Steiner Systems for Topology-Transparent Access Control in MANETs

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Abstract. In this paper we examine the combinatorial requirements of topology-transparent transmission schedules for channel access in mobile ad hoc networks. We formulate the problem as a combinatorial question and observe that its solution is a cover-free family. The mathematical properties of certain cover-free families have been studied extensively. Indeed, we show that both existing constructions for topology-transparent schedules (which correspond to orthogonal arrays) give a cover-free family. However, a specific type of cover-free family – called a Steiner system – supports the largest number of nodes for a given frame length. We then explore the minimum and expected throughput for Steiner systems of small strength, first using the acknowledgement scheme proposed earlier and then using a more realistic model of acknowledgements. We contrast these results with the results for comparable orthogonal arrays, indicating some important trade-offs for topology-transparent access control protocols.

1 Introduction

In any network based on a shared broadcast channel, the means by which access to the channel is controlled has a fundamental impact on the overall network performance. While these networks include satellites and local area networks, our interest is in *mobile ad hoc networks* (MANETs). A MANET is a collection of mobile wireless nodes. What distinguishes a MANET from other wireless networks is that it self-organizes without the aid of any centralized control or any fixed infrastructure. Since the radio transmission range of each node is limited, it may be necessary to forward over multiple hops in order for a packet to reach its destination (as such, MANETs have also been called multi-hop and packet-radio networks). This also offers the opportunity for concurrent transmissions when nodes are sufficiently separated. The challenge in medium access control (MAC) protocols for MANETs is to find a satisfactory trade-off between the two objectives of minimizing delay and maximizing throughput.

Of the myriad of access control techniques, our focus is on topology- transparent approaches. Unlike topology-dependent protocols, which recompute access whenever the network topology changes, a *topology-transparent* protocol acts independently of topology change. One class of protocols which may be viewed as

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topology-transparent is the contention based MAC protocols. Contention based approaches achieve high throughput with a reasonable expected delay but with poor worst-case delay. With increasing interest in multi-media applications, the delay characteristics of contention based MAC protocols do not appear adequate to provide the necessary quality-of-service (QoS) support. While there have been some efforts to make such protocols QoS-aware, in each case the delay guarantee remains probabilistic [1,12,14].

TDMA is an example of a scheduled access control protocol that is trivially topology-transparent. More sophisticated schemes for generating topology-transparent transmission schedules [2,10] depend on two design parameters: N, the number of nodes in the network, and D, the maximum node degree. This creates complex trade-offs between the design parameters and the delay and throughput characteristics of the resulting schedules. For example, while it is often possible to construct schedules that are significantly shorter than TDMA, if the actual node degree exceeds D, the delay guarantee is lost. More exactly, the delay becomes probabilistic rather than deterministic. While the question of what should be done if the protocol fails is important (see [3,16] for some alternatives), we will not address this problem here.

In [16], we observed that existing topology-transparent transmission schedules are instances of orthogonal arrays, and we explored the consequences of this observation on throughput. In this paper we go one step further, looking more carefully at the combinatorial requirements of topology-transparent transmission schedules. This allows us to formulate the problem as a combinatorial question and observe that its solution is a cover-free family. Certain cover-free families have been studied extensively, and rather than derive new mathematical results, we instead show how to use existing results for our application. Our first observation shows that an orthogonal array gives a cover-free family. We then show that a specific type of cover-free family, called a Steiner system, supports the largest number of nodes for a given frame length. We then explore the minimum and expected throughput for Steiner systems of small strength, first using the acknowledgement scheme proposed earlier and then using a more realistic model for acknowledgements. We contrast these results with the results for comparable orthogonal arrays, indicating some important trade-offs for topology-transparent protocols.

The rest of this paper is organized as follows. Section 2 first examines the combinatorial requirements of a topology-transparent transmission schedule, and shows that a cover-free family satisfies the requirements. We also show how cover-free families relate both to orthogonal arrays, and to Steiner systems. In Section 3, we study the selection of parameters of the Steiner system depending on the performance objective of interest. We consider both minimum and expected throughput using an acknowledgment scheme proposed earlier. As well, we introduce a more realistic acknowledgement model and study the resulting frame throughput. We produce our results as a function of neighbourhood size and density, to explore the sensitivity of the actual node degree to the design parameter. Lastly, in Section 4, we summarize and conclude.

