] : \ m_1 \ \textit{not shown} \quad (5.52b)$$

Here again, π indicates an inter-contingency-price at which $\mathbf{L}(m_2)$ is transferred across contingencies. It gives another structure of inter-contingency transfer. There $1/\pi$ indicates not the productivity or accumulation of m_2-resources but the accumulation in bad times of $\mathbf{L}$ units of benefit.

When insurance is supplied in the market and its marginal costs and benefit are linear, this distinction between Eqs. (5.52a) and (5.52b) does not arise; but when **L** displays diminishing returns, how to describe the cost of insurance seems to be ambiguous. Do diminishing returns apply both to quantities reserved and to the lapse of time between adversities, or only to the former? As for comparing Eq. (5.52a) versus Equation (5.52b), the difference seems to lie in "when" the resources transferred across contingencies become productive before (as in Eq. (5.52b)) or after (as in Eq. (5.52a)) the transfer. The novelty of this distinction is due to the fact that when the insurance function is linear, as in market insurance, no such difference arises.

Now, in a sense, the self-insurance function operates during good times to create transfers available only under adversity; so that for the total $\mathbf{L}/\pi$ received in adversity, m_2 was set aside in the good contingency.

5.3.3.2 Salience of Fair Pricing

A second major difference between self-insurance as provided by an entire country and ordinary, market insurance is, that when a whole nation provides insurance to itself, fair pricing would seem to be the standard case and not an outlier just referenced for comparison. Of course nations can make mistakes, have imperfect information etc. But countries in this position are "bargaining with themselves" as to how much insurance and at what price to provide it. They should not in principle have to worry about adverse selection or moral hazard. So they should not give themselves deductibles, "load" prices nor impose or arbitrary insurance limits to control fraud. Of course countries have corruption, rent seeking, and numerous misalignments of incentives to concern them. Nevertheless, it is plausible to assume that the nation as a price maker, not as a price taker, incorporates this actuarially fair condition at her optimization.

5.3.3.3 The Insurance Optimization Problem

Now, similar to Ehrlich and Becker's derivation, expected utility (Eqs. (5.52a) or (5.52b)) is maximized with respect to m_2. This gives Eq. (5.53) as the first order condition corresponding to Eq. (5.52a).

$$\text{FOC:} \quad -p\pi U_Y^1 + (1-p)U_Y^0\mathbf{L}'(m_2/\pi) = 0 \tag{5.53}$$

Equation (5.53) shows the marginal cost of providing **L**, $[p\pi U_Y^1]$, equal to the marginal benefit of providing **L**, $(1-p)U_Y^0\mathbf{L}'$, evaluated at the solution value of m_2/π. If this necessary condition is rewritten as in Eq. (5.54), then its actuarial meaning becomes clear.

$$\frac{U_Y^1}{U_Y^0} = \frac{(1-p)}{p\pi}\mathbf{L}'\left(\frac{m_2}{\pi}\right) \tag{5.54}$$

The RHS there gives the probability weighted marginal insurance receipt under adversity for the last, probability-weighted dollar of premium paid in good times. If self-insurance is actuarially fair (henceforth simply "fair") as we believe should be the paradigm for an entire country, then

$$\pi = (1 - p)\big/ p \tag{5.55}$$

and therefore

$$U_Y^1 \big/ U_Y^0 = \mathbf{L}'(m_2/\pi) \tag{5.56}$$

Equation (5.56) is simply a familiar equality of MRS and Marginal Rate of Transformation, which obtains irrespective of risks $(1-p)$ so long as the price of insurance is fair.

Could insurance proceed so far that outcome in the "bad" state is preferred to the outcome in the "good" state, i.e. so that $U^0 > U^1$? As discussed in 3.3.1. conceivably yes, if the self-insurance function $\mathbf{L}$ is very productive, or the inter-contingency price "super-fair"—i.e. $\pi < (1-p)\mathbf{L}'/p$— then

$$\frac{U_Y^1}{U_Y^0} = \frac{1}{\pi}\left[\frac{(1-p)\mathbf{L}'}{p}\right] > 1 \tag{5.57a}$$

and therefore

$$U_Y^1 > U_Y^0 \Rightarrow U^1 < U^0 \tag{5.57b}$$

However, this sort of reversal is too unlikely to be of interest so we will ignore it. But note, no equivalence is implied between U_Y^1 *and* U_Y^0.

Alternatively, in place of Eq. (5.57a) if the maximand is Eq. (5.52b), then the necessary condition becomes:

$$\text{FOC:} \quad -pU_Y^1 + \frac{(1-p)}{\pi}U_Y^0\mathbf{L}'(m_2) = 0 \tag{5.58}$$

Again, fair self-insurance does not entail equalization of marginal utility of income across contingencies as in the standard economic analysis of fair, market-insurance; nor does it imply that consumption in good and bad times be equal (EB, 1972; Hirshleifer and Riley 1975). Equation (5.58) shows the marginal cost of providing $\mathbf{L}$, i.e.pU_Y^1, "now" equal to the marginal benefit of $\{(1-p)/\pi\}U_Y^0\mathbf{L}'$, evaluated not at m_2/π as before but simply at m_2. When insurance by an entire nation has these properties and is actuarially fair, it is rewritten as in Eq. (5.59) to be compared to Eq. (5.56).

$$U_Y^1 \big/ U_Y^0 = \mathbf{L}'(m_2) \tag{5.59}$$

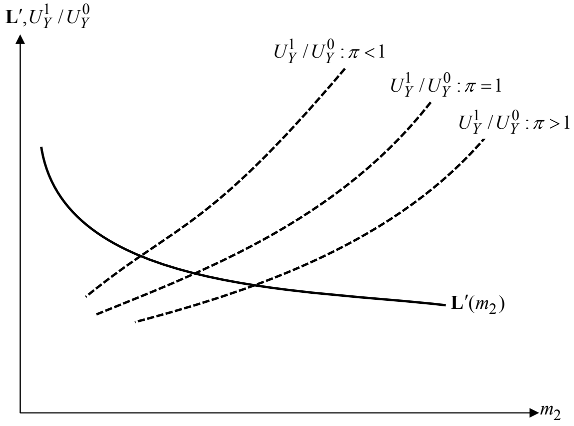


Fig. 5.8 The best choice of m_2 if the self-insurance has the form of Eq. (5.52b). *Source* Authors

Equation (5.59) just as Eq. (5.56) violates the standard result in insurance theory: viz. that fair pricing equalizes marginal utility across contingencies.

If marginal utilities just happened to be equalized at the optimum then consumption would also be equalized, and this would imply $\mathbf{L}(m_2) = p\bar{L}$, in the one case or $\mathbf{L}(m_2/\pi) = p\bar{L}$ in the other. But neither of these conditions is required to obtain even though coverage is fairly priced. The first order optimum "allocates" in a sense the infra-marginal benefits of self-insurance—rents from both decreasing marginal utility of insurance coverage and from increasing marginal cost of providing insurance—across contingencies. But net income need not be equalized. Note again that fairness is not dictated by market pricing but rather by internal calculus of public allocation when a sovereign country "negotiates with its self." This compares with the standard market insurance result that fair rates logically entail "full coverage," or complete protection, and distinguishes fair-market from fair self-insurance.

In any case Eqs. (5.56) and (5.59) inform us of optimal insurance protection and allow us now to compare configurations (5.52a) and (5.52b), assuming self-insurance to be fairly priced. First, note that for $\pi = [(1-p)/p] = 1$, $\mathbf{L}'(m_2/\pi) = \mathbf{L}'(m_2)$, while for $\pi > 1$, $\mathbf{L}'(m_2/\pi) > \mathbf{L}'(m_2)$ and for $\pi < 1$, $\mathbf{L}'(m_2/\pi) < \mathbf{L}'(m_2)$. So if self-insurance has the form of Eq. (5.52b) i.e., $\mathbf{L}(m_2)$, there is an interaction between insurance and protection not present in the case of market insurance. Figure 5.8 shows this. For $\pi < 1$ (and hence risk $(1-p)$ low) the solution value of "m_2" will be less than it is when odds are equal. On the other hand, for $\pi > 1$ (and risk $(1-p)$ high) optimal m_2 will be greater than the best choice of "m_2" when odds are equal (maintaining throughout these comparisons, the equality $\pi = (1-p)/p$). Next suppose that self-insurance has the form of Eq. (5.52a), i.e. $\mathbf{L}(m_2/\pi)$, then there is an additional interaction between insurance and protection. Figure 5.9 illustrates this case. There, when $\pi = (1-p)/p$ varies as a parameter, both the MRS between contingencies shifts and MRT shifts as well so that the overall effect of π on the optimal m_2 cannot be deduced a priori.

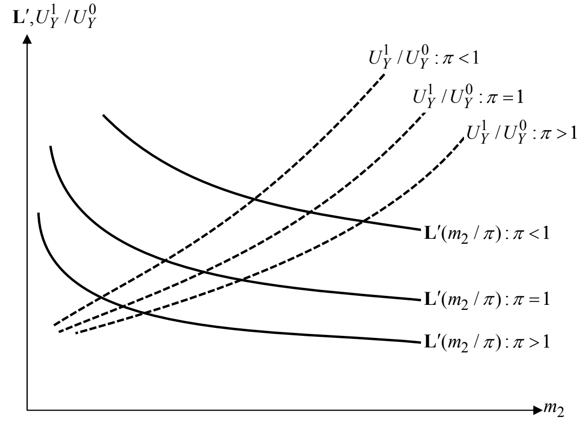


Fig. 5.9 The best choice of m_2 if the self-insurance has the form of Eq. (5.52a). *Source* Authors

5.3.3.4 Effects of Risk Aversion on Self-insurance

Economists have long known of a systematic inter-dependence between risk aversion and provision of market "linear" insurance. Established in theory of market insurance is that insurance will be an inferior good if risk aversion is diminishing (Mossin 1968, Hoy and Robson 1981). To demonstrate that this continues to hold for self-insurance it is the sign of the income effect on insurance spending we must investigate. And these become important especially for the soon-to-be-introduced interactions among members of a group. Specifically, therefore, we want to determine whether m is an inferior or a normal "good". For comparative static results, total differentiation of FOC (5.53) gives:

$$\frac{\partial m_2}{\partial Y} \equiv M_Y = \frac{p\pi U_{YY}^1 - (1-p)U_{YY}^0 \mathbf{L}'}{D} \tag{5.60}$$

where Y is the individual income and D represents the second order condition.

$$\text{SOC:} \quad D = p\pi U_{YY}^1 - (1-p)U_{YY}^0 + (1-p)U_Y^0 \mathbf{L}''/\pi < 0 \tag{5.61}$$

From the SOC we know therefore

$$p\pi U_{YY}^1 - (1-p)U_{YY}^0 \mathbf{L}' < -(1-p)U_Y^0 L''/\pi \text{ and } 0 < -(1-p)U_Y^0 L''/\pi \tag{5.62}$$

Hence the sign of the numerator of Eq. (5.60) is ambiguous. Thus, m_2 as self-insurance may be inferior. The numerator of Eq. (5.60) is actually the difference between the effects of greater income on marginal cost $\Delta MC = MC_{MY} = p\pi U_{YY}^1 < 0$ and marginal benefit $\Delta MB = MB_{MY} = (1-p)U_{YY}^0 L' < 0$. Note for later use that whereas MC_{MY} has elements in the good state of the world, MB_{MY}

refers exclusively to the bad state. Specifically, if the numerator is positive, given the SOC, the sign of Eq. (5.60) is negative, and m_2 becomes inferior. Note if self-insurance is fair and therefore

$$(1-p) = p\pi,$$

then the numerator of Eq. (5.60) becomes $p\pi\left(U^1_{YY} - U^0_{YY}\mathbf{L}'\right)$. That is, when m_2 is inferior, and greater income lowers marginal cost—$\Delta MC = MC_{MY} = p\pi U^1_{YY} < 0$ by more than it reduces marginal benefit—$-\Delta MB = -MB_{MY} = -(1-p)U^0_{YY}\mathbf{L}' > 0$.

This required relationship between MC_{MY} and MB_{MY} can be derived from the risk aversion properties of the utility function, with absolute risk aversion R defined as

$$R = -U_{YY}/U_Y \tag{5.63}$$

or

$$-U_{YY} = RU_Y$$

If risk aversion is increasing $(R_1 > R_0)$, $H = p\pi U^1_{YY} - (1-p)U^0_{YY}\mathbf{L}' < 0$ and the sign of Eq. (5.60) is positive, and vice versa; that is, if risk aversion is increasing (decreasing), m_2 is normal (inferior). This being another approach to Mossin's (1968) result, the numerator of Eq. (5.60) can be rewritten as:

$$\hat{H} = -\left[p\pi R_1 U^1_Y - (1-p)R_0 U^0_Y \mathbf{L}'\right] \tag{5.64}$$

Considering the FOC, we obtain

$$\hat{H} = p\pi U^1_Y(R_0 - R_1) = (1-p)U^0_Y(R_0 - R_1)\mathbf{L}', \tag{5.65}$$

where $\mathbf{L}' > 0$. With the numerator of Eq. (5.61) written as $MC_{MY} - MB_{MY}$ it is understood that both MC_{MY} and MB_{MY} are negative. If $|MC_{MY}| < |MB_{MY}|$ then M_Y and m_2 spent on self-insurance is inferior. Using this notation to write Eq. (5.65) gives

$$\hat{H} = [MC_{MY} - MB_{MY}]$$

and

$$\begin{aligned} MC_{MY} &= -R_1 p\pi U^1_Y \\ -MB_{MY} &= +R_0 p\pi U^1_Y \end{aligned} \tag{5.66}$$

In MC_{MY} the terms p and R interact just as in the case of m_1 spent for self-protection considered below. However, for self-insurance when m_2 and, therefore, $-L$ are optimized this interaction is washed out of the sum of MC_{MY} and $-MB_{MY}$. The change in marginal benefit of more insurance when Y is increased depends only on its impact in one contingency, i.e. on $(1-p)R_0U_Y^0\mathbf{L}'$. When m_2 for insurance is optimized, as shown in Eq. (5.65) U_Y^0 and U_Y^1 are balanced as per Eq. (5.65) so that the independent effect of U_Y^0 and $(1-p)$ are all absorbed in U_Y^1 and p or vice versa. When similar analysis is performed on self-protection, the FOCs do not cancel out the interdependence between R and p, so that our conclusions for the two cases differ.

Generally, we may expect absolute risk aversion to decrease with wealth, so that the amount of insurance purchase declines. It follows that m_2 may well be inferior, a conclusion that parallels the standard case of market insurance (Mossin 1968, Hoy and Robson 1981, Eeckhoudt and Gollier 2000), even though here, optimal and fair self-insurance with diminishing returns need not equalize incomes or marginal utilities across contingencies. Thus the crucial determinants of inferiority are actuarial fairness, and optimization, not market based liner pricing.

5.3.3.5 Remark

The concept and measurement of actuarially fair pricing is crucial as a benchmark throughout the analysis of insurance. This remark dwells briefly on an ambiguity in definition of actuarial fairness that arises when insurance is self-provided with diminishing returns.

Our main analysis assumes one simple relationship between premiums paid, πm_2, during good times and $\mathbf{L}(m_2)$ benefits received during bad times, where the variable of choice is units of coverage m_2. As pointed out, we could cast the problem in terms of units of expenditure (where we define $x_2 = \pi m_2$, "x" for" expenses," and where $m_2 = x/\pi$) such that the variable of choice is x. Here $1/\pi$ would indicate the efficiency of resource transfer across contingencies. But that suggests that in place of Eq. (5.51) we could write *either* Eq. (5.52a) *or* Eq. (5.52b):

$$W = pU^1[Y - \pi m_2] + (1-p)U^0\left[Y - \left\{\overline{L} - \mathbf{L}(m_2)\right\}\right] : \ m_1 \ \textit{not shown} \quad (5.51)$$

$$W = pU^1[Y - x_2] + (1-p)U^0\left[Y - \left\{\overline{L} - \mathbf{L}(x_2/\pi)\right\}\right] : \ m_1 \ \textit{not shown} \quad (5.52a')$$

$$W = pU^1[Y - x_2] \ + (1-p)U^0\left[Y - \left\{\overline{L} - \frac{1}{\pi}\mathbf{L}(x_2)\right\}\right] : \ m_1 \ \textit{not shown} \quad (5.52b')$$

When transfer of resources across contingencies is written as in Eq. (5.52a) it becomes obvious on inspection that every unit of x_2 may *not* transfer into just exactly x_2/π units to the bad contingency. Once this is recognized then the idea of "fair" pricing becomes ambiguous.

In Eq. (5.52b)′ $1/\pi$ indicates not the productivity or accumulation of x_2−resources but the accumulation in bad times of $\mathbf{L}$ units of benefit. Now, in a sense, the $\mathbf{L}$−function operates during good times to create transfers available only under adversity; so that for the total $\mathbf{L}/\pi$ received in adversity, x_2 was set aside in the good contingency.

One way to think about this is to assume there is a steady state. We can then frame self-insurance in terms of changes in, or alternative parameters for this steady state. Normalizing the notation of note 1, let $w = 1$, $d = n$. Here is the steady state. Every $n + 1$ years there is a crisis, requiring that the self-insurance accumulation be utilized. Each year from 1 to n, \$ m_2 is set aside in anticipation of year $n + 1$. That is "self-insurance" is fair. At year $n + 2$ the process starts over again, and on. Therefore, $p = n/(n+1)$, and $(1 - p) = 1/(n+1)$. $\pi = 1/n$ so that $m_2/\pi = nm_2$. Changes in the risk of adversity then are given by changes in n. If n is large adversity happens only rarely so that $(1 - p)$ is small, and if n is small, the risk of adversity $(1 - p)$ is high. We ignore discounting, and uncertainty.

Case I: Here the total insurance consumed in year $n + 1$ is $n\mathbf{L}(m)$. Here diminishing returns apply to each year's insurance savings individually so that every new year the country begins a new decreasing returns process.
Case II: Here the total insurance coverage consumed in year $n + 1$ is $\mathbf{L}(nm)$. Here m_2−savings set aside each year generate less marginal return than the same m savings of the year before.

In both I and II, greater m_2 produces more diminishing returns, although in different manner and to different degree. Note however, that with case II, diminishing returns, in addition to being greater for larger values of m_2, are more severe the rarer the emergency (higher value of n).

When insurance is supplied in the market and its marginal costs and benefit are linear, this distinction between Eqs. (5.52a)′ and (5.52b)′ does not arise or it doesn't matter; but when $\mathbf{L}$ displays diminishing returns how to describe the cost of insurance seems to be ambiguous. Do diminishing returns apply both to quantities reserved and to the lapse of time between adversities, or only to the former? Between Eq. (5.52a)′ versus Equation (5.52b)′, the difference seems to lie in "when" the resources transferred across contingencies become productive—before (as in Eq. (5.52b)′) or after (as in Eq. (5.52a)′) the transfer. The novelty of this distinction, or the fact that it seems to have been overlooked, may be because when the insurance function is linear, as in market insurance, no such difference arises.

For either of these options (5.52a)′ or (5.52b)′ the idea of actuarial fairness is still important, but its implications with respect to outcomes vary. For example, if marginal utilities just happened to be equalized at the optimum such that $\mathbf{L}' = 1$, then consumption would also be equalized. It turns out that this would imply $\mathbf{L}(m_2) = (1 - p)\bar{L}$, in the case of Eq. (5.52b)′ or $\mathbf{L}(x_2/\pi) = (1 - p)\bar{L}$ in the case of Eq. (5.52a)′. Although symmetric to market insurance neither of these conditions is required to obtain even when coverage is fairly priced—not required because there is no necessity that $\mathbf{L}' = 1$. In either case FOCs inform us of optimal protection and allow comparison Eqs. (5.52a)′ and (5.52b)′, assuming self-insurance to be fairly priced. Generally optimal provision may be greater under formulation (5.52a)′ or (5.52b)′ depending on the manner in which p, $(1 - p)$ influence MRS and MRT in each formulation. Moreover, the benefit in the bad contingency might depend in a more complicated manner on resources set aside in good times than either Eqs. (5.52a)′ or (5.52b). Then we would want to write $\mathbf{L} = \mathbf{L}(\pi, p, x_2)$ and the definition of fair insurance correspondingly more ambiguous.

5.3.4 Interaction Between Self-protection and Self-insurance

5.3.4.1 Self-protection

Now to return to our flood protection anecdote, we introduce in Sect. 5.3.2.1 that the country can reduce the frequency of flooding by building flood-barriers, dikes, channels etc. The frequency of flooding depends on the height of its flood barriers, and the relationship between cost of dikes and frequency of flood is known with certainty i.e. $p(M_1)$ in our mathematical model. Thus the second risk management instrument to consider is self-protection with m_1 spent to reduce the chance of a bad event, $1 - p$, i.e. to decrease what we call "baseline risk of $[1 - p(0)]$". Our "baseline risk" corresponds to what is sometimes referred to as "background risk" in economics of insurance analyses. Background risk distinguishes "independent" background risk where $p(0)$ is not influenced by the value of $L(0)$ as in our model here, versus "non-independent" background risk where $p(0)$ and $L(0)$ are interdependent, and asks how the choice of protection or insurance varies with the independence property (See McGuire and Becker 2006, and Schlesinger 2000).

As explained in Sect. 5.2, for self-protection the issue of normality-inferiority is substantially more involved than for self-insurance. A priori, many risk reduction or self-protection functions are plausible: quadratic, exponential, logistic etc. One expects $p' > 0$ throughout for all of these; while p'' may vary, we assume here that $p'' < 0$ throughout. Whatever the form of $p(M)$ it becomes natural and useful to re-write Eq. (5.7) as

$$\widehat{W} = \widehat{W}(C, m_1) \tag{5.67}$$

where this time L, m_2, and $\mathbf{L}$ are now taken to be parameters, in contrast to Eq. (5.42). Functions $\tilde{W}_i(C, m_2)$ *and* $\widehat{W}_i(C, m_1)$ are different, but the difference is implicit in each model, so the notational distinction will be omitted henceforth.

We now desire to extend this analysis developed for a special example to the more general case of increasing costs for self-insurance. That is, we desire to apply these insights to the case where insurance benefit is non-linear, and actuarial fairness interacts with diminishing returns. To begin we repeat Eq. (5.52a) now including both variables m_1 and m_2 and inserting the condition for actuarial fairness, $\pi = [(1 - p)/p]$ directly:

$$W = p(m_1)U^1[Y - m_1 - m_2] + (1 - p(m_1))U^0\left[Y - m_1 - \left\{\overline{L} - \mathbf{L}\left[\frac{m_2 p(m_1)}{1 - p(m_1)}\right]\right\}\right] \tag{5.52a}$$

The FOC for determining optimal expenditure on self-protection becomes:

$$p'(U^1 - U^0) - \left[pU_Y^1 + (1 - p)U_Y^0\right] + \frac{U_Y^0 p' \mathbf{L}' m_2}{1 - p} = 0. \tag{5.68}$$

We can characterize this optimality condition on the provision of self-protection saying that there are "direct" marginal benefits in the form of the gain in utility $p'(U^1 - U^0)$, "direct" marginal costs $\left[pU_Y^1 + (1 - p)U_Y^0\right]$, and "indirect" benefits comprised of, an unambiguous gain from the increase in insurance coverage for the same m_2 premium paid stemming from the lower price implied by lowering risk $(1 - p)$.

$$\text{SOC} \quad J = p''(U^1 - U^0) - 2p'(U_Y^1 - U_Y^0) + \left[pU_{YY}^1 + (1 - p)U_{YY}^0\right] - \frac{U_{YY}^0 p' \mathbf{L}' m_2}{1 - p} + \frac{U_Y^0 p'' \mathbf{L}' m_2}{1 - p} + \frac{U_Y^0 (p')^2 m_2}{(1 - p)^2}\left(\mathbf{L}'' m_2 - \mathbf{L}'\right) < 0 \tag{5.69}$$

Note that without further specification these SOCs need not always hold for mutual self-protection. However, we assume the SOC is satisfied; then taking total differentiation of FOC (5.68) gives:

$$\frac{\partial m_1}{\partial Y} = -\frac{p'\left(U_Y^1 - U_Y^0\right) - \left[pU_{YY}^1 + (1 - p)U_{YY}^0\right] + U_{YY}^0 p' \mathbf{L}' \frac{m_2}{1-p}}{J} \tag{5.70}$$

Equation (5.69), assuming the SOC actually obtains, determines the sign of the denominator in Eq. (5.70) as negative at an optimum. But again the sign of the numerator is ambiguous, and the normality or inferiority of m_1 depends on this numerator, just as in the self-protection model. The numerator of Eq. (5.70) can be written $MB_{MY} - MC_{MY}$, with $MB_{MY} = p'(U_Y^1 - U_Y^0) < 0$ and $MC_{MY} = pU_{YY}^1 + (1 - p)U_{YY}^0 - U_{YY}^0 p' \mathbf{L}' \frac{m_2}{1-p} < 0$. Now, however, there is an interaction between risk aversion R and baseline probability, so the normality/inferiority of m_1 is more involved than in the case of self-insurance. Here if absolute risk aversion is increasing/decreasing and $(1 - p)$ is initially low/high, m_1 will be normal/inferior, while if absolute risk aversion is decreasing/increasing and $(1 - p)$ is low/high, m_1 becomes inferior/normal

Section 5.2 has demonstrated that there is a critical-crossover probability p^* such that if risk aversion is increasing M switches from normal to inferior while if risk aversion is decreasing M switches from inferior to normal as this value p^* is crossed. In a generalized analysis of risk taking and insurance, Eeckhoudt and Gollier (2000) commented on the fact that a decline in first order stochastic risk may increase an optimizing agent's optimal exposure to risk (p. 122) considering it to be a "puzzle." This relation between RA and p accounts for one source of such a "puzzle."

One objection to this analysis might be our assumption that the loss function $L(M)$ is independent of wealth, Y. In fact, one might argue that the richer an agent/country the greater its loss from adversity. We could adjust our formulation to include this effect by writing, for both self-protection and self-insurance, replacing the loss, $L(M)$, with a proportional reduction of income.

$$W = p(m)U^1(Y - m) + [1 - p(m)]U^0(\alpha Y - m) : 0 < \alpha < 1 \tag{5.71}$$

This formulation implies a single crossover probability—not constant p^* but rather $p = p(\alpha)$—such that income effects change sign when probability changes and are neutral at $p(\alpha)$.

5.3.4.2 A Special Example: Linear Fair Market Insurance (LFI)

First of all, as a benchmark case, let us consider the linear fair insurance in a market. That is, we assume that the self-insurance function is linear and the small agent regards the price of insurance as fixed. This case could be relevant for a small agent in an insurance market. Then, we write:

$$C^0 = Y - m_1 - \left[\bar{L} - (m_2/\pi)\right], \tag{5.72}$$

Then welfare becomes

$$W = p(m_1)U^1[Y - m_1 - m_2] + (1 - p(m_1))U^0\left[Y - m_1 - \left(\bar{L} - \frac{m_2}{\pi}\right)\right] \tag{5.73}$$

where m_2 and m_1 are as previously defined and $1/\pi$ the unit price of insurance coverage. Optimal self-insurance, and maximization of Eq. (5.73) with respect to m_2 yields as

$$\frac{\partial W}{\partial m_2} \equiv \hat{T}(m_2) = -p\pi U_Y^1 + (1 - p)U_Y^0. \tag{5.74}$$

Since the agent regards the price of insurance as fixed, we have

$$\frac{dW}{dm_1} \equiv \tilde{T}(m_1) = p'(U^1 - U^2) - \left(pU_Y^1 + (1 - p)U_Y^0\right) \tag{5.75}$$

Now, suppose $\hat{T}(m_2) = 0$. Then, from Eq. (5.74) and the actuarially fair condition (5.47) we know

$$U_Y^0 = U_Y^1 \text{and hence} U^0 = U^1$$

Substituting this into Eq. (5.75), we have $\tilde{T}(m_1) = -U_Y^1 < 0$. Consequently, if Eq. (5.74) and the actuarially fair condition (5.47) hold, we get $\tilde{T}(m_1) = -U_Y^1 < 0$.

On the other hand, suppose $\tilde{T}(m_1) = 0$. Then from Eq. (5.75) we know

$p'(U^1 - U^0) = \left[pU_Y^1 + (1-p)U_Y^0\right] > 0$ and hence $U^1 > U^0$

Considering this, we know $\hat{T}(m_2) = -pU_Y^1 \;\; + \; pU_Y^0 > 0$.

Define curve $\tilde{T}(m_1) = 0$ as the locus of (m_1, m_2) which satisfies

$$\tilde{T}(m_1) = p'(U^1 - U^0) - \left[pU_Y^1 + (1-p)U_Y^0\right] = 0$$

And define curve $\hat{T}(m_2) = 0$ as the locus of (m_1, m_2) which satisfies

$$\hat{T}(m_2) = -pU_Y^1 \;\; + \; pU_Y^0 = 0.$$

Then we obtain the following results.

If $\hat{T}(m_2) = 0$, then $\tilde{T}(m_1) < 0$.

If $\tilde{T}(m_1) = 0$, then $\hat{T}(m_2) > 0$.

We can never find values along Eq. (5.74) = 0 for which Eq. (5.75) = 0, since whenever Eq. (5.74) = 0, then Eq. (5.75) < 0. Therefore, at every point along the curve for Eq. (5.74) = 0, the agent wants to *reduce* m_1 and thus increase risk. If it were possible to increase risk so much that $p < 0$ and $(1-p) > 1$, and be compensated for this increase in risk by negative m_1, the agent will want to do this.

Everywhere along the curve $\tilde{T}(m_1) = 0$, we see that $dW/dm_2 > 0$, meaning that the agent would want to insure more at each value of m_2. Thus it follows that the agent will not provide both types of security at the same time, and instead would normally provide $m_2 > 0$ only.

5.3.5 *General Case: Self-Insurance and Self-Protection*

5.3.5.1 Optimization Problem

According to the story in our introductory narrative in Sect. 5.3.2.1 the island nation subject to flooding, can reduce its frequency by building flood-barriers, dikes, channels etc. and can reduce the magnitude of loss by stockpiles, say of food and other necessities, self-insuring at an actuarially fair price, $\pi = (1-p)/p$. Here we focus on how these two instruments interact with each other. In particular, we show how the effectiveness of insurance plus protection varies crucially with the quality of information shared within the government and of cooperation among insurance and protection measures, programs, or bureaus.

Since agents in a group or countries in an alliance may spend on both insurance and protection, we should be interested in how any one agent's incentives interact with respect to the two instruments when both are available. Specifically, taking fair insurance to be the norm, we reasonably assume that countries anticipate that their expenditures on self-protection influence the price of self-insurance. Then, we ask "how does this anticipation influence the optimal choice of m_1 and therefore of $p(m_1)$?" In other words, suppose the government knows all these relationships as shown by Eqs. (5.52a, b) with certainty. It knows the effect of dike height and cost on frequency, and knows it can self-insure at fair prices.

