

Lapping Pattern, Stock Length, and Shop Drawing of Beam Reinforcements of an RC Building

Yung-Hsiang Chen¹ and Tun-Kai Yang²

Abstract: The purpose of this study is to develop an automation platform that particularly emphasizes the beam reinforcements of a continuous beam of an RC building and automatically checks all designs to ensure that the arrangements of reinforcements comply with the American Concrete Institute code. The platform can establish the types of location patterns of lap splices (called lapping patterns) for the longitudinal reinforcements; calculate the length of every reinforcement; arrange the same sized reinforcements by length in decreasing order; choose the proper stock lengths for bar cutting; and determine the best lapping pattern with optimal stock length by the trial and error method or the linear programming approach, which can yield the most economical way to supply a proper stock length for cutting. Finally, the platform can automatically generate the shop drawing and schedule the reinforcements, which may be very useful for building construction and management. DOI: 10.1061/(ASCE)CP.1943-5487.0000303. © 2014 American Society of Civil Engineers.

Author keywords: Lapping pattern; Stock length; Cutting pattern; Shop drawing; Automation platform; Trial and error method; Linear programming approach.

Introduction

When bidding on projects, a construction firm has limited working days to estimate the amount of material according to structural design drawings and to evaluate the budget. In addition, during the construction stage, field engineers need to estimate and supply materials to each floor level or work zone of a building based on the construction progress. Usually, all of this work is performed by manual labor and is, therefore, very time consuming, especially with respect to determining the required amount and the remainder of reinforcements. This is not only a waste of money and work-force, but often results in many mistakes. New technology in software has been introduced into the practice of civil engineering to improve efficiency and accuracy.

Most previously published articles and research have focused on the application of computer programs in structural analysis (*ETABS*; *SAP2000*; *Midas*), whereas studies regarding reinforcements in construction are restricted to a few specific and simple issues. These studies include determining the arrangement of reinforcements based on the fixed locations of lap splices and proposing a simple and inexpensive reinforcement cutting process based on a schedule and a specific stock length for only one type of reinforcement arrangement, to meet the practical needs of construction (Fan 2001; Yang 2002). Some efforts have also been made to address the so-called cutting stock problem to minimize cutting loss (Coulimis 1990; Gilmore and Gomory 1961, 1963; Haessler 1975; Salem et al. 2007; Shahin and Salem 2004; Vahrenkamp 1996; Yuen and Richardson 1995). The field construction of reinforcements is a complicated, tedious, and labor-consuming job.

Therefore, most studies conducted on this subject focus on mere segments of the matter, rather than addressing an integrated methodology. Recently, some computer programs (Eastman et al. 2008; Tekla Structures) have been established that can integrate the design data and automatically generate the design drawings, but some work relies on the engineer's experience and judgment. This paper aims to establish an automation platform to integrate input data, design drawings, and construction performance, particularly for beam reinforcements. Regarding the construction of reinforcements in an RC building, beam elements are rather more complicated and difficult to handle than other structural components. This is primarily because there are many options for positioning the longitudinal reinforcement lap splices and because the bar sizes are usually larger than #6, which means that they must be cut from those supplied with stock lengths of 12, 13, or 18 m. Determining the most economical way to supply the stock length for cutting is an important subject presented in this paper.

Automation Platform

The automation platform of the shop drawing and schedule of the beam reinforcements of an RC building are shown in Fig. 1; the content of this platform consists of two phases. The primary job of the first phase is to set up a database via *Access* containing the basic information data file based on material properties and American Concrete Institute (ACI) building codes [ACI 315-99 and 318-99 (ACI 2004a, b)], and the other is the design information data file based on the structural design of an RC building. The design information of each beam element of an assigned element code can be keyed in through the window, as shown in Fig. 2. A program via *VB.NET* can automatically check whether the arrangements of reinforcements comply with the ACI code. The drawings of the cross-sectional diagrams of a beam element, including the longitudinal reinforcements and stirrups, can be obtained by *AutoCAD* via visual basic for applications (VBA) (Mcauley 2000). Based on the database established in the first phase, the second phase contains the following important tasks achieved by an integrated program via *VB.NET*: choosing the lapping patterns of the longitudinal

¹Professor, Civil Engineering, National Taiwan Univ., 1, Roosevelt Rd., Section 4, Taipei 106, Taiwan, R.O.C.

²Senior Civil Engineer, Yen Tjing Ling Industrial Research Institute, National Taiwan Univ., 162, Heping East Rd., Section 1, Taipei 106, R.O.C. (corresponding author). E-mail: potatto7@gmail.com

Note. This manuscript was submitted on August 7, 2012; approved on March 12, 2013; published online on March 14, 2013. Discussion period open until August 31, 2014; separate discussions must be submitted for individual papers. This paper is part of the *Journal of Computing in Civil Engineering*, © ASCE, ISSN 0887-3801/04014028(9)/\$25.00.