2 Cover-Free Families, Orthogonal Arrays, and Steiner Systems

Rather than starting with the existing constructions for topology-transparent transmission schedules, let us instead begin anew by turning the problem of generating a topology-transparent transmission schedule into a combinatorial question. Assume that time is divided into discrete units called *slots* and *frames* are a fixed number n of slots. Suppose that each node $i, 1 \leq i \leq N$, in the network is assigned a transmission schedule $S_i = s_1 s_2 \dots s_n$ with n slots (i.e., one frame). If $s_j = 1, 1 \leq j \leq n$, then a node may transmit in the slot j, otherwise it is silent (and could receive).

In designing a topology-transparent transmission schedule with design parameters N, the number of nodes in the network and D, the maximum node degree, we are interested in the following combinatorial property. For each node, we want to guarantee that if a node i has at most D neighbours its schedule S_i guarantees a collision-free transmission to each neighbour.

Let us treat each schedule S_i as a subset T_i on $\{1, 2, \dots, n\}$ by assigning the elements of the subset to correspond to the positions in the schedule, i.e., $j \in T_i$ if $s_j = 1$ in S_i, $j = 1, \dots, n$ (in essence, S_i is the characteristic vector of the set T_i). Now, the combinatorial problem to ask is for each node i to be given a subset T_i with the property that the union of D or fewer other subsets cannot contain T_i. Expressed mathematically, if T_j, $j = 1, \dots, D$, are D neighbours of i ($T_j \neq T_i$), then we require that

$$\left(\bigcup_{j=1}^{D} T_j \right) \not\supset T_i.$$

This is precisely a D *cover-free family*. These are equivalent to disjunct matrices [6] and to certain superimposed codes [7]; see [5].

Let us first observe that the existing constructions for topology-transparent transmission schedules [2,10] which, as we showed in [16] correspond to an orthogonal array, give a cover-free family.

2.1 An Orthogonal Array Gives a Cover-Free Family

Let V be a set of v symbols, usually denoted by $0, 1, \dots, v-1$.

Definition 1. *A $k \times v^t$ array A with entries from V is an* orthogonal array *with v levels and strength t (for some t in the range $0 \leq t \leq k$) if every $t \times v^t$ subarray of A contains each t-tuple based on V exactly once*[1] *as a column. We denote such an array by $OA(t, k, v)$.*

Table 1 shows an example from [9] of an orthogonal array of strength two with $v = 4$ levels, i.e., $V = \{0, 1, 2, 3\}$. Pick any two rows, say the third and the fourth. Each of the sixteen ordered pairs $(x, y), x, y \in V$ appears the same number of times, once in this case.

[1] Here, we assume the index $\lambda = 1$.

Table 1. Orthogonal array $OA(2,4,4)$.

0	0	0	0	1	1	1	1	2	2	2	2	3	3	3	3
0	1	2	3	0	1	2	3	0	1	2	3	0	1	2	3
0	1	2	3	1	0	3	2	2	3	0	1	3	2	1	0
0	1	3	2	3	2	0	1	2	3	1	0	1	0	2	3

In our application, each column gives rise to a transmission schedule. Each column intersects every other in fewer than t positions. For example, the first and the eighth column intersect in no positions, while the first and the second column intersect in a zero in the first position.

The importance of this intersection property is as follows. Select any column. Since any of the other columns can intersect it in at most $t-1$ positions, any collection of D other columns has the property that our given column differs from all of these D in at least $k - D(t-1)$ positions. Provided this difference is positive, the column therefore contains at least one symbol appearing in that position, not occurring in any of the D columns in the same position. In our application this means that at least one collision-free slot to each neighbour exists when a node has at most D neighbours. Thus, as long as the number of neighbours is bounded by D, the delay to reach each neighbour is bounded, even when each neighbour is transmitting. Clearly, the orthogonal array gives a D cover-free family.

