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Basilio Lenzo *Editor*

Vehicle Dynamics

Fundamentals and Ultimate Trends



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Basilio Lenzo
Editor

Vehicle Dynamics

Fundamentals and Ultimate Trends

 Springer

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Preface

Since the invention of the world's first motor vehicle, more than 100 years ago, automobiles have been widely accepted in our society with the progress of modern industry. The study and understanding of vehicle dynamics have always played a crucial role in the design of vehicles, with the aim of guaranteeing safety and stability as well as good performance. The recent advent of electric vehicles and the future perspective of widespread autonomous cars have posed further interesting challenges for the vehicle dynamicist. Nonetheless, the importance of the basics should never be underestimated—after all, essentially a vehicle behaviour is described by second Newton's law, $F = ma$.

With these motivations, an international course was organised at CISM in 2019, with a team of lecturers including two eminent academics, two experienced researchers, and two industrial representatives. The aim of this book—which collects contributions from the lecturers—is to recall the fundamentals of vehicle dynamics and to present and discuss the state of the art of ultimate trends in the field, including torque vectoring control, vehicle state estimation, and autonomous driving.

The first chapter, “[Fundamentals on Vehicle and Tyre Modelling](#)”, discusses the equations of motion for a generic vehicle, including roll and pitch dynamics as well as the vertical travel of the sprung mass. Then, the chapter deals with the fundamentals of tyre modelling, with a detailed analysis of the existing tyre models. Finally, a new linear tyre model with varying parameters is presented, which is at the same time simple—which makes it suitable for control purposes—and accurate in that it represents combined tyre-road interactions (longitudinal and lateral).

The second chapter, “[Vehicle Steering and Suspension Kinematics/Compliance and Their Relationship to Vehicle Performance](#)”, is dedicated to the analysis of the key peculiarities of vehicle steering and suspension systems. After looking into tyre behaviour, the chapter dives into wheel end architecture and suspension kinematics, with the analysis of the suspension design parameters (camber, caster, kingpin, etc.) and their effects on vehicle behaviour, together with suspension compliance. Steering kinematics and compliance are discussed, along with the relationships between weight transfer, kinematics, and compliance. The final part deals with the interesting concept of tyre utilisation.

In the third chapter, “[Tyre Mechanics and Thermal Effects on Tyre Behaviour](#)”, the structure of the tyre and the mechanisms involved in tyre-road interaction are analysed. The effect of temperature on the tyre behaviour is then examined. Detailed thermal models are presented, accounting for heat generation—due to both the tyre-road tangential interactions and the tyre cyclic deformation during rolling—and heat exchange at different levels. Finally, the chapter discusses the factors involved in tyre wear and the different approaches proposed so far to model it.

The second part of the book, devoted to ultimate trends in vehicle dynamics, begins with the chapter “[Torque Vectoring Control for Enhancing Vehicle Safety and Energy Efficiency](#)”. The chapter first presents the principle of torque vectoring and the general framework of a torque vectoring system. Different approaches to define the vehicle reference yaw rate and reference sideslip angle are compared, with a focus on the design of the full vehicle cornering response and the definition of driving modes selectable by the driver. Various control methodologies and torque distribution strategies are discussed, along with their implications for vehicle safety and energy efficiency.

After introducing the motivations for the need of state estimators, the fifth chapter, “[State and Parameter Estimation for Vehicle Dynamics](#)”, presents an accurate analysis of the principles of observers and estimators and the methods to implement them. The well-known Kalman filter is contextualised and presented with rigour, together with its variants. The important concept of observability is dealt with, including its physical interpretation and the support of enlightening examples. A specific estimation methodology is discussed in detail, able not only to estimate relevant vehicle states (such as the sideslip angle), but also tyre parameters.

The sixth chapter, “[Automated Driving Vehicles](#)”, is dedicated to the future of vehicles: autonomous driving. After an introduction on the role of the driver, the chapter discusses the key aspects of sensor fusion, including typical sensor characteristics and requirements, and how to use them to obtain a reliable representation of the environment. Assuming the availability of such representation, the problem of motion planning is dealt with, resulting in the desired driving path and speed profile. Then, the chapter discusses control techniques for the vehicle to follow the desired path and to track the desired speed profile. Finally, verification, validation, and safety issues are addressed.

I would like to thank all the authors for their precious contributions. I trust this book will be a valid resource for graduate students and researchers in the field of vehicle dynamics and control.

Padua, Italy

Basilio Lenzo

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Fundamentals on Vehicle and Tyre Modelling



Moad Kissai

Abstract Many books provide vehicle dynamics and tire dynamics equations. Most of them are either too complex or too simplified for global Vehicle Motion Control (VMC). This chapter is focusing on the couplings that exist between longitudinal, lateral and vertical dynamics at both the vehicle and tire level. The equations provided are simple but sufficient for global VMC. Several research papers and industrial patents tend to provide a control strategy for a standalone system. Simplified car models are taken into consideration from the beginning. As we are steering towards global VMC, more complex models are needed. Here, we start from complex equations that are simplified enough to facilitate control synthesis while capturing the required couplings for a coordinated control. Results show how this simplified global dynamics equations are close enough to more complex high-fidelity models. These equations should be therefore used for the next generation of global VMC.

Car manufacturers and equipment suppliers are constantly proposing new attractive subsystems to stand out from their competitors. Recently, a large interest has been given particularly to automated vehicles. Automation promises indeed safer and smarter vehicles. Several researches have been carried out on one hand in robotic vision, sensor fusion, decision algorithms, big data management, and others. On the other hand, car manufacturers are looking closely on the over-actuation of the vehicle itself (Shyrokau & Wang, 2012; Soltani, 2014; Sriharsha, 2016). Indeed, giving the vehicle new features such as the ability of steering the rear wheels (Seongjin, 2015), distributing the brake torques or/and the engine torques differently between left and right tires (Siampis et al., 2013) and so on, can expand the vehicle's performance and generate new motion possibilities and car behaviors. This can be actually achieved provided that a global chassis control strategy can be designed.

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The development of such strategies can be structured using the “V” development process. This approach is well-accepted for mechatronic systems development although it originates from system engineering and software development (Soltani, 2014). This method consists of feedback steps starting from requirement definition and ending up with in-vehicle validation. A Model Based Design (MBD) methodology is selected as it has proven its effectiveness along with the “V” development process in control system development (Nicolescu & Mosterman, 2010). Even if this approach may differ slightly according to the control architecture adopted, the main steps remain:

- **System Modeling:** Describing mathematically the physical representations of the system dynamics. This is a key step in an MBD design methodology due to the fact that the control logic developed is closely related to the model itself,
- **Controllers Synthesis:** Developing the different algorithms required to control the vehicle dynamics based on the system modeling,
- **Coordination Strategy Development:** as the overall system is over-actuated, coordination between subsystems should be ensured in order to satisfy the high-level demands.

In this chapter, we will mainly focus on the first step, namely, system modeling. More specifically, we will separate the dynamics of the vehicle only, and those of the tire, as it is the only effector of ground vehicles and deserves to be given a special attention.

1 Global Vehicle Modeling

Since today’s passenger cars are more and more over-actuated, a bicycle model would probably not be any longer sufficient to describe vehicle dynamics. Here instead, a global four-wheeled vehicle model will be developed. We particularly put the spotlight on the internal couplings that may arise in case of simultaneous maneuvers, as braking when turning. These couplings may lead to interactions between subsystems, which may result in internal conflicts. For a proper construction of vehicle motion equations, we adopt the ISO 8855-2011 shown in Fig. 1.

In order to take into account the dynamic couplings, vertical load transfer, influence of suspensions and so on, the vehicle will be broken down into two supposedly undeformable masses: the **sprung mass**,¹ and the **unsprung mass**,² as Fig. 2 shows.

In addition, to take into account the differences between the influence of the front axle and the rear axle (especially for a 4 Wheel Steering (4WS) vehicle), the unsprung mass is also decomposed into two supposedly undeformable masses. We then have $\Sigma = S_s + S_{uf} + S_{ur}$, with:

- Σ : the overall vehicle of mass M and Center of Gravity (CoG) G ,

¹ Includes the vehicle body, engine, passengers and so on.

² Includes the wheels, suspensions, brakes and so on.

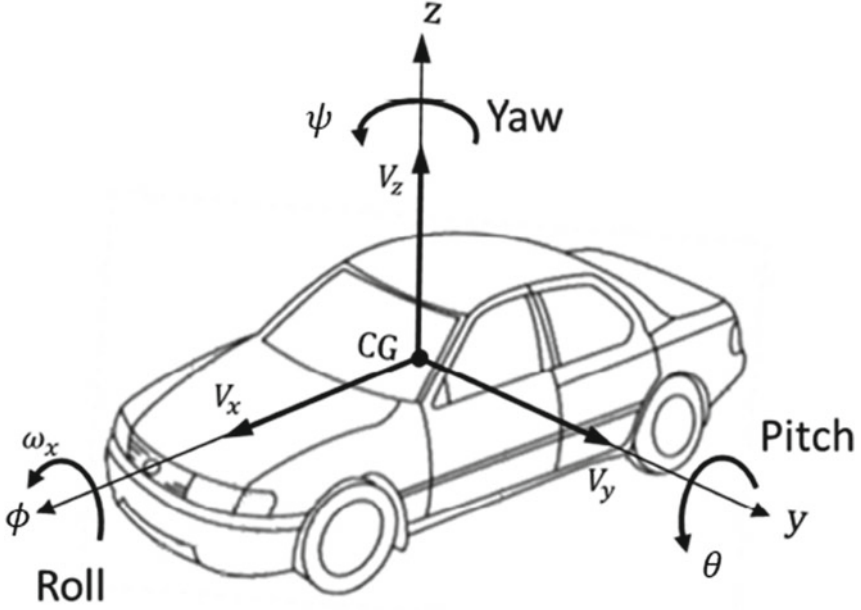


Fig. 1 Vehicle Axis System (ISO 8855-2011)

- S_s : the sprung mass of mass M_s and CoG G_s ,
- S_{uf} : the front unsprung mass of mass M_{uf} and CoG G_{uf} ,
- S_{ur} : the rear unsprung mass of mass M_{ur} and CoG G_{ur} .

1.1 Vehicle Dynamics

We define the dynamic torsor of the vehicle at the point G as follows:

$$\{\mathcal{D}(\Sigma/\mathcal{R}_g)\}_G = \left\{ \vec{\delta}(G, \Sigma/\mathcal{R}_g) = \int_{\forall P \in \Sigma} \overset{M\vec{\Gamma}(G/\mathcal{R}_g)}{\vec{G}P} \wedge \vec{\Gamma}(P/\mathcal{R}_g) dm \right\}_G \quad (1)$$

with:

- $\mathcal{R}_g$: the inertial frame of reference,
- $\vec{\Gamma}$: the acceleration vector,
- $\vec{\delta}$: the dynamic moment.

We define also the exterior contact efforts as follows:

$$\{\mathcal{A}(\overline{\Sigma} \rightarrow \Sigma)\}_G = \left\{ \begin{array}{c} \vec{F}(\overline{\Sigma} \rightarrow \Sigma) \\ \vec{M}(G, \overline{\Sigma} \rightarrow \Sigma) \end{array} \right\}_G \quad (2)$$

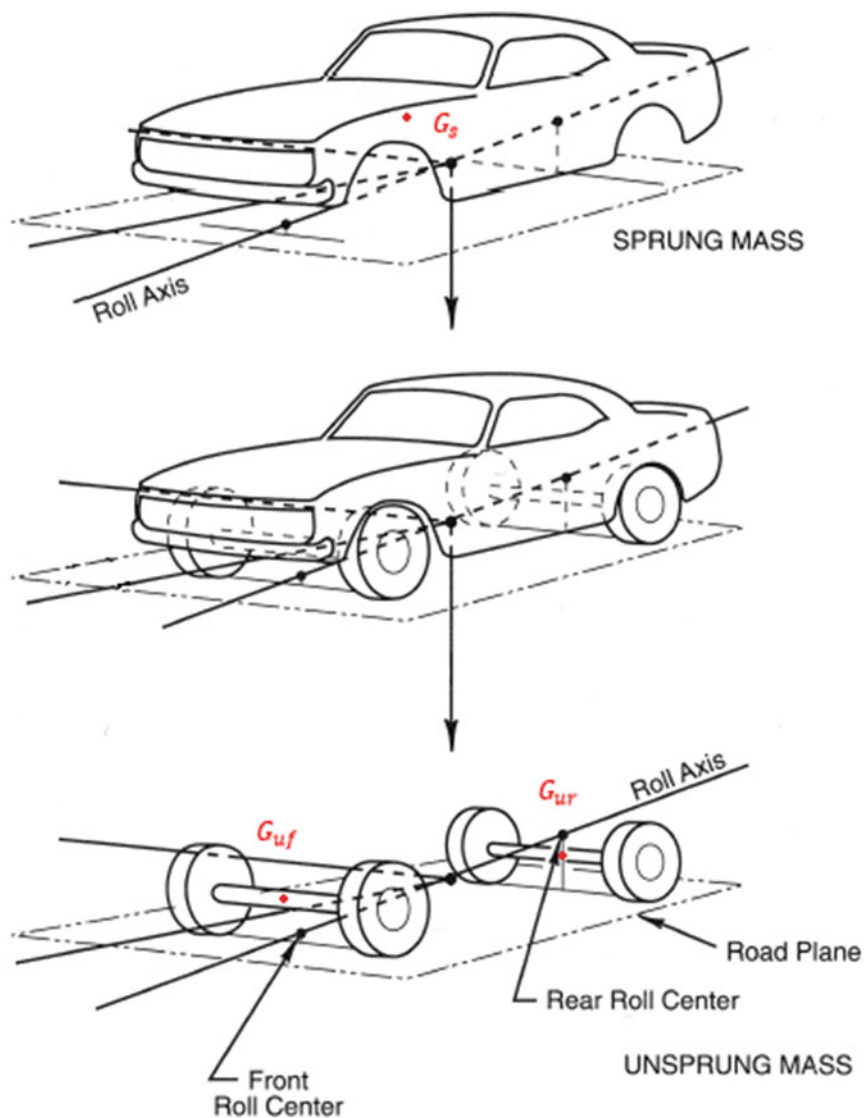


Fig. 2 The sprung and unsprung masses decomposition

with:

- $\overline{\Sigma}$: the complement of the system Σ ,
- $\vec{F}$: the exterior efforts vector,
- $\vec{M}$: the exterior efforts moment vector.