To begin we repeat Eq. (5.52a) now including both variables m_1 and m_2 and inserting the condition for actuarial fairness, $\pi = [(1-p)/p]$ directly:

$$W = p(m_1)U^1[Y - m_1 - m_2] + (1 - p(m_1))U^0\left[Y - m_1 - \left\{\overline{L} - \mathbf{L}\left[\frac{m_2 p(m_1)}{1 - p(m_1)}\right]\right\}\right] \tag{5.52a}$$

As derived in Eq. (5.53),—assuming that the government accepts m_1 as a parameter and thus disregards any anticipated change in $p(m_1)$ and therefore in π—the FOC with respect to m_2 gives

$$S = -U_Y^1[Y - m_1 - m_2] \;\; + \mathbf{L}'U_Y^0\left[Y - m_1 - \left\{\overline{L} - \mathbf{L}\left[\frac{m_2 p(m_1)}{(1 - p(m_1))}\right]\right\}\right] = 0 \tag{5.76}$$

A further gain or loss is due to the change in m_2 induced by the marginal increase in m_1. This last effect depends on the risk aversion parameter and declines in importance with risk, p. We can see this effect from the implicit relation between m_1 and m_2 represented in (5.76). Taking the total differentiation of (5.76), we obtain

$$\frac{dm_2}{dm_1} = -\frac{\partial S/\partial \mathrm{m}_1}{\partial S/\partial \mathrm{m}_2}$$
$$= -\frac{U_{YY}^1 + U_{YY}^0\mathbf{L}'\left\{-1 + p'\mathrm{m}_2\mathbf{L}'\left[\frac{1}{(1-p)} + \frac{p}{(1-p)^2}\right]\right\} + U_Y^0\mathbf{L}''m_2 p'\left[\frac{1}{(1-p)} + \frac{p}{(1-p)^2}\right]}{U_{YY}^1 + U_{YY}^0(\mathbf{L}')^2\frac{p}{(1-p)} + \frac{p}{(1-p)}U_Y^0\mathbf{L}''} \tag{5.77}$$

We can simplify Eq. (5.77) noting that the denominator must be negative and using the definition $R = -U_{YY}/U_Y$, where R is the coefficient of risk aversion. Then

$$\frac{dm_2}{dm_1} > 0$$

if and only if

$$-R_1U_Y^1 + R_0U_Y^0\mathbf{L}' - R_0U_Y^0\mathbf{L}'\left\{p'm_2\mathbf{L}'\left[\frac{1}{(1-p)^2}\right]\right\} + U_Y^0\mathbf{L}''m_2p'\left[\frac{1}{(1-p)^2}\right] > 0 \tag{5.78}$$

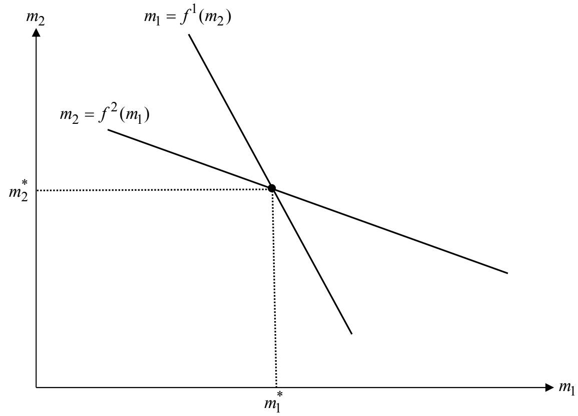


Fig. 5.10 Optimal self-insurance and self-protection. *Source* Authors

Then from the FOC of Eq. (5.68), under constant risk aversion the first two terms of the second inequality of (5.78) cancel (or under increasing risk aversion these two terms sum to a negative amount), so that in either of these two cases it follows that increases in expenditures on protection reduce optimized expenditures on self-insurance, that is $dm_2/dm_1 < 0$.

The FOC for determining optimal expenditure on self-protection m_1 becomes:

$$p'\left(U^1 - U^0\right) - \left[pU_Y^1 + (1-p)U_Y^0\right] + \frac{U_Y^0 p' \mathbf{L}' m_2}{1-p} = 0. \tag{5.79}$$

Therefore, we know that the optimum can be derived *and implemented* from solving these two equations:

$$\frac{\partial W}{\partial m_1} = \phi^1(m_1, m_2) = 0 :\Rightarrow m_1 = f^1(m_2) \tag{5.80}$$

and

$$\frac{\partial W}{\partial m_2} = \phi^2(m_1, m_2) = 0 :\Rightarrow m_2 = f^2(m_1) \tag{5.81}$$

Equation (5.80) gives us m_1^* and that value of m_1 plus Eq. (5.81) gives us m_2^*. We could draw the curves $m_1 = f^1(m_2)$ and $m_2 = f^2(m_1)$—as in Fig. 5.10—in space $m_1 - m_2$. At their intersection we would have the true optimal values of m_1^* and m_2^*.

5.3.5.2 Bureaucratic Coordination

Now to see the effects of compartmentalization, suppose, rather than a unitary decision maker, there are an insurance branch and a protection branch within the government each charged with providing its own type of security m_2 and m_1 respectively. If we assume full information, then that government would understand its optimization condition to be as shown in Eqs. (5.80) and (5.81), and would strive to implement m_1^* and m_2^*. How could it do that? Here are some approaches to that question.

As to bureaucratic coordination, it is useful to consider the following cases.

Case A. Black Box Approach: Just assume that the government makes all the calculations, derives the answer and instructs the insurance branch to spend m_2^* and the dike-building protection branch to spend m_1^*.

Case B. Central Control + Decentralized Execution: The government informs each branch of its correct reaction function, i.e. $m_1 = f^1(m_2) \;\; and \;\; m_2 = f^2(m_1)$. Note that the first of these incorporates the effect of m_1 on π. From any starting point in Fig. 5.10, each branch moves toward its correct reaction function, until the intersection and therefore, the optimum, of m_1^* and m_2^* is reached.

Case C. Hierarchical Execution. The government entrusts one branch to make a final "all at once decision." This avoids the zig-zag iteration implied by case B. Instead, the government gives the function $m_2 = f^2(m_1)$ to the protection branch or the function $m_1 = f^1(m_2)$ to the insurance branch. The leader-branch that receives the reaction function then implements it and solves for its own optimum after which the follower-branch just follows its reaction function.

For example, we assume that $\mathbf{L}(m_2/\pi) = m_2/\pi$ and that the central authority could give the responsibility for coordination to the protection branch, which would calculate its benefit/cost function as

$$\frac{dW}{dm_1} = p'(U^1 - U^0) - \left[pU_Y^1 + (1-p)U_Y^0\right] + (1-p)U_Y^0\left[\frac{m_2 p'}{(1-p)^2} + \frac{p}{1-p}\frac{df^2(m_1)}{dm_1}\right] = 0 \tag{5.82}$$

with the implied dependence of m_2 on m_1 given by $m_2 = f^2(m_1)$ from the insurance branch FOC as above. So

$$\frac{df^2(m_1)}{dm_1} = -\frac{\partial^2 W/\partial m_1 \partial m_2}{\partial^2 W/\partial (m_2)^2} = -\frac{p'\left(U_Y^0 - U_Y^1\right) + p\left[U_{YY}^1 + U_{YY}^0\left\{-1 + \frac{p' m_2}{(1-p)^2}\right\}\right]}{p\left[U_{YY}^1 + U_{YY}^0 \frac{p}{(1-p)}\right]} \tag{5.83}$$

From $U_Y^1 = U_Y^0$ it follows $U^1 = U^0$ and $U_{YY}^1 = U_{YY}^0$ at the optimum. Therefore, Eq. (5.83) simplifies to

$$\frac{df^2(m_1)}{dm_1} = -\frac{p' m_2}{(1-p)} \tag{5.84}$$

To sum up, if a nation, the large agent, may spend on both insurance and protection with full information, we expect that both m_1 and m_2 be provided. Thus, we can avoid the difficulty of corner solutions where only m_2 is provided by the small agent with linear technology and as a price taker.

5.3.6 Mutual Provision of Public Goods

We now consider the case where two countries A and B provide public goods in forms of spending on self-insurance and self-protection. We will investigate this case more fully in Chap. 6. We have shown that in the case of linear technology of market insurance, it is difficult to provide both types of spending. Thus, we only consider the more plausible case where the technology is decreasing and each country may provide the self-insurance. In this case, each country could provide both types of public goods. However, as shown in Sect. 5.3.5, each type of security spending becomes inferior if we expect absolute risk aversion to decrease with wealth.

As shown in Sect. 5.2 (see also Ihori and McGuire (2007) and Kerschbamer and Puppe (1998)), if the public good is inferior, the Nash equilibrium solution becomes unstable and we eventually have a corner solution where only the one country provides the public goods. Intuition is as follows. Suppose one country (the home country) provides security. This would produce a positive externality effect on another country (the foreign country), and hence its effective income rises. Due to the inferiority of public goods, the foreign country would then decrease its provision of the public goods. This in turn yields a negative externality on the home country, which then would react further to raise its provision. Thus, at a Nash equilibrium only the home country alone provides the public goods.

Now suppose both types of security spending are inferior. Since there are two types of public goods and each type is provided only by one country, if these are inferior goods, we have two possibilities.

Case I: Centralized specialization where the one country, say, A provides both types of public goods, while the other country, B, provides none. Suppose country A provides both m_1 and m_2, while B provide none. Here country B free-rides on the provision by A, and the, welfare of country B is higher than welfare of A.
Case II: Decentralized specialization where the one country, say, A, provides one type of public good, while the other country, B, provides another type. Suppose country A provides m_1, while country B provides m_2. Country B partially free-rides on the provision of m_1 by country A, and country A also partially free-rides on the provision of m_2 by B. Now, the welfare of B could be higher or lower than the welfare of A. In short, welfare comparisons become ambiguous.

To sum up, in a world of mutual provision public goods by two countries both insurance and protection are provided to both the two countries, which is to say spending on security becomes a public good. However, either of these goods may become inferior with the result that each type of the security spending is only provided by one country and the other may free ride. Under these conditions the total level of provision will be too little and cooperation among countries notably difficult. Chapter 6 will clarify this difficulty in more detail using numerical examples.

5.4 Conclusion

We have extended the VPG model for analysis of group behavior when risk is a collective bad, indivisibly shared by all members of a group, and its control, therefore, becomes a collective good. Such analysis necessarily includes the effects of increasing-costs/diminishing-returns in public good provision. We have incorporated this effect here by taking preferences over cost inputs as primitive objects—which allows exploitation of the "summation finance" features of the problem. Effectively this innovative definition of the public good as the aggregate of costs contributed by all agents or countries taken together allows infra-marginal effects of increasing costs to be folded into income effects.

Since all interactions between agents in an alliance are mediated through income effects, we have focused on these to show when a public good, M, is inferior and when it is normal. This important property of group interaction we show follows from properties of agents' preferences with respect to risk. When it is risk control that is the public good we show that interactions between preferences—characterized as high versus low absolute risk aversion, and increasing, constant or decreasing risk aversion—and objective risk levels will decisively influence the security interaction among group members. This leads to surprising new properties of Cournot behavior and equilibria in risk control. Since the conventional literature deals mainly with the implications of absolute risk aversion for public goods inferiority, we have conducted an excursion, testing the validity of these results for the case of relative risk aversion. Using the CRRA utility function for decreasing *relative* risk aversion i.e. $U(Y) = Y^{1-\alpha}/(1-\alpha)$, Ihori and McGuire (2007) showed that when $p = 1$ the public good of risk reduction is likely to be inferior, while when $p = 0$ it is likely to be normal. This, analysis, therefore, indicates that just as in the canonical case developed in the text, in the case of declining relative risk aversion, when $R_0 > R_1$ there is a critical risk p^* such that for $p > p^*$ the good is inferior, while for $p < P^*$ M is normal.

Section 5.2 has demonstrated that adding new members and/or economic growth may produce "an inferior good barrier" and if this occurs, further increases in an alliance's membership will not reduce the probability of a bad outcome by providing the public good M. Moreover, systematic patterns of change to and from normality/inferiority are to be expected. In fact, for any configuration there will be a critical risk that together with other inputs determines a crossover point from normality to inferiority or vice versa. Such crossover values moreover, will define barriers to risk improvement and even make growth in group membership a cause for decline in public good provision.

Our analysis along this line has shown that goods inferiority is much less unlikely when collective risk control is at stake than in the run-of-the-mill VPG example.

Accordingly, instabilities in Nash-Cournot outcomes and absence of interior solutions are altogether more likely than in the received textbook case. Reflection suggests that this analysis is bad news for managing multi-country interactions in risk reduction. There are indeed many world risk problems where collective action is needed, with voluntary provision being the minimal level of such "cooperation." Multinational disease control, coping with terror threats, environmental risks is often quite close to pure public goods. But if this analysis is correct, any complacency which our old friend the VPG model can induce is quite out of place here, since stable VPG behavior is highly vulnerable to breakdown when the object is risk control.

The second half of this chapter, Sect. 5.3, has investigated several types of preparation available to expected utility maximizing agents faced with "costs of emergency", self-insurance and self-protection. Self-insurance provided to itself by a large entity such as a nation differs in two important respects from standard market insurance. First of all, self-insurance differs from standard market insurance in that self-insurance function should show diminishing returns or increasing costs. Second, the agent, or country, may incorporate the actuarially fair condition at its optimization. We have investigated how these provisions of self-insurance and self-protection become public goods, and have demonstrated an inherent potential for an "unstable outcome" when perfect specialization in provision of these public goods obtains.

If a small agent provides security spending in an insurance market as a price taker, then the standard result holds. Namely, with fair linear pricing, complete coverage (net of premiums) is purchased from resources available in the good contingency. We have used this benchmark result to compare with self-insurance by an entire country, and also in an examination of the effect of insurance on the choice of protection. It has been shown that if a nation provides its own self-insurance, complete coverage (net of premiums) may not be purchased from resources available in the good contingency due to decreasing productivity of spending on self-insurance. Moreover, if a small agent tries to provide both self-insurance and self-protection, it will not provide both at the same time under a linear price-taker technology. Insurance provision by nations could solve this difficulty since self-insurance is most likely produced by decreasing returns technology and at optimization is provided at actuarially fair price-conditions. However, when we consider combined mutual provisions of the public goods by nations, we may have yet another difficulty due to inferiority.

If we expect absolute risk aversion to decrease with wealth, we will also expect both types of public goods to be inferior, and hence the Nash equilibrium would be given by corner solutions. In such a case we observe specialization in provision of public goods. We have shown that in the case of centralized specialization the free-riding country always gains, while in the case of decentralized specialization both countries free-ride on each other and welfare comparison becomes ambiguous. In either case, the total level of security spending is too little and cooperation between countries in the provision of security spending is difficult. We have thus demonstrated an inherent potential for unstable conflicts among allies with respect to the public-(i.e. joint or non-rival) provision against adversity.

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Chapter 6
Exploitation Hypothesis and Numerical Calculations

6.1 Introduction

Today, more countries than ever face the risk of a disastrous event. Examples of such events include invasion, terrorist attacks, environmental disasters, and infectious disease pandemics. Governments need not passively accept risks that production and/or consumption will decline in unwelcome situations. They try to manage these risks through joint operations in alliances.

As discussed in the previous chapters, Ehrlich and Becker (1972) identified several types of preparations available to utility maximizing agents faced with what we call "costs of emergency." These costs consist of any mix of (a) probability of loss and (b) magnitude of loss (hereafter together referred to as "risk profile"). Among such preparations are (a) "self-insurance" to compensate for or reduce the magnitude of loss and (b) "self-protection" to reduce the probabilities of loss.

Going beyond individual countries, when insurance/protection is provided as a pure public good by nations in an allied group, all allies can enjoy the positive spillovers from such goods, although freeriding incentives remain in our framework. It should be stressed that public goods are not necessarily provided by both countries. In some cases, a single country provides a public good and its allies ride free. In our two-country model, when a public good is provided by two countries, we call that situation a "multilateral provision of public good." When a single country provides the public good, we call the situation a "unilateral provision of public good." Because we investigate two different types of public goods, there are at least four cases to consider. As discussed in this chapter, the configuration of the contributors—who contributes to which public goods—is critically important in the burden sharing of risk management.

Table 6.1 presents examples corresponding to the four provision statuses of public goods. In the top left cell, self-protection and self-insurance are multilaterally provided. In top right cell, self-protection is provided unilaterally, whereas self-insurance is provided multilaterally. For example, the allies in the North

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Table 6.1 Examples of multilateral/unilateral provision of risk management public goods

		Self-protection	
		Multilateral provision	Unilateral provision
Self-insurance	Multilateral provision	US/UK alliance	NATO
	Unilateral provision	Prevention of pandemic	US/Developing country alliance

Source Authors

Atlantic Treaty Organization (NATO) can be divided into two groups. One group of countries provides both nuclear deterrence and conventional forces to prevent invasion. The other group of countries provides only conventional forces. The former group includes the US and UK, whereas the latter group includes Germany and Italy, among others. When we consider the former group as an alliance, the self-protection and self-insurance of this alliance are provided multilaterally, corresponding to the top left cell. If we consider NATO as an alliance that consists of these two groups and assume each group is an individual agent, the self-protection of the alliance is provided unilaterally by the former group, whereas self-insurance is provided multilaterally by both groups, corresponding to the top right cell. In the bottom right cell, self-protection and self-insurance are provided unilaterally. This case corresponds to the US/developing country relationship. In the bottom left cell, self-protection is multilaterally provided but self-insurance is unilaterally provided. Examples of this case include the prevention of emerging infectious disease pandemics. The probability of a pandemic depends on vaccination and other efforts conducted by countries throughout the world. However, the containment and treatment of the disease itself can be achieved only in the country in which the outbreak occurs.

Although the framework of Ehrlich and Becker (1972) is applicable to many security issues in international risk management, utilization of the Ehrlich and Becker framework to model collective improvements to the entire "risk profiles" as international public goods is sparse, with the exception of works by Sandler (1992, 1997, 2005), Lohse et al. (2012), and Muermann and Kunreuther (2008). In particular, economists' voluntary public good (VPG) models, such as those of Bergstrom et al. (1986) and Boadway and Hayashi (1999), have not been well extended to understand incentives and behaviors of sovereign agents desiring to manage risks along multiple channels analogous to Ehrlich and Becker's (1972) self-protection and self-insurance in a multi-country setting. In the context of international risk management, Sandler and Hartley (1999) provided a comprehensive review of the political economy of NATO. However, they used the conventional voluntary provision of the public good model and joint production model to analyze burden sharing in NATO. Ihori et al. (2014) investigated burden sharing in NATO with a VPG model in which Ehrlich and Becker's (1972) self-protection and self-insurance are included as international public goods within the alliance based on numerical simulations. Thus far, it appears that the theory explaining how allies non-cooperatively share the burden of provision of self-insurance and self-protection public goods has not been well developed.

The purpose of this chapter is twofold. First, we theoretically analyze simultaneous optimization of both self-protection and self-insurance by two countries facing the common risk of a disastrous event to derive simple analytical rules in the distribution of contributions. In this analysis, we provide a new perspective from the theory of collective risk management to reveal how allies share the burden of self-insurance and self-protection public goods. Second, we utilize our model to conduct numerical simulations of burden sharing in NATO from 1970 to 2010. In this simulation, we show that whether the conventional exploitation hypothesis holds depends on the risk profile that NATO faces. Our calculated results closely simulate the actual development of the military spending to Gross Domestic Product (GDP) ratio.

The organization of this chapter is as follows. In Sect. 6.2, we first review the conventional exploitation hypothesis using a conventional model of the voluntary provision of public goods, which we discussed in Chap. 3. Next, we theoretically investigate burden sharing in the two-country risk management model developed in Chap. 4. We explore how the difference in income and loss determine the difference in contributions made by the two countries. In Sect. 6.3, we conduct some numerical simulations of the scenario corresponding to the actual burden sharing in NATO from the Cold War era to the 21st century. Section 6.4 concludes.

6.2 Exploitation Hypothesis and Neutrality Results

In this section, we investigate the exploitation hypothesis and the neutrality of income redistribution. In Sect. 6.2.1, we review the conventional exploitation hypothesis. In Sect. 6.2.2, we use the collective risk management model developed in Chap. 5 to investigate burden sharing in the alliance. In Sect. 6.2.3, we review the plausibility of the neutrality results of income redistribution in our model. In this section and the following sections of this chapter, we assume that each country is a unitary decision maker maximizing its expected utility subject to its budget constraints. We assume that when a country chooses its allocation of private consumption and contributions to self-insurance and self-protection public goods, it considers the strategy of the other country as given in a Nash conjecture.

6.2.1 Conventional Exploitation Hypothesis

Buchholz and Sandler (2016) reviewed the conventional exploitation hypothesis established by Olson (1965) and established alternative exploitation hypotheses. They summarized the conventional exploitation hypothesis as follows:

> This hypothesis essentially means that, in a Nash equilibrium of voluntary public good provision, the rich (better endowed) agents make larger contributions to the public good

> than the poor agents so that the rich agents, in a certain sense, are "exploited" in the strategic context of the contribution game.
>
> (Buchholz and Sandler 2016, pp. 103–104)

Next, they showed that exploitation may arise not only because players are different in their endowments, but also in their preferences, co-benefits of contribution, or productivity of the public good. The conventional exploitation hypothesis focuses on the difference in the endowments.

In this subsection, we review the conventional exploitation hypothesis.[1] We assume that players are identical except for their endowments.

We consider a two-country economy in which there is a private and public good. The two countries are indexed by A and B. We assume that the utility functions of the two countries are identical and that both goods are normal goods. The welfare of country A, W^A, is given as:

$$W^A = V(C^A, G), \tag{6.1}$$

where $V(.)$ is the utility function, C^A is the private good consumption of A, and G is the provision of the public good. We assume that $V(.)$ is twice continuously differentiable, increasing and strictly concave in all arguments. The budget constraint of country A is given as:

$$Y^A = C^A + g^A, \tag{6.2}$$

where Y^A is the national income of country A and g^A is country A's contribution to the public good. The utility and the budget of country B are expressed by replacing the superscript A with B in Eqs. (6.1) and (6.2). Without loss of generality, we assume that country A's national income is higher than B's:

$$Y^A > Y^B. \tag{6.3}$$

The provision of the public good is given as the total of contributions. Formally, we assume:

$$G = g^A + g^B. \tag{6.4}$$

Following Bergstrom et al. (1986), it is easy to show that the private good consumption of both countries becomes identical. Using Eq. (6.4), the budget constraint of country A is rewritten as:

$$c^A + G = Y^A + g^B. \tag{6.5}$$

[1]Please refer to Buchholz and Sandler (2016) for a comprehensive review of the conventional exploitation hypothesis.

The right-hand side of Eq. (6.5) is the "full income" of country A. Using Eq. (6.5), we rewrite country A's maximization problem as:

$$\max_{C^A, G} V(C^A, G)$$
$$\text{s.t.} \;\; c^A + G = Y^A + g^B \text{ and } G \geq g^B.$$

Solving this maximization problem, we obtain country A's demand for the public good as:

$$G = \phi(Y^A + g^B). \tag{6.6}$$

Because both goods are normal goods, function ϕ is monotonic increasing. Next, we have the inverse function of ϕ and obtain:

$$Y^A + g^B = \phi^{-1}(G). \tag{6.7}$$

Subtracting G from both sides, we have:

$$Y^A - g^A = \phi^{-1}(G) - G.$$

Because function ϕ is identical between countries, we obtain:

$$Y^A - g^A = Y^B - g^B. \tag{6.8}$$

Rearranging Eq. (6.8), we derive:

$$g^A - g^B = Y^A - Y^B, \tag{6.9}$$

which means that a high-income country contributes more to the public good than a low-income country and that the difference in contribution is exactly equal to the difference in income. If Eq. (6.9) does not hold, the utilities of the players are not identical. We revisit this possibility in our framework later.

6.2.2 Analytical Framework

In this section, we extend the model we developed in Chap. 5 to a two-country model. With this model, we investigate burden sharing in the collective risk management of an alliance.

We consider a two-country economy of countries A and B. We assume that there are two contingent states of the world: a good state ("1") and a bad state ("0"). The expected utility of country A is given as:

$$W^A = pU(C^{1A}) + (1-p)U(C^{0A}), \tag{6.10}$$

where W^A represents country A's expected utility, p is the chance of a good state, $U(C^{1A})$ is A's realized utility in a good state, $U(C^{0A})$ is its realized utility in a bad state, C^{1A} is its consumption in a good state, and C^{0A} is its consumption in a bad state. The variable $(1-p)$ may be interpreted as the risk of trade interruption, disease outbreak, environmental calamity, or war.

We assume that both countries A and B have the same preferences. If we assume that both countries have different preferences, the difference itself may cause exploitation. Therefore, to avoid complexity, we assume identical preferences. The utility function $U(.)$ is also assumed to be the same whether the state of the world is good or bad. We assume that $U(.)$ is twice continuously differentiable, strictly increasing, and strictly concave. Following Ihori et al. (2014), we denote the first and second derivatives of the utility function with U_Y and U_{YY}, respectively. We then obtain:

$$U_Y(C) \equiv \frac{dU(C)}{dC} > 0 \text{ and } U_{YY}(C) \equiv \frac{d^2U(C)}{d(C)^2} < 0. \tag{6.11}$$

We also assume the utility function satisfies the Inada condition:

$$\lim_{C\to 0} U_Y(C) = +\infty \text{ and } \lim_{C\to +\infty} U_Y(C) = 0. \tag{6.12}$$

The probability of the good state depends on the provision of a self-protection public good, which is assumed to be the sum of the contributions made by both countries. Formally, we assume the following:

$$p = p(M_1), \tag{6.13}$$

where $M_1 = m_1^A + m_1^B$ is the provision of the self-protection public good, m_1^A is country A's contribution to the self-insurance public good, m_1^B is country B's contribution to that public good, and $p(.)$ is the probability function. We also assume that the probability of a good state increases with the provision of the self-protection public good and shows diminishing returns:

$$p' \equiv \frac{dp}{dM_1} > 0, \quad p'' \equiv \frac{d^2p}{dM_1^2} < 0. \tag{6.14}$$

Country A's budget constraints in good and bad states are respectively given as:

$$C^{1A} = Y^A - m_1^A - m_2^A, \tag{6.15}$$

$$C^{0A} = Y^A - \bar{L}^A - m_1^A + \mathrm{L}(s), \tag{6.16}$$

where Y^A denotes country A's national income, m_1^A is country A's allocation to self-protection, m_2^A is the self-insurance premium paid by country A in the good state, $\overline{L}^A$ is country A's loss in the bad state, L(s) is the self-insurance benefit in the bad sate, and s is the self-insurance input in the bad state. Without loss of generality, we assume that country A is endowed with the same or higher national income than B:

$$Y^A \geq Y^B. \tag{6.17}$$

We also assume that C^{1A} and C^{0A} are non-negative.

The self-insurance benefit is formalized as follows. We assume that both the self-protection and self-insurance benefits are public goods in this alliance. The benefit function L(s) is assumed to be increasing and concave. We also assume that the marginal product of the self-insurance benefit is lower than the unity.[2] To summarize, we assume the following:

$$\mathrm{L}' \equiv \frac{d\mathrm{L}}{ds} \in (0, 1),\ \ \mathrm{L}'' \equiv \frac{d^2\mathrm{L}}{ds^2} \in (-\infty, 0]. \tag{6.18}$$

The self-insurance input, s, is given as the total self-insurance premium divided by the price of self-insurance. We assume that the price of self-insurance follows actuarial fairness. Thus, we have:

$$s = \frac{M_2}{\left(\frac{1-p}{p}\right)} = \frac{pM_2}{1-p}, \tag{6.19}$$

where $M_2 = m_2^A + m_2^B$ is the total payment of the self-insurance premiums paid by both countries. Substituting Eq. (6.19) into Eq. (6.16), we obtain:

$$C^{0A} = Y^A - \bar{L}^A - m_1^A + \mathrm{L}\left(\frac{pM_2}{1-p}\right). \tag{6.20}$$

Country B's utility function is defined as:

$$W^B \ = pU(C^{1B}) + (1-p)U(C^{0B}), \tag{6.21}$$

where W^B is its expected welfare, C^{1B} is its consumption in the good state, and C^{0B} is its consumption in the bad state. Country B's budget constraints are defined as:

$$C^{1B} = Y^B - m_1^B - m_2^B, \tag{6.22}$$

[2]As shown in the next section, the optimal self-insurance allocation implies that if $\mathrm{L}' > 1$, the consumption in the good state is lower than that in the bad state, $C^{1A} < C^{0A}$. To preclude this counter-intuitive case, we assume $\mathrm{L}' \leq 1$.

$$C^{0B} = Y^B - \bar{L}^B - m_1^B + \mathrm{L}\left(\frac{pM_2}{1-p}\right), \tag{6.23}$$

where Y^B denotes its national income, $\bar{L}^B$ is its loss in the bad state, m_1^B is its contribution to the self-protection public good, and m_2^B is its self-insurance premium payment.

6.2.3 Individual Optimization

Let us consider the maximization problem for country A. The government of country A maximizes its expected utility, Eq. (6.10), subject to its budget constraints, and Eq. (6.13), with respect to its allocation to self-insurance and self-protection, m_1^A and m_2^A.

The first order condition for interior optimal self-protection is given as:

$$\frac{\partial W^A}{\partial m_1^A} = p'\left(U^{1A} - U^{0A}\right) - \left\{pU_Y^{1A} + (1-p)\left(1 - \frac{\partial \mathrm{L}}{\partial M_1}\right)U_Y^{0A}\right\} = 0, \tag{6.24}$$

where we omit the argument of the utility function as follows:

$$U^i \equiv U(C^i),\ U_Y^i \equiv U'(C^i),\ U_{YY}^i \equiv U''(C^i) \text{ for } i = 0A,\ 1A,\ 0B,\ 1B. \tag{6.25}$$

Equation (6.24) represents the first order condition for the optimal expenditure on self-protection. The first term of the right-hand side of this equation is the marginal benefit of self-protection, which represents how much the expected welfare increases with the marginal increase in the probability of the good state. The second term is the marginal cost of self-protection. Country A allocates its resources to self-protection not only in the good state but also in the bad state. In the good state, one additional unit of self-protection reduces private consumption in the state by exactly one unit. Also, in the bad state, one additional unit of self-protection directly reduces private consumption by one unit. Indirectly, it reduces the price of self-insurance, increases the self-insurance input, and increases the self-insurance benefit by $\partial \mathrm{L}/\partial M_1$. For notational simplicity, we define the marginal impact of a self-protection purchase on the self-insurance benefit as μ. We then obtain:

$$\mu \equiv \frac{\partial \mathrm{L}}{\partial M_1} = \frac{M_2 p' \mathrm{L}'}{(1-p)^2}. \tag{6.26}$$

Using Eq. (6.26), we rewrite Eq. (6.24) as:

$$\frac{\partial W^A}{\partial m_1^A} = p'\left(U^{1A} - U^{0A}\right) - \left\{pU_Y^{1A} + (1-p)(1-\mu)U_Y^{0A}\right\} = 0. \tag{6.27}$$

The first order condition for an interior optimal value of self-insurance is given as:

$$\frac{\partial W^A}{\partial m_2^A} = p\big(\mathrm{L}'U_Y^{0A} - U_Y^{1A}\big) = 0, \tag{6.28}$$

which implies the following: if country A increases its payment of the self-insurance premium, m_2^A, by one unit, country A's receipt of the self-insurance benefit increases by $\mathrm{L}'p/(1-p)$ units. Thus, a one-unit increase in the self-insurance payment decreases the utility in the good state by U_Y^{1A} and increases the utility in the bad state by $\mathrm{L}'pU_Y^{0A}/(1-p)$. Because the probability of the good state is p, the probability-weighted expected marginal cost of one unit of a self-insurance purchase is pU_Y^{1A}. Similarly, because the probability of the bad state is $1-p$, the expected marginal benefit of the increasing self-insurance is $p\mathrm{L}'U_Y^{0A}$. Equation (6.28) implies that the expected marginal benefit is equal to the expected marginal cost.

In our model, self-insurance and self-protection are public goods in the alliance. We note that each country may choose to contribute nothing to these public goods. In other words, m_1^A and m_2^A may become zero. From the Kuhn-Tucker condition of utility maximization, the left-hand side (LHS) of Eq. (6.27) is non-negative if $m_1^A = 0$ and the LHS of Eq. (6.28) is non-negative if $m_2^A = 0$.

The second order conditions for utility maximization are given as:

$$\frac{\partial^2 W^A}{\partial (m_1^A)^2} < 0, \tag{6.29}$$

$$\frac{\partial^2 W^A}{\partial (m_2^A)^2} < 0, \tag{6.30}$$

$$\frac{\partial^2 W^A}{\partial^2 m_1^A}\frac{\partial^2 W^A}{\partial^2 m_2^A} - \left(\frac{\partial^2 W^A}{\partial m_1^A \partial m_2^A}\right)^2 > 0. \tag{6.31}$$

We assume that Eqs. (6.29), (6.30), and (6.31) hold.

Solving the first order conditions, we obtain country A's best response functions as follows:

$$m_1^A = m_1^A(m_1^B, m_2^B, Y^A, \overline{L}^A), \tag{6.32}$$

$$m_2^A = m_2^A(m_1^B, m_2^B, Y^A, \overline{L}^A). \tag{6.33}$$

6.2.4 Nash Equilibrium

We define a Nash equilibrium in our model as a vector $\big(m_1^{A*}, m_2^{A*}, m_1^{B*}, m_2^{B*}\big)$ satisfying the following system of equations:

$$m_1^{A*} = m_1^A(m_1^{B*}, m_2^{B*}, Y^A, \bar{L}^A),$$
$$m_2^{A*} = m_2^A(m_1^{B*}, m_2^{B*}, Y^A, \bar{L}^A),$$
$$m_1^{B*} = m_1^B(m_1^{A*}, m_2^{A*}, Y^B, \bar{L}^B),$$
$$m_2^{B*} = m_2^B(m_1^{A*}, m_2^{A*}, Y^B, \bar{L}^B).$$

We denote the total of country A's contributions to two public goods in a Nash equilibrium as $m^{A*}(\equiv m_1^{A*} + m_2^{A*})$. Similarly, we denote country B's total contribution as $m^{B*}(\equiv m_1^{B*} + m_2^{B*})$. Variables corresponding to the Nash equilibrium are denoted by an asterisk.