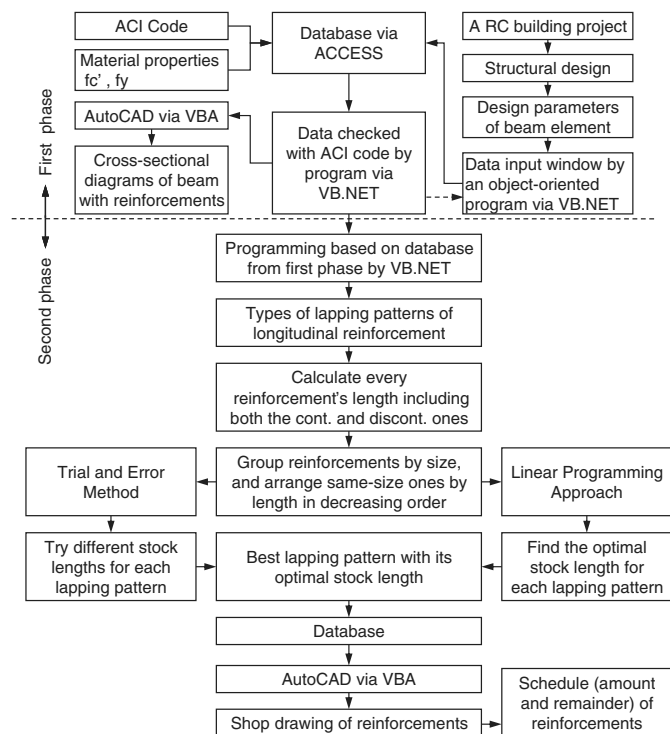


Fig. 1. Automation platform for shop drawing and scheduling the beam reinforcements of an RC building

reinforcements; calculating the lengths of all reinforcements; grouping all reinforcements by size and length; finding the best lapping pattern with the optimal stock length for cutting; generating the shop drawing; and scheduling the reinforcements. The continuous longitudinal reinforcements in a beam element have different options for lapping, which results in various lapping patterns for a continuous beam. The optimal stock length cut for each lapping pattern can be obtained either by the trial and error method or by the linear programming approach presented in the paper. By comparing the required amount of the stock length used for cutting,

the best lapping pattern with the optimal stock length can be determined. Finally, the shop drawing of beam reinforcements is automatically plotted by *AutoCAD* via *VBA*.

This paper uses Visual Studio.NET (*VS.NET*) (Fulcher 2010) as a tool to establish the integrated program platform. *VS.NET* contains several programming languages, including *VBA*, *VB.NET*, Visual C++.NET, Visual C#.NET, and ASP Web. Therefore, it can be used to develop a program with multiple languages. The equipment of the .NET framework function allows programs developed in *VS.NET* to be integrated easily into a network. Suitable programs for Windows and the Internet can be easily and efficiently constructed by using *VB.NET*, which also has object-oriented programming languages and includes features such as inheritance, encapsulation, and polymorphism. In addition, *VB.NET*, with its *VBA* language, can be coupled with other software programs such as *AutoCAD* and *Access* (Elmasri and Navathe 2003). Because *VBA* and *VB.NET* have the same programming language, language conversion can be avoided entirely. *Access* is a relational database management system that can link many tables together via the same name and is characterized by its data inquiry and managing functions.

Database System

A database containing two types of data files is established in the first phase of the automation platform, as follows:

1. The basic information data file contains the properties of material and the specifications of the ACI code; and
2. The design information data file for each beam element consists of the element code and the design parameters, including the material properties, beam size, and details of reinforcements.

First, 61 design parameters are tentatively assigned to each beam element as follows: (1) element code, (2) compressive strength of concrete (f'_c), (3) and (4) yielding stress of the longitudinal reinforcement and stirrup (f_y) of longitudinal reinforcements and stirrups, (5)–(7) beam dimensions, (8)–(49) information about the two-layer top and bottom longitudinal reinforcements, (50) and (51) size and number of web longitudinal reinforcements, (52)–(59) information about stirrups, (60) and (61) widths of the

Dir.	(60) Col. width	(13) 250	(1) Beam Code 5FX2B1	(14) 250	(61) Col. width
Y Dir.	110	(52) 220	Beam Size (6)X(7)	(53) 220	110
Story			70 X 90		
Col. Axis		(54) (55)	(56) (57)	(58) (59)	
X2		#4 @ 8	#4 @ 8	#4 @ 8	
(2) f'_c	Cont. (8) - (9)	(10) LS 0	(11) LS 1	(12) LS 0	
2747	#10 - 6	Discont. (15)-(16)	Discont. (17)-(18)	Discont. (19)-(20)	
(3) f_y of LR	Cont. (21) - (22)	#10 - 2	#10 - 0	#10 - 2	
41202	#10 - 0	Discont. (23)-(24)	Discont. (25)-(26)	Discont. (27)-(28)	
(4) f_y of stir.	Web (50) - (51)	#10 - 5	#10 - 0	#10 - 6	
27468	#4 - 2	#10 - 4	#10 - 0	#10 - 5	
LR Info.	Cont. (29) - (30)	Discont. (33)-(34)	Discont. (35)-(36)	Discont. (37)-(38)	
Layer	#10 - 0	#10 - 2	#10 - 0	#10 - 2	
Size	Cont. (39) - (40)	Discont. (44)-(45)	Discont. (46)-(47)	Discont. (48)-(49)	
#10	#10 - 6	(41) LS 1	(42) LS 0	(43) LS 1	
Save		(31) 250	(32) 250		
Draw		(5) 696.5			

Fig. 2. Data input window for 5FX1B1 beam element (unit: cm; N/cm^2 ; $1 \text{ kgf}/\text{cm}^2 = 9.81 \text{ N}/\text{cm}^2$)