The generalization of the fundamental law of dynamics is then (Pommier & Berthaud, 2010):

$$\{\mathcal{D}(\Sigma/\mathcal{R}_g)\}_G = \{\mathcal{A}(\overline{\Sigma} \rightarrow \Sigma)\}_G \quad (3)$$

This gives two fundamentals laws:

- The dynamic resultant theorem (linear motion):

$$M\vec{\Gamma}(G/\mathcal{R}_g) = \vec{F}(\overline{\Sigma} \rightarrow \Sigma) \quad (4)$$

- The dynamic moment theorem (angular motion):

$$\vec{\delta}(G, \Sigma/\mathcal{R}_g) = \vec{M}(G, \overline{\Sigma} \rightarrow \Sigma) \quad (5)$$

Moreover, the decomposition approach adopted allows us to partition the calculations using the torsor's properties:

$$\{\mathcal{D}(\Sigma/\mathcal{R}_g)\}_G = \{\mathcal{D}(S_s/\mathcal{R}_g)\}_G + \{\mathcal{D}(S_{uf}/\mathcal{R}_g)\}_G + \{\mathcal{D}(S_{ur}/\mathcal{R}_g)\}_G \quad (6)$$

1.2 Dynamic Torsor Calculation

1.2.1 Linear Equations of Motion

Because the CoG of the sprung mass can move with respect to the unsprung mass, it is simpler to first consider a fixed point to establish the equations of motion and then deduce the motion of the CoG (Noxon, 2012). Here, we consider the roll center that we note "O". The velocity at this point is noted:

$$\vec{V}(O/\mathcal{R}_g) = V_{O_x}\vec{i} + V_{O_y}\vec{j} \quad (7)$$

where $\vec{i}$ and $\vec{j}$ are the unit vectors of the chassis frame in the longitudinal and lateral direction respectively.

Because the chassis frame moves with respect to the inertial frame, and because the unsprung mass do not exhibit any roll nor pitch motions, we have:

$$\begin{aligned}\vec{\Gamma}(O/\mathcal{R}_g) &= \begin{vmatrix} \dot{V}_{O_x} \\ \dot{V}_{O_y} \\ 0 \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ \dot{\psi} \end{vmatrix} \wedge \begin{vmatrix} V_{O_x} \\ V_{O_y} \\ 0 \end{vmatrix} \\ &= (\dot{V}_{O_x} - \dot{\psi}V_{O_y})\vec{i} + (\dot{V}_{O_y} + \dot{\psi}V_{O_x})\vec{j}\end{aligned}\quad (8)$$

The same procedure can be applied to the points G_s , G_{uf} and G_{ur} . Regarding the front unsprung mass, we get:

$$\begin{aligned}\vec{V}(G_{uf}/\mathcal{R}_g) &= \begin{vmatrix} V_{O_x} \\ V_{O_y} \\ 0 \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ \dot{\psi} \end{vmatrix} \wedge \begin{vmatrix} l_f \\ 0 \\ h_f \end{vmatrix} \\ &= V_{O_x}\vec{i} + (\dot{V}_{O_y} + \dot{\psi}l_f)\vec{j}\end{aligned}\quad (9)$$

where h_f is the vertical distance between O and G_{uf} . Regarding the acceleration:

$$\begin{aligned}\vec{\Gamma}(G_{uf}/\mathcal{R}_g) &= \left. \frac{d\vec{V}(G_{uf}/\mathcal{R}_g)}{dt} \right|_{\mathcal{R}_g} \\ &= \vec{\Gamma}(O/\mathcal{R}_g) + \begin{vmatrix} 0 \\ 0 \\ \ddot{\psi} \end{vmatrix} \wedge \begin{vmatrix} l_f \\ 0 \\ h_f \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ \dot{\psi} \end{vmatrix} \wedge \left(\begin{vmatrix} 0 \\ 0 \\ \dot{\psi} \end{vmatrix} \wedge \begin{vmatrix} l_f \\ 0 \\ h_f \end{vmatrix} \right) \\ &= (\dot{V}_{O_x} - \dot{\psi}V_{O_y} - l_f\dot{\psi}^2)\vec{i} + (\dot{V}_{O_y} + \dot{\psi}V_{O_x} + l_f\ddot{\psi})\vec{j}\end{aligned}\quad (10)$$

In the same way, we find for the rear unsprung mass:

$$\begin{cases} \vec{V}(G_{ur}/\mathcal{R}_g) = V_{O_x}\vec{i} + (V_{O_y} - \dot{\psi}l_r)\vec{j} \\ \vec{\Gamma}(G_{ur}/\mathcal{R}_g) = (\dot{V}_{O_x} - \dot{\psi}V_{O_y} + l_r\dot{\psi}^2)\vec{i} + (\dot{V}_{O_y} + \dot{\psi}V_{O_x} - l_r\ddot{\psi})\vec{j} \end{cases}\quad (11)$$

Regarding the sprung mass, the equations are slightly more complicated as the vehicle's body turns with respect to the unsprung mass. Roll and pitch angles, noted respectively ϕ and θ , are introduced. The relationship between the vehicle's body frame and the chassis frame is:

$$\begin{pmatrix} \vec{i}_s \\ \vec{j}_s \\ \vec{k}_s \end{pmatrix} = \begin{pmatrix} \cos \theta & 0 & -\sin \theta \\ \sin \phi \sin \theta & \cos \phi & \sin \phi \cos \theta \\ \cos \phi \sin \theta & -\sin \phi & \cos \phi \cos \theta \end{pmatrix} \begin{pmatrix} \vec{i} \\ \vec{j} \\ \vec{k} \end{pmatrix}\quad (12)$$

We then obtain:

$$\vec{V}(G_s/\mathcal{R}_g) = \begin{vmatrix} V_{O_x} - \dot{\theta}(l_s \sin \theta + h_s \cos \phi \cos \theta) + \dot{\phi}h_s \sin \phi \sin \theta - \dot{\psi}h_s \sin \phi \\ V_{O_y} + \dot{\phi}h_s \cos \phi + \dot{\psi}(l_s \cos \theta - h_s \cos \phi \sin \theta) \\ \dot{\theta}(l_s \cos \theta - h_s \cos \phi \sin \theta) - \dot{\phi}h_s \sin \phi \cos \theta \end{vmatrix} \quad (13)$$

where l_s and h_s are the horizontal and vertical distances between G_s and O respectively.

However, due to the transformation in Eq. (12), the acceleration equations obtained are very large to be exposed in this chapter. Nevertheless, we can propose at this point several simplifications. We suppose relatively small values of the roll and pitch angles and angular velocities with respect to yaw dynamics. This gives:

$$\left\{ \begin{array}{l} \sin \phi \approx \phi \quad \cos \phi \approx 1 \quad \sin \theta \approx \theta \quad \cos \theta \approx 1 \\ \phi \theta \approx 0 \quad \dot{\phi} \dot{\theta} \approx 0 \quad \dot{\phi}^2 \approx 0 \quad \dot{\theta}^2 \approx 0 \end{array} \right\} \quad (14)$$

In addition, to be able to apply the dynamic resultant theorem (4), we have to bring the calculation to a single point: G . To do so, we make use of the definition of the center of mass:

$$\vec{OG} = \frac{\sum_i m_i \vec{OG}_i}{\sum_i m_i} \quad (15)$$

Therefore, with the simplifications in (14):

$$\begin{aligned} M \vec{\Gamma}(G/\mathcal{R}_g) &= M_s \vec{\Gamma}(G_s/\mathcal{R}_g) + M_{uf} \vec{\Gamma}(G_{uf}/\mathcal{R}_g) + M_{ur} \vec{\Gamma}(G_{ur}/\mathcal{R}_g) \\ &= \begin{vmatrix} M(\dot{V}_{O_x} - \dot{\psi}V_{O_y}) - \ddot{\theta}M_s(l_s\theta + h_s) - \ddot{\psi}M_s h_s \phi + \dot{\psi}^2 M h_g \theta - 2\dot{\phi}\dot{\psi}M_s h_s \\ M(\dot{V}_{O_y} + \dot{\psi}V_{O_x}) + \ddot{\phi}M_s h_s - \ddot{\psi}M h_g \theta - \dot{\psi}^2 M_s h_s \phi - 2\dot{\theta}\dot{\psi}M_s(l_s\theta + h_s) \\ \ddot{\theta}M_s(l_s - h_s\theta) - \ddot{\phi}M_s h_s \phi \end{vmatrix} \end{aligned} \quad (16)$$

where h_g is the horizontal distance between O and G .

1.2.2 Angular Equations of Motion

The dynamic moment, defined in any point A , is deduced from the “angular moment” noted σ using the definition (Pommier & Berthaud, 2010):

$$\vec{\delta}(A, S/\mathcal{R}_g) = \left. \frac{d\vec{\sigma}(A, S/\mathcal{R}_g)}{dt} \right|_{\mathcal{R}_g} + M \vec{V}(A/\mathcal{R}_g) \wedge \vec{V}(G/\mathcal{R}_g) \quad (17)$$

By choosing $A = G$, this definition is simplified:

$$\vec{\delta}(G, \Sigma/\mathcal{R}_g) = \left. \frac{d\vec{\sigma}(G, \Sigma/\mathcal{R}_g)}{dt} \right|_{\mathcal{R}_g} \quad (18)$$

Again, we first calculate the angular momentum with respect to the reference point O and for each undeformable mass apart. The definition of the angular momentum applied to the front unsprung mass is as follows (Pommier & Berthaud, 2010):

$$\vec{\sigma}(O, S_{uf}/\mathcal{R}_g) = M_{uf} O\vec{G}_{uf} \wedge \vec{V}(O/\mathcal{R}_g) + \overline{I_{uf}}(O, S_{uf}) \cdot \vec{\Omega}_c \quad (19)$$

where I_{uf} is the inertia tensor of the mass S_{uf} . Its definition applied to any vector $\vec{u}$ at the point O is:

$$\overline{I_{uf}}(O, S_{uf}) \cdot \vec{u} = - \int_{P \in S_{uf}} \left[\vec{OP} \wedge (\vec{OP} \wedge \vec{u}) \right] dm \quad (20)$$

$\vec{\Omega}_c$ is the angular velocity vector, which, in this case, contains only the yaw rate.

Using the theorem of Huygens-Steiner (Pommier & Berthaud, 2010), we obtain:

$$\overline{I_{uf}}(O, S_{uf}) = \begin{pmatrix} I_{x_{uf}} + M_{uf}h_{uf}^2 & 0 & -I_{xz_{uf}} + M_{uf}l_f h_{uf} \\ 0 & I_{y_{uf}} + M_{uf}(l_f^2 + h_{uf}^2) & 0 \\ -I_{xz_{uf}} + M_{uf}l_f h_{uf} & 0 & I_{z_{uf}} + M_{uf}l_f^2 \end{pmatrix} \quad (21)$$

with $I_{x_{uf}}$, $I_{y_{uf}}$, and $I_{z_{uf}}$ are the inertia moment in the longitudinal, lateral, and vertical direction with respect to the point G_{uf} respectively. The zeros are due to the fact that S_{uf} is symmetric with respect to the plane (G_{uf}, x, z) . The additional terms are due to the theorem of Huygens-Steiner and the fact that the expressions have been brought to the point O . We finally get:

$$\vec{\sigma}(O, S_{uf}/\mathcal{R}_g) = \begin{vmatrix} M_{uf}h_{uf}(V_{O_y} + l_f\dot{\psi}) - I_{xz_{uf}}\dot{\psi} \\ -M_{uf}h_{uf}V_{O_x} \\ M_{uf}l_f(V_{O_y} + l_f\dot{\psi}) + I_{z_{uf}}\dot{\psi} \end{vmatrix} \quad (22)$$

Using the same procedure we can find:

$$\vec{\sigma}(O, S_{ur}/\mathcal{R}_g) = \begin{vmatrix} M_{ur}h_{ur}(V_{O_y} - l_r\dot{\psi}) - I_{xz_{ur}}\dot{\psi} \\ -M_{ur}h_{ur}V_{O_x} \\ -M_{ur}l_r(V_{O_y} - l_r\dot{\psi}) + I_{z_{ur}}\dot{\psi} \end{vmatrix} \quad (23)$$

For the sprung mass, the equations are more complicated because of the relationship (12) and because the angular velocity vector is more sophisticated:

$$\vec{\Omega}(S_s/\mathcal{R}_c) = \dot{\phi}\vec{i}_s + \dot{\theta}\vec{j}_s + \dot{\psi}\vec{k} \quad (24)$$

where $\mathcal{R}_c$ is the vehicle's body frame. With a non-zero lateral acceleration, the pitch axis is inclined. The same remark stands for the roll axis in case of a longitudinal acceleration. The yaw axis remains the same. We then have:

$$\vec{\Omega} (S_s/\mathcal{R}_g) = \begin{vmatrix} \dot{\phi} \cos \theta + \dot{\theta} \sin \phi \sin \theta \\ \dot{\theta} \cos \phi \\ -\dot{\phi} \sin \theta + \dot{\theta} \sin \phi \cos \theta + \dot{\psi} \end{vmatrix} \quad (25)$$

The expression of the angular moment in this case is too long, and its derivative (to obtain the dynamic angular moment) is even longer. The same simplifications as in (14) can be applied to moderate the results.

In addition, we should again bring the expressions to the point G :

$$\begin{cases} \vec{\sigma} (G, S_s/\mathcal{R}_g) = \vec{\sigma} (O, S_s/\mathcal{R}_g) + M_s \vec{V} (O/\mathcal{R}_g) \wedge \vec{OG} \end{cases} \quad (26)$$

$$\begin{cases} \vec{\sigma} (G, S_{uf}/\mathcal{R}_g) = \vec{\sigma} (O, S_{uf}/\mathcal{R}_g) + M_{uf} \vec{V} (O/\mathcal{R}_g) \wedge \vec{OG} \end{cases} \quad (27)$$

$$\begin{cases} \vec{\sigma} (G, S_{ur}/\mathcal{R}_g) = \vec{\sigma} (O, S_{ur}/\mathcal{R}_g) + M_{ur} \vec{V} (O/\mathcal{R}_g) \wedge \vec{OG} \end{cases} \quad (28)$$

and using again the torsor properties:

$$\vec{\sigma} (G, \Sigma/\mathcal{R}_g) = \vec{\sigma} (G, S_s/\mathcal{R}_g) + \vec{\sigma} (G, S_{uf}/\mathcal{R}_g) + \vec{\sigma} (G, S_{ur}/\mathcal{R}_g) \quad (29)$$

Noting I_{ik} the inertia moment in the direction i of the mass S_k with respect to its CoG, and I_{ijk} the inertia moment in the plan ij of the mass S_k with respect to its CoG, we obtain the dynamic moment at the point G :

$$\left\{ \begin{aligned} \delta_{G_x} = & \ddot{\phi} (I_{x_s} + I_{xz_s} \theta + M_s h_s^2) + \dot{\phi} \dot{\psi} M_s h_s \phi (2h_s \theta - l_s) \\ & + \ddot{\theta} \phi [I_{x_s} \theta - I_{xz_s} + M_s l_s (l_s \theta + h_s)] \\ & + \dot{\theta} \dot{\psi} M_s \left[(l_s^2 - h_s^2) - 4l_s h_s \theta \right] \\ & - \ddot{\psi} \left\{ I_{xz_s} + I_{xz_{uf}} + I_{xz_{ur}} - M_{uf} h_{uf} l_f + M_{ur} h_{ur} l_r - M_s \left[(l_s^2 - h_s^2) \theta + l_s h_s \right] \right\} \end{aligned} \right. \quad (30)$$

$$\left\{ \begin{aligned} \delta_{G_y} = & \ddot{\theta} (I_{y_s} + M_s l_s^2) \end{aligned} \right. \quad (31)$$

$$\left\{ \begin{aligned} \delta_{G_z} = & -\ddot{\phi} [I_{z_s} \theta + I_{xz_s} - M_s h_s (l_s - h_s \theta)] + \ddot{\theta} \phi [I_{z_s} - I_{xz_s} \theta + M_s (l_s^2 - l_s h_s \theta)] \\ & + 2\dot{\phi} \dot{\psi} M_s h_s \phi (l_s \theta + h_s) - \dot{\theta} \dot{\psi} M_s \left[2\theta (l_s^2 - h_s^2) + 2l_s h_s \right] \\ & + \ddot{\psi} [I_{z_s} + I_{z_{uf}} + I_{z_{ur}} + M_{uf} l_f^2 + M_{ur} l_r^2 + M_s (l_s - h_s \theta)^2] \end{aligned} \right. \quad (32)$$

1.3 Exterior Forces Torsor Calculation

This torsor shows the influence of the exterior forces, for example tire forces, on the chosen isolated system. In order to show the influence of the suspensions, we should isolate only the sprung mass where the suspension forces are at the exterior of the studied system. Let us consider Fig. 3.