Depending on whether the contribution made by each country is positive or zero, we can consider 16 types of Nash equilibria. Table 6.2 summarizes the classifications. The first five types are equilibria in which the equilibrium amounts of both public goods are positive. In an interior equilibrium, both countries contribute to both types of public goods. In a self-insurance freeriding equilibrium, one country contributes to both public goods, whereas the other country contributes only to self-protection. In a self-protection freeriding equilibrium, one country contributes to both public goods, whereas the other country contributes only to self-insurance. In a decentralized specialization equilibrium, one country contributes only to self-protection and the other country contributes only to self-insurance. In a centralized specialization equilibrium, one country contributes to both public goods and the other country does not contribute to either. The last two types of Nash equilibria are those in which both countries contribute only to one public good: a self-protection specialization equilibrium, in which they contribute only to the self-protection public good, and a self-insurance specialization equilibrium, in which they contribute only to the self-insurance public good.

In the following, we investigate the nature of burden sharing in these equilibria except for centralized specialization because it is clear that in centralized specialization, one country bears the entire burden of the provision of public goods in this Nash equilibrium. We assume that country A contributes to both public goods in the self-insurance and self-protection freeriding equilibria and that in the decentralized specialization equilibrium, country A specializes in self-protection and country B in self-insurance. The reasoning we follow can be applicable to the equilibrium in which country B contributes to both public goods and country A freerides in either public good and the equilibrium in which country A specializes in self-insurance and country B in self-protection.

6.2.5 Exploitation Hypothesis

In this subsection, we examine how countries share the burden of risk management. In our framework, countries differ both in their income and in their loss in the bad state. These two differences may cause various types of exploitation. The conventional exploitation of the high-income agent by the low-income agent might not always

Table 6.2 Classifications of Nash equilibria

	m_1^{A*}	m_1^{B*}	m_2^{A*}	m_2^{B*}
Interior equilibrium	+	+	+	+
Self-insurance freeriding equilibrium	+	+	+	0
	+	+	0	+
Self-protection freeriding equilibrium	+	0	+	+
	0	+	+	+
Decentralized specialization equilibrium	+	0	0	+
	0	+	+	0
Centralized specialization equilibrium	+	0	+	0
	0	+	0	+
	+	0	0	0
	0	+	0	0
	0	0	+	0
	0	0	0	+
Self-protection specialization equilibrium	+	+	0	0
Self-insurance specialization equilibrium	0	0	+	+
No contribution equilibrium	0	0	0	0

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hold. In addition, because we consider two different public goods, we may have corner solutions. In equilibria with corner solutions such as the self-insurance freeriding equilibrium, burden sharing may be different from that in an interior equilibrium. In this subsection, we will show that in an interior equilibrium, the difference in contribution is equal to that of the income difference, as in the conventional model. However, in equilibria with corner solutions such as the self-insurance or self-protection freeriding equilibrium, the difference in the contribution in our model between countries may be higher or lower than the difference in income.

The organization of this subsection is as follows. We first define different levels of exploitation: heavy, conventional, and light. Next, we consider five types of Nash equilibria in turn. We investigate the first order conditions of the Nash equilibrium and explore how the contributions of public goods in the equilibrium depend on the national income and the loss in the bad state.

6.2.5.1 Level of Exploitation

We define five levels of exploitation based on the levels of consumption. In our model, consumption in a good state is equal to income minus the contribution to two public goods. If the high-income country (country A) contributes more than the low-income country (country B) and if the difference in contribution is higher than that in income, the high-income country consumes less than the low-income country in a good state. Remembering our assumption that $Y^A \geq Y^B$ in Eq. (6.17), we define the following five levels of exploitation:

Definition We classify the Nash equilibrium of our model as follows:

(1) If the vector of total contributions made by countries A and B in a Nash equilibrium, (m^{A*}, m^{B*}), satisfies $m^{A*} > m^{B*}$ and $m^{A*} - m^{B*} > Y^A - Y^B$, we claim that the equilibrium exhibits a *heavy* exploitation of the high-income country by the low-income country.
(2) If the vector satisfies $m^{A*} > m^{B*}$ and $m^{A*} - m^{B*} = Y^A - Y^B$, we claim that the equilibrium exhibits a *conventional* exploitation of the high-income country by the low-income country.
(3) If the vector satisfies $m^{A*} > m^{B*}$ and $m^{A*} - m^{B*} < Y^A - Y^B$, we claim that the equilibrium exhibits a *light* exploitation of the high-income country by the low-income country.
(4) If the vector satisfies $m^{A*} = m^{B*}$ and if we have $Y^A = Y^B$, we claim that the equilibrium exhibits *no* exploitation.
(5) Otherwise, we claim that the equilibrium exhibits an exploitation of the low-income country by the high-income country.

Let us start with the first definition of exploitation. If country A contributes more to the public goods and if the difference in contribution is higher than the difference in income, the high-income country (A) consumes less in the good state than the low-income country (B). We refer to this case as a "heavy exploitation."

The second definition of exploitation is lighter than the first. If country A contributes more to the public goods and if the difference in contribution is exactly the same as the difference in income, both countries consume the identical amount in the good state. We call this a "conventional exploitation" of the high-income country by the low-income country in the sense that country A consumes the same amount as country B despite country A being endowed with a higher income than B.

The third definition is lighter than the second. Suppose country A contributes more than country B and the difference in contribution is lower than the difference in income. Also, country A consumes more than country B. Because country A contributes more to the public good than country B, we interpret country B as exploiting country A. Thus, we call this a "light exploitation" of country A by country B.

The fourth claim defines the non-existence of exploitation. This is a symmetric case in which both countries are endowed with an identical income and contribute an identical amount to the public goods.

The final definition is exploitation of country B by country A. If both public goods are not normal goods, we might observe this case.

Almost all equilibria contain an "exploitation" of one state by another. The only exception is the symmetric case. We now move our focus from the existence of exploitation to its degree. As shown next, our model exhibits various degrees of exploitation depending on the type of Nash equilibria.

Finally, we compare our definition with Olson and Zeckhauser's (1966) definition of exploitation. Olson and Zeckhauser (1966) and studies following them focused on the ratio of military expenditures to GDP. They suppose that country B exploits country A if the ratio of total contributions to the income of country A is higher than that of B. Heavy and conventional exploitations in our terminology correspond to the exploitations of Olson and Zeckhauser (1966). Let us suppose that $m^{A*} - m^{B*} \geq Y^A - Y^B$. We then obtain:

$$m^{A*} - m^{B*} \geq Y^A - Y^B \geq \frac{m^{B*}}{Y^B}\left(Y^A - Y^B\right) = \frac{m^{B*}Y^A}{Y^B} - m^{B*},$$

where the second inequality follows from the assumption that $m^{B*} \leq Y^B$. We then obtain:

$$m^{A*} \geq \frac{m^{B*}Y^A}{Y^B},$$

which implies

$$\frac{m^{A*}}{Y^A} \geq \frac{m^{B*}}{Y^B}. \tag{6.34}$$

Light exploitation in our terminology does not necessarily correspond to the exploitation of Olson and Zeckhauser (1966). If we have $m^{A*} - m^{B*} < Y^A - Y^B$ and if $m^{A*} - m^{B*} \geq \left(Y^A - Y^B\right)\frac{m^{B*}}{Y^B}$, we obtain Eq. (6.34). Thus, the low-income country exploits the high-income country in the sense of Olson and Zeckhauser (1966). However, if $m^{A*} - m^{B*} \leq \left(Y^A - Y^B\right)\frac{m^{B*}}{Y^B}$, we do not obtain Eq. (6.34). Rather, we obtain $\frac{m^{A*}}{Y^A} \leq \frac{m^{B*}}{Y^B}$, which means that the high-income country exploits the low-income country.

To summarize, if we have either heavy or conventional exploitation in our terminology, Olson and Zeckhauser's (1966) exploitation hypothesis holds. If we have light exploitation, Olson and Zeckhauser's (1966) exploitation hypothesis does not necessarily hold.

6.2.5.2 Interior Equilibrium

Let us assume that both countries contribute to both public goods in the Nash equilibrium.[3] We begin this section by discussing the applicability of the derivation of the

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[3]Cornes and Itaya (2010) argued that if two public goods are voluntarily provided in a two-player economy and both players have different preferences, there almost surely does not exist a Nash equilibrium in which both players simultaneously contribute to both public goods. Dasgupta and Kanbur (2005) independently found the theoretically identical result to that of Cornes and Itaya

conventional exploitation hypothesis to our model. Next, we derive two propositions. The first proposition (Proposition 6.1) shows that country A's consumption is equal to country B's consumption in the Nash equilibrium. The second proposition (Proposition 6.2) shows how the self-protection and self-insurance contributions made by the allies vary with the national incomes and the losses in the bad event of the two countries.

As shown in Sect. 6.2.1, the demand of a country for a public good corresponds one-to-one with the full income of the country in the conventional model. In Sect. 6.2.1, Eq. (6.6) shows that function ϕ determines country A's demand for the public good based on its full income. Using the inverse demand function, we obtain country A's full income as a function of the public good provision as in Eq. (6.7), which implies that country A's private good consumption depends only on the provision of the public good. Because both countries consume the same amount of the public good, their private good consumptions are identical.

However, we cannot necessarily apply the procedure we employed in Sect. 6.2.1 to our model with self-protection and self-insurance. Countries' demands for self-protection and self-insurance public goods depend not only on their income but also on their losses in the bad state. In other words, their demands for the public goods are determined by a combination of national income and emergency loss. Different combinations of income and loss may result in the same demand for the public goods in an interior solution.

In the following, we impose additional assumptions on the preferences to establish the equivalence of consumptions of the two countries. The result is summarized in the following proposition:

Proposition 6.1 *Suppose that all elements of the Nash equilibrium vector of contributions,* $(m_1^{A*}, m_2^{A*}, m_1^{B*}, m_2^{B*})$*, are positive and that*

$$\mathrm{L}' R^{0A} > R^{1A} \text{ and } \mathrm{L}' R^{0B} > R^{1B}, \tag{6.35}$$

where $R^i \equiv -U_{YY}(C^i)/U_Y(C^i)$ *for* $i = 0A,\ 1A,\ 0B,\ 1B$ *is the absolute risk aversion when consumption is* C^i*. We then have the following:*

$$C^{0A*} = C^{0B*} \text{ and } C^{1A*} = C^{1B*} \tag{6.36}$$

Proof In this proof, we solve the first order conditions of the optimal self-protection and self-insurance to define the optimal consumptions in the good and bad states as a function of provisions of public goods. Because both countries consume the identical amounts of public goods, their consumptions also become identical.

First, we rewrite the first order conditions of self-insurance and self-protection contributions, Eqs. (6.27) and (6.28), as follows:

(2010). Their claim is not applied to our model because players in our model face two contingent budget constraints. We provide a detailed discussion on the applicability of Cornes and Itaya (2010)'s claim in the appendix of this chapter.

$$p'\big(U(C^{1A}) - U(C^{0A})\big) - \big\{pU_Y(C^{1A}) + (1-p)(1-\mu)U_Y(C^{0A})\big\} = 0, \quad (6.37)$$

$$\mathrm{L}'U_Y(C^{0A}) - U_Y(C^{1A}) = 0. \quad (6.38)$$

In the conventional model of the voluntary provision of public goods, players choose their private good consumption and contribution to the public good. In our model, governments do not directly choose their private good consumptions or public good provisions. Thus, it is not straightforward that we can solve the system of Eqs. (6.37) and (6.38) to obtain C^{0A} and C^{1A} as functions of M_1 and M_2.

The procedure of solving Eqs. (6.37) and (6.38) is as follows. First, we solve Eq. (6.38) to obtain the optimal consumption in the good state as a function of the consumption in the bad state and provision of public goods. Second, we substitute the optimal consumption in the good state for the first order condition of the optimal self-protection to obtain the optimal consumption in the bad state as a function of the provision of public goods. Finally, we show that because public goods for country A are identical to those for country B in the Nash equilibrium, the consumption of country A is also identical to that of country B.

The first process is to derive the optimal consumption in the good state as a function of consumption in the bad state and the provision of public goods. We rewrite Eq. (6.38) as:

$$\mathrm{L}'\left(\frac{pM_2}{1-p}\right)U_Y(C^{0A}) - U_Y\big(C^{1A}\big) = 0. \quad (6.39)$$

We interpret Eq. (6.39) as an implicit function of C^{1A} for any given C^{0A}, M_1 and M_2. Because $U_{YY} < 0$, we derive the optimal consumption function in the good state as follows:

$$C^{1A} = e(C^{0A},\ M_1,\ M_2). \quad (6.40)$$

Because the utility function of country B is identical to that of A, we also obtain:

$$C^{1B} = e(C^{0B},\ M_1,\ M_2). \quad (6.41)$$

Function $e(.)$ has the following properties. From Eq. (6.39), we obtain:

$$\frac{U_Y^A(C^{1A})}{U_Y^A(C^{0A})} = \mathrm{L}'. \quad (6.42)$$

Because we assume $\mathrm{L}' < 1$ and $U_{YY} < 0$, we obtain:

$$e(C^{0A},\ M_1,\ M_2) = C^{1A} > C^{0A}. \quad (6.43)$$

By definition, it follows that:

$$U_Y(e(C, M_1, M_2)) = \mathrm{L}'\left(\frac{pM_2}{1-p}\right)U_Y(C) \text{ for any } C. \tag{6.44}$$

We then obtain the partial derivative of function $e(.)$ as[4]:

$$\frac{\partial e(C, M_1, M_2)}{\partial C} = \frac{\mathrm{L}'U_{YY}(C)}{U_{YY}(e(C, M_1, M_2))} > 0. \tag{6.45}$$

The second process is to obtain the consumption in the bad state as a function of the provisions of the public goods. We rewrite Eq. (6.37) as:

$$\frac{U(C^{1A}) - U(C^{0A})}{U_Y(C^{0A})} = \frac{p\mathrm{L}' + (1-p)(1-\mu)}{p'}. \tag{6.46}$$

Substituting Eq. (6.40) into Eq. (6.46), we obtain:

$$\frac{U(e(C^{0A}, M_1, M_2)) - U(C^{0A})}{U_Y(C^{0A})} = \frac{p\mathrm{L}' + (1-p)(1-\mu)}{p'}. \tag{6.47}$$

The left-hand side (LHS) of Eq. (6.47) represents the marginal benefit of the increasing probability of the good state and the right-hand side (RHS) is its marginal cost. We take the partial differentiation of the LHS, utilize Eq. (6.35), and obtain the following[5]:

$$\frac{\partial}{\partial C^{0A}}\frac{U(e) - U(C^{0A})}{U_Y(C^{0A})} = \frac{-(U(e) - U^{0A})U_{YY}^{0A}}{(U_Y^{0A})^2} + \frac{1}{U_Y^{0A}}\left\{U_Y(e)\frac{\partial e}{\partial C^{0A}} - U_Y^{0A}\right\}. \tag{6.48}$$

Thus, the first term on the RHS of Eq. (6.48) is positive. The sign of the brackets of the second term on the RHS is given as follows:

$$U_Y(e)\frac{\partial e}{\partial C^{0A}} - U_Y^{0A} = U_Y^{1A}\frac{\mathrm{L}'U_{YY}^{0A}}{U_{YY}^{1A}} - U_Y^{0A} = U_Y^{0A}\left(\frac{\mathrm{L}'R^{0A}}{R^{1A}} - 1\right) > 0. \tag{6.49}$$

The last inequality of Eq. (6.49) is derived from our assumption in Eq. (6.35).

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[4]Using Eq. (6.39), we obtain:

$\frac{\partial e(C,M_1,M_2)}{\partial C} < 1$ if and only if $-\frac{U_{YY}(C)}{U_Y(C)} < -\frac{U_{YY}(e(C,M_1,M_2))}{U_Y(e(C,M_1,M_2))}$.

Thus, the slope of the curve of function $e(.)$ in $C^{0A} - C^{1A}$ space is steeper (more gradual) than the unity if the absolute risk aversion increases (decreases).

[5]From Eq. (6.43), the first term on the RHS of Eq. (6.48) is non-negative. Substituting Eq. (6.45) into the second term and using our assumption, Eq. (6.35), we obtain:

$U_Y(e)\frac{\partial e}{\partial C^{0A}} - U_Y^{0A} = U_Y(e)\frac{\mathrm{L}'U_{YY}^{0A}}{U_{YY}(e)} - U_Y^{0A} > 0.$

The final process is to apply a similar reasoning to the first order condition of country B to obtain:

$$\frac{U(e(C^{0B}, M_1, M_2)) - U(C^{0B})}{U_Y(C^{0B})} = \frac{p\mathrm{L}' + (1-p)(1-\mu)}{p'}, \tag{6.50}$$

where the LHS of Eq. (6.50) increases with C^{0B}. Comparing Eq. (6.47) with Eq. (6.50), we obtain the following condition for the Nash equilibrium level of consumption in the bad sate:

$$\frac{U(e(C^{0A*}, M_1^*, M_2^*)) - U(C^{0A*})}{U_Y(C^{0A*})} = \frac{U(e(C^{0B*}, M_1^*, M_2^*)) - U(C^{0B*})}{U_Y(C^{0B*})}. \tag{6.51}$$

Because the LHS of Eq. (6.51) increases with C^{0A} and the RHS increases with C^{0B}, we obtain:

$$C^{0A*} = C^{0B*}. \tag{6.52}$$

Substituting Eq. (6.52) into Eqs. (6.40) and (6.41), we have $C^{1A*} = C^{1B*}$.

The proof is summarized as follows. From the first order condition of the optimal self-insurance, we can define country A's consumption in the good state as a function of its consumption in the bad state. In the proof of Proposition 6.1, this function is denoted by $e(.)$. Because we assume that both countries have the same utility function, $e(.)$ also determines country B's consumption in the good state based on B's consumption in the bad state. Using $e(.)$, the marginal benefit of the increasing probability of the good state is represented as a function of the consumption in the bad state. Because the marginal cost of the increasing probability of the good state is identical between the two countries, the marginal benefit and the consumption in the bad state must also be identical between the two countries. Using $e(.)$, it follows that A's consumption in the good state is identical to B's.

Condition (6.35) of Proposition 6.1 means that the absolute risk aversion of both countries is sufficiently decreasing with consumption. Suppose this condition is not satisfied. In this case, the sign of the LHS of Eq. (6.48) may change from positive to negative (or from negative to positive) because the first term of Eq. (6.48) is positive, but the second term is negative. Thus, the shape of the LHS of Eq. (6.47) in $C^{0A} - C^{1A}$ space may be a U-shape (or an inverted U-shape), which means that Eq. (6.47) may have multiple solutions. There may exist two or more values of C^{0A} satisfying Eq. (6.47). Therefore, Eq. (6.51) does not necessarily imply the equivalence of C^{0A} and C^{0B}.

Because Eq. (6.35) itself is a sufficient condition, we may present an example of the utility function satisfying this equation. If we assume that the utility function is a constant relative risk aversion function such that $U(C) = C^{1-\theta}/(1-\theta)$, where $\theta \in (0, 1)$ is the relative risk aversion, we obtain:

$$\frac{\mathrm{L}'R^{0A}}{R^{1A}} = \frac{\mathrm{L}'C^{1A}}{C^{0A}} \text{ for any } C^{1A} \text{ and } C^{0A}.$$

Thus, Eq. (6.35) is satisfied in the neighborhood of the equilibrium.[6] Additionally, if $\bar{L}^A$ is sufficiently high and if L' is close to unity, Eq. (6.35) is satisfied for any C^{1A} and C^{0A} that country A can purchase under its budget constraints.

Proposition 6.1 implies that the differences in national income and loss in the bad state are canceled out by the contributions to both public goods. Substituting the budget constraints of countries A and B into Eq. (6.36), we obtain:

$$Y^A - m_1^{A*} - m_2^{A*} = Y^{B*} - m_1^{B*} - m_2^{B*}, \tag{6.53}$$

$$Y^A - \bar{L}^A - m_1^{A*} = Y^B - \bar{L}^B - m_1^{B*}. \tag{6.54}$$

Equation (6.53) is derived from the equality of the consumption in the good state, whereas Eq. (6.54) is derived from the equality of the consumption in the bad state. We therefore immediately have the following proposition:

Proposition 6.2 *Suppose that all elements of the Nash equilibrium vector of contributions,* $(m_1^{A*}, m_2^{A*}, m_1^{B*}, m_2^{B*})$*, are positive and that Eq. (6.35) is satisfied. We then have the following:*

$$(m_1^{A*} + m_2^{A*}) - (m_1^{B*} + m_2^{B*}) = Y^A - Y^B, \tag{6.55}$$

$$m_1^{A*} - m_1^{B*} = (Y^A - \bar{L}^A) - (Y^B - \bar{L}^B), \tag{6.56}$$

$$m_2^{A*} - m_2^{B*} = \bar{L}^A - \bar{L}^B. \tag{6.57}$$

Proposition 6.2 provides three insights on the burden sharing of risk management. Equation (6.55) implies that the country with the higher income of the two allies expends more on the security of the alliance than the other country. Equation (6.56) implies that the country with the higher disposable income in the bad state—income net of loss in the bad state—contributes more to self-protection than the other. Equation (6.57) implies that the country facing the greater loss in the bad state of the two purchases more self-insurance than the other.

For example, consider an alliance that consists of two countries endowed with an identical national income and assume that country A is adjacent to a country hostile to the alliance, whereas country B is a far distance from the hostile country. In this case, country A will suffer heavier damage than country B if a bad state occurs.

[6] From Eq. (6.39), we obtain $C^{1A}/C^{0A} = (L')^{-1/\theta}$, which implies $\mathrm{L}'R^{0A}/R^{1A} = (\mathrm{L}')^{1-(1/\theta)} > 1$ because $\mathrm{L}' < 1$ and $\theta < 1$.

Proposition 6.2 implies that in such an alliance, country A will contribute more to self-insurance than B, whereas country A will contribute less to self-protection than B.

6.2.5.3 Self-insurance Freeriding Equilibrium

Let us consider a Nash equilibrium in which country A contributes to both public goods, but country B contributes only to the self-protection public good. In this case, we show that country A's consumption is not greater than country B's and that country A shares a disproportionally heavier burden than country B. Formally, we derive the following proposition:

Proposition 6.3 *Suppose that all elements of the Nash equilibrium vector of contributions,* $(m_1^{A*}, m_2^{A*}, m_1^{B*}, m_2^{B*})$*, satisfy* $m_1^{A*} > 0, m_2^{A*} > 0, m_1^{B*} > 0, m_2^{B*} = 0$ *and Eq. (6.35). We then have the following:*

$$C^{0A*} \leq C^{0B*}, \tag{6.58}$$

$$m_1^{A*} - m_1^{B*} \geq (Y^A - \bar{L}^A) - (Y^B - \bar{L}^B), \tag{6.59}$$

which implies that:

$$m_1^{A*} + m_2^{A*} - m_1^{B*} > Y^A - Y^B + \bar{L}^B - \bar{L}^A. \tag{6.60}$$

Additionally, it follows that:

$$C^{1A*} < C^{1B*} \text{ and } W^{A*} < W^{B*} \text{ if } \bar{L}^A \leq \bar{L}^B. \tag{6.61}$$

Proof This proof consists of two steps. First, we investigate the first order conditions of individual optimization for both countries to show Eq. (6.58). Next, we substitute the budget constraints of the countries into Eq. (6.58) and derive Eqs. (6.59), (6.60), and (6.61).

We show that the consumption of country A in the bad state is no higher than that of B. Because country B does not contribute to self-insurance, we obtain:

$$\frac{\partial W^B}{\partial m_2^B} = p(\mathrm{L}' U_Y(C^{0B}) - U_Y(C^{1B})) \leq 0,$$

which implies that:

$$\mathrm{L}' U_Y(C^{0B}) \leq U_Y(C^{1B}). \tag{6.62}$$

From Eq. (6.44), we obtain:

$$\mathrm{L}' U_Y(C^{0B}) = U_Y(e(C^{0B}, M_1, M_2)). \tag{6.63}$$

Substituting Eq. (6.63) into Eq. (6.62), we obtain:

$$U_Y(e(C^{0B}, M_1, M_2)) \leq U_Y(C^{1B}).$$

Because U_Y is a decreasing function, we obtain:

$$e(C^{0B}, M_1, M_2) \geq C^{1B}. \tag{6.64}$$

From the first order condition for country B's optimal self-protection, we have:

$$\frac{\partial W^B}{\partial m_1^B} = p'(U(C^{1B}) - U(C^{0B})) - \left\{pU_Y(C^{1B}) + (1-p)(1-\mu)U_Y(C^{0B})\right\} = 0. \tag{6.65}$$

Substituting Eq. (6.62) into Eq. (6.65), we have:

$$\begin{aligned} p'(U(C^{1B}) - U(C^{0B})) &= pU_Y(C^{1B}) + (1-p)(1-\mu)U_Y(C^{0B}) \\ &\geq \left\{p\mathrm{L}' + (1-p)(1-\mu)\right\}U_Y(C^{0B}), \end{aligned}$$

which implies:

$$\frac{U(C^{1B}) - U(C^{0B})}{U_Y(C^{0B})} \geq \frac{p\mathrm{L}' + (1-p)(1-\mu)}{p'}. \tag{6.66}$$

Substituting Eq. (6.64) into Eq. (6.66), we obtain:

$$\frac{U(e(C^{0B}, M_1, M_2)) - U(C^{0B})}{U_Y(C^{0B})} \geq \frac{U(C^{1B}) - U(C^{0B})}{U_Y(C^{0B})} \geq \frac{p\mathrm{L}' + (1-p)(1-\mu)}{p'}. \tag{6.67}$$

Comparing Eq. (6.47) with Eq. (6.67), we have Eq. (6.58). Substituting the budget constraints of the countries in the bad state into Eq. (6.58), we have Eq. (6.59). Using Eq. (6.59), we rewrite the consumption of country A in the good state to obtain the following[7]:

$$C^{1A*} = Y^A - m_1^{A*} - m_2^{A*} \leq C^{1B*} + \bar{L}^A - \bar{L}^B - m_2^{A*} < C^{1B*} + \bar{L}^A - \bar{L}^B, \tag{6.68}$$

[7]From Eq. (6.59), we obtain $m_1^{A*} \geq m_1^{B*} + Y^A - \bar{L}^A - Y^B + \bar{L}^B$. Substituting this inequality in the RHS of Eq. (6.15), we obtain $C^{1A} \leq Y^B - m_1^{B*} + \bar{L}^A - \bar{L}^B - m_2^{A*}$. Remembering $m_2^{B*} = 0$ and using Eq. (6.22), we have Eq. (6.68).

which implies that $C^{1A*} < C^{1B*}$ if $\bar{L}^A - \bar{L}^B \leq 0$. It immediately follows that $W^{A*} < W^{B*}$.

Proposition 6.3 implies that when country B freerides in self-insurance, country A will share a disproportionally heavy burden compared to its disposable income in the bad state. As shown in Eq. (6.59), the difference in the contribution to self-protection of the two countries is greater than, or at least equal to, that in the disposable income in the bad state. As a result, country A's consumption in the bad state is no higher than that of country B. Additionally, if country B's loss in the bad state is higher than A's, the expected welfare of country A is no higher than that of B.

Similarly, if country A contributes only to the self-protection public good and if country B contributes to both public goods, country B will share a disproportionally heavy burden compared to the difference in disposable income between the two countries.

6.2.5.4 Self-protection Freeriding Equilibrium

Let us consider a Nash equilibrium in which country A contributes to both public goods but country B contributes only to the self-insurance public good. We will show that country B's consumption and expected welfare are no higher than A's. Formally, we derive the following proposition:

Proposition 6.4 *Suppose that all elements of the Nash equilibrium vector of contributions,* $(m_1^{A*}, m_2^{A*}, m_1^{B*}, m_2^{B*})$*, satisfy* $m_1^{A*} > 0, m_2^{A*} > 0, m_1^{B*} = 0, m_2^{B*} > 0$ *and Eq. (6.35). We therefore have the following:*

$$C^{0B*} \leq C^{0A*},\ C^{1B*} \leq C^{1A*},\ W^{B*} \leq W^{A*}, \tag{6.69}$$

$$m_1^{A*} + m_2^{A*} - m_2^{B*} \leq Y^A - Y^B, \tag{6.70}$$

$$m_1^{A*} - m_1^{B*} \leq (Y^A - \bar{L}^A) - (Y^B - \bar{L}^B). \tag{6.71}$$

Proof First, we investigate the first order conditions of individual optimization for both countries to show Eq. (6.69). Because countries A and B contribute to the self-insurance public good, we obtain:

$$C^{1A} = e(C^{0A}, M_1, M_2) \text{ and } C^{1B} = e(C^{0B}, M_1, M_2). \tag{6.72}$$

We substitute Eq. (6.72) into the first order condition for country A's optimal self-protection and obtain:

$$\frac{U(e(C^{0A}, M_1, M_2)) - U(C^{0A})}{U_Y(C^{0A})} = \frac{p\mathrm{L}' + (1-p)(1-\mu)}{p'}. \tag{6.73}$$

Because country B does not contribute to self-protection, its optimal condition becomes:

$$\frac{U(e(C^{0B}, M_1, M_2)) - U(C^{0B})}{U_Y(C^{0B})} \leq \frac{p\mathrm{L}' + (1-p)(1-\mu)}{p'}. \tag{6.74}$$

Comparing Eq. (6.73) with Eq. (6.74), we obtain:

$$\frac{U(e(C^{0B*}, M_1^*, M_2^*)) - U(C^{0B*})}{U_Y(C^{0B*})} \leq \frac{U(e(C^{0A*}, M_1^*, M_2^*)) - U(C^{0A*})}{U_Y(C^{0A*})}.$$

From Eq. (6.48), we have $C^{0B*} \leq C^{0A*}$. Using Eq. (6.72), we have $C^{1B*} \leq C^{1A*}$. We therefore immediately obtain $W^{B*} \leq W^{A*}$. Substituting the budget constraints of the countries into Eq. (6.69), we have Eqs. (6.70) and (6.71).

Proposition 6.4 implies that when country B freerides in its provision of self-protection, its expected welfare is lower than country A's. This result is explained as follows. Because both countries A and B contribute to self-insurance, their choices of consumption in the good and bad states, (C^{0A}, C^{1A}) and (C^{0B}, C^{1B}), are located on the expansion path, $C^1 = e(C^0, M_1, M_2)$, in (C^0, C^1) space, which represents the individual optimal condition for self-insurance. If Eq. (6.35) holds, the marginal benefit of increasing the probability of the good state, $(U(C^1) - U(C^0))/U_Y(C^0)$, increases with C^0. Remembering that country B freerides in self-protection and that both countries A and B face the same marginal cost of increasing the probability, we conclude that country B's marginal benefit of increasing the probability, $(U(C^1) - U(C^0))/U_Y(C^0)$, must be no more than that of country A. Thus, C^{0B} is no more than C^{0A}, which means that the difference in the disposable income in the bad state is not necessarily cancelled out by the self-protection contribution. In other words, the difference in the self-protection contribution between countries A and B is no more than the difference in the income net of the loss in the bad state. Additionally, C^{1A} is higher than C^{1B} because $C^1 = e(C^0, M_1, M_2)$ increases with C^0. From the budget constraints of the two countries in the good state, it follows that the difference in total security expenditures between the two countries is no higher than that in their national income. Thus, we conclude that country A shares a light burden compared to the difference in income between the two countries.

Finally, we claim that the reasoning developed in Proposition 6.4 is applicable when country A contributes only to the self-insurance public good and country B contributes to both public goods. In such a case, country B will share a disproportionally light burden compared to the difference in disposable income between the two countries.