Table 1. Design Parameters for Beam Elements

(Code) Parameter						
(1) Beam code Material properties: (2) f_c' (N/cm ²) (3) f_y (N/cm ² , LR) (4) f_y (N/cm ² , stirrup) Beam size: (5) Span (cm) (6) Width (cm) (7) Depth (cm)						
Region	Continuous longitudinal reinforcement (LR)			Discontinuous LR of the left side	Discontinuous LR of the middle	Discontinuous LR of the right side
Upper LR of top	(8) Size, (9) Number			(13) Scope of splicing (cm)		(14) Scope of splicing (cm)
	(10) LS at left	(11) LS at middle	(12) LS at right	(15) Size, (16) Number	(17) Size, (18) Number	(19) Size, (20) Number
Lower LR of top	(21) Size, (22) Number			(23) Size, (24) Number	(25) Size, (26) Number	(27) Size, (28) Number
Upper LR of bottom	(29) Size, (30) Number			(31) Scope of splicing (cm)		(32) Scope of splicing (cm)
				(33) Size, (34) Number	(35) Size, (36) Number	(37) Size, (38) Number
Lower LR of bottom	(39) Size, (40) Number			(44) Size, (45) Number	(46) Size, (47) Number	(48) Size, (49) Number
	(41) LS at left	(42) LS at middle	(43) LS at right			
Web LR: (50) Size (51) Number (each side)						
	L	M	R			
	(52) Scope of left side (cm)		(53) Scope of right side (cm)			
	(54) Size, (55) Spacing (cm)	(56) Size, (57) Spacing (cm)	(58) Size, (59) Spacing (cm)			
Stirrup						
Column width: (60) Left (61) Right						

Note: LS represents possible lapping splice of LR; LS = 1 yes, LS = 0 no, 1 kgf/cm² = 9.81 N/cm².

left and right columns, respectively. The definitions of the design parameters of beam elements are listed in Table 1.

Design parameters 1–61 for each beam element of a continuous beam are easily recorded by using the input window designed by *VB.NET* (an object-oriented program), as shown in Fig. 2. These design parameters are input at the corresponding positions on a beam element to avoid potential input mistakes. A program designed via *VB.NET* can automatically test whether the cover and spacing of reinforcements comply with the ACI code.

Beam Cross-Sectional Diagrams with Reinforcements

Longitudinal Reinforcements

Longitudinal reinforcements consist of top and bottom reinforcements. Because of the limit of the beam width, they are usually divided into two layers (or rows). The upper or lower layer (or the lower or upper layer) of the top or the bottom reinforcements is usually the continuous (or discontinuous) layer. If this is the case, reinforcements on the upper and lower layers need to align with each other.

Web Longitudinal Reinforcements

Web longitudinal reinforcements lie between the top and the bottom longitudinal reinforcements, and symmetrically along both sides of the cross section of a beam. Usually, they are distributed evenly between the top and the bottom longitudinal reinforcements with equal spacing and anchored by their development length into the columns at both ends of a beam element.

Stirrups

The arrangement of the stirrups along each beam span can be divided into three regions (left, middle, and right), and the corresponding cross-sectional diagrams of each beam element with reinforcements can be plotted automatically.

Lapping Patterns of Longitudinal Reinforcements for an n-Span Continuous Beam

An *n*-span continuous beam consists of two end beam elements and several middle elements. Users enter an assigned code for the continuous beam, which provides the information about the number of beam elements and the beam location; i.e., the floor level, the reference axial line (usually the column line).

Arrangement of Longitudinal Reinforcements

The longitudinal reinforcements in each beam element include continuous and discontinuous reinforcements. Top continuous longitudinal reinforcements, usually described as negative reinforcements, lie in the upper part of the beam; bottom continuous longitudinal reinforcements, described as positive reinforcements, are placed at the lower part of the beam. All longitudinal reinforcements are hook-anchored inside columns at both ends of the continuous beam and primarily move continuously straight from one beam element to the next. Discontinuous longitudinal reinforcements in each beam element span include the top negative reinforcements at both end regions and the bottom positive reinforcements in the middle region. If two adjacent beam elements have different numbers and sizes of longitudinal reinforcements and different beam sizes so that reinforcements cannot continue, they should be hook anchored or straight-through anchored into the columns.

Lap Splice of Longitudinal Reinforcements

The lapping length of continuous reinforcements should follow the ACI code, and the lapping location (i.e., the center of the lapping length, also called the lapping point) for the top continuous negative reinforcements must be in the middle portion of each beam element and with net span in the range of $1/4$ to $3/4$. The lapping point for the bottom continuous positive reinforcements must be within $1/4$ of the net span from either column. The lapping points of adjacent continuous reinforcements should be staggered at least 60–75 cm; in this paper, 75 cm is adopted for the design. The lapping points for adjacent bottom reinforcements are usually staggered by the lines of the interior column.

Discontinuous reinforcements usually have short lengths; thus, bars of stock lengths (e.g., 12–18 m) can be used for cutting. Therefore, in general, lap splices are not required for the construction of discontinuous reinforcements. However, because of the limits of the stock lengths, lap splices are commonly used in the construction of continuous reinforcements. A simple rule is that in the design, there should be as few lap splices as possible. Each continuous reinforcement can choose to lap or not lap at the lapping points in each beam element, thus generating many different lapping patterns. The total number of possible variations depends on the number of spans. Usually, the lap splices of bottom continuous positive reinforcements are not in the vicinity of the exterior columns. Therefore, to reduce the number of lap splices and the amount of reinforcements, these bottom reinforcements are usually not lapped adjacent to the exterior columns. The total number of possible lapping points for top continuous reinforcements in an n -span continuous beam is n , and that for bottom reinforcements is $n - 1$. Given that at every lapping location, there are two options (lapping or not lapping), the top reinforcements have a total of 2^n combinations and the bottom reinforcements have a total of 2^{n-1} combinations. The events of lapping or not lapping for reinforcements are all independent of one another, thus yielding a total number of possible lapping patterns equal to $2^n \times 2^{n-1}$ or 2^{2n-1} . Among them, “all lapping” is the simplest case. All lapping is easier for construction because it has shorter reinforcements. However, because of the increasing number of lapped points, the amount of reinforcements will increase, making this option less economical and, as a result, rarely adopted in practice.

Trial and Error Method

It has been found that there are, at most, 2^{2n-1} lapping patterns for the continuous reinforcements of an n -span continuous beam.