If we consider the overall system Σ , the exterior forces would be:

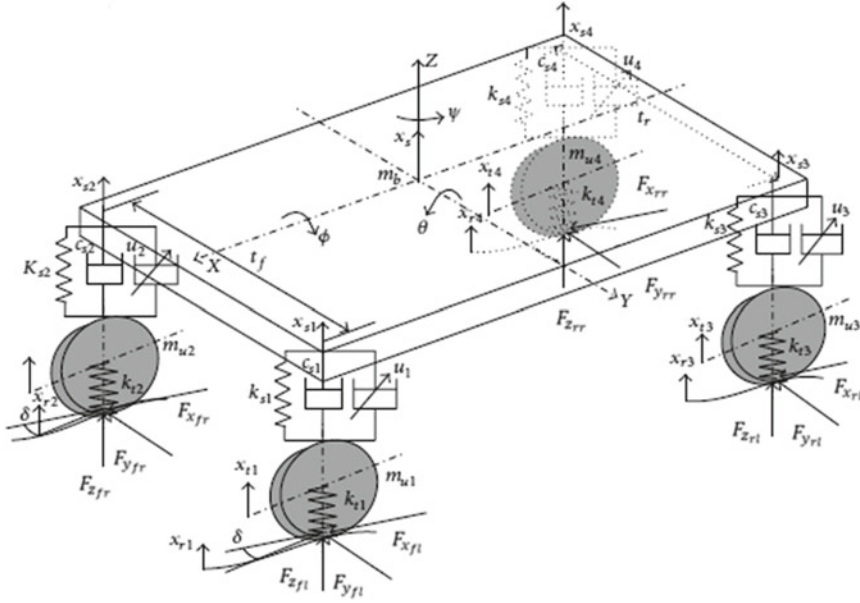


Fig. 3 14-Degrees of Freedom (DoF) vehicle dynamic model (adapted from Zhao & Qu, 2014)

- $F_{x_{i,j}}$: $i - j$ longitudinal tire force,³
- $F_{y_{i,j}}$: $i - j$ lateral tire force,
- $F_{z_{i,j}}$: $i - j$ vertical tire force,
- $\bar{P}$: the vehicle's weight.

Notice that we do not take into account the aerodynamic forces for example. These forces are not controllable and would be considered as disturbances. They should be rejected by a robust control strategy. Next, we apply the fundamental law of dynamics (3) on the overall system Σ first.

1.3.1 The Dynamic Resultant Theorem

For the linear motion, we apply Eq. (4) using the formulas obtained in Eq. (16):

³ Where “ i ” is front or rear, and “ j ” is right or left.

$$\left\{ \begin{aligned} M(\dot{V}_{O_x} - \dot{\psi}V_{O_y}) - \ddot{\theta}M_s(l_s\theta + h_s) - \ddot{\psi}M_s h_s \phi + \dot{\psi}^2 M h_g \theta - 2\dot{\phi}\dot{\psi}M_s h_s \\ = (F_{x_{f,l}} + F_{x_{f,r}}) \cos \delta_f + (F_{x_{r,l}} + F_{x_{r,r}}) \cos \delta_r \\ - (F_{y_{f,l}} + F_{y_{f,r}}) \sin \delta_f - (F_{y_{r,l}} + F_{y_{r,r}}) \sin \delta_r \end{aligned} \right. \quad (33)$$

$$\left\{ \begin{aligned} M(\dot{V}_{O_y} + \dot{\psi}V_{O_x}) + \ddot{\phi}M_s h_s - \ddot{\psi}M h_g \theta - \dot{\psi}^2 M_s h_s \phi - 2\dot{\theta}\dot{\psi}M_s(l_s\theta + h_s) \\ = (F_{y_{f,l}} + F_{y_{f,r}}) \cos \delta_f + (F_{y_{r,l}} + F_{y_{r,r}}) \cos \delta_r \\ + (F_{x_{f,l}} + F_{x_{f,r}}) \sin \delta_f + (F_{x_{r,l}} + F_{x_{r,r}}) \sin \delta_r \end{aligned} \right. \quad (34)$$

$$\left\{ \begin{aligned} \ddot{\theta}M_s(l_s - h_s\theta) - \ddot{\phi}M_s h_s \phi = Mg - F_{z_{f,l}} - F_{z_{f,r}} - F_{z_{r,l}} - F_{z_{r,r}} \end{aligned} \right. \quad (35)$$

with g is the standard gravity acceleration.

1.3.2 The Dynamic Moment Theorem

Let us consider:

- t_f, t_r : the front and rear track of the vehicle respectively,
- h_O : the height of the center of the roll/pitch⁴ axis,
- $\sum M_z$: the influence of the self-aligning moments of the tires.

We apply Eq. (5) using the formulas obtained in Eqs. (30)–(32) to calculate the angular motion:

$$\left\{ \begin{aligned} \delta_{G_x} &= Mgh_g\phi + \frac{t_f}{2}(F_{z_{f,l}} - F_{z_{f,r}}) + \frac{t_r}{2}(F_{z_{r,l}} - F_{z_{r,r}}) \\ &\quad - (h_O + h_g)[(F_{y_{f,l}} + F_{y_{f,r}}) \cos \delta_f + (F_{y_{r,l}} + F_{y_{r,r}}) \cos \delta_r] \end{aligned} \right. \quad (36)$$

$$\left\{ \begin{aligned} \delta_{G_y} &= Mgh_g\theta + l_f(F_{z_{f,l}} + F_{z_{f,r}}) - l_r(F_{z_{r,l}} + F_{z_{r,r}}) \\ &\quad + (h_O + h_g)[(F_{x_{f,l}} + F_{x_{f,r}}) \cos \delta_f + (F_{x_{r,l}} + F_{x_{r,r}}) \cos \delta_r] \end{aligned} \right. \quad (37)$$

$$\left\{ \begin{aligned} \delta_{G_z} &= l_f[(F_{y_{f,l}} + F_{y_{f,r}}) \cos \delta_f + (F_{x_{f,l}} + F_{x_{f,r}}) \sin \delta_f] \\ &\quad - l_r[(F_{y_{r,l}} + F_{y_{r,r}}) \cos \delta_r + (F_{x_{r,l}} + F_{x_{r,r}}) \sin \delta_r] \\ &\quad + \frac{t_f}{2}[(F_{x_{f,l}} - F_{x_{f,r}}) \cos \delta_f - (F_{y_{f,l}} - F_{y_{f,r}}) \sin \delta_f] \\ &\quad + \frac{t_r}{2}[(F_{x_{r,l}} - F_{x_{r,r}}) \cos \delta_r - (F_{y_{r,l}} - F_{y_{r,r}}) \sin \delta_r] + \sum M_z \end{aligned} \right. \quad (38)$$

⁴ Supposed the same in this chapter for further simplification.

1.4 The Sprung Mass Dynamics

Regarding roll dynamics, pitch dynamics and the pure vertical dynamics, the vehicle's body should be isolated. This enables the introduction of the suspension forces. In case of active suspensions, as it is the case in Fig. 3, we have (Zhao & Qu, 2014):

$$\left\{ \begin{array}{l} F_{s_{f,l}} = k_{s_{f,l}} (z_{p_{f,l}} - z_{s_{f,l}}) + c_{s_{f,l}} (\dot{z}_{p_{f,l}} - \dot{z}_{s_{f,l}}) \\ \quad - \frac{k_{\phi_f}}{2t_f} \left(\phi - \frac{z_{p_{f,l}} - z_{s_{f,l}}}{2t_f} \right) + u_{f,l} \end{array} \right. \quad (39)$$

$$\left\{ \begin{array}{l} F_{s_{f,r}} = k_{s_{f,r}} (z_{p_{f,r}} - z_{s_{f,r}}) + c_{s_{f,r}} (\dot{z}_{p_{f,r}} - \dot{z}_{s_{f,r}}) \\ \quad + \frac{k_{\phi_f}}{2t_f} \left(\phi - \frac{z_{p_{f,r}} - z_{s_{f,r}}}{2t_f} \right) + u_{f,r} \end{array} \right. \quad (40)$$

$$\left\{ \begin{array}{l} F_{s_{r,l}} = k_{s_{r,l}} (z_{p_{r,l}} - z_{s_{r,l}}) + c_{s_{r,l}} (\dot{z}_{p_{r,l}} - \dot{z}_{s_{r,l}}) \\ \quad + \frac{k_{\phi_r}}{2t_r} \left(\phi - \frac{z_{p_{r,l}} - z_{s_{r,l}}}{2t_r} \right) + u_{r,l} \end{array} \right. \quad (41)$$

$$\left\{ \begin{array}{l} F_{s_{r,r}} = k_{s_{r,r}} (z_{p_{r,r}} - z_{s_{r,r}}) + c_{s_{r,r}} (\dot{z}_{p_{r,r}} - \dot{z}_{s_{r,r}}) \\ \quad - \frac{k_{\phi_r}}{2t_r} \left(\phi - \frac{z_{p_{r,r}} - z_{s_{r,r}}}{2t_r} \right) + u_{r,r} \end{array} \right. \quad (42)$$

where:

- z_{p_i} : vertical travel of tires,
- z_{s_i} : vertical travel of suspensions,
- k_{s_i} : suspension's stiffness,
- c_{s_i} : suspension's damping,
- k_{ϕ_f}, k_{ϕ_r} : the front and rear anti-roll bars stiffness respectively,
- u_{s_i} : control forces of the active suspensions.

Using again the same theorems (4) and (5) and the same simplifications in (14), we can get:

$$\left\{ \begin{array}{l} \ddot{\phi} (I_{x_s} + I_{xz_s} \theta) + \ddot{\theta} \phi (I_{x_s} \theta - I_{xz_s}) - \ddot{\psi} I_{xz_s} = M_s g h_s \phi \\ \quad + \frac{t_f}{2} (F_{s_{f,l}} - F_{s_{f,r}}) + \frac{t_r}{2} (F_{s_{r,l}} - F_{s_{r,r}}) \\ \quad - (h_o + h_s) [(F_{y_{f,l}} + F_{y_{f,r}}) \cos \delta_f + (F_{y_{r,l}} + F_{y_{r,r}}) \cos \delta_r] \\ \quad - (h_o + h_s) [(F_{x_{f,l}} + F_{x_{f,r}}) \sin \delta_f + (F_{x_{r,l}} + F_{x_{r,r}}) \sin \delta_r] \end{array} \right. \quad (43)$$

$$\left\{ \begin{array}{l} \ddot{\theta} I_{y_s} = M_s g h_s \theta + l_f (F_{s_{f,l}} + F_{s_{f,r}}) - l_r (F_{s_{r,l}} + F_{s_{r,r}}) \\ \quad + (h_o + h_s) [(F_{x_{f,l}} + F_{x_{f,r}}) \cos \delta_f + (F_{x_{r,l}} + F_{x_{r,r}}) \cos \delta_r] \\ \quad - (h_o + h_s) [(F_{y_{f,l}} + F_{y_{f,r}}) \sin \delta_f + (F_{y_{r,l}} + F_{y_{r,r}}) \sin \delta_r] \end{array} \right. \quad (44)$$

$$M_s [\ddot{\theta} (l_s - h_s \theta) - \ddot{\phi} h_s \phi] = M_s g - F_{s_{f,l}} - F_{s_{f,r}} - F_{s_{r,l}} - F_{s_{r,r}} \quad (45)$$

1.5 Model Simplification and Validation

The vehicle equations of motion developed in this chapter are tedious to implement. Nevertheless, they still can be used for control strategies validation. However, from a control synthesis viewpoint, these equations are inadequate. The models should be simplified enough to deduce control strategies, but not too simple to avoid losing important information. This is why we have chosen starting from a complex model and then simplify it, rather than doing the inverse and select models such as the bicycle model. This latter for example would be insufficient for simultaneous operation control as braking in a corner for example.

First, we validate the initial vehicle model. To do so, we use as a reference a high-fidelity vehicle model provided by Simcenter Amesim[®]. Figure 4 illustrates the 15 DoF chassis selected. Complex axle kinematics are used to model the specific joint between the sprung and unsprung masses.