Many techniques are known for constructing orthogonal arrays, usually classified by the essential ideas that underlie them. There is a classic construction based on Galois fields and finite geometries; both Chlamtac and Faragó [2] and Ju and Li [10] use this construction implicitly though neither observed that they were constructing an orthogonal array. They both employ $OA(t, v, v)$'s when v is a prime power. They therefore restrict attention to the case when $k = v$ (forcing all frame lengths to be v^2 unnecessarily), and indeed by not permitting that $k > v$ they do not obtain the best delay guarantees. The restriction of v to prime powers is also not required, as orthogonal arrays exist for these cases, e.g., $OA(2, 7, 12)$, but k is not as large as v in general.

In the same way that allowing different parameters for orthogonal arrays allows more flexibility in the corresponding schedules, relaxing the parameters further and asking for a cover-free family allows more flexibility yet.

2.2 Steiner Systems

Cover-free families have been studied extensively, most frequently with the objective of maximizing the number of sets in the family. In our application, this corresponds to maximizing the number of nodes, so this is certainly a parameter of interest.

There is a celebrated result of Erdös, Frankl, and Füredi [8] that established bounds on the size of a cover-free family (see also, [13,15] and Theorem 7.3.9 in [6]). Specifically, they established that the extreme value on the size, if achievable, is realized by a Steiner system. Hence in terms of the application, for a

Table 2. Steiner system $S(2, 4, 13)$.

0	0	0	0	1	1	1	2	2	3	3	4	5
1	2	4	6	2	5	7	3	6	4	7	8	9
3	8	5	a	4	6	b	5	7	6	8	9	a
9	c	7	b	a	8	c	b	9	c	a	b	c

given number of nodes and a given maximum number of neighbours, Steiner systems achieve the shortest frame length of all cover-free families. Thus, they provide not only a solution to our problem, but indeed the best solution in terms of frame length.

Definition 2. *Given three integers t, k, v such that $2 \leq t < k < v$, a* Steiner system $S(t, k, v)$ *is a v-set V together with a family $\mathcal{B}$ of k-subsets of V (blocks) with the property that every t-subset of V is contained in exactly one block.*

Table 2 shows an example from [4] of a Steiner system on the 13-set $V = \{0, 1, \ldots, 9, a, b, c\}$ together with a family of $\frac{v(v-1)}{k(k-1)} = \frac{13 \cdot 12}{4 \cdot 3} = 13$, 4-subsets of V (the columns). This Steiner system has the property that every 2-subset of V, $\{x, y\}, x, y \in V, x \neq y$ is contained in exactly one column.

While a substantial amount is known about the existence of Steiner systems, in general their existence is not settled [4]. As with orthogonal arrays, there are constructions from finite fields.

Reasons that Steiner systems are of interest for constructing topology- transparent transmission schedules include:

1. Steiner systems admit shorter schedules than orthogonal arrays. This is important since in addition to achieving high throughput, the delay bound is improved. We discuss this issue at length in Section 3.
2. Steiner systems are denser than orthogonal arrays. They can support a larger number of nodes for a given schedule (frame) length.

3 Steiner System Parameter Trade-Offs

The essential difference between the existing constructions of topology- transparent schedules is in the selection of parameters. In [2], the focus is on frame length while in [10] the focus is on throughput.

In [2], an $OA(t, v, v)$ is found for the first $v^t \geq N$, $t \geq 2$, where v is a prime power. In the paper, the interest is in minimizing the frame (or schedule) length in order to minimize delay. The parameters are selected to find a schedule provably shorter than TDMA.

In [10], it is argued that the parameters chosen satisfy the condition on delay but do not maximize minimum throughput. In particular, it is possible to achieve higher minimum throughput at the expense of longer frame length. They select an $OA(t, v, v)$ where $v = 2(t-1)D$ if $\sqrt[t]{N} \leq 2(t-1)D$, and $v = \sqrt[t]{N}$ otherwise.

Intuitively, while Chlamtac and Faragó [2] strive to get *one* free slot per frame, Ju and Li [10] aim to get *many* free slots per frame.