6.2.5.5 Decentralized Specialization Equilibrium

Let us consider a Nash equilibrium in which country A unilaterally contributes to the self-protection public good and country B unilaterally contributes to the self-insurance public good. We will show that country B's consumption in the bad state is no greater than country A's. Formally, we derive the following proposition:

Proposition 6.5 *Suppose that all elements of the Nash equilibrium vector of contributions,* $(m_1^{A*}, m_2^{A*}, m_1^{B*}, m_2^{B*})$*, satisfy* $m_1^{A*} > 0, m_2^{A*} = 0, m_1^{B*} = 0, m_2^{B*} > 0$ *and Eq. (6.35). We then have the following:*

$$C^{0A*} \geq C^{0B*}, \tag{6.75}$$

$$m_1^{A*} \leq (Y^A - \bar{L}^A) - (Y^B - \bar{L}^B), \tag{6.76}$$

which implies that:

$$m_1^{A*} - m_2^{B*} < Y^A - Y^B + \bar{L}^B - \bar{L}^A. \tag{6.77}$$

Additionally, it follows that:

$$C^{1A*} > C^{1B*} \text{ and } W^{A*} > W^{B*} \text{ if } \bar{L}^A \geq \bar{L}^B. \tag{6.78}$$

Proof This proof consists of two steps. First, we investigate the first order conditions of individual optimization for both countries to show Eq. (6.75). Next, we substitute the budget constraints of the countries into Eq. (6.75) and derive Eqs. (6.76), (6.77), and (6.78).

Because country A does not contribute to self-insurance, we obtain:

$$\frac{\partial W^A}{\partial m_2^A} = p(\mathrm{L}' U_Y(C^{0A}) - U_Y(C^{1A})) \leq 0. \tag{6.79}$$

From the definition of function $e(.)$, we have:

$$U_Y(e(C^{0A}, M_1, M_2)) = \mathrm{L}' U_Y(C^{0A}). \tag{6.80}$$

Substituting Eq. (6.80) into Eq. (6.79), we obtain:

$$U_Y(e(C^{0A}, M_1, M_2)) \leq U_Y(C^{1A}). \tag{6.81}$$

Because U_Y is a decreasing function, we obtain:

$$e(C^{0A}, M_1, M_2) \geq C^{1A}. \tag{6.82}$$

Substituting Eqs. (6.80) and (6.82) into Eq. (6.27), we obtain:

$$\frac{U(e(C^{0A}, M_1, M_2)) - U(C^{0A})}{U_Y(C^{0A})} \geq \frac{U(C^{1A}) - U(C^{0A})}{U_Y(C^{0A})} \geq \frac{p\mathrm{L}' + (1-p)(1-\mu)}{p'}. \tag{6.83}$$

Because country B contributes only to self-insurance, we have:

$$e(C^{0B}, M_1, M_2) = C^{1B} \tag{6.84}$$

and

$$\frac{U(C^{1B}) - U(C^{0B})}{U_Y(C^{0B})} \leq \frac{p\mathrm{L}' + (1-p)(1-\mu)}{p'}. \tag{6.85}$$

Substituting Eq. (6.84) into Eq. (6.85), we obtain:

$$\frac{U(e(C^{0B}, M_1, M_2)) - U(C^{0B})}{U_Y(C^{0B})} \leq \frac{p\mathrm{L}' + (1-p)(1-\mu)}{p'}. \tag{6.86}$$

Comparing Eq. (6.83) with Eq. (6.86), we have Eq. (6.75). Substituting the budget constraints of the countries in the bad state into Eq. (6.75), we have Eq. (6.76). Substituting the budget constraints of countries in the good state into Eq. (6.76), we obtain[8]:

$$C^{1A*} \geq C^{1B*} + \bar{L}^A - \bar{L}^B + m_2^{B*} > C^{1B*} + \bar{L}^A - \bar{L}^B. \tag{6.87}$$

From Eq. (6.87) and the budget constraints of the countries, we have Eq. (6.77). We then obtain $C^{1A*} > C^{1B*}$ and $W^{A*} > W^{B*}$ if $\bar{L}^A \geq \bar{L}^B$.

Proposition 6.5 implies that if country A specializes in self-protection and country B specializes in self-insurance, country A will consume no less than country B in a bad state. Additionally, if country A's loss in the bad state is greater than B's, the expected welfare of country A is higher than that of B. Moreover, we claim that the reasoning developed in Proposition 6.5 is applicable when country A contributes only to the self-insurance public good and country B contributes only to the self-protection public good.

6.2.5.6 Summary

Let us briefly summarize the results presented in Propositions 6.1–6.5. Table 6.3 compares the burden sharing of countries in terms of total security expenditures, $m_1^{A*} + m_2^{A*}$ and $m_1^{B*} + m_2^{B*}$, in four types of Nash equilibria. Table 6.4 compares the consumptions of the countries.

[8]We derive the second inequality of Eq. (6.87) from our assumption that $m_2^{B*} > 0$.

Table 6.3 Comparison of total security expenditures

Propositions	Contributor(s) to self-protection public good	Contributor(s) to self-insurance public good	Security expenditure
6.2	A, B	A, B	$(m_1^{A*} + m_2^{A*}) - (m_1^{B*} + m_2^{B*}) = Y^A - Y^B$
6.3	A, B	A	$m_1^{A*} + m_2^{A*} - m_1^{B*} > Y^A - Y^B + \bar{L}^B - \bar{L}^A$
6.4	A	A, B	$m_1^{A*} + m_2^{A*} - m_2^{B*} \le Y^A - Y^B$
6.5	A	B	$m_1^{A*} - m_2^{B*} < Y^A - Y^B + \bar{L}^B - \bar{L}^A$

Source Authors

As summarized in Table 6.3, we showed that if the Nash equilibrium is interior, or each country contributes to both public goods, the difference in total security expenditures is precisely equal to the difference in national income. As shown in Proposition 6.1, country A's consumptions are identical to those of country B. This situation is interpreted as an exploitation of a high-income country, or the so-called "rich," by a low-income country, or the so-called "poor." This result corresponds to the standard exploitation hypothesis in voluntary provision models.

When one country (e.g., country A) contributes to both public goods and the other country (B) freerides in self-insurance, the exploitation of the "rich" (A) by the "poor" (B) is strengthened. As shown in Proposition 6.3, the difference in total security expenditures is greater than the difference in the disposable income in the bad state. If country B's loss in the bad state is as high or higher than that of country A, the difference in the total security expenditures is greater than that in national income. In other words, if country A is endowed with a higher income than B, country A contributes to security much more than B. The difference in contribution is more

Table 6.4 Comparison of consumption of allies

Propositions	Contributor(s) to self-protection public good	Contributor(s) to self-insurance public good	Consumption in the bad state	Consumption in the good state	Expected welfare
6.1	A, B	A, B	$C^{0A*} = C^{0B*}$	$C^{1A*} = C^{1B*}$	$W^{A*} = W^{B*}$
6.3	A, B	A	$C^{0A*} \le C^{0B*}$	$C^{1A*} < C^{1B*}$ and $W^{A*} < W^{B*}$ if $\bar{L}^A \le \bar{L}^B$	
6.4	A	A, B	$C^{0A*} \ge C^{0B*}$	$C^{1A*} \ge C^{1B*}$	$W^{A*} \ge W^{B*}$
6.5	A	B	$C^{0A*} \ge C^{0B*}$	$C^{1A*} > C^{1B*}$ and $W^{A*} > W^{B*}$ if $\bar{L}^A \ge \bar{L}^B$.	

Source Authors

than that in income. In this sense, the “poor” exploits the “rich” more than it does in the interior equilibrium.

In the self-protection freeriding equilibrium in which one country (e.g., country A) contributes to both public goods and the other country (B) freerides in self-protection, the exploitation of the “rich” by the “poor” is mitigated. As shown in Proposition 6.4, the difference in total security expenditures is either as much as or less than that in the national income. As a result, country A consumes either more than or as much as country B in both states. The expected welfare of country A is either higher than or as high as that of B. In this sense, the “poor” exploits the “rich” less than it does in the interior equilibrium.

Finally, in the decentralized specialization equilibrium, we also observe the mitigated exploitation of the “rich” (country A) by the “poor” (B). As shown in Proposition 6.5, the difference in security expenditures is less than that in the disposable income in the bad state. If country A’s loss is either as much as or more than country B’s loss, the difference in security expenditures is lower than that in national income. Therefore, country A’s expected welfare is higher than B’s. In this sense, the exploitation of the “rich” by the “poor” is mitigated compared to the internal Nash equilibrium.

6.2.6 Neutrality Results

As we discussed in Chap. 3, Shibata (1971) and Warr (1983) showed that in a model in which players voluntarily contribute to a conventional public good, any income redistribution among contributors does not affect the consumption of private goods and provision of public goods. This neutrality result was extended by Kemp (1984), Bergstrom et al. (1986), and Cornes and Schweinberger (1996) into the voluntary provision of two or more public goods.

In this subsection, we investigate whether this neutrality result still holds in our risk management model. Unlike the conventional public good model, our model presupposes two states of the world in which two public goods affect the expected welfare in different ways. The self-protection public good reduces the risk of the bad state, whereas the self-insurance public good reduces the loss in the bad state.

We will show that when the self-protection public good is provided by both countries, any marginal income redistribution is cancelled out by changes in the contributions to the self-insurance public good and the redistribution does not affect the private good consumption or provision of the public good. In the following, we consider two types of Nash equilibria—the interior equilibrium and self-insurance freeriding equilibrium—to show the neutrality of income redistribution among contributors in our model. The Nash equilibrium is represented by a vector of four unknown variables, $(m_1^{A*}, m_2^{A*}, m_1^{B*}, m_2^{B*})$. We define the gradient vectors of the first derivative of the expected welfare function, $\mathbf{V}_1^A, \mathbf{V}_2^A, \mathbf{V}_1^B, \mathbf{V}_2^B$, as follows:

$$\mathbf{V_1^A} = \left(W_{11}^A, W_{12}^A, W_{11B}^A, W_{12B}^A\right)^t,\ \mathbf{V_2^A} = \left(W_{12}^A, W_{22}^A, W_{21B}^A, W_{22B}^A\right)^t,$$
$$\mathbf{V_2^B} = \left(W_{11A}^B, W_{12A}^B, W_{11}^B, W_{12}^B\right)^t,\ \mathbf{V_2^B} = \left(W_{21A}^B, W_{22A}^B, W_{12}^B, W_{22}^B\right)^t,$$

where

$$\begin{aligned} W_{ij}^A &\equiv \frac{\partial^2 W^A}{\partial m_i^A \partial m_j^A},\ W_{ijB}^A \equiv \frac{\partial^2 W^A}{\partial m_i^A \partial m_j^B},\ W_{iY}^A \equiv \frac{\partial^2 W^A}{\partial m_i^A \partial Y^A}, \\ W_{ij}^B &\equiv \frac{\partial^2 W^B}{\partial m_i^B \partial m_j^B},\ W_{ijA}^B \equiv \frac{\partial^2 W^B}{\partial m_i^B \partial m_j^A},\ W_{iY}^B \equiv \frac{\partial^2 W^B}{\partial m_i^B \partial Y^B}, \end{aligned} \quad \text{for } i = 1,\ 2.$$

We then have the following proposition:

Proposition 6.6 *Suppose that there is either a unique interior equilibrium or unique self-protection freeriding equilibrium and that the gradient vectors of the first derivative of the expected welfare function are linear independent. Suppose also that one unit of income of country B is transferred to country A, $dT = dY^A = -dY^B$. We therefore have:*

$$\frac{dm_1^{A*}}{dT} = 1,\ \frac{dm_1^{B*}}{dT} = -1,\ \frac{dm_2^{A*}}{dT} = \frac{dm_2^{B*}}{dT} = 0.$$

Proof We provide the proof of the proposition of an interior equilibrium. A similar reasoning establishes the neutrality of income transfers in a self-insurance freeriding equilibrium.

The vector representing the interior equilibrium, $(m_1^{A*}, m_2^{A*}, m_1^{B*}, m_2^{B*})$, satisfies the first order conditions of country A, which are given as Eqs. (6.27) and (6.28). It also satisfies the following first order conditions of country B:

$$\frac{\partial W^B}{\partial m_1^B} = p'\left(U^{1B} - U^{0B}\right) - \left\{pU_Y^{1B} + (1-p)(1-\mu)U_Y^{0B}\right\} = 0, \tag{6.88}$$

$$\frac{\partial W^B}{\partial m_2^B} = p\left(\mathrm{L}'U_Y^{0B} - U_Y^{1B}\right) = 0. \tag{6.89}$$

We consider a marginal transfer of income from country B to country A, $dT = dY^A = -dY^B$. Taking the total differences of Eqs. (6.27), (6.28), (6.88), and (6.89), we obtain:

$$\begin{pmatrix} \mathbf{V_1^{At}} \\ \mathbf{V_2^{At}} \\ \mathbf{V_1^{Bt}} \\ \mathbf{V_2^{Bt}} \end{pmatrix} \begin{pmatrix} dm_1^{A*} \\ dm_2^{A*} \\ dm_1^{B*} \\ dm_2^{B*} \end{pmatrix} = \begin{pmatrix} -W_{1Y}^A \\ -W_{2Y}^A \\ W_{1Y}^B \\ W_{2Y}^B \end{pmatrix} dT. \tag{6.90}$$

We denote the matrix on the LHS of Eq. (6.90) as M. Comparing the second partial derivatives of the expected welfare function, we have the following:

$$W_{11B}^A = W_{11}^A + W_{1Y}^A, \tag{6.91}$$

$$W^A_{21B} = W^A_{12} + W^A_{2Y}, \tag{6.92}$$

$$W^B_{11A} = W^B_{11} + W^B_{1Y}, \tag{6.93}$$

$$W^B_{21A} = W^B_{12} + W^B_{2Y}. \tag{6.94}$$

Substituting Eqs. (6.91), (6.92), (6.93), and (6.94) into Eq. (6.90), we obtain:

$$\begin{pmatrix} W^A_{11} & W^A_{12} & W^A_{11} + W^A_{1Y} & W^A_{12B} \\ W^A_{12} & W^A_{22} & W^A_{12} + W^A_{2Y} & W^A_{22B} \\ W^B_{11} + W^B_{1Y} & W^B_{12A} & W^B_{11} & W^B_{12} \\ W^B_{12} + W^B_{2Y} & W^B_{22A} & W^B_{12} & W^B_{22} \end{pmatrix} \begin{pmatrix} dm^{A*}_1 \\ dm^{A*}_2 \\ dm^{B*}_1 \\ dm^{B*}_2 \end{pmatrix} = \begin{pmatrix} -W^A_{1Y} \\ -W^A_{2Y} \\ W^B_{1Y} \\ W^B_{2Y} \end{pmatrix} dT. \tag{6.95}$$

Because $\mathbf{V^A_1},\ \mathbf{V^A_2},\ \mathbf{V^B_1},\ \mathbf{V^B_2}$ are linear independent, we obtain $|M| \neq 0$. We then use the Cramer's rule to have:

$$\frac{dm^{A*}_1}{dT} = 1, \frac{dm^{B*}_1}{dT} = -1, \frac{dm^{A*}_2}{dT} = \frac{dm^{B*}_2}{dT} = 0.$$

Proposition 6.6 claims that the income redistribution of the two countries does not affect the vector of private goods consumptions and public goods provisions as long as both countries contribute to both public goods after the redistribution. The redistribution of income is completely cancelled out by the changes in the self-protection contribution. The assumption that the gradient vectors are linear independent corresponds to the condition in a conventional voluntary provision model that the reaction functions are different. In our model, we assume that both countries have identical preferences. Moreover, this neutrality result can be extended to an economy in which both countries have different preferences.

Finally, let us discuss the relationship between the exploitation hypothesis and the neutrality result. Proposition 6.2 shows that in an interior equilibrium, the difference in self-protection contributions is equal to the difference in income. If we redistribute income from the low-income country to the high-income country, the difference in income expands, which results in the expansion of the difference in self-protection. This result is consistent with our neutrality result that the recipient of the redistribution increases its contribution to the self-protection public good, whereas the donor reduces its contribution. The relationship between the exploitation hypothesis and the neutrality result in the self-insurance freeriding equilibrium is more complicated than in the interior equilibrium. Proposition 6.3 shows that the difference in the self-protection contributions is as much as or more than the difference in income, net of the loss in the bad state. This might seem inconsistent with the neutrality result. However, Proposition 6.3 is derived not from the marginal propensity to the contribution to self-protection but because country B is freeriding in self-insurance. We also note that we can obtain no self-insurance freeriding equilibrium by redistribution of income from any interior equilibrium. This is because any income redistribution in an interior equilibrium does not alter the self-insurance contribution. The demand for self-insurance is based on the loss in the bad state. The country facing a greater loss in the bad state is likely to demand more self-insurance than the country facing a lesser loss. Redistribution of income does not affect self-insurance purchases.

6.3 Numerical Simulations

6.3.1 *Specification for Simulation*

We conduct several numerical simulations of burden sharing among allies to highlight several changes in security spending in NATO. We specify the form of the utility function with the constant relative risk aversion (CRRA) function:

$$U(C) = \frac{C^{1-\theta}}{1-\theta}, \tag{6.96}$$

where θ is a parameter of constant relative risk aversion.

Following Ihori et al. (2014), we specify the risk function as:

$$p(M_1) = p_0 + (1 - p_0)\frac{M_1}{M_1 + (p_e/p_0)}, \tag{6.97}$$

where $p_0(> 0)$ is the baseline probability of the good state, which is the probability of the good state when no country provides a self-protection public good, and $p_e(> 0)$ is a parameter representing the strength of the opponent. The baseline probability p_0 represents the probability that the opponent does not take an aggressive stance due to, for example, domestic conflict. The opponent takes an aggressive stance with the probability $1 - p_0$. Whether the alliance suffers loss is determined by the Tullock's contest success function. The aggressive effort made by the opponent is represented by p_e/p_0. The deterrence effort by the alliance is given by the provision of the self-protection public good. The alliance wins the contest with probability $M_1/(M_1 + (p_e/p_0))$. If the alliance wins, the allies enjoy the good state. However, if the alliance loses, the allies endure the bad state.

The risk function reduces to the following:

$$p(M_1) = \frac{M_1 + p_e}{M_1 + (p_e/p_0)},$$

which satisfies the following properties:

$$p(0) = p_0,\ p_0 \le p(M_1) < 1,\ p' = \frac{p_e}{(M_1 + p_e/p_0)}\left(\frac{1}{p_0} - 1\right) \text{for any } M_1 \ge 0,$$

Table 6.5 Parameter values

p_0	p_e	ϕ	θ
0.25	1	0.9	0.9

Source Authors

and

$$\lim_{M_1 \to \infty} p(M_1) = 1.$$

We specify the self-insurance benefit function as a linear function:

$$\mathrm{L}(s) = \phi s, \tag{6.98}$$

where $\phi \in (0,\ 1)$ is a marginal loss reduction from one unit of purchased self-insurance. We therefore obtain $\mathrm{L}' = \phi,\ \mathrm{L}'' = 0$.

Table 6.5 summarizes the chosen values of the parameters when we conduct our simulation. The values are determined the same as in Ihori et al. (2014).

6.3.2 *Income Redistribution Scenario*

In this subsection, we report the results of numerical simulations to examine the impact of income redistribution on military expenditures. We begin with the result of a symmetric economy simulation in which both countries are endowed with 50 units of national income and face the risk of a 10-unit loss in a bad state. Next, we report the results of two groups of numerical simulations. The first group assumes the redistribution of income from country B to country A. We reduce country B's income by 0.05 units, increase A's income by the same units, and derive the Nash equilibrium.[9] We repeat this process 100 times. The final income of country A is 55 units and B's is 45 units.

In the second group of simulations, we consider the simultaneous redistribution of income and loss. We transfer not only income of the country but also its loss in a bad state. We reduce country B's income and its loss in a bad state by 0.05 units, increase A's income and loss by the same units, and calculate the values of security expenditures in a Nash equilibrium and other endogenous variables. We also repeat this process 100 times. For each simulation result, we calculate the difference

[9]The derivation of the Nash equilibrium is as follows. We construct a system of equations consisting of a first order condition and corner condition. Next, we check if the inequality constraints are satisfied. For example, we numerically solve a system of equations, $\frac{\partial W^A}{\partial m_1^A} = 0,\ \frac{\partial W^A}{\partial m_2^A} = 0,\ \frac{\partial W^B}{\partial m_1^B} = 0,\ \frac{\partial W^B}{\partial m_2^B} = 0$, and check whether the solution satisfies $m_1^{A*} > 0, m_2^{A*} > 0, m_1^{B*} > 0, m_2^{B*} > 0$. If the solution satisfies the condition, the solution is considered an interior equilibrium. We conduct this procedure for all 16 types of Nash equilibria.

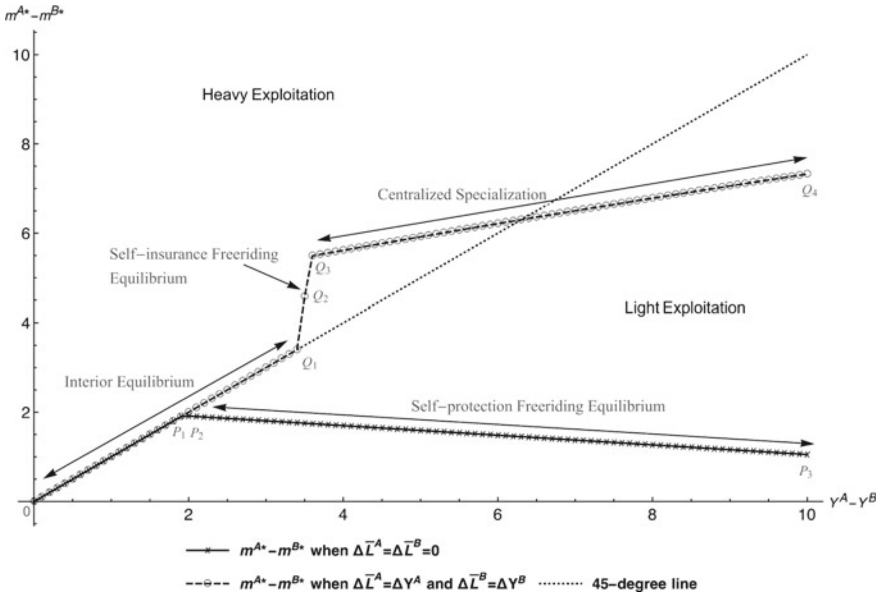

Fig. 6.1 Results of numerical simulations of income redistribution. *Source* Authors

between security expenditures, $m^{A*} - m^{B*}$, and the income differential, $Y^A - Y^B$, and plot a point in a $(Y^A - Y^B) - (m^{A*} - m^{B*})$ plane, which is illustrated as Fig. 6.1.

Figure 6.1 represents two curves with markers and a 45° line. The horizontal axis of this figure shows the gap in income and the vertical axis shows the gap in security expenditures. The curve with "x" markers shows the results of simulations of income redistribution. The curve with "o" markers represents the results of simulations of simultaneous redistribution of income and loss. The dotted line is the 45° line.

6.3.2.1 Income Redistribution

The curve with "x" markers starts from the origin. The origin, or point O, represents the results of a simulation of a symmetric economy. Because both countries are endowed with the same income and face the same amount of loss in the bad state in the origin, their security expenditures become identical, which implies that the gap $(m^{A*} - m^{B*})$ is zero. As we redistribute country B's income to A, the gap in security expenditures increases with the gap in income. On the segment OP_1, we derive an interior equilibrium in each simulation. At point P_2, country B stops providing its contribution to the self-protection public good. Next, we obtain the self-protection freeriding equilibrium. Along the segment P_2P_3, the income redistribution reduces the gap in security expenditures. The gap in security expenditures is then less than

Table 6.6 Results of the numerical simulation of income redistribution

1	Point in Fig. 6.1	O	P_1	P_2	P_3
2	Y^A	50	50.95	51	55
3	Y^B	50	49.05	49	45
4	$\bar{L}^A$	10	10	10	10
5	$\bar{L}^B$	10	10	10	10
6	m_1^{A*}	0.961	1.911	1.924	2.041
7	m_2^{A*}	1.734	1.734	1.729	1.197
8	m_1^{B*}	0.961	0.011	0.000	0.000
9	m_2^{B*}	1.734	1.734	1.738	2.185
10	$m^{A*}(= m_1^{A*} + m_2^{A*})$	2.695	3.645	3.652	3.237
11	$m^{B*}(= m_1^{B*} + m_2^{B*})$	2.695	1.745	1.738	2.185
12	W^{A*}	14.619	14.619	14.621	14.753
13	W^{B*}	14.619	14.619	14.618	14.476
14	W^{A*}/W^B	1.000	1.000	1.000	1.019
15	p^*	0.493	0.493	0.494	0.503
16	C^{1A*}	47.305	47.305	47.348	51.763
17	C^{0A*}	42.079	42.079	42.117	46.044
18	C^{1B*}	47.305	47.305	47.262	42.815
19	C^{0B*}	42.079	42.079	42.041	38.085
20	$m^{A*} - m^{B*}$	0.000	1.900	1.914	1.052
21	$Y^A - Y^B$	0.000	1.900	2.000	10.000
22	m^{A*}/Y^A	0.054	0.072	0.072	0.059
23	m^{B*}/Y^B	0.054	0.036	0.035	0.049

Source Authors

that in income. Thus, we observe the light exploitation of country A by country B in this segment.

Table 6.6 reports the detailed results of simulations corresponding to points O, P_1, P_2, P_3 in Fig. 6.1. Row 1 describes the point in the figure corresponding to the simulation. Rows 2–5 report the setting of incomes and losses in a bad state and rows 6–23 report the equilibrium levels of security expenditures, expected welfare, the probability of the good state, consumptions, the gap in security expenditures, the gap in income, and the ratio of security expenditures to national income.

Comparing columns O and P_1, we observe that after we redistribute 0.95 units of income from country B to A, the self-insurance purchases by both countries are not affected and the income redistribution is completely cancelled out by the changes in self-protection. As shown in Proposition 6.2, the purchases of self-insurance remain unchanged because we fix the losses in the bad state. On the contrary, the transfer of income alters the disposable income in the bad state ($Y^A - \bar{L}^A$ and $Y^B - \bar{L}^B$). Because the gap in self-protection contributions is equal to the gap in disposable income, the income redistribution is cancelled out by the changes in the self-protection contributions.

Column P_2 represents the self-protection freeriding equilibrium. Compared with column P_1, the gap in security expenditures ($m^{A*} - m^{B*}$) increases by 0.014 units and the income gap ($Y^A - Y^B$) increases by 0.1 units. The comparison between P_1 and P_2 also reveals that country A increases its contribution to the self-protection public good. Due to the negative income effect of self-insurance, country A reduces its purchase of self-insurance and country B raises its purchase. As shown in Proposition 6.3, the private consumption and expected welfare of country B are lower than those of country A.

Further redistribution to country A expands the difference in income but shrinks the gap in security expenditures. Let us compare column P_3 with P_2. Four units of country B's income are redistributed to country A. Country A purchases less self-insurance than before the income redistribution, whereas country B purchases more self-insurance. Although country A increases its contribution to the self-insurance public good, the difference in security expenditures ($m^{A*} - m^{B*}$) reduces and the difference in income increases. As a result, the expected welfare of country A improves and that of country B deteriorates.

6.3.2.2 Simultaneous Redistribution of Income and Loss

Next, let us investigate the impact of the simultaneous redistribution of income and loss. We again start from the symmetric economy in which both countries are endowed with 50 units of income and face 10 units of loss in a bad state. We decrease country B's income and loss and increase country A's income and loss by the same amount. The curve with "o" markers in Fig. 6.1 shows the results of this simulation. When the amount of redistribution is sufficiently low, we derive an interior equilibrium, as shown on the segment OQ_1 in Fig. 6.1. After a further redistribution, we derive the self-insurance freeriding equilibrium as shown in point Q_2. The gap in security expenditures jumps from point Q_1 to point Q_2. This jump is high enough that the equilibrium corresponds in the case of heavy exploitation of country A by country B. The segment Q_2Q_3 corresponds to the results of simulations in which we obtain the centralized specialization equilibrium. On the segment, country A contributes to both self-protection and self-insurance, whereas country B does not contribute at all. The slope of the segment is upward but lower than the unity, which means that the redistribution of income and loss expands the differential in security expenditures and that the marginal increase in the differential in security expenditures is less than the marginal increase in income disparity. As we continue to redistribute, the gap in security expenditures becomes less than the gap in income in the simulation, the result of which corresponds to point Q_4. We therefore observe the light exploitation of country A by country B.

Table 6.7 describes the results of simulations corresponding to points O, Q_1, Q_2, Q_3, and Q_4. Column Q_1 reports the results of the simulation in which country A's income and its loss in a bad state are 1.7 units more than those in column O and country B's income and its loss are 1.7 units less. The security expenditures reported in both columns are classified as an interior equilibrium. The difference in security expenditures is equal to that in income. However, the self-protection contributions made by both countries are identical between the two columns. If we redistribute income, the self-protection contributions cancel out the income redistribution. However, Proposition 6.2 implies that any simultaneous redistribution of income and loss in interior equilibrium does not change the difference in self-protection contributions. Proposition 6.2 also shows that the difference in self-insurance purchases is equal to the difference in the loss in a bad state. We notice that both countries adjust their self-insurance purchases so as to cancel out the redistribution of income and loss.

Point Q_2 corresponds to the self-insurance freeriding equilibrium. Proposition 6.3 shows that country A consumes less than country B in the bad state if country A contributes to both public goods and country B contributes only to the self-protection public good. Let us compare column Q_2 with Q_1. Country A increases its contributions to self-protection and self-insurance. The difference in security expenditures is more than that in income. Compared with Q_1, the income gap, $Y^A - Y^B$, increases by 0.1 unit and the security expenditure gap, $m^{A*} - m^{B*}$, increases by 1.081 units. The consumption of country A in good and bad states reduces, whereas country B's consumption increases in both states. The expected welfare of A is lower than that of B. We assume $\bar{L}^A > \bar{L}^B$ in the simulation shown in column Q_2 but Proposition 6.3 shows that country A's consumption in the good state is less than country B's if country A's loss is less than country B's. The condition $\bar{L}^A < \bar{L}^B$ is a sufficient condition. Even if country A's loss is greater than B's, country A's welfare may be lower than country B's.

We further redistribute income and loss in the bad state from point Q_2 to obtain point Q_3. Point Q_3 is the centralized specialization in which country A contributes to both public goods and country B contributes nothing. Compared with Q_2, country A increases its self-protection contribution by 0.439 units and self-insurance purchases by 0.045 units, whereas its income increases only by 0.05 units. The security expenditure gap is 5.492 and the income gap is 3.6. The expected welfare of country A is lower than that of country B. Because country A's income is higher than country B's, it seems paradoxical. However, we should remember that country A's loss in the bad state is also higher than country B's. Thus, country A is more vulnerable to the disastrous event than B. Because of this vulnerability, country A demands more self-insurance than B. Country A's affluent provision of self-insurance not only enables country B to freeride in self-protection but also reduces country B's demand for self-protection. Then, country B freerides in self-protection, which raises country A's provision of that public good. At this point, we observe the heavy exploitation of country A by country B. This exploitation is caused not only because country A

Table 6.7 Results of the simulations of income-and-loss redistribution

1		O	Q_1	Q_2	Q_3	Q_4
2	Y^A	50	51.7	51.75	51.8	55
3	Y^B	50	48.3	48.25	48.2	45
4	$\bar{L}^A$	10	11.7	11.75	11.8	15
5	$\bar{L}^B$	10	8.3	8.25	8.2	5
6	m_1^{A*}	0.961	0.961	1.501	1.940	2.732
7	m_2^{A*}	1.734	3.434	3.507	3.552	4.592
8	m_1^{B*}	0.961	0.961	0.426	0.000	0.000
9	m_2^{B*}	1.734	0.034	0.000	0.000	0.000
10	$m^{A*}(= m_1^{A*} + m_2^{A*})$	2.695	4.395	5.007	5.492	7.324
11	$m^{B*}(= m_1^{B*} + m_2^{B*})$	2.695	0.995	0.426	0.000	0.000
12	W^{A*}	14.619	14.619	14.602	14.588	14.641
13	W^{B*}	14.619	14.619	14.637	14.651	14.635
14	W^{A*}/W^{B*}	1.000	1.000	0.998	0.996	1.000
15	p^*	0.493	0.493	0.494	0.495	0.554
16	C^{1A*}	47.305	47.305	46.743	46.308	47.676
17	C^{0A*}	42.079	42.079	41.579	41.192	42.409
18	C^{1B*}	47.305	47.305	47.824	48.200	45.000
19	C^{0B*}	42.079	42.079	42.653	43.133	45.141
20	$m^{A*} - m^{B*}$	0.000	3.400	4.581	5.492	7.324
21	$Y^A - Y^B$	0.000	3.400	3.500	3.600	10.000
22	m^{A*}/Y^A	0.054	0.085	0.097	0.106	0.133
23	m^{B*}/Y^B	0.054	0.021	0.009	0.000	0.000

Source Authors

is endowed with more income than B but also because country A faces a greater loss in the bad state than B.