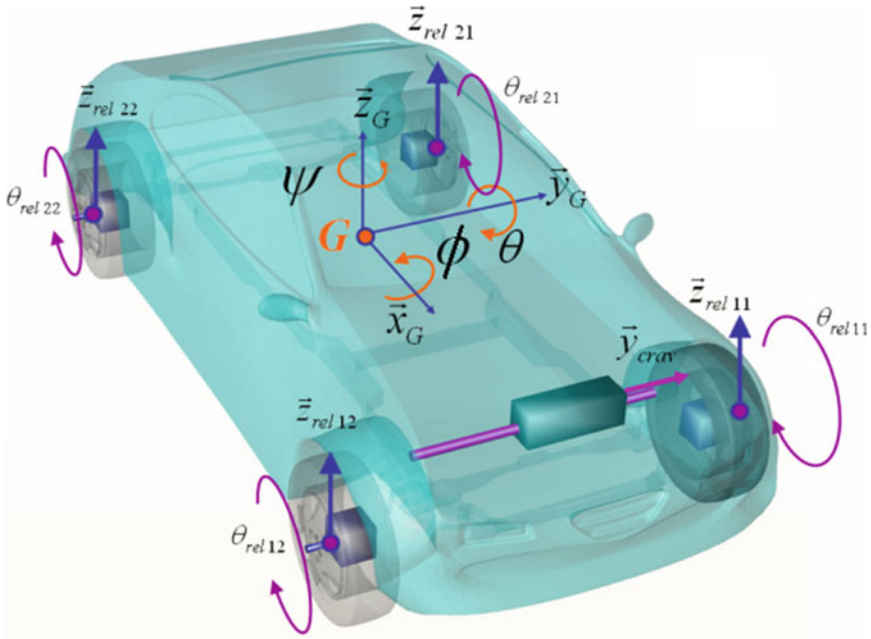


Fig. 4 The 15 DoF chassis provided by Amesim[®]

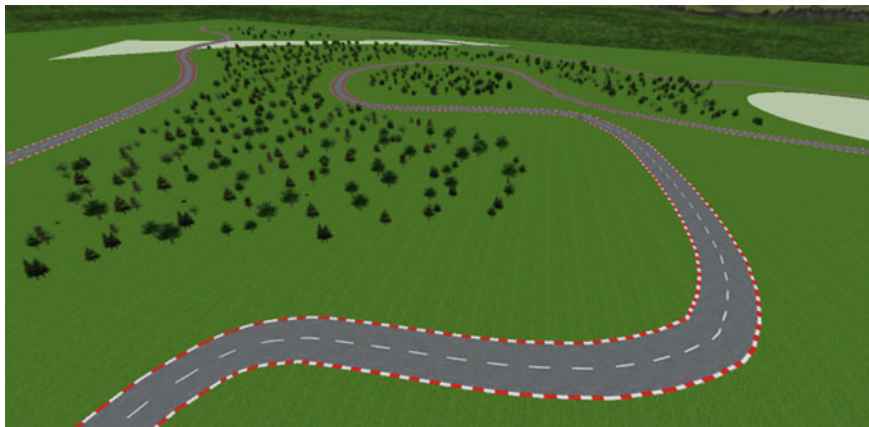


Fig. 5 3D aspect of the Magny-Cours race track with hills area

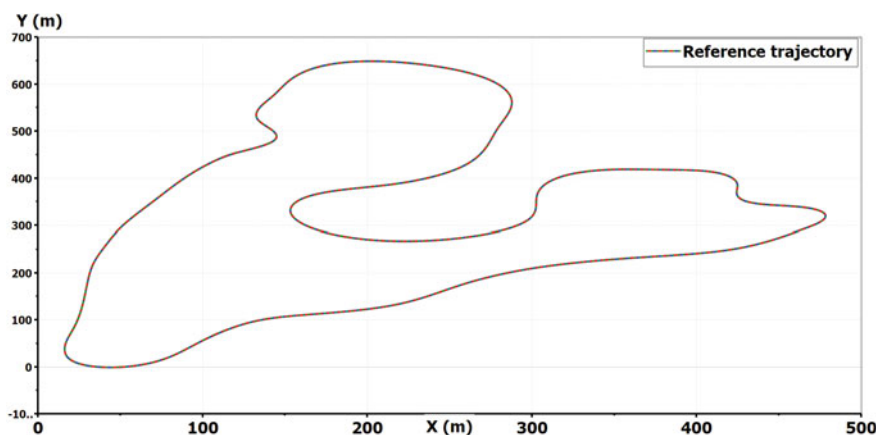


Fig. 6 Magny-Cours trajectory

The procedure is simple. We simulate both the high fidelity vehicle model of Amesim[®] and the model developed previously in several use-cases. We then compare the results of the two models. We identify the order of magnitude of each term in every equation before summing all the components of the equations presented. Then we just simplify the least influencing terms. For a simulation that covers the excitation of all vehicle dynamics, we selected a 3D road reproduced by AmeSim's engineers from a real life race track: the approved International Circuit of Magny-Cours depicted in Fig. 5.

A top view of the Magny-Cours trajectory is illustrated in Fig. 6.

After the aforementioned simplification procedure, we can obtain the following state-space representation:

$$\begin{aligned}
& \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \frac{K_p}{g} + M_s h_s & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{I_{y_s}}{M_s h_s} \phi & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{I_z}{I_z} \phi & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{V}_x \\ \dot{V}_y \\ V_z \\ \dot{V}_z \\ \dot{\phi} \\ \ddot{\phi} \\ \dot{\theta} \\ \ddot{\theta} \\ \dot{\psi} \end{bmatrix} \\
& = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{K_r}{I_{x_s}} - \frac{C_{s_r}}{I_{x_s}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{M - M_s}{M} \frac{K_p}{I_{y_s}} - \frac{C_{s_p}}{I_{y_s}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_x \\ V_y \\ z \\ V_z \\ \phi \\ \dot{\phi} \\ \theta \\ \dot{\theta} \\ \psi \end{bmatrix} \\
& + \begin{bmatrix} \frac{1}{M} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{M} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{M} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{M} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{I_{x_s}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{I_{y_s}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{I_z} & 0 \end{bmatrix} \begin{bmatrix} F_{x_{tot}} \\ F_{y_{tot}} \\ F_{z_{tot}} \\ M_{x_{tot}} \\ M_{y_{tot}} \\ M_{z_{tot}} \end{bmatrix}
\end{aligned} \tag{46}$$

where:

- z : vertical travel of the sprung mass,
- V_z : vertical velocity of the sprung mass,
- K_r : equivalent overall antiroll bar stiffness,
- C_{s_r} : equivalent overall roll suspension damping,

- K_p : equivalent overall pitch suspension stiffness,
- C_{s_p} : equivalent overall pitch suspension damping,
- I_z : yaw inertia moment of the overall vehicle with respect to its CoG,
- $F_{i_{tot}}$: combination of tire forces projected at the axis “ i ”,
- $M_{i_{tot}}$: combination of moments generated by tire forces with respect to the axis “ i ”.

with:

$$\left\{ \begin{array}{l} F_{x_{tot}} = (F_{x_{f,l}} + F_{x_{f,r}}) \cos \delta_f + (F_{x_{r,l}} + F_{x_{r,r}}) \cos \delta_r \\ \quad - (F_{y_{f,l}} + F_{y_{f,r}}) \sin \delta_f - (F_{y_{r,l}} + F_{y_{r,r}}) \sin \delta_r \end{array} \right. \quad (47)$$

$$\left\{ \begin{array}{l} F_{y_{tot}} = (F_{y_{f,l}} + F_{y_{f,r}}) \cos \delta_f + (F_{y_{r,l}} + F_{y_{r,r}}) \cos \delta_r \\ \quad + (F_{x_{f,l}} + F_{x_{f,r}}) \sin \delta_f + (F_{x_{r,l}} + F_{x_{r,r}}) \sin \delta_r \end{array} \right. \quad (48)$$

$$F_{z_{tot}} = F_{z_{f,l}} + F_{z_{f,r}} + F_{z_{r,l}} + F_{z_{r,r}} \quad (49)$$

$$\left\{ \begin{array}{l} M_{x_{tot}} = \frac{t_f}{2} (F_{z_{f,l}} - F_{z_{f,r}}) + \frac{t_r}{2} (F_{z_{r,l}} - F_{z_{r,r}}) \end{array} \right. \quad (50)$$

$$M_{y_{tot}} = l_f (F_{z_{f,l}} + F_{z_{f,r}}) - l_r (F_{z_{r,l}} + F_{z_{r,r}}) \quad (51)$$

$$\left\{ \begin{array}{l} M_{z_{tot}} = l_f [(F_{y_{f,l}} + F_{y_{f,r}}) \cos \delta_f + (F_{x_{f,l}} + F_{x_{f,r}}) \sin \delta_f] \\ \quad - l_r [(F_{y_{r,l}} + F_{y_{r,r}}) \cos \delta_r + (F_{x_{r,l}} + F_{x_{r,r}}) \sin \delta_r] \\ \quad + \frac{t_f}{2} [(F_{x_{f,l}} - F_{x_{f,r}}) \cos \delta_f - (F_{y_{f,l}} - F_{y_{f,r}}) \sin \delta_f] \\ \quad + \frac{t_r}{2} [(F_{x_{r,l}} - F_{x_{r,r}}) \cos \delta_r - (F_{y_{r,l}} - F_{y_{r,r}}) \sin \delta_r] + \sum M_z \end{array} \right. \quad (52)$$

and:

$$\left\{ \begin{array}{l} K_r = K_{\phi_f} + K_{\phi_r} \end{array} \right. \quad (53)$$

$$\left\{ \begin{array}{l} C_{s_r} = 2c_{s_f} \left(\frac{t_f}{2} \right)^2 + 2c_{s_r} \left(\frac{t_r}{2} \right)^2 \end{array} \right. \quad (54)$$

$$\left\{ \begin{array}{l} K_p = 2k_{s_f} l_f^2 + 2k_{s_r} l_r^2 \end{array} \right. \quad (55)$$

$$\left\{ \begin{array}{l} C_{s_p} = 2c_{s_f} l_f^2 + 2c_{s_r} l_r^2 \end{array} \right. \quad (56)$$

where:

- k_{s_f}, k_{s_r} : the front and rear suspension stiffness respectively.⁵
- c_{s_f}, c_{s_r} : the front and rear suspension damping respectively.

By inverting the first matrix, we finally get the state-space representation in the standard form:

⁵ The front suspensions are alike by design. The same remark holds for the rear suspensions.

$$\left\{ \begin{aligned} & \begin{bmatrix} \dot{V}_x \\ \dot{V}_y \\ V_z \\ \dot{V}_z \\ \dot{\phi} \\ \ddot{\phi} \\ \dot{\theta} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{M_s}{M}g & 0 & 0 & V_y \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -V_x & \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & -\frac{K_r}{I_{x_s}} & -\frac{C_{s_r}}{I_{x_s}} & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{MK_p + M_s^2 h_s g}{MI_{y_s}} & -\frac{C_{s_p}}{I_{y_s}} & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{M_s^2 h_s g}{MI_z} \phi & 0 & 0 & \end{bmatrix} \begin{bmatrix} V_x \\ V_y \\ z \\ V_z \\ \phi \\ \dot{\phi} \\ \theta \\ \dot{\theta} \\ \psi \end{bmatrix} \\ & + \begin{bmatrix} \frac{1}{M} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{M} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{I_{x_s}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{M_s h_s}{MI_{y_s}} & 0 & 0 & 0 & \frac{1}{I_{y_s}} & 0 \\ -\frac{M_s h_s}{MI_z} \phi & 0 & 0 & 0 & 0 & \frac{1}{I_z} \end{bmatrix} \begin{bmatrix} F_{x_{tot}} \\ F_{y_{tot}} \\ F_{z_{tot}} \\ M_{x_{tot}} \\ M_{y_{tot}} \\ M_{z_{tot}} \end{bmatrix} \\ & \begin{bmatrix} V_x \\ V_y \\ z \\ \phi \\ \theta \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_x \\ V_y \\ z \\ V_z \\ \phi \\ \dot{\phi} \\ \theta \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \end{aligned} \right. \quad (57)$$

$$\begin{bmatrix} V_x \\ V_y \\ z \\ \phi \\ \theta \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_x \\ V_y \\ z \\ V_z \\ \phi \\ \dot{\phi} \\ \theta \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \quad (58)$$

Regarding the validation procedure, we make use of a driver model provided by Simcenter Amesim[®] and designed using a Model Predictive Control (MPC) algorithm to track the Magny-Cours path with an adapted velocity profile. Simulations for this severe maneuver are shown in Figs. 7, 8, 9, 10, 11 and 12.

The model shows good precision for all states in severe coupled maneuvers. Note that the effect of slopes is taken into account in this model. This model can then be chosen as a starting model for problems related to Global Chassis Control (GCC) synthesis. It is important to start with a complex full vehicle model and then reduce it while justifying each simplification. Starting with a simplified model, as the bicycle

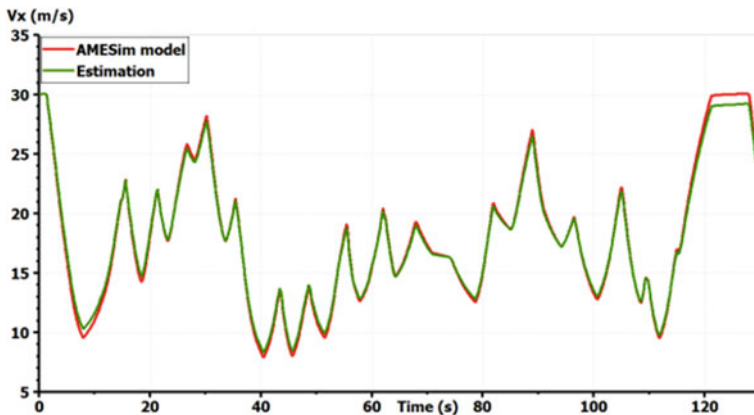


Fig. 7 Vehicle model validation: longitudinal speed

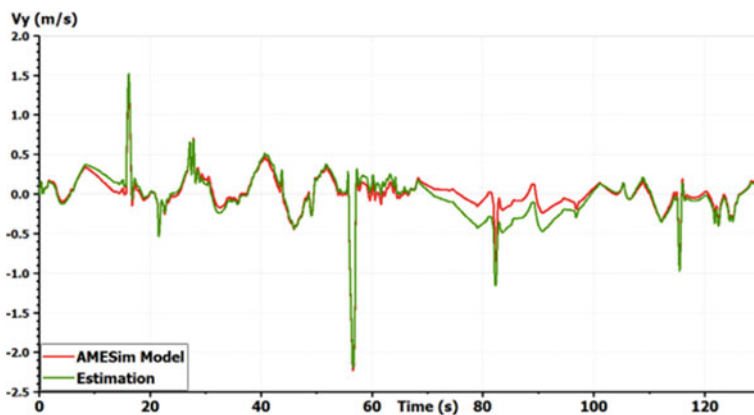


Fig. 8 Vehicle model validation: lateral speed

model, could lead to the ignorance of important dynamics and couplings leading to unexpected emergent behaviors. If only the horizontal motion is concerned, vertical dynamics could be simplified (for the high-level control only) in the control synthesis. However, the vertical forces applied to the tires should always be taken into account as they modify the potential of each tire to drive, brake or steer the vehicle (Pacejka, 2005).