According to the stock lengths and the lapping patterns, a proposed procedure of bar cutting is given as follows:

1. Determine the total possible lapping patterns of continuous reinforcements (i.e., 2^{2n-1} in total);
2. Eliminate inappropriate lapping patterns on the basis that the length of each reinforcement should be no longer than the longest stock length (e.g., 18 m);
3. Choose one of the lapping patterns;
4. Calculate the length of each reinforcement for this lapping pattern. The length of each reinforcement, continuous or discontinuous, is calculated according to the ACI code, including the length of lap and anchorage;
5. Group reinforcements, continuous or discontinuous, by size;
6. Arrange the same sized reinforcements by length in decreasing order;
7. Eliminate any unsuitable stock lengths (12–18 m) that are shorter than the longest reinforcement;
8. Cut reinforcements with a specific stock length of same bar size. Perform cutting one or more times on the reinforcements in an order of decreasing length by each stock length. After every cut, check the remaining bar to determine whether it can still be cut for another reinforcement;
9. When the remaining stock bar cannot be used anymore, it is referred to as remainder;
10. When cutting the longitudinal reinforcements of a specific lapping pattern with different stock lengths, the stock length that yields the minimum required amount used for cutting is called the optimal stock length for this specific lapping pattern and the discontinuous longitudinal reinforcements may also be included in consideration;
11. Compare all lapping patterns with their corresponding optimal stock lengths so that the best lapping pattern with optimal stock length can be obtained, which requires a minimum amount of stock length used for cutting. The scheme to find the optimal stock length cut for a specific lapping pattern is the trial and error method; and
12. If the continuous reinforcements have two or more different sizes, the best lapping pattern with optimal combination of the different sized stock lengths should be found. This case will yield the minimum amount of these different sized stock lengths used for cutting.

Linear Programming Approach

Some studies have been conducted on the so-called cutting stock problem to minimize the cutting loss of a specific stock length, either by the linear programming approach (Coulimis 1990; Gilmore and Gomory 1961, 1963; Haessler 1975; Vahrenkamp 1996; Yuen and Richardson 1995) or by the genetic algorithm approach (Salem et al. 2007; Shahin and Salem 2004). This study attempts to find the optimal stock length to cut for each lapping pattern of the longitudinal reinforcements of a continuous beam by applying the theory of the linear programming (Karlof 2006; LINGO; Nauss 1979; Strayer 1989; Solow 1984) instead of the trial and error method; it also attempts to determine the difference between these two methods. The optimal stock length for cutting will yield the result of the minimum required amount of stock length. Therefore, an objective function, Q , for this purpose is proposed as follows:

$$Q = \sum_{j=1}^{N_1} m_j l_j + \sum_{i=1}^N \left(L - \sum_{j=1}^{N_1} n_{ij} l_j \right) \quad (1)$$

where Q = total required amount of stock length; N = number of required stock length; L = stock length; N_1 = number of different lengths (l_j , $j = 1 \sim N_1$) of the longitudinal reinforcements; m_j = number of j th length (l_j) of the longitudinal reinforcements; and n_{ij} = possible cutting number of the i th ($i = 1 \sim N$) stock length (L) cut for the rest of the j th length (l_j) longitudinal reinforcements after cutting by ($i - 1$)th stock lengths (L), respectively.

The first and second terms on the right-hand side of Eq. (1) represent the total amount of longitudinal reinforcements and the total remainder of stock length after cutting, respectively.

To linearize the nonlinear objective function, Q , as given by Eq. (1), the unknown number, N , the required number of the stock length for cutting, should be temporarily assumed a given number, which will be discussed later. Thus, the unknowns of Eq. (1) include L and n_{ij} values ($i = 1 \sim N$, $j = 1 \sim N_1$), and the values of n_{ij} may be zero or any positive integer. The total number of unknowns of Eq. (1) is $(N \times N_1 + 1)$.

The objective function, Q , defined by Eq. (1) should be constrained by some conditions, called the constrained conditions and described as follows.

The unknown stock length, L , used for cutting should be shorter than an assigned maximum stock length, $L_{\max}$ (e.g., 18 m), and larger than the longest length of the longitudinal reinforcements, $l_{\max}$ (i.e., $l_{\max} \geq l_j$, $j = 1 \sim N_1$), thus

$$l_{\max} \leq L \leq L_{\max} \quad (2)$$

Every term ($i = 1 \sim N$) of the second term on the right-hand side of Eq. (1) should be positive, which means that the remainder of each stock length after cutting should be positive:

$$L \geq \sum_{j=1}^{N_1} n_{ij} l_j, \quad i = 1 \sim N \quad (3)$$

The sum of the possible cutting numbers of all stock lengths ($i = 1 \sim N$, all the same L) cut for the j th length longitudinal reinforcements should be equal to the numbers of these longitudinal reinforcements (m_j):

$$\sum_{i=1}^N n_{ij} = m_j, \quad j = 1 \sim N_1 \quad (4)$$

Therefore, the total equations of the constrained conditions given by Eqs. (2)–(4) are $(2 + N + N_1)$.

The optimal stock length, L , can be determined from Q of Eq. (1) with its constrained conditions of Eqs. (2)–(4) by the linear programming approach. The simplex method (Solow 1984; Strayer 1989) should first be applied by adding some positive or negative slack variables to the constrained equations to obtain the solutions for L and n_{ij} by Gauss elimination. Because all n_{ij} values should be zero or any positive integer, and L may also be an integer if desired, the branch and bound method (Karlov 2006; Nauss 1979) can properly be applied to find L , which will result in the minimum Q , i.e., the minimum amount of stock length for a specific lapping pattern of the longitudinal reinforcements.