We also note that if we transfer country B's income to A but do not transfer the loss in the bad state, the resulting equilibrium does not move to point Q_3. In such a case, we have a new self-insurance freeriding equilibrium in which the income redistribution is cancelled out through an adjustment in the self-protection contribution.

Finally, we investigate point Q_4, where we derive the centralized specialization equilibrium. As shown in Fig. 6.1, the gap in security expenditures expands as we redistribute income and loss from country B to country A. However, the expansion of the gap in security expenditures is slower than the expansion of the gap in income. Thus, the exploitation of country A by country B gradually changes from heavy exploitation to light exploitation. At point Q_4, the security expenditure gap is 7.324 and the income gap is 10, which indicates light exploitation.

The results of the numerical simulations of income redistribution are summarized as follows. The income redistribution from a symmetric economy is initially cancelled out by the changes in self-protection. We have the conventional neutrality result that private goods consumption and public goods provision are not affected by the transfer. When the income gap between the two countries is sufficiently large, we obtain the self-protection freeriding equilibrium. In this equilibrium, the difference in security expenditures between countries A and B is lower than the difference in income. Thus, country A enjoys a higher expected welfare than country B.

When we transfer not only income but also loss in the bad state, the results differ. We increase country A's loss in the bad state by the same amount of income that country A receives from country B. Simultaneously, we reduce country B's loss by the same amount of income that country B gives to country A. When the amount of transfer is sufficiently low, the transfer does not affect the consumption, public goods provision, or expected welfare of the countries. However, the channel that cancels out the redistribution is different from the previously discussed income redistribution. When we redistribute income and loss, the receiver of the transfer increases its self-insurance purchase by the same amount that the country receives and the giver of the transfer reduces its self-insurance purchase by the same amount. When the transfer becomes sufficiently high, we have the self-insurance freeriding equilibrium. In this equilibrium, the receiver expends much more for the security of the alliance than the giver. The difference in security expenditures is higher than that in income. Thus, the consumption of the high-income country is less than that of the low-income country. We further redistribute income and loss; we obtain the centralized specialization in which country A solely provides all public goods. As a result, country A's security expenditures rise and the exploitation of country A by country B increases. We repeat the redistributions of income and loss from country B to A. In response, the security expenditure gap becomes narrower and the difference in security expenditures becomes lower than that in income, which implies that the exploitation of country A by B changes from heavy to light.

6.3.3 NATO Scenario

In this subsection, we report the results of the numerical simulation of the NATO scenario. We consider a two-country configuration in which country A corresponds to the United States (US) and country B is the agglomeration of the four Western European countries: the Federal Republic of Germany (Germany), the United Kingdom (UK), France, and Italy. We simulate the military expenditures of these countries from 1970 to 2016. Because our model is static, we calculate the solution of the Nash equilibrium in each year. In this subsection, we add subscripts to the variables to indicate the year of observation: for example, Y_t^A is the national income of country A in year t.

Table 6.8 Nation GDP of five NATO members

	Country A	Country B				
	United States	Germany	France	United Kingdom	Italy	Country B total
1970	47.80	15.34	10.45	9.98	9.50	45.28
1980	65.29	20.41	14.92	12.31	13.80	61.44
1990	90.64	25.69	19.07	16.43	17.49	78.68
2000	127.13	31.24	23.46	20.95	20.60	96.26
2010	149.64	34.17	26.47	24.41	21.25	106.30
2016	169.20	37.82	28.11	27.58	20.81	114.30

Unit 100 billion constant 2000 USD
Source The World Bank (2018)

We use the observed GDP in 2000 constant price US dollars as the national incomes of the countries. Table 6.8 represents the actual GDPs of the five countries we consider. The unit of the numbers in Table 6.8 is 100 billion US dollars. The numbers reported in the table are rounded to two decimal places. For example, in 1970, the GDP of the US is 47.80 and the GDPs of the four Western European countries are as follows: Germany's is 15.34, France's is 10.45, the UK's is 9.98, Italy's is 9.50, and the total is 45.28. In the simulation, we do not round the numbers. The number used in the simulation is, for example, 47.796843942 as the actual value of the GDP of the US in 1970, Y^A_{1970}.

The values of the loss in the bad state in t, $\bar{L}^A_t$, and $\bar{L}^B_t$ are unknown. We use these parameters to calibrate the model so the ratios of total security expenditures to national income in 1970 (m^{A*}_{1970}/Y^A_{1970} and m^{B*}_{1970}/Y^B_{1970}) coincide with the actual ratio of military expenditures to GDP in 1970. We assume the values of $\bar{L}^A_t$ and $\bar{L}^B_t$ in years after 1970 using a hypothetical growth rate of loss, which is explained next.

The calibration in 1970 is conducted as follows. We add constraints so the ratios of total security expenditures to national income in 1970 (m^{A*}_{1970}/Y^A_{1970} and m^{B*}_{1970}/Y^B_{1970}) are equal to the actual values. Next, we treat $\bar{L}^A_{1970}$ and $\bar{L}^B_{1970}$ as endogenous variables and solve the system equations. Table 6.9 summarizes the actual military spending to GDP ratio of the five countries, rounding the numbers to one decimal place. We assume that the ratios of security expenditures to national income are equal to the observed military expenditures to GDP ratio. Formally, we assume the following:

$$\frac{m^{A*}_{1,1970} + m^{A*}_{2,1970}}{Y^A_{1970}} = \frac{7.42143205029538}{100}, \tag{6.99}$$

$$\frac{m^{B*}_{1,1970} + m^{B*}_{2,1970}}{Y^B_{1970}} = \frac{3.37242790369779}{100}, \tag{6.100}$$

Table 6.9 Military spending to GDP ratio (% of GDP)

	Country A	Country B				
	United States	Germany	France	Italy	United Kingdom	Country B total
1970	7.4	2.9	3.9	2.2	4.6	3.4
1980	4.8	2.8	3.8	1.9	4.5	3.2
1990	5.1	2.4	3.3	2.0	3.6	2.8
2000	2.9	1.4	2.5	2.0	2.1	2.0
2010	4.7	1.4	2.3	1.7	2.4	1.9

Source The World Bank (2018)

where $m_{1,1970}^{A*}$ ($m_{1,1970}^{B*}$) and $m_{2,1970}^{A*}$ ($m_{2,1970}^{B*}$) are country A's (B's) self-protection and self-insurance expenditures in the Nash equilibrium using the national income observed in 1970. We then have six equations—four of which represent the Nash equilibrium, along with Eqs. (6.99) and (6.100)—and six endogenous variables ($m_{1,1970}^{A*}$, $m_{2,1970}^{A*}$, $m_{1,1970}^{B*}$, $m_{2,1970}^{B*}$, $\bar{L}_{1970}^{A*}$, $\bar{L}_{1970}^{B*}$). Next, we solve the system to obtain $\bar{L}_{1970}^{A}$ and $\bar{L}_{1970}^{B}$. As shown in Table 6.2, our model has 16 types of Nash equilibria to be considered depending on whether the nonnegative constraints of contribution hold. Thus, we repeat this procedure for all 16 types of Nash equilibria. Our numerical simulation shows that when the values of $\bar{L}_{1970}^{A}$ and $\bar{L}_{1970}^{B}$ satisfy the following condition, the resulting security expenditures ($m_{1,1970}^{A*}$, $m_{2,1970}^{A*}$, $m_{1,1970}^{B*}$, $m_{2,1970}^{B*}$) satisfy Eqs. (6.99) and (6.100):

$$\bar{L}_{1970}^{A} = 9.4520760690986 \text{ and } \bar{L}_{1970}^{B} = 9.09762222219322.$$

In 1971–2016, we assume that the growth rates of $\bar{L}_t^A$ and $\bar{L}_t^B$ reflect the security environment and economic growth of NATO. Admittedly, the changing security environment affects not only the losses but also the baseline probability of a good state (p_0). In this chapter, we omit the impact on p_0 to avoid complexity. We report how well we simulate the development of actual military spending using assumptions on $\bar{L}_t^A$ and $\bar{L}_t^B$.

Table 6.10 summarizes the assumed growth rate. For the sake of comparison, this table also reports the average growth rates of Y^A and Y^B, which are calculated using the geometric average of the variable.[10] The growth rates are assumed as follows. Reflecting the détente in the 1970s, we assume that the loss in the bad state grows slower than the income. In the 1980s, reflecting the increase in the threat from the Soviet Union after its invasion of Afghanistan, we assume that the growth rates of the loss are slightly higher than that of income. In the 1990s, when the Cold War ended, we assume that the growth rates of the loss are negative. In 2000–2009, which

[10]For example, the average growth rate of Y^A in 1970–1979 is given as $(Y_{1980}^A/Y_{1970}^A)^{1/10} - 1$. The average growth rates of Y^A and Y^B reported in the table are rounded to two decimal spaces.

Table 6.10 Growth rate of loss in the bad state

	$\bar{L}^A_{t+1}/\bar{L}^A_t - 1$	$\bar{L}^B_{t+1}/\bar{L}^B_t - 1$	Average growth rate of Y^A	Average growth rate of Y^B
1970–1979	0.010	0.020	0.032	0.031
1980–1989	0.050	0.040	0.033	0.025
1990–1999	−0.010	−0.010	0.034	0.020
2000–2009	0.040	0.025	0.016	0.010

Source Authors

corresponds to the years after the September 11 attacks, we assume that the growth rates of loss are much higher than the growth rates of income, especially in country A. Using the assumed growth rates in Table 6.10, we obtain all values of $\bar{L}^A_t$ and $\bar{L}^B_t$. Next, we specify the values of all the exogenous variables. We conduct numerical simulations to obtain the Nash equilibrium allocation of our model.

Figure 6.2 illustrates the simulated security expenditures and compares them with the actual security expenditures. The solid curve with "Δ" markers shows the calculated ratio of country A's security expenditures to its income, m^{A*}_t/Y^A_t. The dotted curve with "Δ" markers represents the actual ratio of the US military expenditures to its GDP. The solid curve with "o" markers illustrates the calculated ratio of country B's security expenditures to its income, m^{B*}_t/Y^B_t. The dotted curve with "o" markers represents the actual ratio of the military expenditures of the four European countries—Germany, France, Italy, and the UK—to their GDP. As shown in this figure, the calculated ratio of security expenditures resembles the corresponding actual ratio of military expenditures.

Table 6.11 reports the detailed results of our numerical simulations. Row 1 describes the year when countries determine their contribution to both public goods in our simulation. Rows 2–5 report the setting of the exogenous variables. Rows 6–23 report the equilibrium levels of the endogenous variables and rows 24 and 25 report the actual ratio of the military spending to GDP.

The first column represents the result of the simulation of the scenario corresponding to the actual data in 1970. The ratio of security expenditures to national income of country A (B), m^{A*}_{1970}/Y^A_{1970} (m^{B*}_{1970}/Y^B_{1970}), is calibrated so it is equal to the actual ratio of the military spending of the US (four European countries) to its GDP. As shown in rows 6 and 7, country A contributes to both self-insurance and self-protection public goods, whereas country B specializes in the contribution of the self-insurance public good as shown in rows 8 and 9.

The second column of Table 6.11 reports the results of the numerical simulation using the actual GDP in 1980 and the assumed loss in the bad state. The ratio of country A's security expenditures to its national income decreases from 7.42% in 1970 to 4.63% in 1980 and the ratio of country B's security expenditures decreases from 3.37% in 1970 to 3.08% in 1980. We assume that the loss in the bad state grows slower than the national income during the 1970s. The results of the simulation show that the negative income effect, which we discussed in Chap. 5, dominates in this

Table 6.11 Results of the numerical simulations

1	t	1970	1980	1990	2000	2010
2	Y_t^A	47.80	65.29	90.64	127.13	149.64
3	Y_t^B	45.28	61.44	78.68	96.26	106.30
4	$\bar{L}_t^A$	9.45	10.44	17.01	15.38	22.77
5	$\bar{L}_t^B$	9.10	11.09	16.42	14.85	19.00
6	$m_{1,t}^{A*}$	1.72	2.08	3.39	3.40	5.06
7	$m_{2,t}^{A*}$	1.83	0.94	1.56	0.00	1.68
8	$m_{1,t}^{B*}$	0.00	0.00	0.00	0.00	0.00
9	$m_{2,t}^{B*}$	1.53	1.89	1.96	2.24	2.21
10	$m_t^{A*}(= m_{1,t}^{A*} + m_{2,t}^{A*})$	3.55	3.02	4.96	3.40	6.74
11	$m_t^{B*}(= m_{1,t}^{B*} + m_{2,t}^{B*})$	1.53	1.89	1.96	2.24	2.21
12	W_t^{A*}	14.52	15.03	15.54	16.10	16.35
13	W_t^{B*}	14.50	14.96	15.37	15.66	15.84
14	W_t^{A*}/W_t^{B*}	1.00	1.00	0.99	0.97	0.97
15	p^*	0.48	0.51	0.65	0.52	0.60
16	C_t^{1A*}	44.25	62.27	85.69	123.73	142.90
17	C_t^{0A*}	39.36	55.39	76.22	110.57	127.12
18	C_t^{1B*}	43.75	59.55	76.71	94.02	104.10
19	C_t^{0B*}	38.92	52.97	68.24	83.63	92.60
20	$m_t^{A*} - m_t^{B*}$	2.02	1.13	2.99	1.16	4.53
21	$Y_t^A - Y_t^B$	2.52	3.85	11.97	30.87	43.34
22	m_t^{A*}/Y_t^A (%)	7.42	4.63	5.47	2.67	4.50
23	m_t^{B*}/Y_t^B (%)	3.37	3.08	2.50	2.33	2.08
24	Military spending of US (% of GDP)	7.42	4.83	5.12	2.93	4.67
25	Military spending of Germany, France, Italy, and UK (% of GDP)	3.37	3.17	2.78	1.96	1.90

Note Rows 2–23 are the settings and results of our numerical simulations. Rows 24 and 25 are the actual ratios of military expenditures to their GDP calculated from the data in The World Bank (2018)
Source Authors

period. The high-income country (A) reduces its security expenditures more rapidly than the low-income country (B). As in Ihori et al. (2014), we observe that the burden sharing of the low-income ally increased in the 1970s and the sharing of the high-income ally decreased in the same period. Comparing the calculated ratio of security expenditures to income with the actual military expenditure ratio, we notice that the difference between the two rates is relatively small. The calculated ratio of country A is 0.2% points lower than the observed ratio of the US military expenditures. Similarly, country B's ratio of security expenditures is 0.09% points lower than the actual military spending to GDP ratio.

The third column of Table 6.11 reports the results of the simulation of the scenario corresponding to 1990. We assume that the loss in the bad state grows slightly faster than the national income in the 1980s, as described in Table 6.11. Our results are consistent with the conventional exploitation hypothesis. Row 22 shows that the ratio of country A's security expenditures to its income increases to 5.47% in 1990. This is 0.35% points higher than the actual security expenditure ratio. Unlike country A, country B's security expenditures decline in the 1980s. As shown in rows 23 and 25, B's security expenditures are only 2.5% in 1990, which is lower than the actual military expenditure ratio, 2.78%, by 0.28% points. As in Ihori et al. (2014), the burden sharing of the high-income ally increases in the 1980s, whereas the sharing of the low-income ally decreases in that period.

After the end of the Cold War, the threat from the Eastern bloc had significantly decreased. To simulate this ease of tension, we assume that the loss in the bad state decreases in the 1990s as shown in Table 6.11. Comparing column 1990 and 2000,

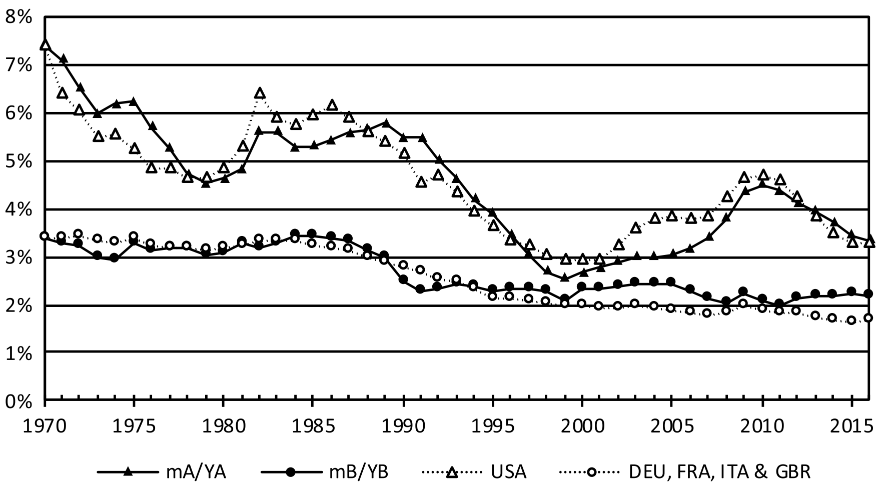

Fig. 6.2 Comparison of the simulated and actual security expenditures. *Note* mA/YA and mB/YB are the simulation results conducted by the author. "USA" and "DEU, FRA, ITA & GBR" are the actual ratios of military expenditures to their GDP calculated from the data in The World Bank (2018). *Source* Authors

we observe that both countries reduce their ratio of security expenditures to national income. Country A reduces its ratio to 2.67%, which is 0.26% points lower than the actual ratio. Country B decreases its ratio to 2.33%, which is 0.37% points higher than the actual ratio. Row 12 shows that the expected welfare of country A increases from 15.54 in 1990 to 16.10 in 2000 by 3.6%. In the same period, country B's expected welfare increases by 1.9%. The decrease in the loss improves the welfare of the high-income ally more than that of the low-income country.

In the 2000s, NATO allies faced threats from terrorist attacks and were engaged in the wars in Afghanistan and Iraq. We assume that country A's loss in the bad state increases from 2000 to 2009 much faster than the rate of economic growth. As shown in rows 22 and 23, the ratio of country A's security expenditures to its national income increases to 4.50%, whereas that of country B's decreases to 2.08%. The actual military expenditures to GDP ratio of the US also increases, whereas that of the Western European countries decreases.

6.4 Conclusion

We have examined the burden sharing in a two-nation alliance managing common risks with voluntarily provided self-insurance and self-protection public goods.

As presented in the first subsection of Sect. 6.2, the exploitation hypothesis in the conventional model shows that the high-income country contributes more to the public good than the low-income country. When both countries have identical preferences, the difference in contribution is exactly equal to that in income and the consumptions of the two countries become identical.

In the rest of Sect. 6.2, assuming identical preferences of the allies, we showed that burden sharing depends on whether the allies contribute to the public goods. In other words, the configuration of the corner or interior solution dominates how allies share the cost of risk management.

We defined five levels of exploitation based on its impact on private good consumption. Without loss of generality, we assumed that country A's income is higher than B's. If the difference in total security expenditures, $m^{A*} - m^{B*}$, is equal to the difference in income, $Y^A - Y^B$, the consumptions of the two countries in a good state are identical because the consumption of a country in a good state is given by its income net of the total security expenditures. We defined this case as conventional exploitation. If the difference in total security expenditures between countries A and B is lower than the difference in income, country A's consumption in the good state is higher than country B's. We defined this case as light exploitation. If the difference in total security expenditures is higher than that in income, country A's consumption in a good state is lower than country B's. We defined this case as heavy exploitation.

First, we investigated burden sharing in an interior Nash equilibrium in which both countries contribute to both public goods. If the absolute risk aversion of the allies sufficiently decreased, we obtained the following seven results: (1) the country with the higher national income, net of the loss in the bad state, contributes more to self-protection than the other country, (2) the country facing the greater loss in the bad state of the two purchases more self-insurance than the other, (3) the higher-income ally expends more for public goods in the total self-protection and self-insurance expenditures than the other, (4) the difference in the total security expenditures is equal to the difference in income (conventional exploitation), (5) the difference in self-protection contributions is exactly equal to the difference in the net income in the bad state, (6) the difference in self-insurance contributions is precisely equal to the difference in the loss in the bad state, and (7) in each state of the world, both countries consume the same amount, enjoy the same level of realized utility, and achieve the same level of expected welfare.

Second, we investigated the self-insurance freeriding equilibrium in which country A contributes to both public goods but country B specializes in self-protection. Under the same condition as in the interior equilibrium, we showed that country A contributes no less to the self-protection public good than B and the difference in contributions to self-protection of the two countries is greater than, or at least equal to, that in the disposable income in the bad state. In this sense, we argue that country A shares a disproportionally heavier burden than country B.

Third, we examined the self-protection freeriding equilibrium in which country A contributes to both public goods but country B specializes in self-insurance. We showed that if the same condition as in the interior equilibrium is satisfied, country B's expected welfare is lower than, or at highest equal to, country A's. In this equilibrium, the difference in contributions is no greater than the difference in income or disposable income. Thus, the high-income ally (country A) consumes no less than the low-income ally (country B) in each state of the world, which implies light exploitation.

Fourth, we investigated the decentralized specialization equilibrium in which country A specializes in self-protection and country B specializes in self-insurance. We showed that if the absolute risk aversion sufficiently decreases with consumption, country A consumes no less than country B in the bad state. Additionally, if country A's loss in the bad state is greater than, or at least equal to, B's, the expected welfare of country A is higher than that of B, which is light exploitation.

Additionally, we showed that when both countries contribute to the self-protection public good, any marginal income redistribution does not affect the consumption of private goods and provision of public goods. This is our version of the well-known neutrality result.

In Sect. 6.3, we conducted numerical simulations. Following Ihori et al. (2014), we specified the form of the utility function with the CRRA function. We also followed the specification of the probability function. The difference from the model developed by Ihori et al. (2014) lies in the budget constraint. From Ihori et al. (2014), we assumed that the allies choose the self-protection contribution and the amount of self-insurance. However, in the model developed by Ihori et al. (2014), the price of self-insurance appears in the budget constraint in the good state, whereas it appears in the budget constraint in the bad state in this chapter. This difference in the treatment of self-insurance affects the price effect of self-protection. In our model, the price of self-insurance depends on the probability of a good state, which depends on the amount of the self-protection public good. An increase in self-protection reduces the price of self-insurance. In Ihori et al. (2014), the reduction in the price of self-insurance increases consumption in the good state but consumption is increased in the bad state in this chapter. However, this difference does not significantly influence the theoretical analysis and the results of the numerical simulations do not qualitatively differ from those in Ihori et al. (2014).

We conducted the numerical simulations of two scenarios. One is a simulation of income redistribution using a hypothetical dataset. The other is a simulation of burden sharing in NATO using an actual dataset.

In Sect. 6.3.1, we reported the results of the numerical simulations of income redistribution. We supposed a symmetric economy and investigated the impact of income redistribution between the two countries. The simulation is classified into two groups. The first group assumes the redistribution of income from country B to country A. In the second group of simulations, we consider the simultaneous redistribution of income and loss. We transfer not only the income of the country but also its loss in a bad state.

The results of the first group are summarized as follows. The income redistribution from a symmetric economy is cancelled out by the changes in self-protection if redistribution is sufficiently low. If the income redistribution is sufficiently high, we obtain the self-protection freeriding equilibrium. In this equilibrium, the difference in security expenditures is lower than the difference in income (light exploitation). Thus, country A enjoys a higher expected welfare than country B.

The results of the second group of numerical simulations are summarized as follows. We increased country A's loss in the bad state by the same amount of income that country A received from country B. Simultaneously, we reduced country B's loss by the same amount of income that country B gives to country A. When the amount of transfer is sufficiently low, the receiver of the transfer increases its self-insurance purchase by the same amount that the country receives and the giver of the transfer reduces its self-insurance purchase by the same amount. Thus, the transfer does not alter the consumption, public goods provision, or expected welfares. When the transfer is sufficiently high, we have the self-insurance freeriding equilibrium in which the receiver expends much more for the security of the alliance than the giver. This is a case of heavy exploitation. When the redistribution is higher than in the case of self-insurance freeriding, we obtain the centralized specialization in which country A solely provides all public goods. Thus, the exploitation of country A by country B is initially heavier than in the self-insurance freeriding equilibrium. However, as the redistribution increases, the exploitation lessens. If the redistribution is sufficiently high, the consumption of country A is more than that of country B, which implies light exploitation.

In Sect. 6.3.3, we conducted a numerical simulation of burden sharing in NATO. We followed Ihori et al. (2014) in the specification of the values of the exogenous variables except for the value of income and loss in the bad state. Irrespective of the difference in model structure, the results of the numerical simulation we reported in Sect. 6.3 are consistent with the simulation of burden sharing in NATO reported by Ihori et al. (2014). From Ihori et al. (2014), we chose hypothetical values as income and loss. In this chapter, we assumed that country A is the US and country B is the aggregate of Germany, France, Italy, and the UK and used the actual GDP of the US as the income of country A and the aggregate income of the four countries as the income of country B. Next, we calibrated the hypothetical value of the loss in the bad state in 1970 using the observed ratio of military spending to GDP. The values of the loss in the year after 1970 were chosen to satisfy our assumption on the growth rate of the loss.

Our numerical simulation of the NATO scenario provided the following results.

First, we assumed that the loss in the bad state had grown slower than the national income during the 1970s, reflecting the reduced strained relations between the Eastern and Western blocs. Next, our simulation showed that the burden sharing of the low-income ally increased in the 1970s and the sharing of the high-income ally decreased in the same period.

Second, reflecting the increase in the threat from the Soviet Union after its invasion of Afghanistan, we assumed that the growth rates of the loss were slightly higher than that of income in the 1980s. Our result was consistent with the conventional exploitation hypothesis. The burden sharing of the high-income ally increased in the 1980s, whereas the sharing of the low-income ally decreased in that period.

Third, in the 1990s, when the Cold War ended, we assumed that the growth rates of the loss were negative. However, our simulation result showed that the security expenditure to income ratio of the high-income ally drastically decreased and the ratio of the low-income ally slightly decreased. Thus, the decrease in the loss caused by the end of the Cold War improved the welfare of the high-income ally more than that of the low-income ally.

Fourth, in 2000–2009, which covers the years after the September 11 attacks, we assumed that the growth rates of loss were much higher than the growth rate of income, especially in country A. Our simulation result showed that the ratio of country A' security expenditures to its national income increased and the ratio of country B's security expenditures decreased. This change in the ratio of security expenditures was consistent with the actual change in the military spending ratio of the NATO allies.

Appendix: Multi-public Goods and Multi-constraint Model

Cornes and Itaya (2010) considered an economy that consists of two players: one private good and two voluntarily provided public goods. The researchers showed that if the players are different in preferences, the number of unknown variables is strictly lower than the number of equations representing the Nash equilibrium in which both players simultaneously make positive contributions to both public goods. Thus, they claimed "there 'almost surely' does not exist a Nash equilibrium in which both players simultaneously make positive contributions to both public goods" (Cornes and Itaya 2010, Proposition 2(i), p. 369). Cornes and Itaya's (2010) proposition is based on the relationship between the number of unknown variables and the number of equations.

The main message of this appendix is as follows. As we will see, because the number of resource constraints for each player can be equal to the number of public goods in a multi-public good economy, as in our model developed in this chapter, their claim does *not* necessarily hold.

The organization of this appendix is as follows. In Section "Generalized Version of Cornes and Itaya's Proposition", we review Cornes and Itaya's (2010) proposition in a more general setting than their model. Their proposition consists of two parts. First, they derived a necessary condition that two agents simultaneously make positive contributions to both public goods in a voluntary contribution in a two public goods game. Second, they showed that the necessary condition is not satisfied if both agents are different in preferences. We extend their result to derive the necessary condition that H agents simultaneously contribute positive amounts to J public goods. This result is a generalized version of Cornes and Itaya's (2010) proposition.

Moreover, we also show that Cornes and Itaya's (2010) proposition does not necessarily hold in a variety of multi-public good models. In Section "Interior Equilibrium in our Model", we discuss whether Cornes and Itaya's (2010) claim applies to our model. In Section "Multi-constraint Model", we investigate a general model in which each player faces many resource constraints in addition to those of public goods. We then derive the corresponding necessary condition, which can be satisfied in general. We provide several examples in which two players with different preferences simultaneously make positive contributions to two public goods in a Nash equilibrium.

Generalized Version of Cornes and Itaya's Proposition

In this section, we extend Cornes and Itaya's (2010) model to an economy consisting of H players. The players are indexed by $h = 1, \ldots, H$. They consume I private goods, indexed by $i = 1, \ldots, I$. They voluntarily provide J public goods, indexed by $j = 1, \ldots, J$. The amount of private good i consumed by player h is denoted by c_i^h. The amount of public good j is represented by G_j. The utility of player h is given as:

$$U^h\left(c_1^h, \ldots, c_I^h, G_1, \ldots, G_J\right), \tag{6.101}$$

where $U^h(.)$ is strictly increasing, strictly quasi concave, and continuously differentiable. The amount of public good j is given as the sum of contributions:

$$G_j = \sum_{h=1}^{H} g_j^h \text{ for } j = 1, \ldots, J, \tag{6.102}$$

where g_j^h is player h's contribution to public good j. The budget constraint of player h is given as:

$$Y^h = \sum_{i=1}^{I} q_i c_i^h + \sum_{j=1}^{J} p_j g_j^h, \tag{6.103}$$

where Y^h represents the exogenously determined income of player h, q_i represents the price of private good i, and p_i represents the marginal cost of the contribution to public good j. We assume that Y^h, q_i, and p_j are positive. In general, Y^h can be interpreted as a resource, which may be transformed into a private good and public good. In this sense, we interpret Eq. (6.103) as the resource constraint of player h. In the next two sections, we introduce additional constraints including the standard budget constraint.

The utility maximization problem of player h is summarized as:

$$\max U^h\left(c_1^h, \ldots, c_I^h,\ G_1, \ldots, G_J\right),$$

subject to Eqs. (6.102) and (6.103) with respect to $c_1^h, \ldots, c_I^h,\ g_1^h, \ldots, g_J^h$. We then have the following proposition, which is a generalized version of Cornes and Itaya's (2010) proposition:

Proposition 6.7 *We suppose that $H > 1,\ J > 1$ and consider H players, I private goods, and J public goods economy. We also assume that each player faces one resource constraint. Except for coincidence, there is no Nash equilibrium in which all players simultaneously contribute positive amounts to all public goods.*

Proof The Lagrangian function of player h's utility maximization problem is defined as:

$$L^h = U^h\left(c_1^h, \ldots, c_I^h, G_1, \ldots, G_J\right) + \lambda^h \left\{ Y^h - \sum_{i=1}^{I} q_i c_i^h - \sum_{j=1}^{J} p_j g_j^h \right\}. \tag{6.104}$$

The first order conditions of an interior solution to $h's$ utility maximization problem are given as:

$$\frac{\partial U^h}{\partial c_i^h} - \lambda^h q_i = 0,\ \text{for } i = 1, \ldots, I, \tag{6.105}$$

$$\frac{\partial U^h}{\partial G_j} - \lambda^h p_j = 0,\ \text{for } j = 1, \ldots, J. \tag{6.106}$$

Solving these conditions for public good 1, we obtain:

$$\lambda^h = \frac{1}{p_1} \frac{\partial U^h}{\partial G_1}. \tag{6.107}$$

Table 6.12 The number of unknown variables and equations in a generalized version of Cornes and Itaya's proposition

Unknown variable		Equations	
Private goods	$H \times I$	First order conditions	$H \times (I + J - 1)$
Public goods	J	Resource constraint	1
Total	$HI + J$	Total	$H(I + J - 1) + 1$

Source Authors

Substituting Eq. (6.107) into Eq. (6.105), we obtain I equations. Substituting Eq. (6.107) into Eq. (6.106) for public good $j = 2, \ldots, J$, we also obtain $J - 1$ equations as follows:

$$\frac{\partial U^h}{\partial G_j} - \frac{p_j}{p_1} \frac{\partial U^h}{\partial G_1} = 0 \text{ for } j = 2, \ldots, J. \tag{6.108}$$

Thus, we obtain $I + J - 1$ equations for each player.

Aggregating the resource constraints of all players, we obtain:

$$\sum_{h=1}^{H} Y^h = \sum_{h=1}^{H} \sum_{i=1}^{I} q_i c_i^h + \sum_{j=1}^{J} p_j G_j. \tag{6.109}$$

Equations (6.105), (6.106), and (6.109) constitute a system of equations representing a Nash equilibrium in which all players contribute positive amounts to all public goods.