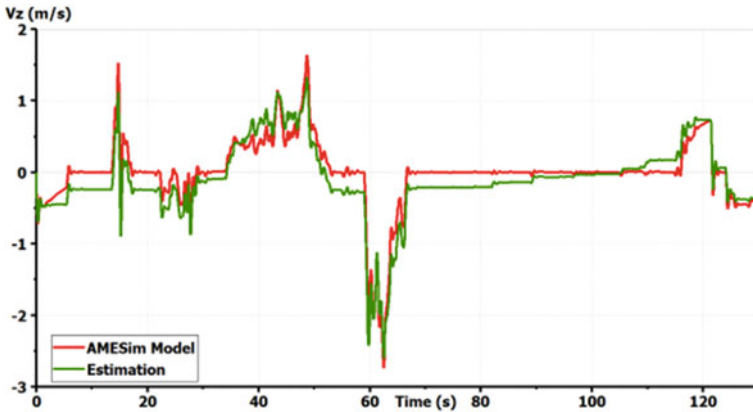


Fig. 9 Vehicle model validation: vertical velocity of the sprung mass

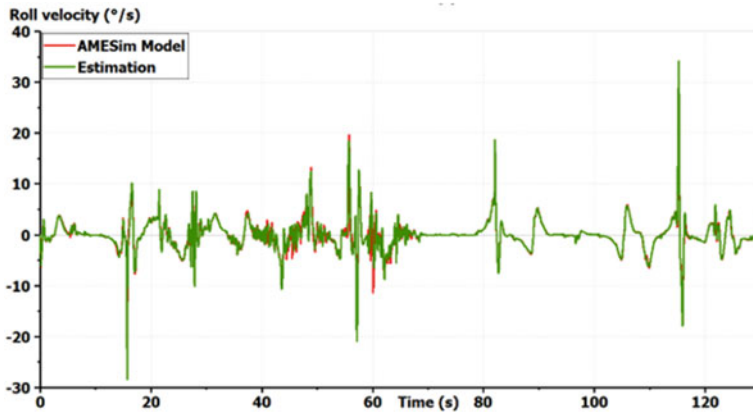


Fig. 10 Validation of the vehicle model: roll velocity

2 Tire Modeling

If the vehicle is equipped at the same time by systems based on lateral tire forces such as the Electric Power Assisted Steering (EPAS) or the 4-Wheel Steering (4WS) system, and others based on longitudinal tire forces such as Anti-lock Braking System (ABS) or the 4-Wheel Driving (4WD) system, the combined slip phenomenon should be taken into account (Pacejka, 2005). From a control synthesis point of view, this requires a tire model giving enough insights to handle coupled operations, for example braking while turning. In this context, the literature is abundant by either empirical models that rely on experimental measures to make simulation more accurate, or complex physical models developed to improve the tire construction by the finite element method. Empirical and semi-empirical models are well-known for their

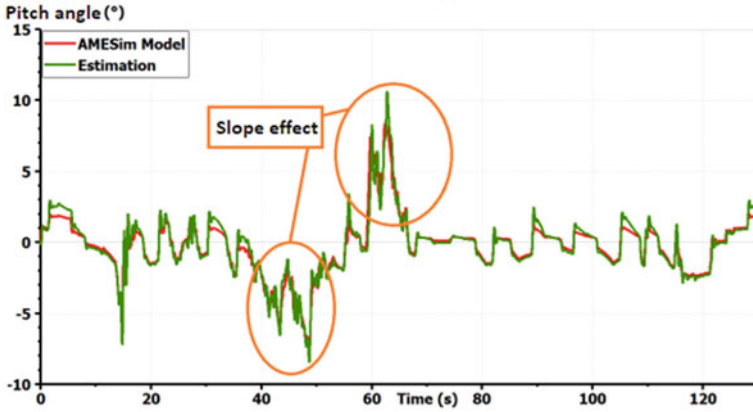


Fig. 11 Vehicle model validation: pitch angle taking into account the slopes

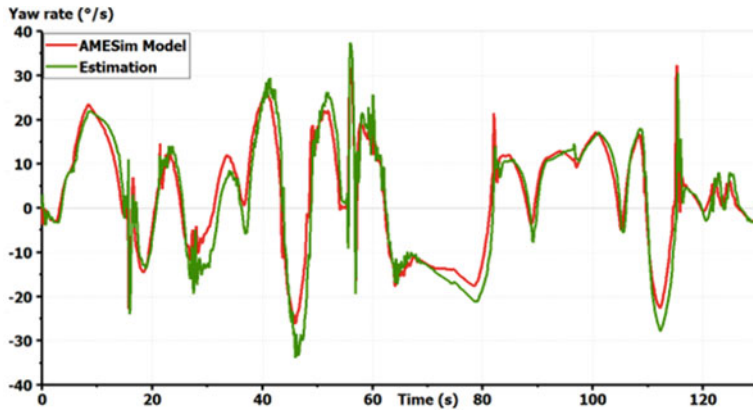


Fig. 12 Vehicle model validation: yaw rate

high-fidelity and accuracy with respect to the reality (Pacejka, 2005), since they are derived from real experiments. However, these models usually depend on identified parameters without much physical significance, which makes them hard to measure or estimate in real-time. This is not suitable for online control problems. Analytic models give a good understanding of tire mechanisms to control the vehicle and foresee its loss of stability. Nonetheless, these models either are not accurate enough for combined slip maneuvers, or they are too complex to be implemented or to use in order to pre-compute a control strategy. We believe that a new tire model especially fitted for GCC should be designed. To do so, we review the most famous tire models that are used in the literature. We compare these models in order to identify the gap that exists with respect to GCC. We will keep our focus on the substantial characteristics that the new tire model should adopt. These characteristics can be summarized as follows:

- The tire model should respect the tire physical fundamentals to give enough insight about the tire behavior. The tire model should be able to depict any perturbation due to tire dynamics and adapt the control strategy. This should not be confused with external disturbances that should be rejected.
- The tire model should describe as precise as possible the combined slip behavior. This is one of the pillars of GCC in combined maneuvers.
- The tire model should be simple enough for controllability issues. The main objective remains developing control algorithms to improve the vehicle motion. The tire model should be easily invertible, and if possible, linear.
- The tire model should depend on a minimum set of parameters that can be measured or estimated in order to favor real-time operations. Also, these parameters should be easily estimated and updated online as fast as possible.

2.1 Tire Physical Fundamentals

In vehicle dynamics, the interface between tires and the road is the one that matters most. We are therefore interested in only the outer layer made of rubber blocks. The rubber is a *viscoelastic* material (Michelin, 2001): the stress is proportional to the deformation (elastic behaviour) and phase-shifted to it (viscous behaviour). To understand this, a closer look into the friction concept is needed. First, we define the tire coordinate system. Same as for the vehicle model, the ISO 8855-2011 depicted in Fig. 13 is adopted:

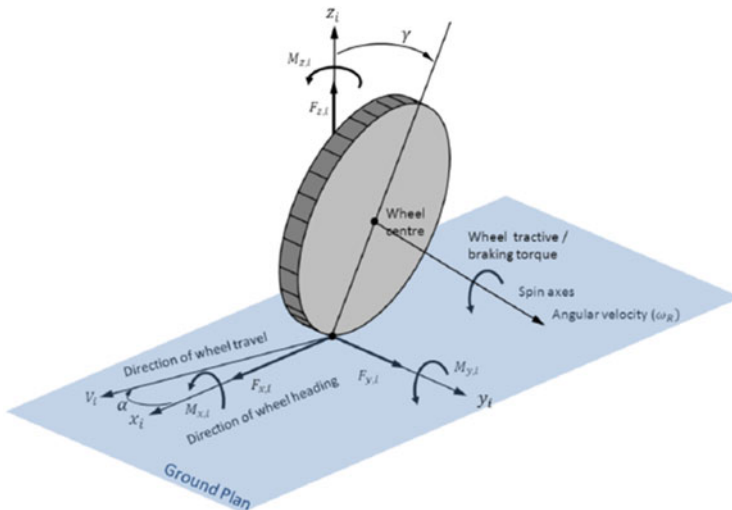


Fig. 13 ISO tire coordinate system

Fig. 14 Indentation phenomenon (Michelin, 2001)

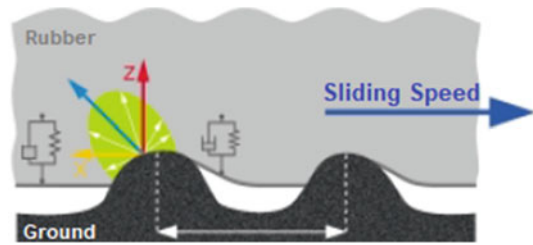
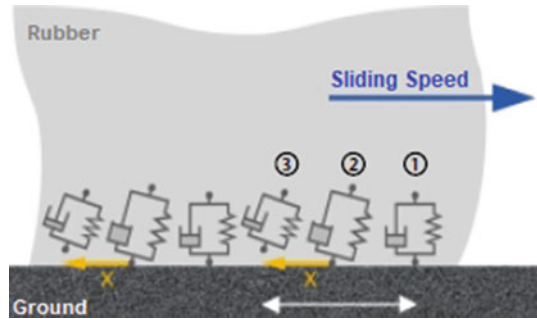


Fig. 15 Adhesion phenomenon (Michelin, 2001)



2.1.1 Friction

Two phenomena characterize the rubber/ground friction: indentation and adhesion (Michelin, 2001). In the indentation phase, the rubber deforms by sliding on the ground asperities. Because of its viscous behaviour, the rubber block does not go back to its initial height immediately on the other side of the asperity.⁶ As Fig. 14 shows, this asymmetry creates a reaction force opposed to the sliding direction.

Regarding the adhesion phase, molecular interactions occur at the rubber/ground interface, called *Van der Waals bonds*. Figure 15 illustrates the different steps: the bonds are formed, stretched, broken, and then reformed further.

2.1.2 Longitudinal Force

Two mechanisms occur when the tire adheres to the ground: shearing and sliding (Michelin, 2001). Shearing means that the rubber block deforms without sliding, which generates a resistance force proportional to the deformation. This phase is also called *pseudo-slip* because the superior rigid plate actually slide with respect to the ground (see Fig. 16). The resistance force continue growing until it reaches its maximum.⁷ This limit depends on the vertical load applied by the vehicle F_z ,

⁶ Hysteresis phenomenon.

⁷ Coulomb friction force.

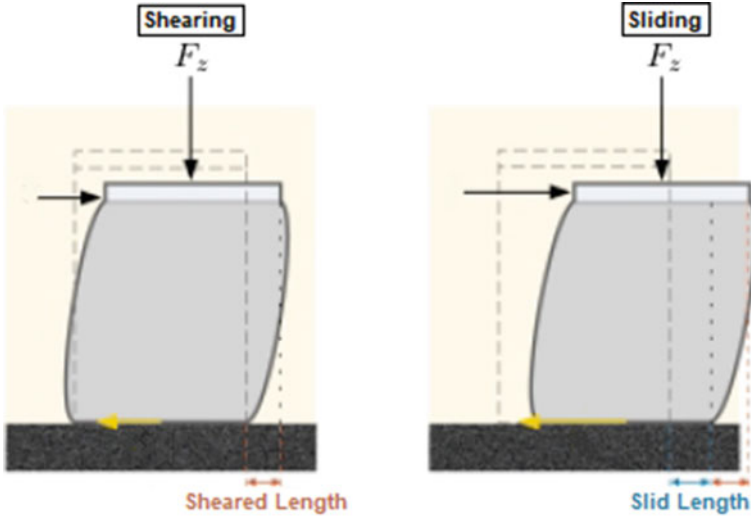


Fig. 16 Friction mechanisms (Michelin, 2001)

the rubber condition, and the ground surface condition. First, let us suppose that the wheel rolls without steering. This generates a longitudinal force:

$$F_{x_{max}} = \mu_x F_z \quad (59)$$

where μ_x is the longitudinal friction coefficient that characterizes the interface rubber/ground condition, and consequently, the adhesion potential. Beyond this limit, the interface rubber/ground can no longer resist and the rubber block starts sliding with respect to the ground (Fig. 16). The tire suffers then from a loss of potential, which may lead to the loss of stability of the vehicle. It is then very important to estimate this maximum value.

The slip κ is defined as the ratio (Maakaroun, 2011)⁸:

$$\kappa = \frac{R\omega - V}{\max(R\omega, V)} \quad (60)$$

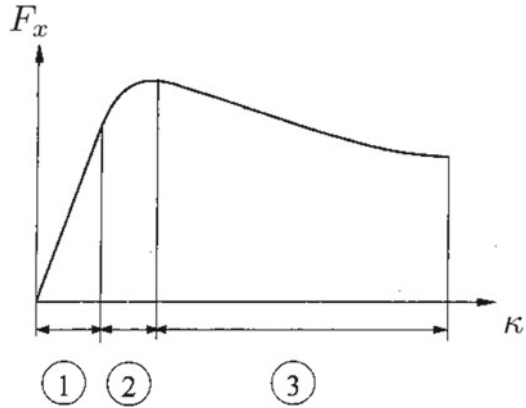
where:

- R: wheel's dynamic radius,
- ω : wheel's angular velocity,
- V: vehicle's speed.

$\kappa > 0$ indicates an acceleration situation while $\kappa < 0$ indicates a braking one. Figure 17 shows the longitudinal force as a function of the slip. Three regions are distinguished:

⁸ The $\max$ function is used to avoid singularity.

Fig. 17 Longitudinal force variation with respect to the slip (Halconruy, 1995)



- Region 1: the curve is linear and increasing. The stress is mainly due to the rubber deformation (pure shearing).
- Region 2: the curve is nonlinear, increasing and ends up reaching the maximum $\mu_x F_z$. A portion of the contact area begins to slide.
- Region 3: the curve is decreasing. There is total sliding of the tire. The tire behaviour is unstable.

The tire overall behaviour is highly nonlinear.

2.1.3 Lateral Force

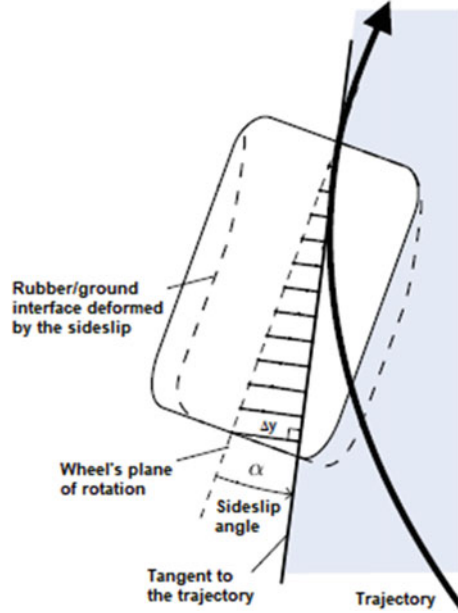
When cornering, the vehicle is subjected to a centrifugal force. To maintain the vehicle on its trajectory, the rubber/ground interface should provide centripetal force of equal value to the centrifugal force. The wheels are then directed towards not the trajectory but rather the inside of the turn (Michelin, 2001). This introduces an offset between the wheel's plan of rotation and trajectory of the wheel's center. This offset is called the side-slip. It induces friction between the tire and the road which generates the centripetal transverse force. The side-slip angle is defined as (Michelin, 2001):

$$\alpha = -\arctan\left(\frac{V_y}{V_x}\right). \quad (61)$$

In the literature, α is also called sometimes simply the slip angle. Figure 18 illustrates this phenomenon.

The lateral force follows the same mechanisms as the longitudinal one. At the entrance of the contact area, the rubber blocks remain vertical with respect to the ground. As they progress towards the back of the contact area, they deform laterally to follow the trajectory until reaching their maximum. This limit has the same

Fig. 18 The side-slip
(Michelin, 2001)



formulation as the previous one:

$$F_{y_{max}} = \mu_y F_z \quad (62)$$

Beyond this maximum, the rubber blocks begin to slide. Here, μ_y is the lateral friction coefficient. Because of the tire's geometry, there is no reason for μ_y to be of the same value than μ_x . Consequently $F_{y_{max}} \neq F_{x_{max}}$.