In Eq. (1), Q is essentially nonlinear; therefore, an iterative calculation should be conducted to obtain the convergent solution of N , the number of the optimal stock lengths for cutting. N may initially be assumed to be a result obtained by the trial and error method. In general, it may be better to assume N for a longer length or for a length close to the longest stock length, for the iterative calculation.

The solutions of Eqs. (1)–(4) by the linear programming approach provide the unknowns of L , N , and a set of n_{ij} values

($i = 1 \sim N$, $j = 1 \sim N_1$, respectively). The result of a set of n_{ij} values can provide the information about a cutting pattern of the stock length. The formulation of Eqs. (1)–(4) apparently has nothing to do with cutting pattern; generally, the solution of a set of n_{ij} values may not be unique and should depend on the accuracy of the iterative calculation of L . This means that there may be different cutting patterns, (n_{ij} values, $i = 1 \sim N$, $j = 1 \sim N_1$) for the same L . A cutting pattern proposed by the trial and error method to cut the stock length in an order of decreasing length of the longitudinal reinforcements may also be one of the solution of n_{ij} values of Eqs. (1)–(4); this may be the most simple and practical pattern in application. More discussions on the cutting pattern will be given by an example in this paper.

Optimal Shop Drawing and Schedule of Beam Reinforcements

The automation platform is shown in Fig. 1, in which where *AutoCAD* and *VBA*, in conjunction with the database, can produce the shop drawing and schedule for the best lapping pattern cut by its optimal stock length. The detail of each longitudinal reinforcement, either continuous or discontinuous, including the length, lap splice, lapping point, and hook anchorage, can all be determined and shown in the shop drawing. Additionally, the schedule of reinforcements can also be determined, including longitudinal reinforcements, stirrups, and amount and remainder of stock length used for cutting.

Example and Discussion

A continuous beam on the fifth floor along the vertical axial line, X2, of a 25-story RC building is used as an example for application. This beam is a four-span continuous beam with unequal spans of 806.5, 689, 615, and 1,180 cm, respectively. From the bottom up, they are assigned as 5FX2B1–B4. The compressive strength of concrete, f'_c , is 2,750 N/cm² (280 kgf/cm²), the yielding stress of the longitudinal reinforcements, f_y , is 41,200 N/cm² (4,200 kgf/cm²), and that of the stirrup is 27,500 N/cm² (2,800 kgf/cm²).

The 61 design parameters (1–61) and their corresponding information for the beam element 5FX2B1 are shown in Fig. 2 as an example of the data input. The three cross-sectional diagrams with reinforcements of this beam element can be plotted automatically.

There are a total of 128 possible lapping patterns for continuous reinforcements (i.e., $2^{2 \times 4 - 1}$). The program can automatically eliminate the lapping patterns that do not meet the stock lengths (e.g., 12–18 m). Types of lapping patterns of longitudinal reinforcements cut by the possible integral stock lengths are shown in Table 2. There are 12 lapping patterns for the stock length of 18 m, three patterns for the stock length of 17 m, and two patterns for each of the stock lengths of 16 and 15 m. Among them, Type 1 refers to all possible cases of lapping. The stock length of less than 15 m is ignored because this stock length is shorter than the lengths of the reinforcements in all lapping patterns. Schedules (amount and remainder) of all longitudinal reinforcements, including the discontinuous reinforcements cut by the optimal stock length with their corresponding lapping patterns, are automatically generated and shown in Tables 3 and 4. The best lapping pattern is 12, with its optimal stock length of 18 or 17.95 m, which is also shown in the tables. The results shown in Tables 3 and 4 were obtained by both the trial and error method and the linear programming approach and have been verified; they are very consistent with each other. The 18-m long stock length is finally chosen for cutting, which

Table 2. Types of Lapping Patterns of Longitudinal Reinforcements Cut by the Possible Integral Stock Lengths

Type	Lapping pattern	Longest length of LR (m)	Stock length (m)
1	$\bar{1} \bar{2} \bar{3} \bar{4}$	14.55	15, 16, 17, 18
2	$\bar{1} \bar{2} \bar{3}$	17.60	18
3	$\bar{1} \bar{2} \bar{3} \bar{4}$	16.60	17, 18
4	$\bar{1} \bar{2} \bar{3} \bar{4}$	14.55	15, 16, 17, 18
5	$\bar{1} \bar{2} \bar{3}$	17.60	18
6	$\bar{1} \bar{2} \bar{3}$	17.60	18
7	$\bar{1} \bar{2} \bar{3} \bar{4}$	17.45	18
8	$\bar{1} \bar{2} \bar{3}$	17.60	18
9	$\bar{1} \bar{2} \bar{3} \bar{4}$	17.45	18
10	$\bar{2} \bar{3} \bar{4}$	17.45	18
11	$\bar{1} \bar{2} \bar{3}$	17.60	18
12	$\bar{2} \bar{3}$	17.60	18

Note: $\bar{i}$ = lapping of the top reinforcements at the middle range of the i th span; i = lapping of the bottom reinforcements at either side of the i th interior column; LR = longitudinal reinforcement.

results in the reinforcement amount of 5.636 t and a remainder of 0.437 t. All of the web longitudinal reinforcements are of sizes #4 and #5; therefore, they are excluded when cutting.