Table 6.12 summarizes the number of unknown variables and that of equations. Comparing the total numbers, we obtain:

$$\{H(I + J - 1) + 1\} - (HI + J) = (H - 1)(J - 1).$$

If there is more than one player and more than one public good, the number of equations in the interior Nash equilibrium exceeds the number of unknown variables.

Interior Equilibrium in Our Model

In this subsection, we show that the number of unknown variables coincides with that of the equations in our model. The expected welfare maximization problem of country A is rewritten as follows:

$$\max W^A = pU(C^{1A}) + (1 - p)U(C^{0A}),$$

subject to

$$
\begin{aligned}
C^{1A} &= Y^A - m_1^A - m_2^A, \\
C^{0A} &= Y^A - \bar{L}^A - m_1^A + \mathrm{L}(s), \\
p &= p(M_1), \\
s &= \frac{pM_2}{1-p}, \\
M_1 &= m_1^A + m_1^B, \\
M_2 &= m_2^A + m_2^B.
\end{aligned}
$$

The first order conditions for the interior solution are given by:

$$
\begin{aligned}
\frac{\partial W^A}{\partial m_1^A} =& p'(M_1)\Big(U(C^{1A}) - U(C^{0A})\Big) \\
& - \left\{ p(M_1)U_Y(C^{1A}) + (1-p(M_1))\left(1 - \frac{\partial \mathrm{L}}{\partial M_1}\left(\frac{p(M_1)M_2}{1-p(M_2)}\right)\right)U_Y(C^{0A}) \right\} = 0, \\
\frac{\partial W^A}{\partial m_2^A} =& p(M_1)\left(\mathrm{L}'\left(\frac{p(M_1)M_2}{1-p(M_2)}\right)U_Y(C^{0A}) - U_Y(C^{1A})\right) = 0
\end{aligned}
$$

We note that the unknown variables in the first order conditions are consumptions (C^{0A}, C^{1A}) and provisions of both public goods (M_1, M_2). Combining the budget constraints of both countries, we obtain:

$$
\begin{aligned}
C^{1A} + C^{1B} &= Y^A + Y^B - M_1 - M_2, \\
C^{0A} + C^{0B} &= Y^A + Y^B - \bar{L}^A - \bar{L}^B - M_1 + 2\mathrm{L}\left(\frac{p(M_1)M_2}{1-p(M_2)}\right).
\end{aligned}
$$

Table 6.13 summarizes the number of unknown variables and equations of an interior Nash equilibrium. As summarized in this table, we have six unknown variables and six equations. Thus, Cornes and Itaya's (2010) claim does not hold in our model of two public goods. Because our model includes two states of the world and two resource constraints corresponding to the respective state of the world, we have as many equations as unknown variables. In the next section, we generalize this finding.

Table 6.13 Number of unknown variables and equations in our model

Unknown variable		Equations	
Consumption	4	First order conditions	4
Public goods	2	Resource constraint	2
Total	6	Total	6

Source Authors

Multi-constraint Model

In this section, we extend the model presented in Section "Generalized Version of Cornes and Itaya's Proposition" to a multi-constraint model in which the number of resource constraints is equal to the number of public goods.

We replace the budget constraint of player h, Eq. (6.103), with the following constraints:

$$Y_k^h = \sum_{i=1}^{I} q_{ik} c_i^h + \sum_{j=1}^{J} p_{jk} g_j^h, \text{ for } k = 1, \ldots, J, \tag{6.110}$$

where Y_k^h, q_{ik}, p_{ik} are positive constants and their interpretations depend on the setting of the model. For example, when we consider a household production model, k is the type of the resource, Y_k^h is player h's endowment of the type k resource, q_{ik} is the units of the type k resource required for one unit of production of private good i, and p_{jk} represents the units of the type k resource required for one unit of contribution to public good j. We note that the number of h's resource constraints, Eq. (6.110), is equal to the number of public goods, J. We define a matrix of p_{jk} as P:

$$P \equiv \begin{pmatrix} p_{11} & \cdots & p_{1J} \\ \vdots & \ddots & \vdots \\ p_{J1} & \cdots & p_{JJ} \end{pmatrix}.$$

We also assume that:

$$|P| \neq 0. \tag{6.111}$$

To summarize, we revise the utility maximization problem of player h as:

$$\max U^h\left(c_1^h, \ldots, c_I^h, G_1, \ldots, G_J\right),$$

subject to Eqs. (6.102) and (6.110) with respect to $c_1^h, \ldots, c_I^h,\ g_1^h, \ldots, g_J^h$. We then have the following proposition:

Proposition 6.8 *We consider H players, I private goods, and J public goods economy. We also assume that the number of each player's resource constraints is J and the matrix of coefficients of contributions in resource constraints, P, is full rank. The number of equations of the interior Nash equilibrium is then always equal to the number of unknown variables. In other words,* Cornes and Itaya's (2010) *claim is not applicable to this situation.*

Proof The Lagrangian function can be defined as:

$$\tilde{L}^h = U^h\big(c_1^h, \ldots, c_I^h, G_1, \ldots, G_J\big) + \sum_{k=1}^{J} \lambda_k^h \left(Y_k^h - \sum_{i=1}^{I} q_{ik} c_i^h - \sum_{j=1}^{J} p_{jk} g_j^h \right). \tag{6.112}$$

The first order conditions of an interior solution are given as:

$$\frac{\partial \tilde{L}^h}{\partial c_i^h} = \frac{\partial U}{\partial c_i^h} - \sum_{k=1}^{J} \lambda_k^h q_{ik} = 0, \text{ for } i = 1, \ldots, I, \tag{6.113}$$

$$\frac{\partial \tilde{L}^h}{\partial g_j^h} = \frac{\partial U^h}{\partial G_j} - \sum_{k=1}^{J} \lambda_k^h p_{jk} = 0, \text{ for } j = 1, \ldots, J. \tag{6.114}$$

Equation (6.114) is rewritten as the following matrix form:

$$P \begin{pmatrix} \lambda_1^h \\ \vdots \\ \lambda_J^h \end{pmatrix} = \begin{pmatrix} \frac{\partial U^h}{\partial G_1} \\ \vdots \\ \frac{\partial U^h}{\partial G_J} \end{pmatrix}. \tag{6.115}$$

Because we assume that matrix P is full rank, we have:

$$\begin{pmatrix} \lambda_1^h \\ \vdots \\ \lambda_J^h \end{pmatrix} = P^{-1} \begin{pmatrix} \frac{\partial U^h}{\partial G_1} \\ \vdots \\ \frac{\partial U^h}{\partial G_J} \end{pmatrix}. \tag{6.116}$$

Substituting Eq. (6.116) into Eq. (6.113), we obtain:

$$\frac{\partial U}{\partial c_i^h} - (q_{i1}, \ldots, q_{iJ}) P^{-1} \begin{pmatrix} \frac{\partial U^h}{\partial G_1} \\ \vdots \\ \frac{\partial U^h}{\partial G_J} \end{pmatrix} = 0, \text{ for } i = 1, \cdots, I. \tag{6.117}$$

Thus, we have $H \times I$ first order conditions for all players.

Finally, we aggregate the resource constraints, Eq. (6.110), and obtain J resource constraints:

$$\sum_{h=1}^{H} Y_k^h = \sum_{h=1}^{H} \sum_{i=1}^{I} q_{ik} c_i^h + \sum_{j=1}^{J} p_{jk} G_j, \;\; j = 1, \ldots, J. \tag{6.118}$$

Table 6.14 The number of unknown variables and equations in the model developed in Sect. 6.2

Unknown variable		Equations	
Private goods	$H \times I$	First order conditions	$H \times I$
Public goods	J	Resource constraints	J
Total	$HI + J$	Total	$HI + J$

Source Authors

Table 6.14 summarizes the number of variables and equations. As shown in this table, when each player faces as many resource constraints as the number of public goods, the number of unknown variables is equal to that of the equations.

Proposition 6.8 is intuitively explained as follows. On one hand, the system of equations representing an interior Nash equilibrium consists of the following three types of equations: (1) the first order conditions with respect to private goods, (2) the first order conditions with respect to contributions, and (3) the resource constraints. On the other hand, this system of equations reduces to a system of equations of unknown variables representing private goods and public goods. In general, the number of equations can exceed the number of unknown variables because the first order conditions with respect to contributions do not have corresponding variables in the system of equations, which consists of variables representing private goods and public goods. However, when the number of players' resource constraints is equal to the number of public goods, the first order conditions with respect to contributions are solved with respect to Lagrange multipliers. These equations are then substituted in the first order conditions of private goods. Thus, the number of equations becomes equal to that of unknown variables.

This result establishes the inapplicability of Cornes and Itaya's (2010) claim—when agents are different in preferences, there almost surely does not exist a Nash equilibrium in which all players simultaneously make positive contributions to all public goods—to a situation in which each of the players faces as many resource constraints as the number of public goods.

Examples of Multi-resource Constraint Models

In this section, we present several examples in which each player faces as many resource constraints as the number of public goods. These include:

(1) *States-of-the-world model*: The players face several contingent states of the world. The public goods in different states are treated as different public goods.[11]

[11]This chapter and Ihori et al. (2014) investigated another type of a states-of-the-world model, which consists of two states of the world and two public goods.

(2) *International public goods provided by fragmented national government model*: Each national government is divided into as many administrative departments as international public goods and the departments are so bureaucratic that they have virtually independent budget constraints.
(3) *Household production model*: The players are endowed with several factors of production. They produce private goods and public goods using these factors. The player has as many resource constraints as the number of production factors.[12]
(4) *Dynamic model with liquidity constraint*: The players voluntarily provide a public good in each period. The public goods provided in different periods are treated as different public goods. Due to the liquidity constraint, the budget constraints of each player do not reduce to a single inter-temporal budget constraint.

Example of State-of-the-World Model: Clear Water Supply

Let us consider two contingent states: D and W. State D has a dry climate and state W has a wet climate. The probability of state D is p and the probability of state W is $1 - p$.

There are two households: $h = 1,\ 2$. They consume private goods and clear water. We assume that clear water is provided as a public good and the marginal cost of clear water depends on the state of the world. The utility of household h is given as:

$$u^h = pU^h(c_D^h, G_D) + (1 - p)U^h(c_W^h, G_W),$$

where c_D^h and c_W^h are consumptions, G_D and G_W are provisions of clear water, and the subscripts represent the state of the world. The amounts of public goods supplied are given as:

$$G_D = g_D^1 + g_D^2, \tag{6.119}$$

$$G_W = g_W^1 + g_W^2, \tag{6.120}$$

where $g_D^h, g_W^h (h = 1,\ 2)$ are household h's contributions to the provision of clear water in the two states. The budget constraint of household h is given as:

$$y^h = c_D^h + p_D g_D^h, \tag{6.121}$$

[12]Cornes and Schweinberger (1996) assumed that public goods are produced by households with several production factors. However, they also assumed that households buy private goods in the market with the income they make by selling the factors. Thus, each household in their model faces only one budget constraint except for corner solutions.

$$y^h = c_W^h + p_W g_W^h, \tag{6.122}$$

where y^h is fixed income and p_D and p_W are the marginal costs of clear water ($p_D > p_W > 0$). The number of resource constraints for each player is equal to that of the public goods.

Example of International Public Goods Provided by Fragmented National Governments Model

We assume that there is one private good, one national public good, and two international public goods. For example, the national public good is national defense and the international public goods are scientific knowledge and international peacekeeping operations.

The world consists of H countries, indexed by h. The welfare of country h is given as:

$$u^h = U^h(c_1^h, c_2^h, G_1, G_2),$$

where c_1^h is private good consumption, c_2^h is national defense, G_1 is scientific knowledge, and G_2 is the international peacekeeping operation. The amounts of international public goods are given as:

$$G_1 = g_1^1 + g_1^2, \tag{6.123}$$

$$G_2 = g_2^1 + g_2^2, \tag{6.124}$$

where g_1^h, $(h = 1,\ 2)$ is country h's contribution to scientific knowledge and g_2^h, $(h = 1,\ 2)$ is country h's contribution to international peacekeeping operations.

Country h's national defense and contribution to peacekeeping operations are carried out by its ministry of defense. We assume that the military budget is fixed for a political reason. Country h's resource constraints are therefore given as:

$$Y^h = c_1^h + g_1^h + \bar{B}^h, \tag{6.125}$$

$$\bar{B}^h = c_2^h + g_2^h, \tag{6.126}$$

where Y^h is the fixed national income and $\bar{B}^h$ is the fixed military budget. The number of resource constraints for each player is equal to that of public goods.

Example of Household Production Model: Capital and Labor

Assume an economy consists of two countries, one private good, two public goods, and two factors of production. The countries are indexed by $h = 1,\ 2$. The welfare of country h is given as:

$$u^h = U\left(c^h, G_1, G_2\right),$$

where c^h is h's private good consumption and G_1 and G_2 are public goods. The amounts of public goods are given as:

$$G_1 = g_1^1 + g_1^2, \tag{6.127}$$

$$G_2 = g_2^1 + g_2^2, \tag{6.128}$$

where g_j^h $(h,\ j = 1,\ 2)$ represents h's contribution to public good j. The country produces private goods and public goods with two factors—capital and labor—with linear production technology. The resource constraints of country h are given as:

$$Y_K^h = q_K c^h + p_{K1} g_1^h + p_{K2} g_2^h, \tag{6.129}$$

$$Y_L^h = q_L c^h + p_{L1} g_1^h + p_{L2} g_2^h, \tag{6.130}$$

where Y_K^h is h's capital endowment, Y_L^h is h's labor endowment, and $q_K, p_{K1}, p_{K2}, q_L, p_{L1}, p_{L2}$ are parameters representing the production technology.

In general, if every public good requires one intrinsic factor of production and all public goods are different in their intrinsic factors, the number of resource constraints for each player is equal to that of public goods.

Example of Dynamic Model

Let us consider the T-period model. Periods are index by $t = 1, \ldots, T$. This economy consists of H households, indexed by $h = 1, \ldots, H$. Households consume a private good and voluntarily contribute to a public good in every period. The current utility of household h in period t is given as:

$$U^h(c_t^h, G_t),$$

where c_t^h is h's private good consumption and G_t is the public good provision. The amount of the public good supplied at period t is given as:

$$G_t = \sum\nolimits_{h=1}^{H} g_t^h, \tag{6.131}$$

where g_t^h is h's contribution to the public good. The discount utility of household h is given as:

$$u^h = \sum_{t=1}^{T} \delta^{t-1} U^h\left(c_t^h, G_t\right),$$

where δ is the discount factor. We assume that households can neither lend nor borrow. The budget constraint at period t is:

$$Y_t^h = c_t^h + g_t^h. \tag{6.132}$$

The number of resource constraints of each household is T, which is equal to the number of public goods. We should remember that the condition we derived here is a necessary condition of the existence of the Nash equilibrium in which all players make positive contributions to all public goods. Even if each player faces as many resource constraints as the number of public goods, some players might not make positive contributions to some public goods in corner solutions.

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Chapter 7
Threat Misestimations and the Role of NGOs

7.1 Introduction

In this chapter, we investigate the impacts on collective risk management of mistakes made in estimating the severity of a threat. In general, in the real world, it is difficult for policymakers to make precise estimates of threats to their country. Although the governments intend to collect precise information on threats, the collected information can contain significant errors and is also often biased in bureaucratic administrative processes. Moreover, interest groups try to influence the evaluation of the information and the policymaking process through lobbying.

Using the alliance model developed in this book, we examine how misestimation of a threat affects burden-sharing among allies. Allies may "over"-estimate the threat so that the estimated level is higher than the true level of the threat. We will show that if an ally overestimates the threat to an alliance, it may contribute more to the public goods than when its estimates are accurate. Although a threat when "over"-estimated is higher than the true one, the welfare of the allies resulting from this mistake may be higher than if estimation were correct. The free-riding characteristic of voluntary public goods provision could be counteracted so as to attain the socially optimal level. (The prefix OVER of our term "overestimation" merely means that the estimated value is higher than the true parameter.)

Accordingly, we explore the possibility that overestimation (or more generally mistakes in estimation) of the threat may actually improve the welfare of the allies. We argue that there may exist a socially optimal level of overestimation of threat, which in Nash equilibrium can induce governments to voluntarily provide just the socially optimal amount of a self-protection public good. Although the terminology is somewhat contradictory, "socially optimal overestimation" means that in a second-best economy where private provision of the public good is under-provided the erroneous estimation higher than the true one can result in a socially optimal provision of the public good.

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Existing literature on the voluntary provision of public goods reports that "ambiguity" in the contribution by other players mitigates the free-riding effect. Eichberger and Kelsey (2002) investigated the relationship between externality and ambiguity. They defined ambiguity as follows: "Ambiguity refers to situations in which individuals have to make decisions when the relevant probabilities are unknown (Eichberger and Kelsey 2002, p.437)".[1] They showed that the amount of a public good voluntarily provided by players increases with ambiguity. Bailey et al. (2005) also showed that if ambiguity concerning the contribution by other players persists, the provision of public good increases with the population of an economy. Following the conventional literature on the private provision of public goods, these two pieces of research assumed that the provision of a public good is given as the sum of contributions. Unlike them, Kelsey and le Roux (2017) constructed a two-player game in which players voluntarily contribute to the provision of a public good, and the benefit of the public good is given by best-shot or weakest-link technologies.[2] They suppose that players are ambiguity-averse and show that under a best-shot technology, high ambiguity over other player's contribution causes both players to contribute the highest level to the public good, whereas under a weakest-link technology, high ambiguity causes them to contribute the lowest level to the public good.[3] Overall, in the context of alliance collective risk management, previous research suggests that, if allies are uncertain about the security expenditures of other allies, they contribute more to the security of the alliance.

However, that allies are uncertain mainly about the security expenditures of other allies is not a straightforward assumption. It seems more plausible to assume that allies are uncertain about the threat to their security more than others' security expenditures. We, therefore, prioritize uncertainty over threat magnitude itself and investigate a situation wherein allies may over/underestimate the threat to their alliance. And for the sake of simplicity, we make the contrary assumption that each ally is certain about the contributions made by other allies.

We formulate misestimations concerning threat as a bias in the estimate of loss in a bad event. Our models in Chaps. 5 and 6, show security spending to affect the welfare of allies through two channels. First, a preventive force of security spending affects loss in a bad state. For example, preventive military power could reduce damage to the alliance in case of war. Second, bellicosity affects the probability of a bad state. When security spending is more provocative, a bad state will more likely occur. As between these two channels, it is more straightforward to investigate over/underestimates of a loss. Moreover, the analytical implications of the second channel are more

[1]For the details of the definition of ambiguity, please refer to Eichberger and Kelsey (2002).

[2]Hirshleifer (1983) introduced the concept of the best-shot and weakest-link technologies. Under a best-shot technology, collective provision of the public good equals the maximum of contributions made by any single player. Under a weakest-link technology this provision equals the minimum of individual contributions.

[3]Though Kelsey and le Roux (2017) mathematically defined "ambiguity-averse," Eichberger and Kelsey (2014) intuitively explain the meaning of "ambiguity-averse" as follows: if individuals are ambiguity-averse, "they would pay some positive amount of money to avoid a situation where probabilities are poorly defined (Eichberger and Kelsey 2014, p.484)."

complicated. We, therefore, concentrate on the first issue by extending our models in Chaps. 5 and 6 to incorporate misestimations about the magnitude of loss in the bad state.

Because of such threat misestimates, countries may overprovide or underprovide public goods for international security. For example, consider the Iraq War. Before the breakout of the war, the US insisted that Saddam Hussein was hiding weapons of mass destruction. However, the US could not find the weapons after its invasion of Iraq. If Saddam had owned such weapons and *was* connected to terrorists, his potential threat could have been devastating; in short, the US government overestimated Saddam's threat. We can also find other examples of the overestimation of an opponent's threat, such as the Cold War era's "Missile Gap" in the late 1950s. On the other hand, examples of threat underestimation in our history could include a well-known fact: in the 1930s, the UK and other European countries underestimated a remilitarized Germany under Adolf Hitler.

The first half of this chapter investigates how misestimates concerning a threat affect burden-sharing in an alliance. In Sect. 7.2, we incorporate overestimation of the magnitude of loss in the bad state of our basic model of Chap. 6. We show that if two allies overestimate their loss in the bad state, and if the estimate of one ally deviates more from the true loss than that of the other, then if the Nash equilibrium is interior the former ally consumes more than the latter in the bad state. Moreover, we show that difference in total security expenditures between allies is not influenced by misestimations although the compositions of those security expenditures are influenced. If a high-income ally overestimates its loss compared with a low-income ally, the former contributes less to self-protection and more to self-insurance than the latter. Then, the difference in self-protection (or in self-insurance) between the two allies becomes smaller (larger) than it is in the absence of overestimation.

In the second half of this chapter, we conduct a "normative analysis of misestimation." Our question is: can misestimation maximize the utilitarian social welfare of individual allies? In a one-country world, misestimations can "hinder" a government from achieving an optimal supply of security. However, in a world of alliances, voluntary provision results in sub-optimal-collective-provision of the public good (at the non-cooperative Nash solution) because the nature of a public good itself entails a free-riding problem. Thus, a threat overestimation could stimulate public good provision and mitigate the free-riding.

In Sect. 7.3 we simplify our investigation by considering a single-public-good model wherein allies voluntarily provide a self-protection public good only. Here we discuss whether threat overestimation can induce countries to provide a socially optimal amount of public good in total.

In Sect. 7.4 we investigate the influence of interest groups on threat estimates. Although various interest groups may affect the government in the process of threat estimation, we hypothetically assume the existence of an international NGO (Non-Governmental Organization) located outside the alliance which can affect governments' behaviors when they estimate losses. The reason we focus on international NGOs outside the alliance is twofold: one is their increasing presence in the international policy arena, which we discuss below, and the other is analytical simplicity. If we assume that an agent inside the alliance can influence the estimation of loss, the welfare analysis should also consider the payoff to that agent, which would require a much more complicated analysis than ours.

In the realm of international affairs, NGOs are, in fact, providers of international public goods. For decades, they have contributed to various public goods, such as education, public healthcare, and international development assistance.[4] They have also indirectly stimulated international public goods by persuading governments to provide more of them. For example, they participate in international conferences and facilitate discourse in line with their objectives. In the domain of international security, some NGOs have played an essential role in developing arms control treaties, such as the Convention on the Prohibition of the Use, Stockpiling, Production and Transfer of Anti-Personnel Mines and on their Destruction (which is known as the *Ottawa Treaty*). However, their participation in international security has been relatively limited compared with developing assistance and public health.

In this chapter, we shed light on the normative role of NGOs in the latter case. That is, we focus on the normative role of NGOs as agents improving governments' decision-making in non-cooperative Nash equilibrium by providing somewhat biased information. We use the term "NGO" as organizations whose objective is to solve global issues, such as climate change or a pandemic of infectious diseases. For example, an international NGO that protects the global environment would have an incentive to induce allies' policymakers to pessimistically estimate environmental emergency losses. Then if policymakers mistakenly estimate the cost of emergency to be higher than the true cost, they may choose a more stringent environmental policy than they would in the absence of overestimation.

In a conventional model of voluntary provision of public goods, each player does not consider the external benefit of its provision enjoyed by other players. All players together, therefore, provide an inefficiently small amount of public good. If a planner supervises the players of the voluntary provision game, collects tax from them, and pays a matching grant for the provision of public goods, he/she can induce players to provide the socially optimal amount. However, we cannot utilize this policy instrument in the case of international risk management because no "World Government" exists that can compulsorily collect a tax from allied countries. In place of such "World Government," international NGOs (intentionally or unintentionally) might pessimistically bias the government's estimation of the loss in a bad state, thereby increasing the equilibrium level of self-protection, and improve the welfare of all allies.

This chapter consists of five sections. In Sect. 7.2, we extend the basic model of Chap. 6 to include overestimation of loss in the bad state. This allows us to investigate how bias in threat estimates affects burden-sharing among allies. Generally, overestimation of one variable (loss in a bad state) alone may not induce a government to allocate socially optimal levels of two variables (self-insurance and self-protection). In Sect. 7.3, we thus simplify the multi-variable model by omitting self-insurance

[4] Besley and Ghatak (2001, 2017) present a theory of the relationship between a government and an NGO in the provision of public goods.

and investigate whether overestimation could generate a socially optimal amount of the self-protection. With this simplification, we will show that choosing an appropriate level of overestimation may induce countries to choose a socially optimal level of self-protection. In Sect. 7.4, we extend our model to introduce a hypothetical international NGO's optimizing behavior. We assume that it may effectively influence the level of mis-estimate and that its objective is not to maximize the welfare of the allied countries but to maximize its own payoff. Then, we discuss the possibility that NGOs bias the ally's estimation of the loss and improve the welfare of the allies. Section 7.5 concludes this chapter.

7.2 Exogenous Misestimation and Exploitation Hypothesis

Although each government maximizes the expected welfare of its representative household, a government's estimate of loss could be influenced by external organizations, such as NGOs and lobbyist groups. For example, environmental protectionists might extremely emphasize the threat of climate change, while arms-producing companies may emphasize the threat of terrorist attacks. In the following sections, we incorporate overestimation (or underestimation) of the magnitude of loss in a bad state into our basic model in Chap. 6 and then investigate how the misestimation influences allies' burden-sharing in risk management.

7.2.1 Analytical Framework

In the following, we construct our alliance model with misestimation. Mainly we describe the maximization problem for the government of country A only. We could easily describe a similar maximizing problem for country B by replacing the superscript A with B.

With misestimation, the government of country A maximizes its welfare based on biased information of loss in the bad state. We name the objective of maximization as the "estimated expected" welfare of country A. We here use the term "estimated" in the sense that country A does not know the true loss in the bad state but estimates it when deciding its budget allocation. We assume that both the true loss and the estimated loss are exogenously given. This "estimated expected" welfare of country A is then given by the following:

$$\tilde{W}^A = pU(\tilde{C}^{1A}) + (1 - p)U(\tilde{C}^{0A}), \tag{7.1}$$

where $\tilde{W}^A$ is country A's estimated expected utility, $\tilde{C}^{1A}$ is country A's estimated consumption if the state of the world is good, $\tilde{C}^{0A}$ is A's estimated consumption if the state is bad, $U(.)$ is a utility function, and p is the probability of the good state. We denote the value of $U(.)$ as follows:

$$\tilde{U}^{iA} \equiv U(\tilde{C}^{iA}) \text{ for } i = 0, 1.$$

We assume that the utility function, $U(.)$, is increasing and concave[5]:

$$\tilde{U}_Y^{iA} \equiv dU(\tilde{C}^{iA})/d\tilde{C}^{iA} > 0, U_{YY}^{iA} \equiv d^2\tilde{U}(\tilde{C}^{iA})/d(\tilde{C}^{iA})^2 < 0, (i = 0, 1). \quad (7.2)$$

We also assume the utility function satisfies the Inada condition:

$$\lim_{C \to 0} dU(C)/dC = +\infty \text{ and } \lim_{C \to +\infty} dU(C)/dC = 0. \quad (7.3)$$

As in Chap. 6, the probability of the good state, p, increases with the provision of self-protection. The self-protection purchased by both countries raises the probability of the good state with that benefit of increasing probability of the good state being nonrival and nonexcludable. As in the basic model of Chap. 6, self-protection is an international public good. Its amount is given by the sum of the contributions of allied countries as follows:

$$M_1 = m_1^A + m_1^B, \quad (7.4)$$

where M_1 is the total amount of the self-protection public good, m_1^A is country A's contribution to the public good, and m_1^B is country B's contribution. The probability of the good state is given by:

$$p = p(M_1), \quad (7.5)$$

where

$$p''(M_1) > 0, p''(M_1) < 0, p(M_1) \in (0, 1) \text{ for any } M_1 \geq 0. \quad (7.6)$$

When the government of country A determines its self-insurance and self-protection expenditures, its budget constraint is given as follows:

$$\tilde{C}^{1A} = Y^A - m_1^A - m_2^A; \quad (7.7)$$

$$\tilde{C}^{0A} = Y^A - \tilde{L}^A - m_1^A + \mathrm{L}(s), \quad (7.8)$$

[5]In the notation of the derivative of the utility function, we use subscript ***Y***. As defined below, ***Y*** represents the unconditional income of the country. The reason why we use ***Y*** instead of C is that we focus on the income effect throughout this paper.

where Y^A is country A's income, $\tilde{L}^A$ is its estimated loss in the bad event, m_2^A, its contribution to the self-insurance public good, $\mathrm{L}(s)$ is the self-insurance benefit in the bad state and s is the self-insurance input in the bad state. The estimated loss may be formulated as the sum of the true loss and the bias in estimation:

$$\tilde{L}^A = \bar{L}^A + \alpha^A, \tag{7.9}$$

where $\bar{L}^A$ is the true loss in the bad state and α^A measures the overestimation of that loss (which is given exogenously in this section). If A underestimates its loss, we obtain $\alpha^A < 0$. The government of A presupposes $\tilde{L}^A$ as the true estimated loss when it chooses its contributions to the public good.

The self-insurance benefit function $\mathrm{L}(s)$ is increasing and concave. We assume that the marginal product or benefit of self-insurance is not higher than unity. Thus, we have

$$\mathrm{L}' \equiv \frac{d\mathrm{L}}{ds} \in (0, 1), \; L'' \equiv \frac{d^2\mathrm{L}}{ds^2} \in (-\infty, 0]. \tag{7.10}$$

Self-insurance input, s, is the total self-insurance premium divided by the price of self-insurance. As in Chap. 6, we assume that the price of self-insurance is actuarially fair. Then, we have

$$s = \frac{M_2}{\left(\frac{1-p}{p}\right)} = \frac{pM_2}{1-p}, \tag{7.11}$$

where $M_2 = m_2^A + m_2^B$ is the total payment of the self-insurance premiums paid by both countries and the denominator of the right-hand side (RHS) of (7.11), $(1-p)/p$, is the actuarially fair price of self-insurance.

The time structure of this model is as follows. Both A and B simultaneously determine the allocations of their endowments to self-insurance and self-protection. Then, the state of the world is stochastically determined. If the state is bad, the true loss is revealed. The *ex post* budget constraints of country A are thus given by the following:

$$C^{1A} = Y^A - m_1^A - m_2^A; \tag{7.12}$$

$$C^{0A} = Y^A - \bar{L}^A - m_1^A + \mathrm{L}(s), \tag{7.13}$$

where C^{1A} is A's consumption in the good state, C^{0A} is consumption in the bad state, and $\bar{L}^A$ is country A's true actual (or *ex post*) loss in the bad state.[6]

[6]The value of C^{1A} is identical to that of $\tilde{C}^{1A}$. We distinguish $\tilde{C}^{1A}$ from C^{1A} to clarify that $\tilde{C}^{1A}$ represents the consumption in a good state estimated in the first period, while C^{1A} is the realized consumption in the second state.

7.2.2 Individual Optimization and Nash Equilibrium

In this subsection, we derive the conditions for individual optimization and define "Nash equilibrium with misestimation." The expected welfare maximization problem is identical to that analyzed in Chap. 6, except that $\bar{L}^A$ is substituted for $\tilde{L}^A$. The first order condition for interior self-protection is given as follows:

$$\frac{\partial \tilde{W}^A}{\partial m_1^A} = p'\left(\tilde{U}^{1A} - \tilde{U}^{0A}\right) - \left\{p\tilde{U}_Y^{1A} + (1-p)\left(1 - \frac{\partial \mathrm{L}}{\partial M_1}\right)\tilde{U}_Y^{0A}\right\} = 0. \quad (7.14)$$

The first order condition for an interior self-insurance is given as follows:

$$\frac{\partial \tilde{W}^A}{\partial m_2^A} = p\left(L'\tilde{U}_Y^{0A} - \tilde{U}_Y^{1A}\right) = 0. \quad (7.15)$$

We also assume that the second order conditions for the expected welfare maximization are satisfied:

$$\frac{\partial^2 \tilde{W}^A}{\partial (m_1^A)^2} < 0, \ \frac{\partial^2 \tilde{W}^A}{\partial (m_2^A)^2} < 0 \text{ and } \frac{\partial^2 \tilde{W}^A}{\partial^2 m_1^A}\frac{\partial^2 \tilde{W}^A}{\partial^2 m_2^A} - \left(\frac{\partial^2 \tilde{W}^A}{\partial m_1^A \partial m_2^A}\right)^2 > 0. \quad (7.16)$$

Then, country A's best response function is given as follows:

$$m_1^A = m_1^A(m_1^B, m_2^B, Y^A, \tilde{L}^A), \quad (7.17)$$

$$m_2^A = m_2^A(m_1^B, m_2^B, Y^A, \tilde{L}^A). \quad (7.18)$$

Functions $m_1^A(.)$ and $m_2^A(.)$ are identical to those in Chap. 6, except that $\tilde{L}^A$ in the argument substitutes for $\bar{L}^A$.