2.1.4 Global Friction Force

The longitudinal force can penalize the lateral force. Indeed, the two forces must share the adhesion potential provided by the rubber/ground interface (Michelin, 2001). Because $F_{y_{max}} \neq F_{x_{max}}$, the overall adhesion is delimited by a *friction ellipse* (Pacejka, 2005; Svendenius, 2003; Wong, 2001). The maximum longitudinal force and the maximum lateral tire force cannot be reached simultaneously. This concept is illustrated in Fig. 19.

A longitudinal force request would penalize a lateral force request if the limits of adhesion are reached. Figure 20 shows the impact of the lateral force on the longitudinal force.

Fig. 19 The friction ellipse concept (Wong, 2001)

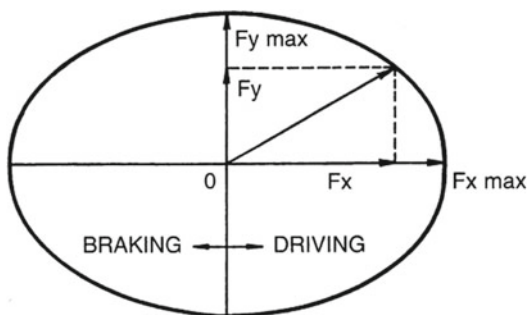
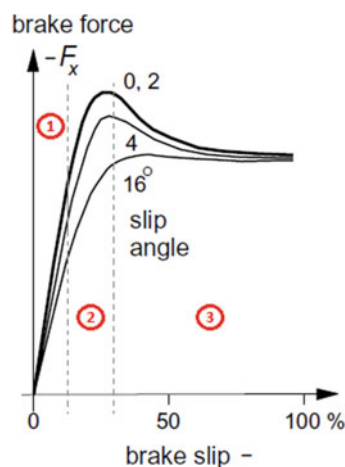


Fig. 20 Combined side force and brake force characteristics (adapted from Pacejka, 2005)



2.2 Tire Behavioural Models

Two major classes can be distinguished: empirical models and theoretical models. We adopt the SAE ISO notation (Svendenius & Wittenmar, 2003) to enable their comparison.

2.2.1 Empirical Models

These models are usually deduced from experimental data or by using similarity methods. In the first approach, regression procedures are used to develop mathematical formulations whose parameters fit best the measured data. The similarity approach uses rather simple distortion and re-scaling methods to develop simpler empirical models.

Holmes Model: Holmes has proposed a particular empirical structure for the lateral force. A vehicle speed dependent model has been developed to express the behaviour

of the force for large slip values (Svendenius, 2003):

$$F_y = a_0 + a_1 V_x + a_2 V_x^2 + a_3 \alpha + a_4 \alpha^2 + a_5 \alpha^3 + a_6 R + a_7 P \quad (63)$$

where:

- R, P : two characteristic constants of the tire,
- $a_{0...7}$: parameters used for curve fitting.

However, the $a_{0...7}$ parameters do not have any physical meaning. Equation (63) characterizes the relation between the lateral force and the side-slip angle, but with minor changes, the same structure can be used to express the longitudinal behaviour of the tire (Svendenius, 2003).

The Magic Formula of Pacejka: Due to its high precision, this model is perhaps the most used model in vehicle dynamics analysis (Soltani, 2014). The tire forces are defined as follows (Pacejka, 2005):

$$Y(X) = y(x) + S_v \quad (64)$$

with:

$$\begin{cases} x = X + S_h \\ y = D \sin [C \arctan \{Bx - E (Bx - \arctan (Bx))\}] \end{cases} \quad (65)$$

$$\quad (66)$$

where:

- Y : F_x (longitudinal force), F_y (lateral force) or M_z (aligning torque),
- X : κ (longitudinal slip) or $\tan \alpha$ (where α is the side-slip angle),

and:

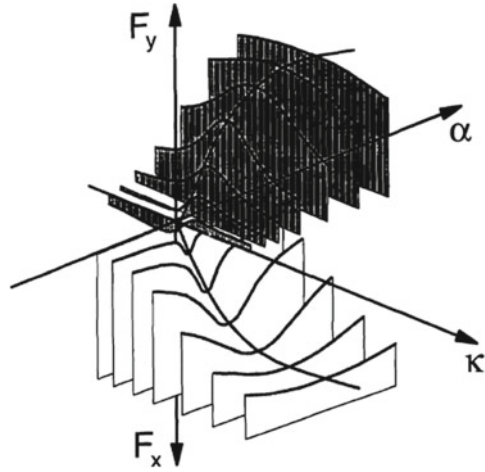
- B : stiffness factor,
- C : shape factor,
- D : peak value,
- E : curvature factor,
- S_H : horizontal shift,
- S_V : vertical shift.

The offsets S_H and S_V appear to occur when ply-steer, conicity effects, and possibly the rolling resistance cause the F_x and F_y curves not to pass through the origin (Pacejka, 2005).

Weighting functions are used to take into account the combined slip:

$$G = D \cos [C \arctan (Bx)] \quad (67)$$

Fig. 21 Forces interactions in a combined slip (Pacejka, 2005)



The parameters have the same signification as before. These weighting functions are multiplied by the original functions (64)–(66) to produce the interactive effects of κ on F_y or α on F_x .

Regarding the lateral force for instance, we have:

$$F_y = G_{y\kappa} F_{y0} + S_{Vy\kappa} \quad (68)$$

with:

$$\begin{cases} G_{y\kappa} = \frac{\cos [C_{y\kappa} \arctan \{B_{y\kappa} (\kappa + S_{Hy\kappa})\}]}{\cos [C_{y\kappa} \arctan \{B_{y\kappa} S_{Hy\kappa}\}]} \\ S_{Vy\kappa} = D_{Vy\kappa} \sin [r_{Vy5} \arctan (r_{Vy6} \kappa)] \end{cases} \quad (69)$$

$$(70)$$

where:

$$\begin{cases} B_{y\kappa} = r_{By1} \cos [\arctan \{r_{By2} (\alpha - r_{By3})\}] \\ C_{y\kappa} = r_{Cy1} \end{cases} \quad (71)$$

$$(72)$$

F_{y0} is the lateral force in case of pure side-slip calculated using Eqs. (64)–(66). $S_{Vy\kappa}$ is caused by the ply-steer phenomenon induced by κ . The different r coefficients are constant curve fitting parameters whose values can be found in the appendix of (Pacejka, 2005). The same procedure is used for the longitudinal force F_x . To illustrate the combined slip effects, Pacejka uses a 3D diagram as shown in Fig. 21.

Brach equations for combined slip: To describe the effects of the combined slip, Nicholas and Comstock have proposed in 1972 the following formulas:

$$\left\{ \begin{array}{l} F_x(\kappa, \alpha) = \frac{F_x(\kappa) F_y(\alpha) \kappa}{\sqrt{\kappa^2 F_y^2(\alpha) + \tan^2(\alpha) F_x^2(\kappa)}} \end{array} \right. \quad (73)$$

$$\left\{ \begin{array}{l} F_y(\kappa, \alpha) = \frac{F_x(\kappa) F_y(\alpha) \tan(\alpha)}{\sqrt{\kappa^2 F_y^2(\alpha) + \tan^2(\alpha) F_x^2(\kappa)}} \end{array} \right. \quad (74)$$

where $F_x(\kappa)$ and $F_y(\alpha)$ are calculated using the Magic Formula in the case of pure slip (64)–(66).

However, we have in this case:

$$\left\{ \begin{array}{l} F_x(\kappa, 0) \neq F_x(\kappa) \\ F_y(0, \alpha) \neq F_y(\alpha) \end{array} \right. \quad (75)$$

$$\left\{ \begin{array}{l} F_y(0, \alpha) \neq F_y(\alpha) \end{array} \right. \quad (76)$$

To solve this, Brach introduced a modification in Brach and Brach (2000):

$$\left\{ \begin{array}{l} F_x(\kappa, \alpha) = \frac{F_x(\kappa) F_y(\alpha)}{\sqrt{\kappa^2 F_y^2(\alpha) + \tan^2(\alpha) F_x^2(\kappa)}} \end{array} \right. \quad (77)$$

$$\left\{ \begin{array}{l} F_y(\kappa, \alpha) = \frac{C_\alpha}{\sqrt{\kappa^2 F_y^2(\alpha) + \tan^2(\alpha) F_x^2(\kappa)}} \end{array} \right. \quad (78)$$

where C_s is the longitudinal stiffness of the tire and C_α is the cornering one.

Kiencke's Model: In Kiencke and Nielsen (2000), the authors use two techniques:

- Calculation of the friction coefficient using Burckhardt extended model,
- Calculation of the different contact points' speed. The displacement of the resultants centre (detachment point) with respect to the vertical projection of the wheel center is evaluated through the forces acting on the tire.

The friction coefficient expression is given by:

$$\mu = (c_1 (1 - e^{-c_2 S}) - c_3 S) e^{-c_4 S V_G} (1 - c_5 F_z^2) \quad (79)$$

where:

- $S = \sqrt{\kappa^2 + \alpha^2}$: the resultant slip,

- V_G : velocity of the vehicle's CoG,
- c_1, c_2, c_3 : parameters depending on the ground surface condition,
- c_4 : parameter depending on the maximal vehicle speed,
- c_5 : parameter depending on the maximal vertical load.

The tire forces are then determined by:

$$\begin{cases} F_x = \mu \frac{F_z}{S} (\kappa \cos(\alpha) - c_{\mu t} \alpha \sin(\alpha)) \\ F_y = \mu \frac{F_z}{S} (c_{\mu t} \alpha \cos(\alpha) + \kappa \sin(\alpha)) \end{cases} \quad (80)$$

$$\begin{cases} F_x = \mu \frac{F_z}{S} (\kappa \cos(\alpha) - c_{\mu t} \alpha \sin(\alpha)) \\ F_y = \mu \frac{F_z}{S} (c_{\mu t} \alpha \cos(\alpha) + \kappa \sin(\alpha)) \end{cases} \quad (81)$$

where $c_{\mu t}$ is a weighting coefficient (varies between 0.9 and 0.95).

We have also:

$$\begin{cases} \mu_x = \mu \frac{\kappa}{S} \\ \mu_y = \mu \frac{\tan(\alpha)}{S} \end{cases} \quad (82)$$

$$\begin{cases} \mu_x = \mu \frac{\kappa}{S} \\ \mu_y = \mu \frac{\tan(\alpha)}{S} \end{cases} \quad (83)$$

The different parameters c_i used by Burckhardt can be found in Kiencke and Nielsen (2000) or Maakaroun (2011). This gives different friction curves with respect to the ground surface condition (see Fig. 22).

2.2.2 Theoretical Models

Based on physical models, theoretical models gives a better understanding of the tire behaviour. More complex models especially designed to improve tire performances related to its construction can be found. In this case, complex finite element based models are usually adopted (Pacejka, 2005). These latter exceed the scope of this chapter. Relatively simple models should be intended for control synthesis.

The Brush Model: As its name may reveal, the tire is assimilated to a set of brush bristles (Fig. 23).

When rolling without sliding, a tread element is assumed to enter the contact zone in a vertical position with respect to the ground (Pacejka, 2005). It remains vertical until it leaves this zone without deforming. In contrast, when $V \neq R\omega$, a horizontal deformation of the element is developed. The base point of this element moves backwards at a speed equal to $R\omega$ with respect to the wheel's axis. The same point moves at a speed called slip⁹ velocity with respect to the ground. The lower part of the element remains attached to the ground. The element adheres to the ground as long as the friction limits allow it. The maximum deformation depends on the coefficient of friction μ , the distribution of the vertical load q_z , and the element

⁹ The slip of the carcass with respect to the ground.

Fig. 22 Friction coefficient for different surface types (Maakaroun, 2011)

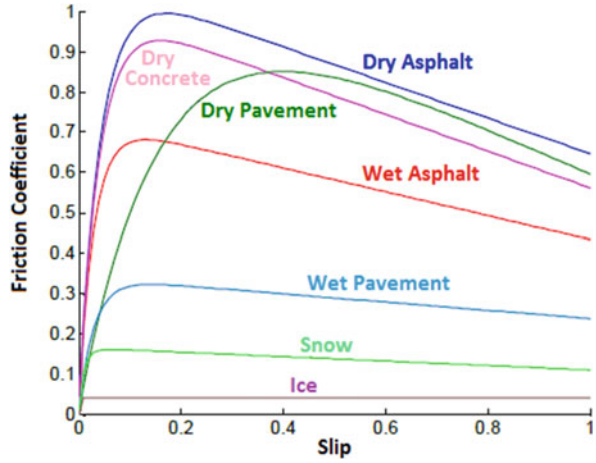
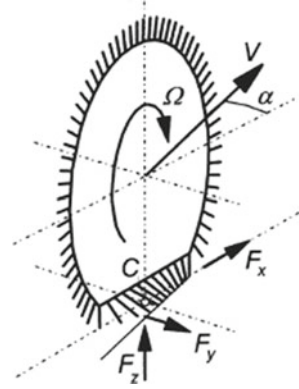


Fig. 23 The Brush model (Pacejka, 2005)



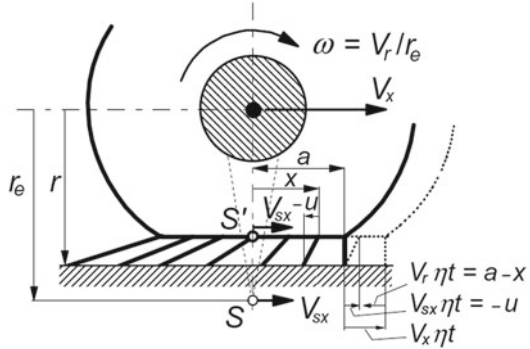
stiffness c_p . Once this maximum is reached, the element begins to slide. As shown in the Fig. 23, this phenomenon begins at the rear of the wheel. The sliding part can be distinguished from the adhering part by an intersection point. When slip increases, this point moves towards the front of the wheel until it reaches its edge, which is considered as total sliding.

Several mathematical formulations were adopted regarding this physical representation (Pacejka, 2005; Svendenius, 2003; Svendenius & Wittenmark, 2003). These formulas differ according to the assumptions considered regarding the tire physical characteristics, namely, the friction coefficient and the vertical load distribution. Here, we suppose a parabolic distribution of vertical load:

$$q_z = \frac{3F_z}{4a} \left[1 - \left(\frac{x}{a} \right)^2 \right] \quad (84)$$

where:

Fig. 24 Side view of Brush model when braking (Pacejka, 2005)



- a : half the contact length,
- x : the tread element coordinate (see Figs. 24 or 25).