The arrangement of reinforcements for the best lapping pattern (12) with its optimal stock length ($L = 18$ m) is shown in Fig. 3,

which provides the beam span, beam dimensions, column width, and information regarding continuous and discontinuous longitudinal reinforcements, web longitudinal reinforcements, and stirrups. Information regarding the longitudinal reinforcements includes the size, length, and number of bars. Fig. 3 also shows the locations of lap splices and the stirrup data, which include the size, number, and spacing in three regions along each beam element. Fig. 3 cannot display detailed information regarding every longitudinal reinforcement, particularly the length and the lapping location. This information is clearly displayed in the shop drawing, as shown in Fig. 4. In Fig. 4, the top and bottom longitudinal reinforcements (including the continuous and discontinuous reinforcements) are partitioned by stirrups. Fig. 4 also shows the size, number, and space of all stirrups. In addition, the web longitudinal reinforcements are shown in the figure, below the bottom reinforcements. The details of the schedules for all kinds of reinforcements, including stirrups, can be generated by the automation platform. The details for longitudinal reinforcements can also be found from the cutting pattern, as shown in Table 5.

The best lapping pattern, 12, has 19 l_j values ($j = 1 \sim N_1$, $N_1 = 19$) of the longitudinal reinforcements, as given in Table 5. The corresponding L and N are 18 m and 49, respectively. The cutting pattern (n_{ij} values, $i = 1 \sim 49$, $j = 1 \sim 19$) of the optimal stock length, 18 m, obtained by the trial and error method is shown in Table 5. Coincidentally, the same result was also achieved by the linear programming approach with an error of 0.1% of L by the iterative calculation. The cutting pattern according to the linear

Table 3. Schedule of Reinforcements for Different Lapping Patterns Cut by the Corresponding Optimal Integral Stock Lengths

Type	Stock length L (#10)				Amount		Remainder	
	L (m)	N	Sum of L (m)	Sum of W (t)	L (m)	W (t)	L (m)	W (t)
1	16	57	912	5.827	848.3	5.253	63.7	0.407
2	18	51	918	5.886	842.4	5.383	75.6	0.483
3	17	53	901	5.757	836.55	5.346	64.45	0.412
4	16	56	896	5.725	838.75	5.360	57.25	0.366
5	17	54	918	5.886	843	5.387	75	0.479
6	17	54	918	5.886	825	5.272	93	0.594
7	17	54	918	5.886	839.05	5.362	78.95	0.504
8	17	54	918	5.886	829.05	5.298	88.95	0.568
9	17	54	918	5.886	828.45	5.294	89.55	0.572
10	18	52	936	5.981	825.25	5.273	110.75	0.708
11	18	51	918	5.886	828.95	5.297	89.05	0.569
12 ^a	18	49	882	5.636	813.65	5.199	68.35	0.437

^aThe best lapping pattern cut by its optimal stock length.

Table 4. Schedule of Reinforcements for Different Lapping Patterns Cut by the Corresponding Optimal Stock Lengths

Type	Stock length L (#10)				Amount		Remainder	
	L (m)	N	Sum of L (m)	Sum of W (t)	L (m)	W (t)	L (m)	W (t)
1	16.65	54	899.1	5.745	848.3	5.253	50.8	0.325
2	17.7	51	902.7	5.768	842.4	5.383	60.3	0.385
3	17.6	51	897.6	5.736	836.55	5.346	62.05	0.396
4	15.8	56	884.8	5.654	838.75	5.360	46.05	0.294
5	17.7	51	902.7	5.768	843	5.387	59.7	0.381
6	17.4	51	887.4	5.67	825	5.272	62.4	0.399
7	17.05	52	886.6	5.665	839.05	5.362	47.55	0.304
8	17.7	50	885	5.655	829.05	5.298	55.95	0.358
9	17.7	50	885	5.655	828.45	5.294	56.55	0.361
10	17.6	49	862.4	5.511	825.25	5.273	37.15	0.237
11	17.4	51	887.4	5.67	828.95	5.297	58.45	0.373
12 ^a	17.95	48	861.6	5.506	813.65	5.199	47.95	0.306

^aThe best lapping pattern cut by its optimal stock length.

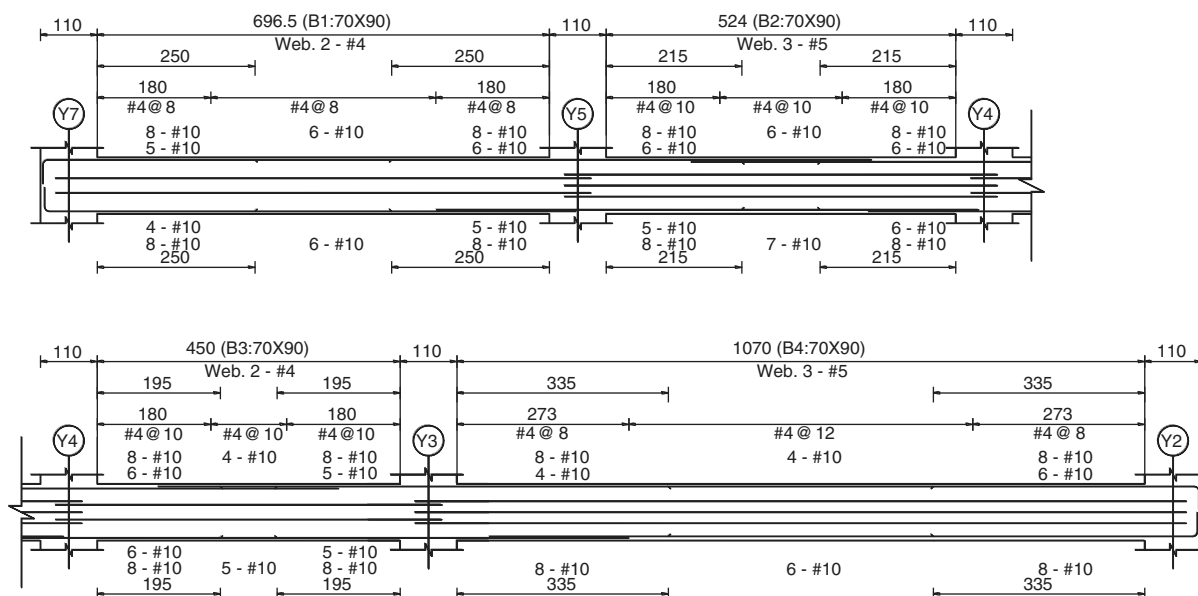


Fig. 3. Arrangement of reinforcements for four-span continuous beam 5FX1B1-B4 (unit: cm)