The Nash equilibrium of this model, $(m_1^{A*}, m_2^{A*}, m_1^{B*}, m_2^{B*})$, is defined as the solution to the following system:

$$m_1^{A*} = m_1^A(m_1^{B*}, m_2^{B*}, Y^A, \tilde{L}^A), \quad (7.19)$$

$$m_2^{A*} = m_2^A(m_1^{B*}, m_2^{B*}, Y^A, \tilde{L}^A), \quad (7.20)$$

$$m_1^{B*} = m_1^B(m_1^{A*}, m_2^{A*}, Y^B, \tilde{L}^B), \quad (7.21)$$

$$m_2^{B*} = m_2^B(m_1^{A*}, m_2^{A*}, Y^B, \tilde{L}^B). \quad (7.22)$$

Without misestimation, or $\tilde{L}^A = \bar{L}^A$ and $\tilde{L}^B = \bar{L}^B$, the Nash equilibrium reduces to that derived in Chap. 6. Variables corresponding to the Nash equilibrium are denoted by an asterisk.

When country A maximizes its expected welfare, it estimates its consumption in both states. The estimated consumptions in the good and bad states (under Nash equilibrium allocations to self-insurance and self-protection) are denoted $\tilde{C}^{1A*}$ and $\tilde{C}^{0A*}$, respectively. They are given as follows:

$$\tilde{C}^{1A*} = Y^A - m_1^{A*} - m_2^{A*}, \tag{7.23}$$

$$\tilde{C}^{0A*} = Y^A - \bar{L}^A - \alpha^A - m_1^{A*} + \mathrm{L}(s^*). \tag{7.24}$$

The true (or *ex post*) consumptions in good and bad states under the Nash equilibrium are defined as C^{1A*} and C^{0A*}, and are given as follows:

$$C^{1A*} = Y^A - m_1^{A*} - m_2^{A*},$$
$$C^{0A*} = Y^A - \bar{L}^A - m_1^{A*} + \mathrm{L}(s^*).$$

We refer to the probability-weighted realized utility as the true expected welfare:

$$W^{A*} = p(M_1^*)U(C^{1A*}) + (1 - p(M_1^*))U(C^{0A*}). \tag{7.25}$$

7.2.3 Exploitation Hypothesis in Misestimation Model

Concentrating on an interior equilibrium, we examine the impact of mistaken estimations on alliance burden-sharing. We showed that if absolute risk aversion is sufficiently decreasing, both countries consume the same amount (Proposition 6.1). In this section, we show that, even if estimates are wrong, both countries consume the same amount in both states of the world from the estimated *ex ante* viewpoint. However, now misestimates affect real *ex post* consumption in the bad state. Then, the true expected welfare is not identical between allies. We also show that bias in estimation will not affect the inter-ally difference in total security expenditure but will affect the resource allocations between self-insurance and self-protection. These results are summarized in the following propositions.

Proposition 7.1 *Suppose that all elements of the Nash equilibrium vector of contributions,* $(m_1^{A*}, m_2^{A*}, m_1^{B*}, m_2^{B*})$*, are positive and that*

$$\mathrm{L}'\tilde{R}^{0A} > \tilde{R}^{1A} \text{ and } \mathrm{L}'\tilde{R}^{0B} > \tilde{R}^{1B}, \tag{7.26}$$

where $\tilde{R}^i \equiv -U_{YY}(\tilde{C}^i)/U_Y(\tilde{C}^i)$ *for* $i = 0A, 1A, 0B, 1B$ *is absolute risk aversion while estimated consumption is* $\tilde{C}^i$*. Then, estimated consumptions in Nash equilibrium satisfy*

$$\tilde{C}^{0A*} = \tilde{C}^{0B*} \; and \; \tilde{C}^{1A*} = \tilde{C}^{1B*}. \tag{7.27}$$

Proof We replace $\bar{L}^A$ and $\bar{L}^B$ in the proof of Proposition 6.1 in Chap. 6 with $\tilde{L}^A$ and $\tilde{L}^B$. Then, the modified proof establishes this proposition.

According to Proposition 7.1, if both countries contribute to both public goods and if the absolute risk aversions of both countries are sufficiently decreasing with consumption, then estimated consumptions become identical regardless of the difference in their national incomes. However, unlike Proposition 6.1, the true expected welfare is not identical between allies as shown in the following proposition.

Proposition 7.2 *Suppose that Eq. (7.26) is satisfied. Suppose also that the Nash equilibrium is an interior solution. Then, we have the following:*

$$C^{1A*} = C^{1B*}, \tag{7.28}$$

$$C^{0A*} - C^{0B*} = \alpha^A - \alpha^B, \tag{7.29}$$

$$W^{A*} > W^{B*} \; if \; and \; only \; if \; \alpha^A > \alpha^B. \tag{7.30}$$

Proof We remember that the Nash equilibrium allocation $(m_1^{A*}, m_2^{A*}, m_1^{B*}, m_2^{B*})$ satisfies the budget constraints given by Eqs. (7.7) and (7.8). Country B's budget constraints are given by replacing superscript A with B in Eqs. (7.7) and (7.8). The Nash equilibrium levels of contributions made by country B (m_1^{B*} and m_2^{B*}) satisfy B's budget constraints. Substituting the budget constraints into Eq. (7.27), we obtain

$$Y^A - m_1^{A*} - m_2^{A*} = Y^B - m_1^{B*} - m_2^{B*}. \tag{7.31}$$

$$Y^A - \bar{L}^A - \alpha^A - m_1^{A*} + \mathrm{L}(s^*) = Y^B - \bar{L}^B - \alpha^B - m_1^{B*} + \mathrm{L}(s^*), \tag{7.32}$$

Substituting Eq. (7.12) into Eq. (7.31), we obtain Eq. (7.28). Substituting Eq. (7.13) into Eq. (7.32), we obtain

$$C^{0A*} - \alpha^A = C^{0B*} - \alpha^B. \tag{7.33}$$

Rearranging Eq. (7.33), we obtain Eq. (7.29).

To show Eq. (7.30), we first assume that $\alpha^A > \alpha^B$. Then, Eq. (7.29) implies that $C^{0A*} > C^{0B*}$. Using Eq. (7.28), we obtain

$$W^{A*} = p(M_1^*)U(C^{1A*}) + (1 - p(M_1^*))U(C^{0A*}) > p(M_1^*)U(C^{1B*}) + (1 - p(M_1^*))U(C^{0B*}) = W^{B*} \tag{7.34}$$

Thus, the sufficient condition of Eq. (7.30) is established. Next, we assume that $W^{A*} > W^{B*}$. Then, we obtain

$$W^{A*} = p(M_1^*)U(C^{1A*}) + (1 - p(M_1^*))U(C^{0A*}) > p(M_1^*)U(C^{1B*}) + (1 - p(M_1^*))U(C^{0B*}) = W^{B*} \tag{7.35}$$

Using Eq. (7.28) and our assumption that $p(M_1) \in (0, 1)$, Eq. (7.35) reduces to

$$U(C^{0A*}) > U(C^{0B*}). \tag{7.36}$$

Since $U' > 0$ and $C^{0A*} = \tilde{C}^{0A*} + \alpha^A$, we have

$$\tilde{C}^{0A*} + \alpha^A > \tilde{C}^{0B*} + \alpha^B. \tag{7.37}$$

Substituting Eq. (7.27) into Eq. (7.37), we obtain the necessary condition of Eq. (7.30).

According to Proposition 7.2, if Nash equilibrium is interior, the country with the larger overestimation bias consumes more in the bad state than the other country. This implies that the country with larger overestimate enjoys a higher true expected welfare. The intuition behind this proposition is simple. As shown in Proposition 7.1, if both countries contribute to both public goods, their estimated consumptions in the bad state will be identical. However, the realized consumption in the bad state is larger than the estimated consumption by the amount of the overestimate of the loss. Thus, the country that makes the bigger mistake and the larger overestimate consumes more in the bad state than the other country. However, in the good state, their estimated consumptions are identical. Hence, their realized consumptions in the good state are also identical because the realized consumption and estimated consumption in the good state are the same. To summarize, a country with the larger bias in estimated loss consumes more than the other country in the bad state, consumes the same as the other country in the good state, and enjoys a higher true expected welfare than the other country.

Proposition 7.1 also has implications for burden-sharing between the two allies, as summarized in the following proposition.

Proposition 7.3 *Suppose that Eq. (7.26) is satisfied. Suppose also that the Nash equilibrium to be an interior solution. Then, we have the following:*

$$(m_1^{A*} + m_2^{A*}) - (m_1^{B*} + m_2^{B*}) = Y^A - Y^B, \tag{7.38}$$

Table 7.1 Parameter values

p_0	p_e	ϕ	θ
0.25	1	0.9	0.9

Source Authors

$$m_1^{A*} - m_1^{B*} = (Y^A - \bar{L}^A) - (Y^B - \bar{L}^B) + \alpha^B - \alpha^A, \tag{7.39}$$

$$m_2^{A*} - m_2^{B*} = \bar{L}^A - \bar{L}^B + \alpha^A - \alpha^B. \tag{7.40}$$

Proof Rearranging Eq. (7.31), we obtain Eq. (7.38). Similarly, rearranging Eq. (7.32), we obtain Eq. (7.39). Combining Eq. (7.38) with Eq. (7.39), we obtain Eq. (7.40).

Proposition 7.3 claims that the difference in total security expenditure between allies does not depend on biases in estimates. As shown in Eq. (7.38), the difference between $m_1^{A*} + m_2^{A*}$ and $m_1^{B*} + m_2^{B*}$ depends only on the gap in national income between allies. The biases in estimates of loss, α^A and α^B, affect the composition of security expenditure. For example, let us suppose that country A overestimates its loss in the bad state more than country B, $\alpha^A > \alpha^B$. This causes m_2^{A*} to be more than m_2^{B*}, while it causes m_1^{A*} to be less than m_1^{B*}. Then, the difference in self-protection contributions is smaller, while that in self-insurance contributions is larger than in the absence of mistaken estimates. The shrinking of the gap in self-protection expenditure is canceled out by the widening of the gap in the self-insurance expenditure. However, differences between countries in their equilibrium levels of total security expenditures are unaffected by their mistakes.

7.2.4 Numerical Examples

In this section, we conduct several numerical simulations to investigate how an overestimate of loss affects burden-sharing in the alliance. We follow our numerical simulation in Chap. 6 in the specification of the utility function and that of the probability function. We also follow Chap. 6 in the setting of parameters. The values of the parameters, which are summarized in Table 7.1, are identical to those in the simulations reported in Chap. 6. Both countries are endowed with identical 50 units of income and face the identical ten units of loss in the bad state.

Table 7.2 summarizes the results of three numerical simulations. Column 1 reports the result of a baseline scenario wherein no misestimates of loss occur. Then, both countries contribute to self-protection and self-insurance public goods. Country A's contribution of self-insurance and self-protection are equal to those made by country B.

Column 2 reports the result of simulation wherein only country A overestimates the loss by 0.1 unit. Compared with column 1, country A increases its self-insurance

Table 7.2 Numerical simulations of exogenous overestimation and burden-sharing

Scenario	1 Baseline	2 Overestimation by A	3 Overestimation by both
Y^A	50	50	50
Y^B	50	50	50
$\bar{L}^A$	10	10	10
$\bar{L}^B$	10	10	10
α^A	0	0.1	0.1
α^B	0	0	0.1
m_1^{A*}	0.961	0.920	0.978
m_2^{A*}	1.734	1.796	1.759
m_1^{B*}	0.961	1.020	0.978
m_2^{B*}	1.734	1.696	1.759
M_1^*	1.923	1.939	1.956
M_2^*	3.467	3.493	3.518
$m^{A*}(= m_1^{A*} + m_2^{A*})$	2.695	2.716	2.737
$m^{B*}(= m_1^{B*} + m_2^{B*})$	2.695	2.716	2.737
$\tilde{W}^{A*}$	14.619	14.619	14.618
$\tilde{W}^{B*}$	14.619	14.619	14.618
W^{A*}	14.619	14.621	14.620
W^{B*}	14.619	14.619	14.620
$m_1^{A*} - m_1^{B*}$	0.000	- 0.100	0.000
$m_2^{A*} - m_2^{B*}$	0.000	0.100	0.000
$m^{A*} - m^{B*}$	0.000	0.000	0.000

Source Authors

contribution and decreases its self-protection contribution. On the contrary, country B decreases its self-insurance and increases its self-protection. As shown in Proposition 7.3, country A's self-protection is smaller than country B's by 0.1 unit, and country A's self-insurance is larger than country B's by 0.1 unit. The total security expenditure of country A is identical to that of country B. Although differences in total security expenditure are not affected by the overestimation, total security expenditure itself increased with the overestimation. In column 1, total security expenditure of country A is 2.695, while that in column 2 is 2.716. As predicted in Proposition 7.2, the true welfare of country A is higher than that of country B in column 2.

Column 3 illustrates the result of the simulation wherein both countries A and B overestimate the loss by 0.1 unit. Then, both countries increase their contributions to both public goods. Each country contributes more to self-insurance and self-protection than they do in the baseline. Furthermore, this overestimate of the loss is Pareto-improving, that is, the true expected welfares of both countries improve. This result highlights the normative role of misestimation.

However, it should be stressed that overestimating the loss alone cannot generally achieve the socially optimal allocation of self-insurance and self-protection because we cannot choose the optimal allocation of two endogenous variables (self-insurance and self-protection) using one exogenous variable (over/under-estimate of loss). In the next section, we simplify our model to a model with a self-protection public good only and conduct a normative analysis of mistakes in estimations to attain the social optimum in the Nash equilibrium.

7.3 A Normative Analysis of Misestimation

As shown in the previous section, overestimation of loss may improve the welfare of the allies when the allies non-cooperatively provide public goods. Here we will further investigate the normative role of misestimation to alleviate the free riding outcome of public good provision. If a country estimates its loss in the bad state to be more than its true magnitude, it may purchase more security instruments to manage that risk. Moreover, suppose there exists a "World Government," which maximizes the social welfare of the alliance even though it cannot control the provision of public goods directly. If it chooses the level of bias in loss estimation such that bias may induce countries to choose socially optimal levels of the public good, optimal levels may be voluntarily chosen in the Nash equilibrium. To summarize, we assume that nature, or the "World Government," determines the bias in estimation and this section intends to investigate whether a socially optimal bias exists. For the sake of simplicity, we omit self-insurance from the alliance model developed in Sect. 7.2 and investigate the impacts of bias in estimation exclusively on self-protection and implied welfare.

7.3.1 Analytical Framework

Modifying our model to one public good model, we investigate how a country responds to a change in an assumed overestimation. Because we omit self-insurance, country A's maximization problem transforms as follows: A determines its provision of self-protection to maximize expected welfare subject to the following budget constraints:

$$\tilde{C}^{1A} = Y^A - m_1^A, \tag{7.41}$$

$$\tilde{C}^{0A} = Y^A - \bar{L}^A - \alpha^A - m_1^A. \tag{7.42}$$

Here, the magnitude of overestimate is exogenously assumed as in Sect. 7.2.

7.3.1.1 First-Best Self-Protection

Then, we define the first-best contribution to self-protection. We here suppose that a "World Government" maximizes the utilitarian social welfare of the two countries. It knows the true magnitude of the loss in the bad state and can arbitrarily choose the amount of self-protection. Then, we define the first-best amount of self-protection, m_1^{A**} and m_1^{B**}, as the solution to the following problem:

$$\max_{m_1^A, m_1^B} W^A + W^B = p(m_1^A + m_1^B)\Big[U(Y^A - m_1^A) + U(Y^B - m_1^B)\Big] \\ + (1 - p(m_1^A + m_1^B))\Big[U(Y^A - \bar{L}^A - m_1^A) + U(Y^B - \bar{L}^B - m_1^B)\Big]. \tag{7.43}$$

As in Chap. 5, the first order condition of this problem is given as follows:

$$\frac{\partial W}{\partial m_1^A} = p'\big(U^{1A} + U^{1B} - U^{0A} - U^{0B}\big) - \big[pU_Y^{1A} + (1-p)U_Y^{0A}\big] = 0, \tag{7.44}$$

where $U^{1i} \equiv U(Y^i - m_1^i)$, $U^{0i} \equiv U(Y^i - \bar{L}^i - m_1^i)$, $U_Y^{1i} \equiv \partial U^{1i}/\partial m_1^i$, $U_Y^{0i} \equiv \partial U^{0i}/\partial m_1^i$ for $i = A, B$. The first term of Eq. (7.44) represents the marginal benefit of self-protection, which is the gain (going from a bad to good state of the world) in social welfare multiplied by the increase in the probability of a good state. The second term represents the marginal costs of self-protection. Because one unit of self-protection reduces country A's consumptions in both states, the marginal cost is given as a probability-weighted sum of marginal utility of consumption. We assume that the second order condition is satisfied, $\partial^2 W/\partial(m_1)^2 < 0$. Because the definition of W does not contain any estimation bias, the first-best optimal amount of self-protection does not depend on the bias.

7.3.1.2 Individual Optimization in the Second-Best Economy

We suppose the "World Government" cannot control the provision of self-protection directly in the second-best economy. It maximizes the social welfare by choosing the level of bias in loss estimation.

We begin with the maximization problem of the individual allies in the second-best economy. Country A's maximization problem is summarized as follows:

$$\max_{m_1^A, m_2^A} \tilde{W}^A = pU(\tilde{C}^{1A}) + (1-p)U(\tilde{C}^{0A}), \tag{7.45}$$

subject to Eqs. (7.4), (7.5), (7.41), and (7.42).

The first order condition of the maximization problem for an interior solution is given as:

$$p'\left(\tilde{U}^{1A} - \tilde{U}^{0A}\right) - \left[p\tilde{U}_Y^{1A} + (1-p)\tilde{U}_Y^{0A}\right] = 0, \tag{7.46}$$

where $\tilde{U}^{iA} \equiv U(\tilde{C}^{iA})$, $\tilde{U}_Y^{iA} \equiv \partial U(\tilde{C}^{iA})/\partial \tilde{C}^{iA}$ for $i = 0, 1$. The first term on the left-hand side (LHS) of Eq. (7.46) shows A's marginal benefit of self-protection, and the second term represents its marginal cost. A's contribution to self-protection benefits both countries A and B. Compared with Eq. (7.44), Eq. (7.46) does not include the marginal benefit of this self-protection for B. Thus, with no overestimation, A's contribution is less than the first-best level.

The second order condition is:

$$\frac{\partial^2 \tilde{W}^A}{\partial (m_1^A)^2} < 0. \tag{7.47}$$

We assume that the second order condition is satisfied. Solving Eq. (7.46), gives A's best response function, which depends on country B's contribution, A's income, and A's estimated loss:[7] $m_1^A(m_1^B, Y^A, \bar{L}^A + \alpha^A)$.

As explained in Chap. 5, Ihori and McGuire (2007) investigated the properties of such a best response function in a model without misestimates. The corresponding properties in our model are almost the same as in theirs, except that expected loss is not accurate, but biased. Taking the total differentiation of Eq. (7.46), gives the following:

$$\begin{aligned}\frac{\partial m_1^A}{\partial m_1^B} &= \frac{-\frac{\partial^2 \tilde{W}^A}{\partial m_1^A \partial m_1^B}}{\frac{\partial^2 \tilde{W}^A}{\partial (m_1^A)^2}} \\ &= \frac{-1}{\frac{\partial^2 \tilde{W}^A}{\partial (m_1^A)^2}}\left\{p''\left(\tilde{U}^{1A} - \tilde{U}^{0A}\right) + 2p'\left(\tilde{U}_Y^{0A} - \tilde{U}_Y^{1A}\right) + p\tilde{U}_{YY}^{0A} + (1-p)\tilde{U}_{YY}^{1A}\right\},\end{aligned} \tag{7.48}$$

$$\frac{\partial m_1^A}{\partial \alpha^A} = \frac{-\frac{\partial^2 \tilde{W}^A}{\partial m_1^A \partial \tilde{L}^A}}{\frac{\partial^2 \tilde{W}^A}{\partial (m_1^A)^2}} = \frac{-1}{\frac{\partial^2 \tilde{W}^A}{\partial (m_1^A)^2}}\left\{p'\tilde{U}_Y^{0A} + (1-p)\tilde{U}_Y^{1A}\right\}. \tag{7.49}$$

Then, we can deduce the following proposition.

Proposition 7.4 *Suppose that country A purchases a positive amount of self-protection. Then, we have the following:*

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[7] Since the estimated loss is biased upwards (or downwards), the best response does not maximize A's true expected welfare. In this sense, A's best response is not objectively "best" in its literal meaning, although it maximizes A's estimated (and biased) expected welfare.

$$\frac{\partial m_1^A}{\partial m_1^B} < 0 \Leftrightarrow p''\left(\tilde{U}^{1A} - \tilde{U}^{0A}\right) - 2p'\tilde{U}_Y^{1A} + p\tilde{U}_{YY}^{1A} + \tilde{U}_Y^{0A}\left(2p' - (1-p)\tilde{R}^{0A}\right) < 0, \quad (7.50)$$

$$\frac{\partial m_1^A}{\partial \alpha^A} > 0 \Leftrightarrow p'\tilde{U}_Y^{0A} + (1-p)\tilde{U}_{YY}^{0A} = \tilde{U}_Y^{0A}\left\{p' - (1-p)\tilde{R}^{0A}\right\} > 0, \quad (7.51)$$

where $\tilde{U}_{YY}^{iA} \equiv \partial^2 U(\tilde{C}^{iA})/\partial(\tilde{C}^{iA})^2$.

Proposition 7.4 shows a comparative statics property of the best response function. Equation (7.50) implies that, if absolute risk aversion in the bad state is sufficiently large, the contributions by countries to self-protection are strategic substitutes in the sense that if country A (B) increases its contribution, B's (A's) national welfare maximizing contribution decreases. A sufficient condition for contributions to be strategic substitutes is that $2p' \leq (1-p)\tilde{R}^{0A}$. Even if this sufficient condition is not satisfied, self-protections may still be strategic substitutes if the first three terms on the LHS of Eq. (7.50) are sufficiently negative.

Equation (7.51) implies that, if the absolute risk aversion is sufficiently small, country A's self-protection increases with its bias in loss estimation. Such an increase in the bias has two effects on A's decision.

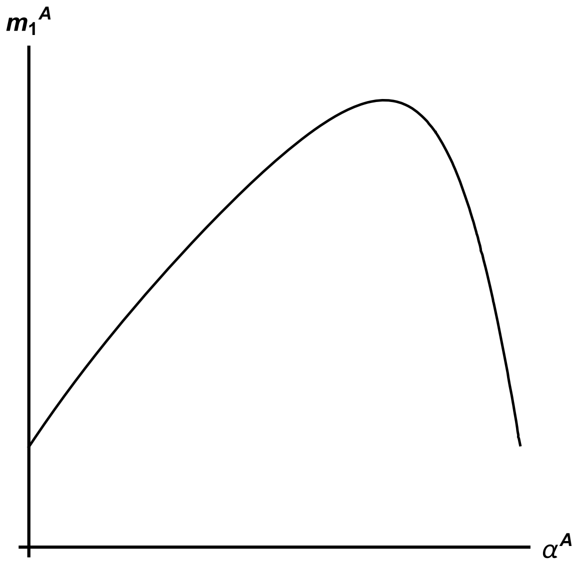
The first effect is a positive effect in the sense that an additional bias increases the demand for self-protection. By reducing calculated utility in the bad state, it increases the welfare gap between the two states and raises the marginal benefit of self-protection.

The second effect is a negative effect in the sense that an increase in the estimated loss reduces the demand for self-protection. The marginal cost of self-protection is given as the probability-weighted sum of marginal utilities in both states of the world because the country pays the cost of self-protection in both states. The additional loss reduces the income in the bad state. If we fix the amount of self-protection purchased, it reduces the private consumption in the bad state. Then, the last unit of private consumption abandoned to purchase the last unit of self-protection has more impact on welfare than before the loss increases. Thus, the marginal cost of self-protection rises when we evaluate the cost in terms of marginal utility. The rise in the marginal cost reduces the demand for self-protection.

The magnitude of the positive effect depends on marginal utility, whereas the negative effect depends on the second derivative of utility. Thus, comparing these two effects, if country A's absolute risk aversion is sufficiently small, the positive dominates the negative. Then, an increase in the estimated loss from the bad event will raise A's purchase of self-protection.

For example, assume the utility function is a constant relative risk aversion (CRRA) function. For such functions, absolute risk aversion, $\tilde{R}^{0A}$, decreases with the consumption in the bad state. Since estimated consumption in the bad state decreases with estimated losses, absolute risk aversion increases with the bias in loss estimation. When the bias is sufficiently small, $\tilde{R}^{0A}$ is so low that $\partial m_1^A/\partial\alpha^A$ is positive. However, when the bias is sufficiently large, $\tilde{R}^{0A}$ becomes so high that $\partial m_1^A/\alpha^A$ is negative. Therefore, as shown in Fig. 7.1, the curve representing m_1^A in (α^A, m_1^A)-space has an inverted-U shape.

Fig. 7.1 Self-protection and bias in loss estimation. *Source* Authors



Let us give an example wherein a pessimistic bias in loss estimation may improve the welfare of allies. Assume that the magnitude of the loss in the bad state is not devastating and that absolute risk aversion decreases with income, as in the CRRA utility function. Then, the absolute risk aversion is sufficiently small in the bad state such that an increase in bias of loss estimation may raise the provision of self-protection and improve the expected welfare.[8]

To summarize, when the utility function shows decreasing risk aversion, overestimation of loss (or positive bias in the loss estimation) may improve the welfare if income net of loss in the bad state is sufficiently large. On the other hand, if net income is sufficiently small, underestimation of the loss (or negative bias in the loss estimation) may improve the welfare.

Unlike in Sect. 7.2, here we cannot obtain a simple relationship between both countries' contributions, such as Eqs. (7.38), (7.39), and (7.40). Note that Propositions 6.1 and 7.1 depend on the fact that both countries contribute to the self-insurance public good. When a country purchases self-insurance, the ratio of marginal utility in the good state to that in the bad state is equal to the marginal insurance benefits. If country A contributes to self-protection, we rewrite the first order condition for self-protection as:

$$p'\frac{\left(\tilde{U}^{1A}-\tilde{U}^{0A}\right)}{\tilde{U}_Y^{0A}}=p\frac{\tilde{U}_Y^{1A}}{\tilde{U}_Y^{0A}}+(1-p). \tag{7.52}$$

[8]If the loss in the bad state is small, the absolute risk aversion is small because the income net of loss is large and the absolute risk aversion decreases with net income.

The LHS of Eq. (7.52) represents the marginal benefit of self-protection, and the RHS represents the marginal cost. If country A contributes to both self-protection and self-insurance, the ratio, $\tilde{U}_Y^{1A}/\tilde{U}_Y^{0A}$, in the first term on the RHS equals the marginal benefit of self-insurance. If both countries contribute to both self-protection and self-insurance, the RHS becomes identical for both countries. Then, consumption must be identical as well. However, if both countries do not contribute to self-insurance, the RHS is not necessarily identical for the two, so that in this case, we would not obtain a simple equation, such as Eqs. (7.38), (7.39), or (7.40).

7.3.2 Nash Equilibrium

We consider Nash equilibrium of our game wherein countries A and B non-cooperatively provide shared self-protection public good. We define the Nash equilibrium as a bundle of purchases of self-protection, (m_1^{A*}, m_1^{B*}), which satisfies the following equations:

$$m_1^{A*} = m_1^A(m_1^{B*}, Y^A, \tilde{L}^A), \tag{7.53}$$

$$m_1^{B*} = m_1^B(m_1^{A*}, Y^B, \tilde{L}^B). \tag{7.54}$$

Nash equilibrium level of self-protection and realized consumption of country A are given as:

$$M_1^* = m_1^{A*} + m_1^{B*}. \tag{7.55}$$

$$C^{1A*} = Y^A - m_1^{A*}, \tag{7.56}$$

$$C^{0A*} = Y^A - \bar{L}^A - m_1^{A*}. \tag{7.57}$$

The realized utility of households in country A is given as follows:

$$U^{1A*} = U(C^{1A*}) \text{ and } U^{0A*} = U(C^{0A*}). \tag{7.58}$$

We refer to the “true” probability weighted realized utility as the true expected welfare:

$$W^{A*} = pU^{1A*} + (1-p)U^{0A*}. \tag{7.59}$$

However, because (in Nash equilibrium) A maximizes its <u>estimated</u> expected welfare for any given level of B’s contribution to self-protection, A does not maximize its <u>true</u> expected welfare. Nevertheless, the pessimistic bias in loss estimation provides additional incentives to contribute to self-protection and might cancel out free-riding incentives in both countries, just enough to lead to the first-best level of the true expected welfare.

7.3.3 *Socially Optimal Estimation Bias*

Although any bias in loss estimation leads to an inefficient outcome in a single country model, such a bias may improve the welfare of allies in our alliance model because it mitigates the free-riding incentives. If biases in loss estimation induce allies to contribute more to the self-protection public good than they do in the absence of estimation bias, there may exist an optimal degree of bias such that it could induce allied countries to contribute just exactly the first-best optimal amount of the self-protection public good. We refer to this optimal degree of bias as a "socially optimal degree of estimation bias" and discuss the sufficient condition of its existence in this subsection.

7.3.3.1 Symmetric Countries

For the sake of simplicity, we assume that both countries are endowed with the identical income, each loses the identical amount in case of the bad state and biased by the same level in their estimation of the loss. Then, we have

$$Y^A = Y^B = Y, \bar{L}^A = \bar{L}^B = \bar{L}, \text{ and } \alpha^A = \alpha^B = \alpha, \tag{7.60}$$

where Y is the national income of each country, $\overline{L}$ is the loss in the bad state in each country and α is the overestimation of loss. Then, the first-best self-protection is identical:

$$m_1^{A**} = m_1^{B**} = m_1^{**},$$

where m_1^{**} is the first-best self-protection in this symmetric case. We also limit ourselves to consider a symmetric Nash equilibrium wherein both countries contribute the same amount to the public good. Because the allies' contributions in the Nash equilibrium depend on Y, $\overline{L}$ and α, we denote the contribution to self-protection in the symmetric Nash equilibrium as follows:

$$m_1^{A*} = m_1^{B*} = m_1^*(Y, \bar{L} + \alpha), \tag{7.61}$$

where $m_1^*(Y, \bar{L} + \alpha)$ represents the contribution in a symmetric Nash equilibrium when $Y^A = Y^B = Y$, $\bar{L}^A = \bar{L}^B = \bar{L}$, and $\alpha^A = \alpha^B = \alpha$.

7.3.3.2 Socially Optimal Estimation Bias in the Second-Best Economy

In the second-best economy, the "World Government" cannot directly choose m_1 but can choose α to maximize the welfare of a representative player. In this symmetric case, the "World Government" solves the following problem:

$$\max_{m_1^*, \alpha} W^* = p(2m_1^*)U(Y - m_1^*) + (1 - p(2m_1^*))U(Y - \bar{L} - m_1^*)$$

subject to

$$m_1^* = m_1^*(Y, \overline{L} + \alpha). \tag{7.62}$$

The first order condition is given as the following:

$$\frac{\partial W^*}{\partial m_1^*}\frac{\partial m_1^*}{\partial \alpha} = 0. \tag{7.63}$$

Equation (7.63) holds either if the purchase of self-protection is the first-best optimum ($\partial W^*/\partial m_1^* = 0$), if the estimation bias maximizes the purchase of self-protection ($\partial m_1^*/\partial \alpha = 0$), or both.

However, the first-best contribution of self-protection, m_1^{**}, is not necessarily realized in the second-best setting. We define the maximum of self-protection in the second-best setting, $\bar{m}_1^*$, as

$$\bar{m}_1^* = \max_{\alpha} m_1^*(Y, \bar{L} + \alpha). \tag{7.64}$$

If the first-best provision is larger than this maximum, the "World Government" cannot bias countries' estimate to provide the first-best amount of self-protection. In the following, we refer to m_1^{**} as the first-best self-protection, and $\bar{m}_1^*$ as the maximum self-protection.