In both studies, however, the figure of merit is minimum throughput measured as number of free slots within a frame divided by frame length. To employ such an analysis, a transmitting node must be able to transmit multiple different packets within a frame. How does it decide to transmit a "new" packet? In this environment, it is expected that collisions occur, and topology-transparency dictates that the collisions cannot be anticipated. Hence an acknowledgement scheme is needed. Both schemes based on orthogonal arrays can transmit in two consecutive slots, and indeed must send different packets in these slots to achieve the minimum throughput in their analyses. Both propose an acknowledgement scheme that involves instantaneous acknowledgment of successful receipt without lengthening the slot. Naturally, this is an optimistic assumption to facilitate the analysis. However, the analysis can be misleading if it leads us to seek many free slots in a frame without an acceptable (realistic) acknowledgment scheme. For purposes of comparison, we consider the throughput measures employed in [2,10,16]. We also adopt a more conservative approach.

We consider a more realistic model for acknowledgements. Rather than a slot by slot acknowledgement, we assume we can piggyback an acknowledgement onto a packet sent from the destination. In the worst case, this might require that the sender wait an entire frame. Hence we define *frame throughput* as the throughput achievable on a per frame basis. This properly incorporates the length of the schedule in the throughput calculation.

In this section we investigate three questions:

1. What is the probability of a successful transmission in a frame?
2. What is the expected throughput?
3. What is the expected frame throughput?

All are functions of the number of active transmitters among the neighbours of a node.

Consider a situation with sender $\mathcal{S}$ and receiver $\mathcal{R}$. Let S be a schedule for sender $\mathcal{S}$ and $T_1, \ldots, T_{D-1}$ be the subsets that correspond to the schedules of the other active neighbours of $\mathcal{R}$ (here, we assume the worst case, when all neighbours are transmitting). Let T_D be the subset corresponding to the schedule for $\mathcal{R}$, and assume that $\mathcal{R}$ is also active.

The probability of successful transmission within a frame is just the probability that $\mathcal{S}$ has a slot that does not appear in $T_1, \ldots, T_D$. Expected throughput then, is the expected number of such slots. The frame throughput is the expected number of slots over the frame length. This effectively normalizes the expected throughput by frame length allowing easier comparison between Steiner systems.

We derive these measures analytically but present the derivations elsewhere. The most complex derivation is for expected throughput. We did this for schedules that correspond to orthogonal arrays in [16]. This formulation may be used as a basis to derive expected throughput for schedules that correspond to Steiner systems.

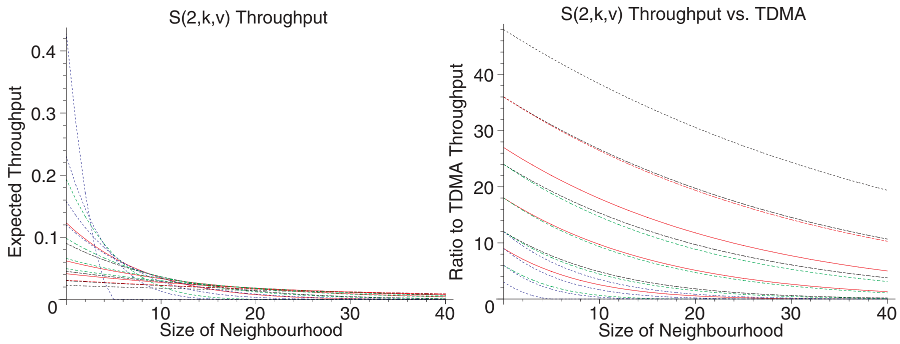

Fig. 1. Expected throughput for (a) $S(2, k, v)$; and (b) versus TDMA, for $k = 3, 6, 9, 12$.

3.1 Numerical Results

The results in this section were obtained using Maple [11], a mathematical software package.

Figure 1(a) plots the expected throughput for $S(2, k, v)$ for $k = 3, 6, 9, 12$ as a function of the number of neighbours. In each of the following cases, $N = v(v - 1)/k(k - 1)$. For $k = 3$, $v = 7, 13, 19, 25$ are considered for $N = 7, 26, 57, 100$ number of nodes, respectively. For $k = 6$, $v = 31, 61, 91, 121$ are considered for $N = 31, 122, 273, 484$ number of nodes, respectively. For $k = 9$, $v = 73, 145, 217, 289$ are considered for $N = 73, 290, 651, 1156$ number of nodes, respectively. Finally, for $k = 12$, $v = 133, 265, 397, 529$ are considered for $N = 133, 530, 1191, 2116$ number of nodes, respectively. In the figure, the y-intercept is given by k/v, and so the curve with the highest y-intercept has the shortest frame length ($k = 3$, $v = 7$). Successive curves with lower y-intercept have successively longer frame length. The shorter the frame, the faster the expected throughput drops to zero. As well, the expected throughput is much more sensitive to changes in neighbourhood size.