Next, we present the brush model formulas for the most frequent cases:

- For pure longitudinal slip (Fig. 24), we find (Svendenius, 2003):

$$- \text{ if } \sigma_x < \frac{3\mu F_z}{C_s}$$

$$F_x = C_s \sigma_x \left[1 - \frac{C_s \sigma_x}{3\mu F_z} + \frac{1}{3} \left(\frac{C_s \sigma_x}{3\mu F_z} \right)^2 \right] \quad (85)$$

$$- \text{ if } \sigma_x \geq \frac{3\mu F_z}{C_s}$$

$$F_x = \mu F_z \quad (86)$$

where $\sigma_x = \frac{\kappa}{1 + \kappa}$ is the “theoretical longitudinal slip”. Note that the tire force does not decrease when it reaches its maximum. We can already see that this model is not accurate in the unstable region. Same remark holds for the following expressions.

- For pure side slip (Fig. 25), we find (Pacejka, 2005; Svendenius, 2003):

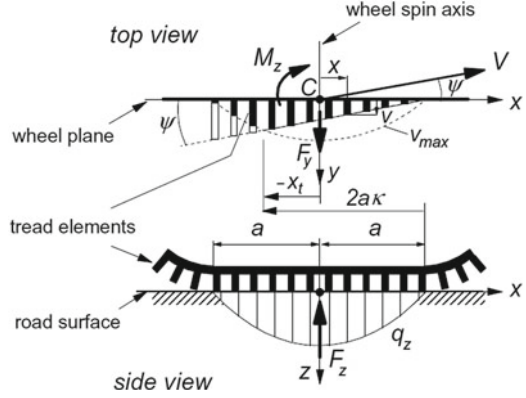
$$- \text{ if } \sigma_y < \frac{3\mu F_z}{C_\alpha}$$

$$F_y = C_\alpha \sigma_y \left[1 - \frac{C_\alpha \sigma_y}{3\mu F_z} + \frac{1}{3} \left(\frac{C_\alpha \sigma_y}{3\mu F_z} \right)^2 \right] \quad (87)$$

$$- \text{ if } \sigma_y \geq \frac{3\mu F_z}{C_\alpha}$$

$$F_y = \mu F_z \quad (88)$$

Fig. 25 Top and side view of Brush model at pure side slip (Pacejka, 2005)



where $\sigma_y = \frac{\tan(\alpha)}{1 + \kappa}$ is the “theoretical lateral slip”.

- For combined slip (Fig. 26), the formulas depend on the formulations considered for the friction and tire stiffness:
 - In the case of an isotropic friction with equal longitudinal and lateral stiffnesses $C_s = C_\alpha = C$, we find (Pacejka, 2005):

$$\text{If } \|\vec{\sigma}\| < \frac{3\mu F_z}{C}$$

$$\|\vec{F}\| = \mu F_z \left[3\theta \|\vec{\sigma}\| - 3(\theta \|\vec{\sigma}\|)^2 + (\theta \|\vec{\sigma}\|)^3 \right] \quad (89)$$

$$\text{If } \|\vec{\sigma}\| \geq \frac{3\mu F_z}{C}$$

$$\|\vec{F}\| = \mu F_z \quad (90)$$

where:

$$\|\vec{\sigma}\| = \sqrt{\sigma_x^2 + \sigma_y^2}$$

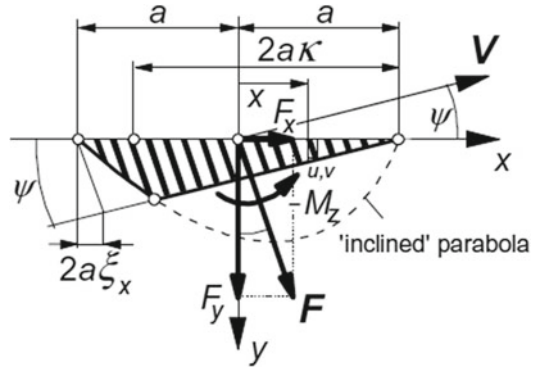
$$\vec{F} = \frac{\vec{\sigma}}{\|\vec{\sigma}\|} \|\vec{F}\|$$

$$\theta = \frac{C}{3\mu F_z}$$

- In the case of an isotropic friction and a rectangular tire/ground contact surface, the longitudinal stiffness is different from the cornering one. Pursuing the same development logic in Svendenius (2003), and considering the engine operation instead of braking operation, we find:

$$\text{If } \psi(\sigma_x, \sigma_y) < 1$$

Fig. 26 Brush model deformation at combined slip (Pacejka, 2005)



$$\left\{ \begin{array}{l} F_x(\sigma_x, \sigma_y) = C_s \sigma_x (1 - \psi(\sigma_x, \sigma_y))^2 \\ \quad + \frac{\sigma_x}{\sqrt{\sigma_x^2 + \sigma_y^2}} \mu F_z \psi^2(\sigma_x, \sigma_y) (3 - 2\psi(\sigma_x, \sigma_y)) \\ F_y(\sigma_x, \sigma_y) = C_\alpha \sigma_y (1 - \psi(\sigma_x, \sigma_y))^2 \\ \quad + \frac{\sigma_y}{\sqrt{\sigma_x^2 + \sigma_y^2}} \mu F_z \psi^2(\sigma_x, \sigma_y) (3 - 2\psi(\sigma_x, \sigma_y)) \end{array} \right. \quad (91)$$

$$\left\{ \begin{array}{l} F_x(\sigma_x, \sigma_y) = C_s \sigma_x (1 - \psi(\sigma_x, \sigma_y))^2 \\ \quad + \frac{\sigma_x}{\sqrt{\sigma_x^2 + \sigma_y^2}} \mu F_z \psi^2(\sigma_x, \sigma_y) (3 - 2\psi(\sigma_x, \sigma_y)) \\ F_y(\sigma_x, \sigma_y) = C_\alpha \sigma_y (1 - \psi(\sigma_x, \sigma_y))^2 \\ \quad + \frac{\sigma_y}{\sqrt{\sigma_x^2 + \sigma_y^2}} \mu F_z \psi^2(\sigma_x, \sigma_y) (3 - 2\psi(\sigma_x, \sigma_y)) \end{array} \right. \quad (92)$$

If $\psi(\sigma_x, \sigma_y) \geq 1$

$$\left\{ \begin{array}{l} F_x(\sigma_x, \sigma_y) = \frac{\sigma_x}{\sqrt{\sigma_x^2 + \sigma_y^2}} \mu F_z \\ F_y(\sigma_x, \sigma_y) = \frac{\sigma_y}{\sqrt{\sigma_x^2 + \sigma_y^2}} \mu F_z \end{array} \right. \quad (93)$$

$$\left\{ \begin{array}{l} F_x(\sigma_x, \sigma_y) = \frac{\sigma_x}{\sqrt{\sigma_x^2 + \sigma_y^2}} \mu F_z \\ F_y(\sigma_x, \sigma_y) = \frac{\sigma_y}{\sqrt{\sigma_x^2 + \sigma_y^2}} \mu F_z \end{array} \right. \quad (94)$$

where $\psi(\sigma_x, \sigma_y) = \sqrt{\left(\frac{C_s \sigma_x}{3\mu F_z}\right)^2 + \left(\frac{C_\alpha \sigma_y}{3\mu F_z}\right)^2}$ represents the adhesion limits when the vertical load distribution is parabolic.

- In the case of an anisotropic friction, interest is given to the friction ellipse. The forces' expressions are a combination of the influence of tread element deformation and the influence of tread element sliding:

$$\left\{ \begin{array}{l} F_x(\sigma_x, \sigma_y) = F_{dx}(\sigma_x, \sigma_y) + F_{sx}(\sigma_x, \sigma_y) \\ F_y(\sigma_x, \sigma_y) = F_{dy}(\sigma_x, \sigma_y) + F_{sy}(\sigma_x, \sigma_y) \end{array} \right. \quad (95)$$

$$\left\{ \begin{array}{l} F_x(\sigma_x, \sigma_y) = F_{dx}(\sigma_x, \sigma_y) + F_{sx}(\sigma_x, \sigma_y) \\ F_y(\sigma_x, \sigma_y) = F_{dy}(\sigma_x, \sigma_y) + F_{sy}(\sigma_x, \sigma_y) \end{array} \right. \quad (96)$$

where the adhesive forces expressions are (Svendenius, 2003):

If $\psi(\sigma_x, \sigma_y) < 1$

$$\begin{cases} F_{dx}(\sigma_x, \sigma_y) = -C_s \sigma_x [1 - \psi(\sigma_x, \sigma_y)]^2 \\ F_{dy}(\sigma_x, \sigma_y) = -C_\alpha \sigma_y [1 - \psi(\sigma_x, \sigma_y)]^2 \end{cases} \quad (97)$$

$$(98)$$

If $\psi(\sigma_x, \sigma_y) \geq 1$

$$F_{dx} = F_{dy} = 0 \quad (99)$$

where $\psi(\sigma_x, \sigma_y) = \sqrt{\left(\frac{C_s \sigma_x}{3\mu_{sx} F_z}\right)^2 + \left(\frac{C_\alpha \sigma_y}{3\mu_{sy} F_z}\right)^2}$ this time, and μ_{sx} and μ_{sy} are the longitudinal and lateral static friction coefficients respectively. The sliding forces consists on the distribution of the vertical load acting on the sliding portion between the longitudinal and lateral directions. This vertical load is expressed as (Svendenius, 2003):

$$F_{sz} = F_z \psi^2(\sigma_x, \sigma_y) [3 - 2\psi^2(\sigma_x, \sigma_y)] \quad (100)$$

There are three methods to describe how the vertical load is distributed in case of an anisotropic friction: *collinear slide forces*, *maximum dissipation rate*, and *slip-projection method* (Svendenius, 2003).

Collinear slide forces:

$$\begin{cases} F_{sx}(\sigma_x, \sigma_y) = -\cos(\beta^-) \mu_{kx} F_{sz} \\ F_{sy}(\sigma_x, \sigma_y) = -\sin(\beta^-) \mu_{ky} F_{sz} \end{cases} \quad (101)$$

$$(102)$$

where $\tan(\beta^-) = \frac{\mu_{kx} v_{sy}}{\mu_{ky} v_{sx}}$, with μ_k is the kinetic friction coefficient and v_s is the sliding speed.

Maximum dissipation rate:

The resulting sliding friction force $\vec{F}_s''$ is the one which maximizes the mechanical work $W = -\vec{v}_s \vec{F}_s''$ under the constraint:

$$\left(\frac{F_{sx}''}{\mu_{kx} F_{sz}}\right)^2 + \left(\frac{F_{sy}''}{\mu_{ky} F_{sz}}\right)^2 \leq 1 \quad (103)$$

This gives:

$$\begin{cases} F_{sx}''(\sigma_x, \sigma_y) = -\cos(\beta') \mu_{kx} F_{sz} \\ F_{sy}''(\sigma_x, \sigma_y) = -\sin(\beta') \mu_{ky} F_{sz} \end{cases} \quad (104)$$

$$(105)$$

$$F_{xd} = C_s \sigma_x + \frac{1}{3} \frac{(C_s \sigma_x)^2}{(d-1) \mu F_z} - \frac{1}{27} \frac{(3d+1) (C_s \sigma_x)^3}{(d-1)^3 (\mu F_z)^2} + \mathcal{O}(\sigma_x^4) \quad (109)$$

$$\begin{aligned} - \text{ If } \sigma_x \geq \frac{3\mu F_z}{C_s} (d+1) \\ F_x = \mu F_z \end{aligned} \quad (110)$$

- The lateral force is more concerned by the *carcass flexibility* (Pacejka, 2005). The total deformation consists on the sum of the bristle and carcass deflections. The carcass deformation is treated as a beam. However, no smooth expression is given when considering this phenomenon.
- Variable friction:

Three cases are distinguished in Svendenius (2003):

1. The friction coefficient is constant but has two different values: μ_s (static) if the tread element is in the adhesion portion and μ_k (kinetic) if the tread element is in the sliding portion,
2. The friction coefficient is linearly dependent on the sliding speed,¹¹
3. The friction coefficient is exponentially dependent of the sliding speed.

– Case of a constant friction:

The force can be expressed as:

$$\begin{aligned} \text{If } \sigma_x < \frac{3\mu_s F_z}{C_s} \\ F_x = C_s \sigma_x + \frac{1}{3} \frac{(\mu_k - 2\mu_s) (C_s \sigma_x)^2}{\mu_s^2 F_z} + \frac{1}{27} \frac{(3\mu_s - 2\mu_k) (C_s \sigma_x)^3}{\mu_s^3 F_z^2} \end{aligned} \quad (111)$$

$$\begin{aligned} \text{If } \sigma_x \geq \frac{3\mu_s F_z}{C_s} \\ F_x = \mu_k F_z \end{aligned} \quad (112)$$

– Case of linear dependency:

Here we simply make the replacement:

$$\mu_k(\sigma_x, v_x) = \mu_0 - n v_x \frac{\sigma_x}{1 + \sigma_x} \quad (113)$$

where $\mu_0 = \mu_s$ and n is a parameter used or curve fitting.

– Case of exponential dependency:

According to de Wit et al. (2001):

$$\mu_k(\sigma_x, v_x) = \mu_k + (\mu_s - \mu_k) \exp \left[\left| \frac{\sigma_x v_x}{(1 + \sigma_x) v_{st}} \right|^\varepsilon \right] \quad (114)$$

¹¹ $v_{sx} = \frac{\sigma_x}{1 + \sigma_x} v_x$.

where v_{st} is the stribeck velocity and ε is a curve fitting parameter. This model requires then four parameters. To reduce parameter numbers, the authors of Sven- denius (2003) consider $\varepsilon = 0.5$ and $v_{st} = 30$ m/s, and introduce a parameter h so they have:

$$\mu_k(\sigma_x, v_x) = \mu h + \mu(1 - h) \exp \left[\left| \frac{\sigma_x v_x}{30(1 + \sigma_x)} \right|^{0.5} \right] \quad (115)$$

The Physical Model of Dugoff: Dugoff developed an analytic model based on the classical analysis of Fiala (Dugo et al., 1969). He assumed a constant friction coefficient and a constant vertical load distribution. These assumptions give:

$$\begin{cases} F_x = C_s \frac{\kappa}{1 - \kappa} \tau \\ F_y = C_\alpha \frac{\tan(\alpha)}{1 - \kappa} \tau \end{cases} \quad (116)$$

$$\quad (117)$$

τ is introduced to take into account the combined slip:

$$\tau = \begin{cases} (2 - \sigma) \sigma & \text{if } \sigma < 1 \\ 1 & \text{otherwise} \end{cases} \quad (118)$$

where:

$$\sigma = \frac{(1 - \kappa) \mu F_z}{2\sqrt{C_s^2 \kappa^2 + C_\alpha^2 \tan^2(\alpha)}} \quad (119)$$

Note that in case of a pure longitudinal slip ($\alpha = 0$), and we have:

$$\lim_{\kappa \rightarrow 1} F_x = \mu F_z \quad (120)$$

The same remark holds for the pure side-slip. Dugoff suppose therefore that the tire force does not experience a loss of potential when it reaches its maximum. However, he pointed out that considering a sliding-speed-dependent coefficient of friction improves the tire force curve in the unstable region.