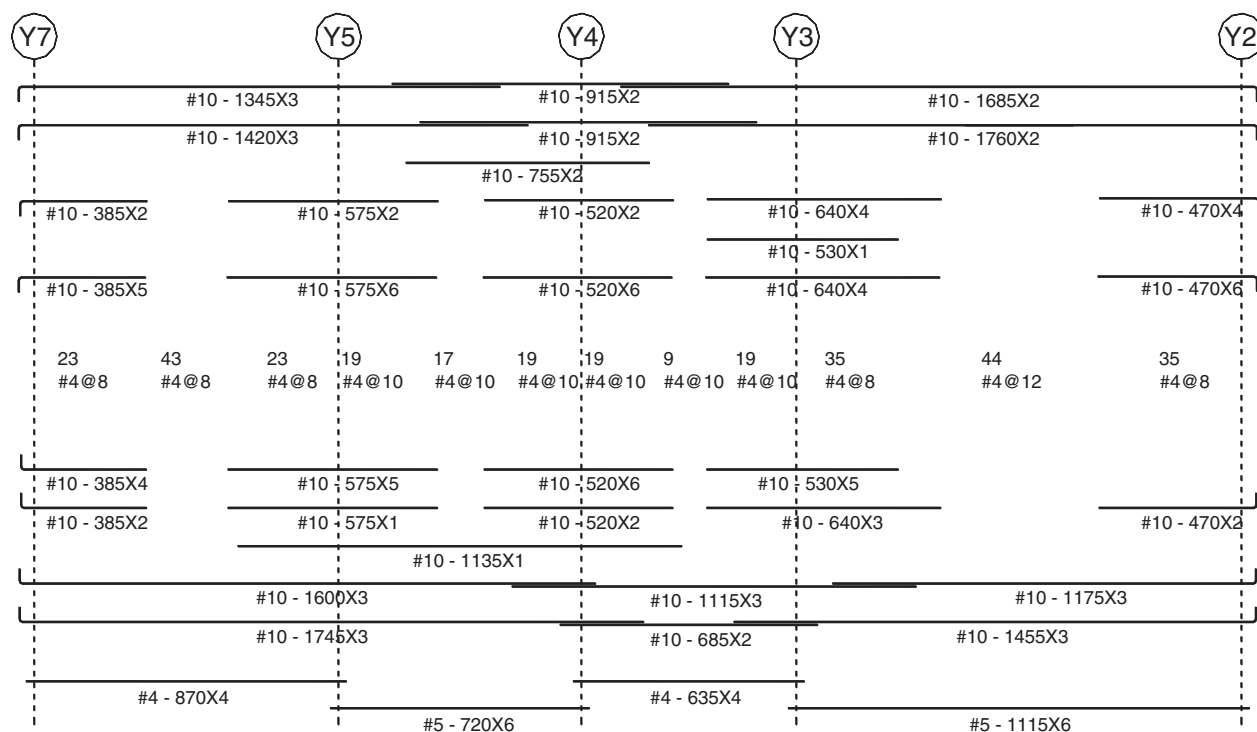


Fig. 4. Shop drawing of reinforcements for four-span continuous beam 5FX1B1-B4 (unit: cm)

programming approach with an error of 1% of the optimal stock length is also shown in Table 5; as shown, only a few terms of n_{ij} (30 of total 49×19) are changed, and indicated by slashes (/). The numbers and the left or right numbers of the slashes represent n_{ij} values of the cutting pattern obtained by the trial and error method or by the linear programming approach with an error of 1% of the optimal stock length. This table also shows that most terms of n_{ij} are zero. The total amount of stock length, i.e., $49 \times 18 = 882$ m, and the 68.35 m remainders are the same for these two different cutting patterns, as shown in Table 5. This also shows that the cutting pattern (a set of n_{ij} values)

might not be unique for the same optimal stock length used for cutting. However, in practice, it is far easier to apply a cutting pattern proposed by the trial and error method to find the optimal stock length cut in order of the decreasing length of the reinforcements.

Conclusions

According to the study described in this paper, the conclusions explained in the following can be drawn.