In Fig. 1(b), we plot the expected throughput for $S(2, k, v)$ for $k = 3, 6, 9, 12$ over the throughput of TDMA with the same frame length, as a function of the number of neighbours. For example, now the curve with the highest y-intercept is $k = 12$, $v = 529$. This Steiner system supports 2116 nodes, so the expected frame throughput is $\frac{k/v}{1/N} = \frac{12}{529} \cdot 2116 = 48$. In other words, in the best case, this Steiner system has expected throughput that is 48 times that of TDMA with the same frame length. When the ratio of expected throughput to the corresponding TDMA is taken, the curve on the left essentially inverts position on the right. This means that longer frames with more opportunities to transmit are better than shorter frames with fewer opportunities to transmit from the perspective of throughput.

Figure 2(a) plots the more conservative frame throughput for $S(2, k, v)$ for $k = 3, 6, 9, 12$ as a function of neighbourhood size, for the same v's as in the previous figure. Now, the y-intercepts correspond to $1/v$ rather than k/v. Again,

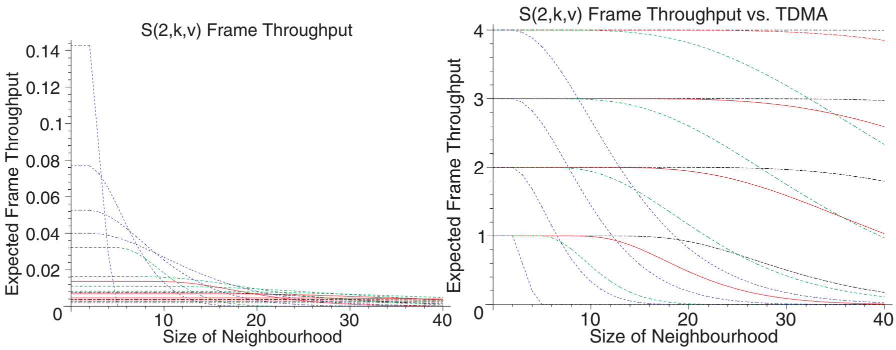

Fig. 2. Frame throughput for (a) $S(2,k,v)$; and (b) versus TDMA, for $k = 3,6,9,12$.

the curves with a shorter frame length have a more pronounced drop than curves with longer frame length. As well, curves with the same k value now show a guarantee (i.e., are horizontal) for up to k neighbours, after which the guarantee degrades.

In Fig. 2(b), we plot the ratio of frame throughput for $S(2,k,v)$ for $k = 3,6,9,12$ over the throughput of TDMA for the same frame length as a function of neighbourhood size, for the same v's as given earlier. Now, we see that the best possible throughput is $\frac{1/v}{1/N} = N/v$ which is $4,3,2,1$ for increasing values of v. Again, the slot guarantee is evident. That is, the curves are horizontal for neighbourhood sizes less than or equal to k and the degrade as the neighbourhood increases. The degradation is slower for the longer frames. The curves whose maximum expected frame throughput equals one correspond to orthogonal arrays $OA(2,v,v)$. Hence it is plainly evident that schedules constructed from Steiner systems are much denser than those constructed from orthogonal arrays, with the potential to yield much higher throughput.

Figure 3(a) plots minimum throughput for $S(2,k,v)$ for $k = 3,6,9,12$ as a function of neighbourhood size for the same values of v as given earlier. Here, the y-intercept is k/v (the same as in Fig. 1), however now the x-intercept is k and is the same for each value of v. This results in the curves dropping to zero much more quickly than in Fig. 1. A curiosity is that the four segments that correspond to the maximum minimum throughput correspond to $S(2,k,v)$ where the smallest frame length v for the given k provides a range of neighbours over which it provides the best minimum throughput. That is, $S(2,3,7)$ and $S(2,12,133)$ are better over a larger range of neighbours than are $S(2,6,31)$ and $S(2,9,73)$.