Consider the following two cases:

Case a: The first-best self-protection is not larger than the maximum self-protection ($m_1^{**} \leq \bar{m}_1^*$).
Case b: The first-best self-protection is larger than the maximum self-protection ($m_1^{**} > \bar{m}_1^*$).

We discuss these two cases separately. In case a, there exists a value of α such that each country voluntarily purchases the first-best self-protection. We refer to this value as the socially optimal estimation bias and denote it by α^{**}. By definition, we obtain

$$m_1^{**} = m_1^*(Y, \overline{L} + \alpha^{**}). \tag{7.65}$$

If the "World Government" chooses α^{**}, each country purchases m_1^{**} units of self-protection and the true expected welfare of each country is maximized.

We denote the level of bias such that the purchase of self-protection is maximized by $\bar{\alpha}^*$. Formally, $\bar{\alpha}^*$ is defined as the solution of the following equation:

$$\bar{m}_1^* = m_1^*(Y, \overline{L} + \bar{\alpha}^*). \tag{7.66}$$

Figure 7.2a represents the relationship between estimation bias and self-protection purchase in case a.[9] The solid curve shows the locus of function $m_1^*(Y, \bar{L} + \alpha)$ in (α, m_1)-space. As in Fig. 7.1, this curve has an inverted-U form. An increase in the bias has positive and negative effects on the demand for self-protection. The positive effect is that the increase in the estimated loss widens the difference between the welfare of both states and increases the marginal benefit of self-protection. The negative effect is that the increasing loss decreases the estimated income net of the loss in the bad state and increases the marginal utility of private consumption in that state. Then, the marginal cost of self-protection evaluated in the marginal utility also rises, and the demand for self-protection declines. If the bias is small (large), the positive (negative) effect exceeds the negative (positive) effect. The self-protection purchase along the solid curve becomes maximum at point B, where the maximal value of self-protection is $\bar{m}_1^*$. The horizontal dashed line represents the first-best optimal amount of self-protection, m_1^{**}. This figure shows a case wherein m_1^{**} is lower than $\bar{m}_1^*$. The dashed line intersects the solid curve at points A and C. The abscissas of these intersections, α_1^{**} and α_2^{**}, represent the levels of estimation bias that make countries voluntarily purchase the first-best optimal amount of self-protection in the Nash equilibrium.

In contrast, the estimation bias maximizing the purchase of self-protection, $\bar{\alpha}^*$, does not maximize the expected welfare of each country. Although $\bar{\alpha}^*$ satisfies Eq. (7.63), it does not maximize the expected welfare because the "World Government" can improve the expected welfare by changing the bias from $\bar{\alpha}^*$ to α_1^{**} or a_2^{**}.

In case b, the first-best self-protection (m_1^{**}) is larger than the maximum self-protection ($\bar{m}_1^*$). Figure 7.2b represents the relationship between self-protection purchase and bias in case b. The solid curve represents the equilibrium self-protection purchase function, $m_1^*(Y, \bar{L}+\alpha)$. The dashed line shows the first-best self-protection. As shown in this figure, there exists no α such that the representative country chooses the first-best self-protection (m_1^{**}). In this case, the sign of $\partial W^*/\partial m_1^*$ is always positive for any $m_1 \leq \bar{m}_1^*$. Thus, the estimation bias maximizing the self-protection purchase ($\bar{\alpha}^*$) also maximizes the expected welfare of the representative country in the second-best setting. However, the welfare of each ally is lower than the first-best level.

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[9]This figure is drawn based on our specification for numerical simulations. Utility function is a CRRA function. The probability of the good state is given by a contest success function developed by Tullock (1967). The parameter values are identical to those in Table 7.3. Please refer to Appendix B for the details of this setting.

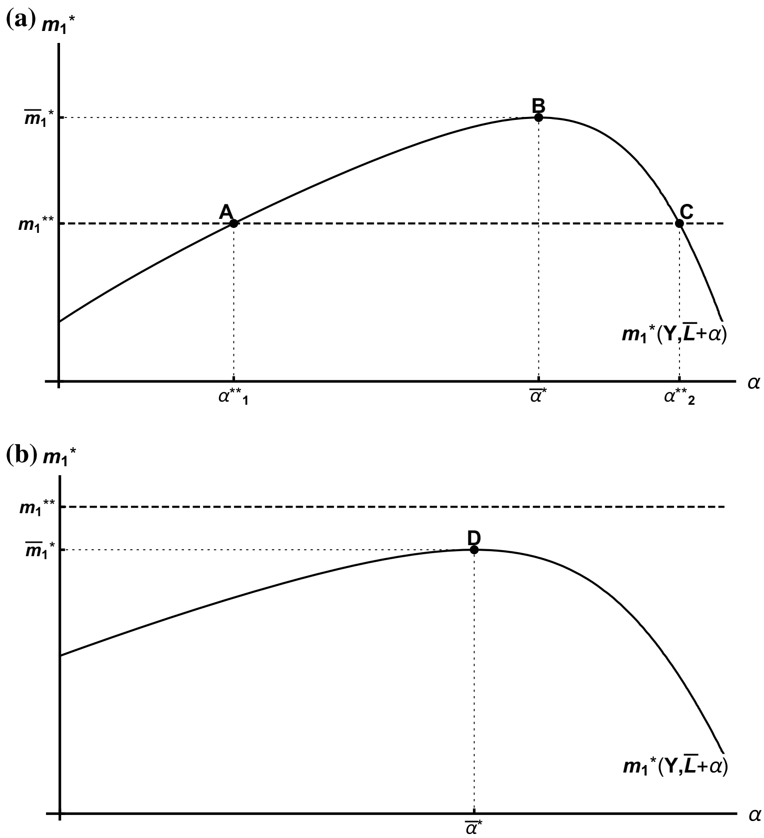

Fig. 7.2 **a** Estimation bias and self-protection purchase in case a. **b** Estimation bias and self-protection purchase in case b. *Source* Authors

The first-best self-protection depends on the income and the loss in the bad state. Fixing the national income, Fig. 7.3 represents how the first-best and maximum self-protections vary with the true loss.[10] The solid curve represents the first-best, and the dashed line represents the maximum self-protection. Because the "World Government" can adjust the bias to maximize the self-protection, the maximum is not influenced by the change in true loss and shown as a horizontal line.

If the true loss is sufficiently small ($\bar{L} \leq \bar{L}_s$), the first-best is zero because the damage from the bad event is too low to purchase self-protection. If the true loss satisfies $\bar{L} \in (\bar{L}_s, \bar{L}_m]$, the first-best is at largest as large as the maximum. In this case, socially optimal estimation bias exists.

If the true loss is in a range ($\bar{L} \in (\bar{L}_m, \bar{L}_b)$), the first-best is higher than the maximum. The intuition is explained as follows. In the first-best allocation, the "World

[10]This figure is drawn based on our specification for numerical simulations in Appendix B.

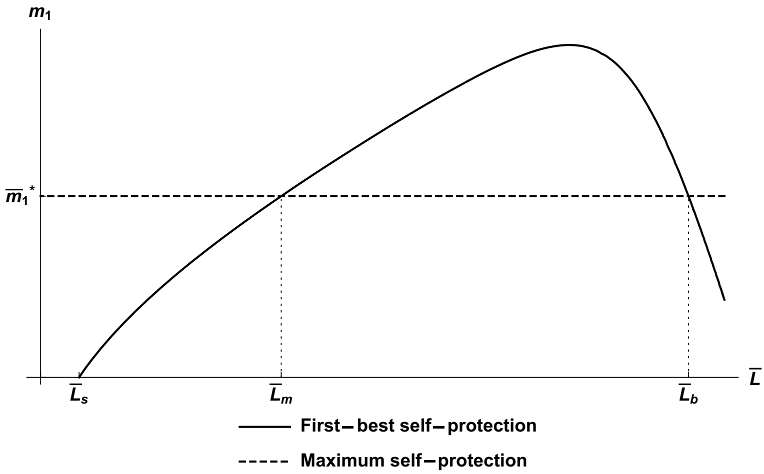

Fig. 7.3 True loss and the relationship between m_1^{**} and $\bar{m}_1^*$. *Source* Authors

Government" takes into account the benefits of self-protection to both countries. In the second-best setting, each country non-cooperatively purchases its self-protection without taking into account the benefit spillover to the other ally. Thus, in the absence of estimation bias, the self-protection is underprovided in the second-best case. In this case, the sufficiently large loss implies that the neglected benefit to other country is substantial. Additionally, because the loss is not extremely large, the negative effect of loss on the marginal cost of self-protection does not dominate the positive effect. Thus, the first-best self-protection becomes larger than the maximum self-protection in this case.

If the true loss is extremely large ($\bar{L} \geq \bar{L}_b$), the maximum self-protection exceeds the first-best self-protection again. In this range, the first-best declines with the true loss because the negative effect of increasing true loss dominates the positive effect. By underestimating the true loss, the "World Government" can increase the income estimate in the bad state and induce countries to purchase the first-best self-protection.

Finally, we conducted several numerical simulations to explore the condition that there exists a degree of estimation-bias that induces countries voluntarily to provide the first-best self-protection. Details of the results are reported in Appendix B. We show an example wherein there exist two values of estimation-bias that give rise to the first-best optimum in the second-best economy.

7.4 Endogenous Misestimation and the Normative Role of NGOs

Although overestimates of loss may improve the welfare of allies in our alliance model, individually the allies have no incentives to overestimate. It is also unlikely that governments will correctly estimate loss from bad events because many agents, such as lobbyists, activists, politicians, bureaucrats, and journalists, often influence those governments' estimates. In this section, therefore, we focus on the influences of international NGOs because they have increased their presence in the formation of foreign policies and international agreements in the last decades.

As an example, let us consider self-protection as diplomatic efforts including foreign aid. Each allied country may offer foreign aid to developing countries not only to make them friendly to the alliance but to prevent a humanitarian crisis in these countries, (which could lead to a surge of refugees to the allied countries). Because of free-riding incentives, such aid would be less than the socially optimal level. International NGOs might extract additional aid or support to the developing world from our allied governments by emphasizing the threat of humanitarian crisis. Our model here posits that these NGOs are trying to increase the governments' self-protection through emphasizing damages in bad states of the world.

Another example is pro-environmental groups. Although evidence of climate change seems to be increasing, several countries have been reluctant to impose strict environmental policies, (such as introducing substantial carbon taxes and restricting construction of coal power plants). From our perspective, these countries free-ride on others' efforts to manage climate change. Pro-environmental groups are trying to change the policy of such countries by emphasizing the impact of climate change. Here we interpret these NGOs' behavior as lobbying to bias governments' estimation of their loss in case of severe climate change. Next we will discuss the impacts of such behavior of NGOs on the welfare of allied countries.

7.4.1 Analytical Framework

We first conduct a positive analysis of the behavior of the NGO. As before, consider two identical allied countries. We incorporate a hypothetical NGO into our model. In actuality, NGOs have various objectives. However, we concentrate on NGOs that benefit from the provision of self-protection. For example, additional foreign aid facilitates the activities of humanitarian-NGOs. Then, we will argue that such NGOs benefit from foreign aid. We assume that through its lobbying efforts the NGO can influence loss estimation by all allied governments. We, therefore, assume that the NGO benefits from the provision of the self-protection public good and that it maximizes its net benefit—defined as its benefit minus its lobbying cost. Also, we assume that the NGO is located outside the alliance, which implies that the governments of the allied countries ignore neither the benefit nor the cost of NGO itself.[11] Even if such NGOs are geographically located within "their" alliance, our analysis is applicable if governments in their decision making ignore benefits and costs to NGOs. This is the simplest theoretical framework to formulate the NGO's behavior so as to derive meaningful analytical results.

The model is time-structured with three periods summarized as follows:

Period 0 An international NGO determines its lobbying efforts in the two allied countries to maximize its net benefit. Lobbying biases the loss estimation by the allied countries with estimated loss of each country increasing with the NGO's lobbying effort in that country.

Period 1 Each country independently and simultaneously determines its purchase of self-protection to maximize its "estimated expected" welfare based on its estimate of losses.

Period 2 The state of the world is stochastically determined based on the self-protection provided in Period 1. The consumptions of households in both countries are realized

As in Sect. 7.3, we assume that both countries are endowed with the identical income, each loses the identical amount in case of the bad state, biased by the same level in their estimation of the loss. Then, we have

$$Y^A = Y^B = Y, \bar{L}^A = \bar{L}^B = \bar{L}, \text{and}\, \alpha^A = \alpha^B = \alpha. \tag{7.67}$$

We also concentrate on a symmetric Nash equilibrium where

$$C^{1A*} = C^{1B*} = C^{1*}, C^{0A*} = C^{0B*} = C^{0*}, \text{and}\, m_1^{A*} = m_1^{B*} = m_1^*. \tag{7.68}$$

7.4.2 *Subgame Perfect Nash Equilibrium*

We solve this game using backward inductions. In Period 2, the state of the world is chosen, where the probability of good or bad states is determined by the provision of self-protection, as per Eq. (7.55). The consumptions of each country in good and bad states, C^{1*} and C^{0*}, are given by Eqs. (7.56) and (7.57).

In Period 1, each country non-cooperatively maximizes its expected welfare. The governments of A and B do not know their true losses in the bad state. Each country estimates its loss as $\bar{L} + \alpha$. Country A estimates its budget constraints in Period 2 as Eqs. (7.41) and (7.42). It maximizes its expected welfare subject to its estimated budget constraints. The first order condition is given by Eq. (7.46). The Nash equilibrium of the game in Period 1 is then given by Eqs. (7.53) and (7.54). As in Sect. 7.3, we denote the Nash equilibrium in Period 1 as $m_1^*(Y, \bar{L} + \alpha)$ because equilibrium

[11] For example, consider an NGO trying to influence public opinion through the Internet from outside the alliance.

allocation to self-protection is given as a function of national income and estimated loss.

In Period 0, the international NGO determines its lobbying efforts to maximize its net benefit. For the sake of simplicity, we assume that the benefit of the NGO is proportional to the amount of self-protection public good and that its lobbying cost is proportional to the level of bias that it creates for the calculus of A and B. The NGO solves the following problem:

$$\max_{\alpha} \Pi = 2\beta m_1^*(Y, \bar{L} + \alpha) - 2c\alpha, \tag{7.69}$$

where Π is its net benefit, β is a coefficient of the benefit of self-protection and c is the constant unit cost of lobbying. The unit cost of lobbying is the cost required to increase a government's loss-estimate by one unit.

The interior first order condition for the NGO is given as follows:

$$\frac{\partial m_1^*}{\partial \alpha} = \frac{c}{\beta}. \tag{7.70}$$

The partial derivatives of $m_1^*(.)$ is obtained from total differentiation of Eqs. (7.53) and (7.54). Details of this derivation are given in appendix A of this chapter. The subgame perfect Nash equilibrium of this model follows as the solution of the system of Eqs. (7.53), (7.54), and (7.70).

Figure 7.4 illustrates the net benefit maximization of the NGO. The solid curve shows the purchase of self-protection in Period 1 as a function of bias in loss estimation. The NGO can choose any point on this curve. In this sense, we can interpret this curve as a production possibility frontier of the NGO. The solid line L is one of the iso-net-benefit lines of the NGO. Along with this line, the net benefit of the NGO is identical. The slope of this line is c/β. Although only one iso-net-benefit line is drawn on Fig. 7.4, there may exist uncountable numbers of these lines. A higher iso-net-benefit line corresponds to a larger net benefit. Point A is a tangent point of an iso-net-benefit line to the possibility frontier. The net benefit of the NGO is maximized at this point. However, in general, the NGO does not necessarily choose the socially optimal estimation bias which induces countries to purchase the first-best self-protection. The NGO chooses the socially optimal estimation bias only if the cost-benefit ratio of lobbying (c/β) coincides with the slope of the possibility frontier evaluated at the socially optimal estimation bias (α^{**}).

Using Fig. 7.4, we briefly investigate the impacts of changes in the cost of lobbying. The unit cost may decrease with further opening of the international policy arena to NGOs. For example, countries can invite NGOs to international conferences of policymakers more often than they do today, which may reduce the unit cost of lobbying. We here investigate the impacts of such a fall in the unit cost of lobbying. When the unit cost of lobbying decreases, the slope of the iso-net-benefit line becomes more gradual. As a result, the tangent point of the iso-net-benefit line to the possibility frontier moves toward the right-hand side, which implies that the NGO

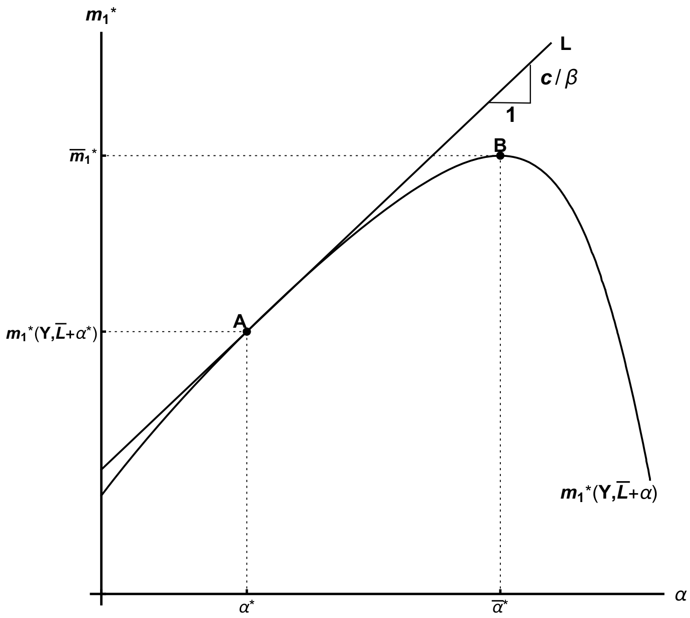

Fig. 7.4 Iso-net-benefit line and possibility frontier for the NGO. *Source* Authors

increases their lobbying activity to induce countries to purchase more self-protection. To summarize, countries can further integrate NGOs into the policy-making process. This integration lowers the lobbying cost of NGOs and induces them to conduct more effort to increase the provision of self-protection, which may improve the welfare of countries.

7.4.3 Role of NGOs

Our result explores the beneficial role of NGOs. If NGOs may influence governments' estimations of loss and induce them to contribute more to public goods, their intervention in policymaking is beneficial, that is, their efforts may improve the welfare of all allies.

National governments and international organizations may facilitate NGOs' activities by lowering their lobbying cost. For example, integrating NGOs into the policy-making process, as in the Ottawa process, may reduce the lobbying cost. If the self-protection is underprovided compared with the first-best level, lowering the lobbying cost may increase the provision of self-protection and improve the welfare of countries.

However, there is a limitation in the role of NGOs. As shown in Figs. 7.2b and 7.3, when the true magnitude of loss is in a certain range, the difference between the social optimal and the Nash equilibrium becomes so wide that any estimation bias cannot induce countries to contribute the socially optimal amount of the public good. In this range, the loss is sufficiently large that the spillover benefit to the other ally is substantial. However, the loss is not large enough for the negative effect of increasing loss to reduce the first-best self-protection. Thus, the first-best self-protection exceeds the maximum self-protection. No matter how much the NGO spreads an extremely pessimistic estimation of the bad state, the first-best provision of self-protection cannot be achieved in this case.

7.5 Conclusion

In this chapter, we have investigated the impacts of misestimates of the severity of a threat on collective risk management. Here, we formulated such misestimation as a bias in the estimation of a loss in the bad event. We addressed the following three questions: (1) How would bias in loss estimation affect burden-sharing among allies? (2) Does there exist a degree of bias that induces governments to voluntarily provide a socially optimal amount of shared self-protection? (3) Could an international NGO voluntarily choose a socially optimal level of lobbying to influence governments' loss-estimates, and thereby induce those governments to provide the socially optimal degree of self-protection?

In Sect. 7.2, we investigated the impacts of overestimates (or underestimates) of loss in the bad state, extending the model developed in Chap. 6. At an interior Nash equilibrium, compared to the other ally, a country that is more biased in estimation consumes more in the bad state. We also showed that the difference in total security expenditure between allies is not influenced by their misestimates. However, the composition of security expenditure is influenced by estimation bias. To elaborate, the difference in self-protection (the difference in self-insurance) between two allied countries is smaller (larger) when estimates are biased than it is in the absence of any estimation bias. This shrinking of the difference in self-protection expenditure is canceled out by the widening of the difference in self-insurance expenditure. Using numerical simulations, we showed that in our two-country model overestimations of losses may improve the welfare of both countries. The over- estimates of losses increase the incentive to contribute, which may cancel out the free-riding incentives in voluntary provision of public good.

In Sect. 7.3, we conducted a normative analysis of misestimation. We simplified our model such that we only considered the voluntary provision of self-protection as a public good. We explored how much bias in loss-estimate is required to achieve maximum utilitarian social welfare when allies follow Nash-Cournot rules. In a one-country model, estimation bias would simply hinder the government in achieving the optimal supply of self-protection. However, in an alliance model, bias in loss estimate may stimulate the provision of the public good and mitigate the free-riding problem in the Nash equilibrium.

In Sect. 7.4, we introduced an international NGO that can affect the loss estimations of governments. We assumed one international NGO located outside the alliance, which lobbies for more pessimistic estimation of the loss-in-the-bad-state as an input to government decision-making. We also assumed that the NGO benefits from the provision of self-protection, that it pays a cost for lobbying, and that it maximizes its net benefit through this lobbying. Then, we investigated the impacts of changes in the cost of lobbying. When the unit cost of lobbying decreases, the NGO increases their lobbying activity to induce countries to purchase more self-protection, which may improve the welfare of countries.

Our analysis in this chapter sheds new light on the normative role of the NGOs. In international risk management, we cannot presuppose the "World Government" that might achieve an optimal provision of the public goods by collecting some tax from the players and paying matching grants for the provision of public goods. However, intentionally or unintentionally, international NGOs may pessimistically bias government estimates of losses in the bad state. This then increases the Nash-equilibrium level of the self-protection and improves the welfare of the allies. To summarize, our result explores the beneficial role of NGOs, that is, as long as the difference between the social optimum and the Nash equilibrium exists, misestimation caused by the lobbying efforts of NGOs could improve the social welfare.

Appendix A: Comparative Statics of Nash Equilibrium in Period 1

In this appendix, we conduct the comparative statics of the Nash equilibrium in Period 1 in the model constructed in Sect. 7.4.1. We take the total differentiation of Eqs. (7.53) and (7.54), and solve the resulting simultaneous equations. Then, we obtain the following:

$$dm_1^{A*} = \frac{1}{D}\left(\frac{\partial m_1^A}{\partial \tilde{L}^A} d\tilde{L}^A + \frac{\partial m_1^A}{\partial m_1^B}\frac{\partial m_1^B}{\partial \tilde{L}^B} d\tilde{L}^B\right), \tag{7.71}$$

$$dm_1^{B*} = \frac{1}{D}\left(\frac{\partial m_1^B}{\partial \tilde{L}^B} d\tilde{L}^B + \frac{\partial m_1^B}{\partial m_1^A}\frac{\partial m_1^A}{\partial \tilde{L}^A} d\tilde{L}^A\right), \tag{7.72}$$

where $D \equiv 1 - \frac{\partial m_1^A}{\partial m_1^B}\frac{\partial m_1^B}{\partial m_1^A}$. We substitute Eqs. (7.48) and (7.49) in Eq. (7.71), and assume $d\tilde{L}^B = 0$ to obtain the following:

$$\frac{\partial m_1^{A*}}{\partial \tilde{L}^A} = \frac{-1}{\Delta}\frac{\partial^2 \tilde{W}^B}{\partial (m_1^B)^2}\frac{\partial^2 \tilde{W}^A}{\partial m_1^A \partial \tilde{L}^A}, \tag{7.73}$$

where $\Delta \equiv \frac{\partial^2 \tilde{W}^A}{\partial (m_1^A)^2} \frac{\partial^2 \tilde{W}^B}{\partial (m_1^B)^2} - \frac{\partial^2 \tilde{W}^A}{\partial m_1^A \partial m_1^B} \frac{\partial^2 \tilde{W}^B}{\partial m_1^B \partial m_1^A}$. Assuming $d\tilde{L}^B = 0$ in Eq. (7.72), we obtain the following:

$$\frac{\partial m_1^{B*}}{\partial \tilde{L}^A} = \frac{1}{\Delta} \frac{\partial^2 \tilde{W}^B}{\partial m_1^B \partial m_1^A} \frac{\partial^2 \tilde{W}^A}{\partial m_1^A \partial \tilde{L}^A}. \tag{7.74}$$

Appendix B: Numerical Examples of the Second-Best Model

In this appendix, we conduct several numerical simulations to consider the following two questions: (1) Whether is there a socially optimal degree of bias in loss estimation that may effectively induce allied countries to contribute the socially optimal amount of self-protection? (2) How low is the unit cost of lobbying enough low for the NGO to cause the socially optimal degree of bias in governments' estimations?

B.1 Specification for Simulations

To conduct numerical analysis, we specify the forms of the functions in our model. As in Chap. 6, we follow Ihori et al. (2014) in the specification. We specify the utility function, $U(.)$, as a CRRA function:

$$U(C) = \frac{C^{1-\theta}}{1-\theta}, \tag{7.75}$$

where θ is the parameter representing the relative risk aversion of the country. We also specify the probability function following Tullock's contest success function:

$$p(M_1) = \frac{M_1 + p_e}{M_1 + (p_e/p_0)}, \tag{7.76}$$

where $p_0(> 0)$ is the baseline probability of the good state, which is the probability of the good state when no country provides any self-protection public good, and $p_e(> 0)$ is a parameter representing the strength of the opponent. The self-insurance benefit function is specified as a linear function:

$$\mathrm{L}(s) = \phi s, \tag{7.77}$$

where $\phi \in (0, 1)$ is a marginal loss reduction from one unit of purchased self-insurance. We therefore obtain $\mathrm{L}' = \phi,\ \mathrm{L}'' = 0$.

For the sake of simplicity, we assume that both countries are identical in their preferences, their national income, their loss in the bad state. Then, we have

$$Y^A = Y^B = Y, \bar{L}^A = \bar{L}^B = \bar{L}, \text{ and } \alpha^A = \alpha^B = \alpha. \quad (7.78)$$

B.2 Socially Optimal Estimation Bias

Table 7.3 presents three numerical examples of our two-period game in Sect. 7.3 wherein the bias in loss estimation is given as a parameter. In this model, the loss in the bad state is 20% of the income. Column 1 of this table reports the solution of the utilitarian social welfare maximization problem. Under the settings of the parameters in this table, the optimal purchase of self-protection is 1.906, which is 3.8% of the income. Column 2 represents the Nash equilibrium allocation of this model wherein no bias in estimation exists. In this case, each country voluntarily purchases 0.714 units of self-protection, which is less than half of the optimal level. Columns 3a and 3b report the allocations in the Nash equilibria in the presence of the socially optimal estimation bias. As shown in Sect. 7.3.3, there are two values of socially optimal estimation bias that induce countries to provide the socially optimal level of the self-protection public good, which are 10.022 in column 3a and 35.546 in column 3b, respectively. In both cases, each country purchases 1.906 units of self-protection, which is socially optimal. The levels of true expected welfare in columns 3a and 3b are equal to those in the socially optimum scenario.

Figure 7.5 shows how the socially optimal estimation bias changes with the true loss in the bad state. The parameters are set as they are in Table 7.3, except for loss. We assume that the true loss, $\bar{L}$, increases from 0 to 20. The horizontal axis represents the loss, while the vertical axis represents the socially optimal estimation bias, $\alpha^{A**} = \alpha^{B**} = \alpha^{**}$. When $\bar{L} \leq 2.5$, the socially optimal bias is zero because the demand for self-protection is so small that the first-best self-protection is zero. When $3 \leq \bar{L} \leq 16.5$ there are two values of socially optimal estimation bias for each given level of loss in the bad state. For example, when $\bar{L} = 10$, the socially optimal estimation biases are 10.022 and 35.546, such as in Table 7.3. The high (low) value of socially optimal bias decreases (increases) with loss. As loss increases, the high and low values converge. When $\bar{L} \geq 17$, socially optimal bias does not exist.

B.3. Endogenous Overestimation

Table 7.4 summarizes the results of numerical simulations of our three-period game in Sect. 7.4 wherein the NGO endogenously determines the bias in the governments' estimations. For comparison, the first two columns are identical to those in Table 7.3. The third column represents the result of a numerical simulation wherein the international NGO chooses the degree of bias to maximize its net benefit. In this column, the unit cost of lobbying is 0.011. In Period 0, the NGO chooses its lobbying activities to make each government estimate a loss in the bad state more than the true loss by 7.617 units. In Period 1, the government of country A purchases 1.648 units of

Table 7.3 Numerical simulations of the socially optimal estimation bias

Scenario	1 Social optimum	2 Nash equilibrium without estimation bias	3a Nash equilibrium with socially optimal estimation bias	3b Nash equilibrium with socially optimal estimation bias
Y^A	50	50	50	50
Y^B	50	50	50	50
$\bar{L}^A$	10	10	10	10
$\bar{L}^B$	10	10	10	10
p_0	0.25	0.25	0.25	0.25
p_e	1	1	1	1
θ	0.9	0.9	0.9	0.9
α^A	n/a	0	10.022	35.546
α^B	n/a	0	10.022	35.546
m_1^{A*}	1.906	0.714	1.906	1.906
m_1^{B*}	1.906	0.714	1.906	1.906
M_1^*	3.811	1.428	3.811	3.811
$\tilde{W}^{A*}$	n/a	14.583	14.434	13.290
$\tilde{W}^{B*}$	n/a	14.583	14.434	13.290
W^{A*}	14.600	14.583	14.600	14.600
W^{B*}	14.600	14.583	14.600	14.600

Source Authors

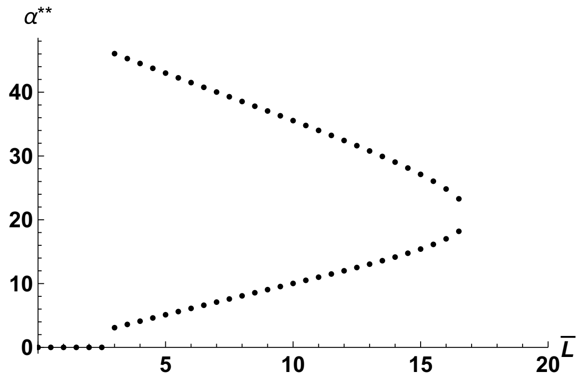

Fig. 7.5 Socially optimal estimation bias and the true loss. *Source* Authors

Table 7.4 Numerical simulations of endogenous overestimation

Scenario	1 Social optimum	2 Nash equilibrium without estimation bias	4 Endogenous overestimation with non-optimal lobbying cost	5 Endogenous overestimation with optimal lobbying cost
Y^A	50	50	50	50
Y^B	50	50	50	50
$\bar{L}^A$	10	10	10	10
$\bar{L}^B$	10	10	10	10
p_0	0.25	0.25	0.25	0.25
p_e	1	1	1	1
θ	0.9	0.9	0.9	0.9
β	n/a	n/a	0.1	0.1
c	n/a	n/a	0.011	0.010
α^A	n/a	0	7.617	10.022
α^B	n/a	0	7.617	10.022
m_1^{A*}	1.906	0.714	1.648	1.906
m_1^{B*}	1.906	0.714	1.648	1.906
M_1^*	3.811	1.428	3.296	3.811
$\tilde{W}^{A*}$	n/a	14.583	14.470	14.434
$\tilde{W}^{B*}$	n/a	14.583	14.470	14.434
W^{A*}	14.600	14.583	14.599	14.600
W^{B*}	14.600	14.583	14.599	14.600
Π^*	n/a	n/a	0.162	0.172

Source Authors

self-protection. Since country B purchases the same amount of self-protection, the provision of the self-protection public good is 3.296 units. Then, the true expected welfare of each country is 14.599, which is lower than the socially optimum level; however, it is still higher than the Nash equilibrium level in the second column. Furthermore, the purchase of self-protection in the third column is smaller than that in the first column. Hence, self-protection is still under-provided. The fourth column of Table 7.4 reports the numerical simulation wherein the marginal cost of lobbying is sufficiently low, such that the NGO chooses the socially optimal estimation bias. Then, the allied countries contribute the socially optimal amounts to self-protection. As shown in the last row of the fourth column, the net benefit of the NGO is more than that in the third column.

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