Table 5. Cutting Pattern

l_j (m)	17.6	17.45	16.85	16	14.55	14.2	13.45	11.75	11.35	11.15	9.15	7.55	6.85	6.4	5.75	5.3	5.2	4.7	3.85		
m_j	2	3	2	3	3	3	3	3	1	3	4	2	2	11	14	6	16	12	13		
	i																			Length (m)	Remainder (m)
j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19		
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17.6	0.4
2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17.6	0.4
3	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17.45	0.55
4	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17.45	0.55
5	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17.45	0.55
6	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	16.85	1.15
7	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	16.85	1.15
8	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	16	2
9	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	16	2
10	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	16	2
11	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	14.55	3.45
12	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	14.55	3.45
13	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	14.55	3.45
14	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	14.2	3.8
15	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	14.2	3.8
16	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	14.2	3.8
17	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	17.3	0.7
18	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	17.3	0.7
19	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	17.3	0.7
20	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	17.5	0.5
21	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1/0	0/1	0	0	0	17.5/17.05	0.5/0.95
22	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1/0	0	0/1	0	0	17.5/16.95	0.5/1.05
23	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	17.75	0.25
24	0	0	0	0	0	0	0	0	0	1	0	0	1/0	0/1	0	0	0	0	0	18/17.55	0/0.45
25	0	0	0	0	0	0	0	0	0	1	0	0	1/0	0	0	0	0/1	0	0	18/16.35	0/1.65
26	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	17.55	0.45
27	0	0	0	0	0	0	0	0	0	0	1	1/0	0	0	0	0/1	0	0	0	16.7/14.45	1.3/3.55
28	0	0	0	0	0	0	0	0	0	0	1	1/0	0	0	0	0	0/1	0/1	0/1	16.7/17.7	1.3/0.3
29	0	0	0	0	0	0	0	0	0	0	1	0	0	1/0	0	0	0	0	0/2	15.55/16.85	2.45/1.15
30	0	0	0	0	0	0	0	0	0	0	1	0	0	1/0	0/1	0	0	0	0	15.55/14.9	2.45/3.1
31	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	1	0	0	18	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	1	0	0	18	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	1	0	0	18	0
34	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2/0	0	0	0	0/3	17.9/17.95	0.1/0.05
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	17.25	0.75
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	17.25	0.75
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	17.25	0.75
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	15.9	2.1
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0/3	3/0	0	0	0	15.9/17.25	2.1/0.75
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	15.6	2.4
41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	15.6	2.4
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0/1	0	0	3/2	0	0	15.6/16.8	2.4/1.2
43	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0/1	3/2	0	0	15.6/15.7	2.4/2.3
44	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	0	0	14.6	3.4
45	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	1	17.95	0.05
46	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	1	17.95	0.05
47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	1	17.95	0.05
48	0	0	0	0	0	0	0	0	0	0	0	0	0/2	0	0	0	0	1/0	3/1	16.25/17.55	1.75/0.45
49	0	0	0	0	0	0	0	0	0	0	0	0/2	0	0	0	0	0	0	4/0	15.4/15.1	2.6/2.9
sum	2	3	2	3	3	3	3	3	1	3	4	2	2	11	14	6	16	12	13	813.65	68.35

Note: n_{ij} values, $i = 1 - 49$, $j = 1 - 19$; $L = 18$ m.

1. A graphical interface allows users to more easily see and review all inputs. The data input window for the beam element developed in this paper can easily be used to input and test all parameters in the corresponding positions on the beam element, and accordingly, they are more likely to detect and avoid mistakes.
2. The shop drawing and scheduled automation of beam reinforcements based on the best lapping pattern with its optimal

stock length clearly display the arrangements and details of beam reinforcements. Therefore, shop drawing and scheduling automation can greatly improve accuracy and efficiency in construction management, particularly for the control of reinforcements.

3. The optimal stock length for a specific lapping pattern can be determined by either the trial and error method or the linear programming approach. The formulation of the objective

function and its constrained conditions, as defined by Eqs. (1)–(4) of the linear programming approach, apparently have nothing to do with the cutting pattern. The results of the optimal stock length obtained by these two methods have been checked and are consistent with each other. Thus, the cutting pattern presented in this paper, of longitudinal reinforcements cut in the order of decreasing length with a stock length specified by the trial and error method, will yield the minimum required amount of the stock length, which may be the most economical way to supply a proper stock length for cutting.

4. The reliability of the trial and error method to find the optimal stock length has been proven by the linear programming approach. The disadvantage of the linear programming approach is that when the dimension ($N \times N_1$) of n_{ij} values shown in Eq. (1) is large, the solution obtained by this method is very time consuming, particularly for a beam system having many possible choices of lapping patterns of the longitudinal reinforcements. Therefore, the trial and error method seems to be the simpler and more practical method in application. Additionally, the trial and error method can provide more information about the lapping patterns with their optimal stock lengths, so it is more flexible than the linear programming approach for an engineer to make or change the decision in design.
5. The cutting pattern, which was proposed by the trial and error method to find the optimal stock length cut in the order of decreasing length of the reinforcements, is very easily applied in practice and reinforcements can also be cut in a more efficient manner. Therefore, it is by no means restricted to beams or other structural components (e.g., column, wall, or slab), but is also applicable to the construction of reinforcements for an entire floor, a work zone, or even an entire RC building or structure.
6. The presented automation platform and the relevant technologies can also be applied to all structural components of an RC building, such as beam, column (Chen and Yang 2009), slab, or wall. First, the design parameters, as the input to the window (Table 1 and Fig. 2 show a beam example) should be defined appropriately for each kind of structural components. Next, the programming process in the first and second phases of the automation platform, as shown in Fig. 1, should be modified according to different purposes.
7. The task of manually entering parameter data is time consuming. The process can be made more efficient by directly obtaining the results from structural analysis containing the design details for beam reinforcements, through the well-known software like *ETABS* or *SAP2000*. The required design parameters of all beam elements, as defined in Table 1, can be extracted from these results, converted to the appropriate file format, and saved for later application. Information about the cutting pattern of optimal stock length, shop drawing, and detail of the schedule of reinforcements can be very useful for the construction of reinforcements. Therefore, the presented automation platform can be linked not only to the early stages of structural analysis and design, but also to the later stage of reinforcement cutting. The program can integrate every aspect of the construction of an RC building or another kind of RC structure, thus achieving chain automation in structural analysis, design, shop drawing, reinforcement cutting process, and stock supply. This will be a topic for future research.

Acknowledgments

The authors sincerely appreciate the continuous financial support from the National Taiwan University, the Republic of China. Special thanks should be due to Prof. Tsung-Wu Lin for his valuable comments and suggestions.

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