Figure 3(b) plots the ratio of minimum throughput for $S(2,k,v)$ for $k = 3,6,9,12$ over TDMA with the same frame length as a function of neighbourhood size for the same v's. Again we see that the curves invert order when the ratio is considered. Specifically, the curve with the highest y-intercept is $S(2,12,529)$ since this is given by $\frac{k/v}{1/N}$ as in Fig. 1. However the x-intercept now corresponds to k as on the figure on the left. Now, the largest v for each k provides the best minimum throughput relative to TDMA.

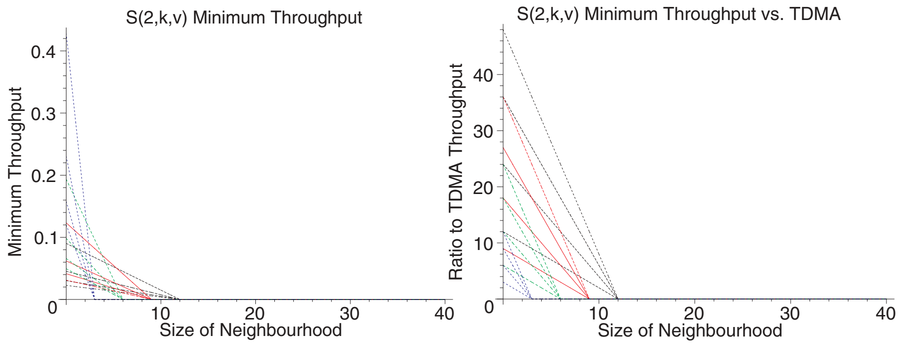

Fig. 3. Minimum throughput for (a) $S(2, k, v)$; and (b) versus TDMA, for $k = 3, 6, 9, 12$.

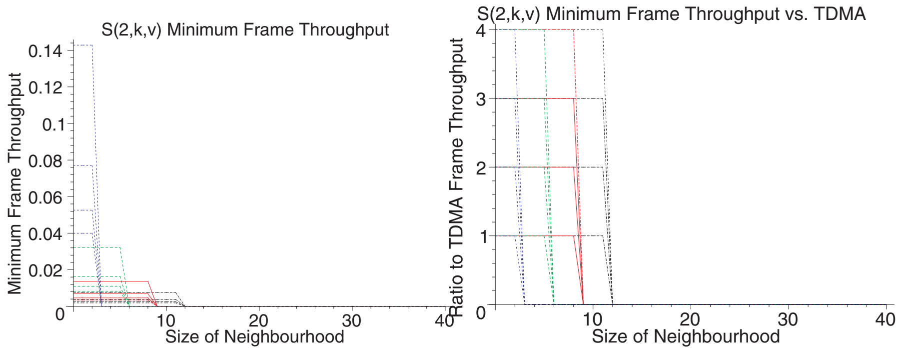

Fig. 4. Minimum frame throughput for (a) $S(2, k, v)$; and (b) versus TDMA, for $k = 3, 6, 9, 12$.

Again, we look at frame throughput, this time the minimum value, in Fig. 4 (for the same k's and v's). Not surprisingly, the minimum frame throughput is lower than when using the more optimistic acknowledgement model. The main difference between this figure and Fig. 2 is the x-intercepts. Here, they correspond to k, clearly showing that with minimum frame throughput, once the neighbourhood exceeds the design parameter, all guarantees are lost immediately. This is also true for the ratio of minimum frame throughput over TDMA with the same frame length (b). This figure also shows that the minimum frame throughput is essentially constant for each k as long as the design parameter is satisfied.

Figure 4 shows us something very important, in addition. Larger Steiner systems give us a minimum frame throughput substantially better than TDMA when the neighbourhood is within the bound. This is in stark constrast with the schemes in [2,10]; they *never* outperform TDMA on minimum frame throughput when orthogonal arrays of strength two are used.

Figure 5 is different from all other figures in that it plots expected throughput versus *density* of the neighbourhood. That is, the x-axis is the percentage of nodes that are neighbours — these are not absolute values, and represent much

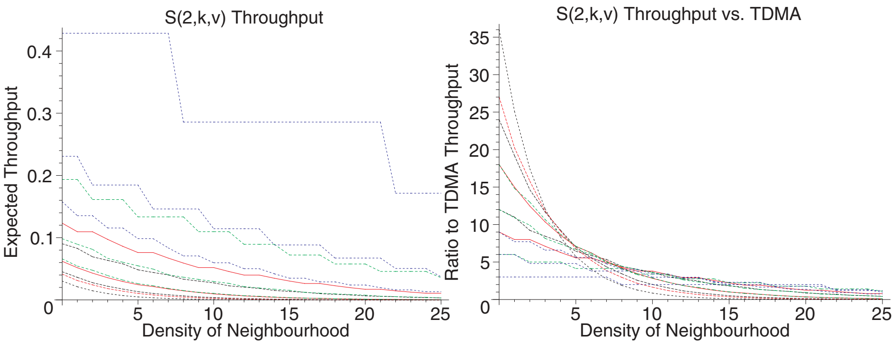

Fig. 5. Throughput versus density for (a) $S(2,k,v)$; and (b) versus TDMA, for $k = 3,6,9,12$.

larger neighbourhood sizes in general. The reason that the curves are jagged is that the closest integer value is taken as the percentage of neighbours, i.e., we do not consider fractional numbers of neighbours. While the figure shows $S(2,k,v)$ for $k = 3,6,9,12$, only the first three values of v for each k are shown since the computations are highly memory and compute intensive. The y-intercepts are the same as in Fig. 1. As a function of neighbourhood density, the expected throughput (a) is more well-behaved than as a function of neighbourhood size. When the ratio of expected throughput to TDMA throughput is considered versus neighbourhood density (b) the curves drop more rapidly as the density increases more rapidly than a linear function.

Finally, Fig. 6 once again plots expected throughput versus neighbourhood size for three Steiner systems that support the same number of nodes, namely $N = 651$ and one orthogonal array that supports a number very close to that (625). Specifically from the top down, the curves correspond to $S(2,3,63)$, $S(2,9,217)$, $S(2,26,651)$ and $OA(2,26,25)$. First, we see that the last two curves are essentially indistinguishable from each other. That is, for all intents and purposes, the $S(2,26,651)$ and $OA(2,26,25)$ give the same performance but the Steiner system supports more nodes. The Steiner system with shorter frame length gives better expected throughput until the neighbourhood is about 20, at which point the curves all cross. Its performance also degrades more rapidly with increasing neighbourhood size.

4 Summary and Conclusions

In this paper, we stepped back and examined anew the combinatorial properties of topology-transparent schedules. The properties were found to correspond precisely to D cover-free families, where D is a design parameter indicating maximum number of neighbours.

Studies of several Steiner systems show the following general trends. Steiner systems admit shorter schedules (frames) than previous cosntructions based on

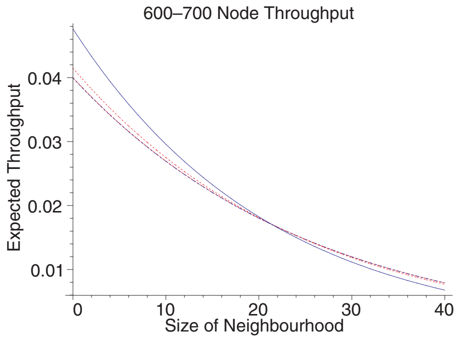

Fig. 6. Expected throughput for Steiner systems for 600-700 nodes.

orthogonal arrays. This is significant for delay sensitive applications such as multi-media. Since Steiner systems are also more dense, they support more nodes for a given frame length and hence achieve higher throughput. While shorter schedules give the best minimum and expected throughput, they also degrade faster as the design parameter D is exceeded. That is, longer schedules are more robust to changes in neighbourhood size. Another general observation is that the Steiner systems that yield longer schedules achieve higher ratios on minimum and expected throughput when compared to TDMA schedules of the same length.

We have characterized the types of solutions topology-transparent transmission schedules require as cover-free families. Using this, along with a more realistic acknowledgement model, we plan to investigate the issue of what to do when the schedule fails due to node mobility causing the design parameter on neighbourhood size to be exceeded. This, together with simulations using mobility models are required to determine how such scheduled topology-transparent protocols compare to contention based protocols.

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