Modified Dugoff Model: To improve Dugoff's model accuracy in the unstable zone, the authors in Bian et al. (2014) adjusted Dugoff's model to Pacejka's using curve fitting methods. This has led them to develop weighting functions to be multiplied by the original model of Dugoff:

- For the longitudinal force:

$$G_s = (1.15 - 0.75\mu_{max}) \kappa^2 - (1.63 - 0.75\mu_{max}) \kappa + 1.27 \quad (121)$$

- For the lateral force:

$$G_{\alpha} = (\mu_{max} - 1.6) \tan(\alpha) + 1.155 \quad (122)$$

As expected the model's accuracy has improved. However, the weighting functions make the model lose its analytic nature.

LuGre Model: “Lu” stands for Lund (Sweden) and “Gre” for Grenoble (France). This model has been developed in a joint cooperation between the Automatic Control Departments of the two cities' universities. The model is expressed as (de Wit et al., 2003):

$$\left\{ \begin{array}{l} \dot{z} = v_s - \frac{\sigma_0 |v_s|}{g(v_s)} z \\ F_x = (\sigma_0 z + \sigma_1 \dot{z} + \sigma_2 v_s) F_z \end{array} \right. \quad (123)$$

$$F_x = (\sigma_0 z + \sigma_1 \dot{z} + \sigma_2 v_s) F_z \quad (124)$$

$$g(v_s) = \mu_k + (\mu_s - \mu_k) \exp\left(-\sqrt{\left|\frac{v_s}{v_{st}}\right|}\right) \quad (125)$$

where:

- z : deflection of the material in the friction surface,
- $g(v_s)$: friction function,¹²
- v_s : sliding velocity,
- v_{st} : stribbeck velocity,
- σ_0 : the rubber longitudinal lumped stiffness,
- σ_1 : the rubber longitudinal lumped damping,
- σ_2 : the viscous relative damping.

There exists several representations of this model. For more information, see Svendenius (2003) and de Wit et al. (2003).

Gim's Model: It is a model that separates the longitudinal slip effects from the side-slip ones (Seddiki et al., 2006). Gim supposes a rectangular contact surface. The model is based on the pressure calculation along the contact surface. The vertical force is calculated through the pressure integration along the contact surface. This gives:

$$\left\{ \begin{array}{l} F_x = C_s \kappa l_n^2 + \mu_x F_z (1 - l_n^2 + 2l_n^3) \\ F_y = C_\alpha \alpha l_n^2 + \mu_y F_z (1 - 3l_n^2 + 2l_n^3) \end{array} \right. \quad (126)$$

$$\left\{ \begin{array}{l} F_x = C_s \kappa l_n^2 + \mu_x F_z (1 - l_n^2 + 2l_n^3) \\ F_y = C_\alpha \alpha l_n^2 + \mu_y F_z (1 - 3l_n^2 + 2l_n^3) \end{array} \right. \quad (127)$$

where:

¹² The friction is assumed to be dependent on the sliding velocity.

$$\begin{cases} C_s = \frac{K_x W}{2} \end{cases} \quad (128)$$

$$\begin{cases} C_\alpha = \frac{K_y W}{2} \end{cases} \quad (129)$$

$$\begin{cases} l_n = 2l_r \left(1 - \frac{2Wl_r^2}{3\mu F_z} \sqrt{K_x^2 \kappa^2 + K_y^2 \alpha^2} \right) \end{cases} \quad (130)$$

with:

- l_r : contact surface length,
- W : contact surface width,
- K_x : longitudinal stiffness per unit area,
- K_y : lateral stiffness per unit area.

2.2.3 Tire Models Comparison

The Magic Formula is widely used by car manufacturers to simulate and validate dynamic performances due to its precision. Its parameters are changed offline to take into account the different road conditions. In this chapter, interest is given more to control synthesis. In this case, we should be able to change the different parameters online. To do this, we should use physical parameters that could be measured or at least estimated. Consequently, in this chapter, we also use the Magic Formula only for validation.

To compare the physical models with Pacejka's model, the Magic Formula parameters should be interpreted in a way to meet a physical meaning. In Pacejka (2005), Pacejka pointed out that the product BCD is equivalent to the stiffness. Indeed, considering Eq. (66), the product BCD represents the tangent at the origin in the same way as C_s for the longitudinal force in Eq. (91) or (116), and C_α for the lateral force in Eq. (92) or (117). For numerical applications, we can use the shape coefficients given in Wong (2001) for a vertical load of 4000 N for example. The stiffness coefficients are then given by the value of the product BCD .

First, we only consider the physical model of Dugoff and the Brush model in the case of an isotropic friction coefficient and a rectangular distribution of the vertical load. The other models either depends on empirical parameters or are too complex to be used for control synthesis. Figure 28 depicts the differences for the longitudinal force in a combined slip with a side-slip angle equal to 3° in the case of the Magic Formula, the Brush model and Dugoff's model. Nevertheless, a global comparison of all models including a new model developed in Kissai et al. (2017) will be depicted later in this chapter.

Although the brush model uses physical parameters, it does not take into account the loss of potential phenomenon when the maximum friction force is reached. In Svendenius (2003), this model has been improved by considering a speed dependent friction, and in Svendenius and Wittenmark (2003), an asymmetric vertical load is

considered to improve the peak value location accuracy. However, the complexity also increases.

Dugoff's model suffers from the same drawback. The force's shape is totally inaccurate in the nonlinear region given that the coefficient of friction and the vertical load distribution are considered constant.

In control synthesis, these criteria are not crucial. As we mentioned, the third region of Fig. 20 is unstable. There is no point to control the vehicle in this region. This behavior should be predicted and avoided. Thereby, the Brush model and Dugoff's model represent good candidates.

These two models respect three out of the four requirements that we proposed. They both respect the tire physical fundamentals, they represent precisely enough the combined slip in the stable region and they are dependent on physical parameters. However, their mathematical formulas are still too complex to manipulate. They should be linearized to be used for control synthesis.

2.3 Tire Models Linearization

Here, the methodology exposed in Kissai et al. (2017) is adopted. The model of Dugoff differs from the brush model by the vertical load distribution assumptions. Dugoff supposes a constant vertical load distribution (Dugoff et al., 1969). In the brush model, a parabolic distribution is considered (Svendenius, 2003). Figure 28 shows however that little improvement is brought by this assumption. But then again, the formulas (91) and (92) are more complex to manipulate than (116) and (117). The brush model equations' derivatives are too heavy to be exposed. Hence, only Dugoff's model linearization is presented. Nevertheless, both results will be illustrated.

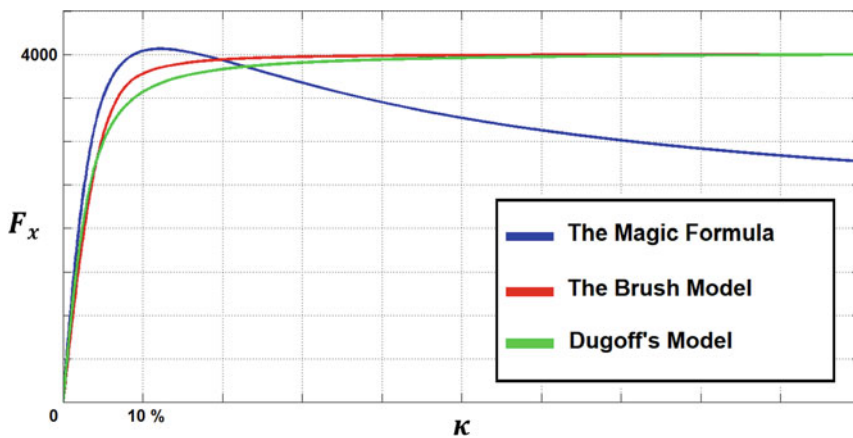


Fig. 28 Comparison of models' longitudinal force shapes in combined slip

According to (118), Dugoff's model has two expressions depending on whether the tire effort has reached its saturation or not. As our concern is the stable part of the tire behavior, it is the expression related to the case where $\sigma < 1$ that we linearize.

2.3.1 Longitudinal Force Linearization

Let κ^* be the stable operating point located in the linear region of the tire force curve (Fig. 20). After replacing the variable τ in Eq. (116) by its expression in Eq. (118), and σ by its expression in (119), we perform a first-order Taylor series approximation of F_x about κ^* :

$$F_x(\kappa) = \left[F_x(\kappa^*) - \frac{dF_x(\kappa^*)}{d\kappa} \bigg|_{\kappa=\kappa^*} \kappa^* \right] + \frac{dF_x(\kappa^*)}{d\kappa} \bigg|_{\kappa=\kappa^*} \kappa \quad (131)$$

As the linear region is concerned, κ^* is very small:

$$(\kappa^* \ll 1) \Rightarrow \left(F_x(\kappa^*) \cong \frac{dF_x(\kappa^*)}{d\kappa} \bigg|_{\kappa=\kappa^*} \kappa^* \right) \quad (132)$$

Therefore:

$$(\kappa^* \ll 1) \Rightarrow \left(F_x(\kappa) \cong \frac{F_x(\kappa^*)}{\kappa^*} \kappa \right) \quad (133)$$

Equation (133) shows that the term $\frac{F_x(\kappa^*)}{\kappa^*}$ corresponds to a longitudinal stiffness. Let us symbolize it C_s^* . Using Dugoff's model equations (116), (118), and (119), we find:

$$C_s^*(\alpha, \mu, F_z) = \frac{4\sqrt{C_s^2 \kappa^{*2} + C_\alpha^2 \tan^2(\alpha)} - (1 - \kappa^*) \mu F_z}{4(C_s^2 \kappa^{*2} + C_\alpha^2 \tan^2(\alpha))} \mu F_z C_s \quad (134)$$

This longitudinal stiffness depends on varying-parameters α , μ , and F_z . These parameters could be estimated online (Kissai et al., 2018). It depends also on constant parameters C_s , C_α , and κ^* . An explicit expression of κ^* will be given afterwards. By setting:

$$\rho = \frac{4\sqrt{C_s^2 \kappa^{*2} + C_\alpha^2 \tan^2(\alpha)} - (1 - \kappa^*) \mu F_z}{4(C_s^2 \kappa^{*2} + C_\alpha^2 \tan^2(\alpha))} \mu F_z \quad (135)$$

It appears that C_s^* is the product of C_s and a coupling term ρ . As a result, the linearized longitudinal force can be rewritten as:

$$F_x(\kappa) = C_s^* \kappa \quad (136)$$

This formulation respects the proposed requirements:

- It follows the physical nature as it depends on α , μ , and F_z ,
- It takes into account the combined slip by means of ρ ,
- It is linear and therefore simple from a control synthesis perspective,
- It depends on parameters obtainable from online estimation.

2.3.2 Lateral Force Linearization

The same reasoning is pursued. Here we suppose an α^* to be a stable operating point in the tire linear region. Again, we have $\alpha^* \ll 1$. We proceed then to the same simplifications as (132) and (4). Moreover, in the vicinity of α^* :

$$(\alpha \ll 1) \Rightarrow (\tan(\alpha) \cong \alpha) \quad (137)$$

Again, a lateral stiffness C_α^* can be defined using Dugoff's model equations (117), (118), and (119):

$$C_\alpha^*(\kappa, \mu, F_z) = \frac{4\sqrt{C_s^2\kappa^2 + C_\alpha^2\alpha^{*2}} - (1 - \kappa)\mu F_z}{4(C_s^2\kappa^2 + C_\alpha^2\alpha^{*2})} \mu F_z C_\alpha \quad (138)$$

The linearized lateral force is expressed as:

$$F_y(\alpha) = C_\alpha^* \alpha \quad (139)$$

The proposed requirements are again fulfilled. The only difference is that C_α^* depends on κ instead of α .

2.4 Dynamic Saturation

The linearized forces developed are indefinitely increasing. As we have mentioned, the overall adhesion is delimited by a “friction ellipse” (see Fig. 19). The tire potential is delimited by the product μF_z . In general, the friction can be anisotropic. The maximum longitudinal force could be therefore superior to the maximum lateral force. To simplify the problem, one may consider an isotropic friction. In this case, we can talk about the “friction circle concept”. Despite this simplification, the same logic is applied. When the tire is only solicited longitudinally, the maximum longitudinal force is equal to μF_z . If the tire is solicited laterally while accelerating or braking, the overall force moves along the friction circle. The maximum overall tire force is still μF_z , but the maximum longitudinal force is penalized. The different forces saturation should be then dynamic:

$$\begin{cases} F_x \leq \sqrt{(\mu F_z)^2 - F_y^2} \\ F_y \leq \sqrt{(\mu F_z)^2 - F_x^2} \end{cases} \quad (140)$$

$$\begin{cases} F_x \leq \sqrt{(\mu F_z)^2 - F_y^2} \\ F_y \leq \sqrt{(\mu F_z)^2 - F_x^2} \end{cases} \quad (141)$$

2.5 Simulation of the Linearized Models

By reassembling Eqs. (134), (136), (138)–(141), we simulate the new linearized tire model and compare it to Pacejka model using Eqs. (64)–(67) in Matlab/Simulink® environment. The linearized Brush-based model is also plotted.

2.5.1 Brush-Based and Dugoff-Based Linearized Models Comparison

Here, we only present the longitudinal force in combined slip with a side-slip angle of 1° (see Fig. 29). The lateral force follows the same behavior but with different parameters.

Although the brush model considers a parabolic vertical load distribution, once linearized, it does not offer much more precision with respect to Dugoff's model. The linearized model based on Dugoff's model provides a better simplicity/precision compromise. This is the reason why it is retained in Kissai et al. (2017).

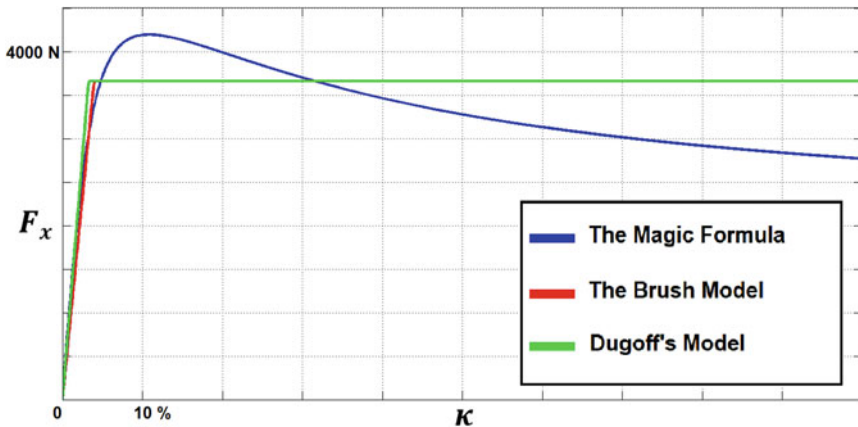


Fig. 29 Comparison of Pacejka model with both linearized models in combined slip

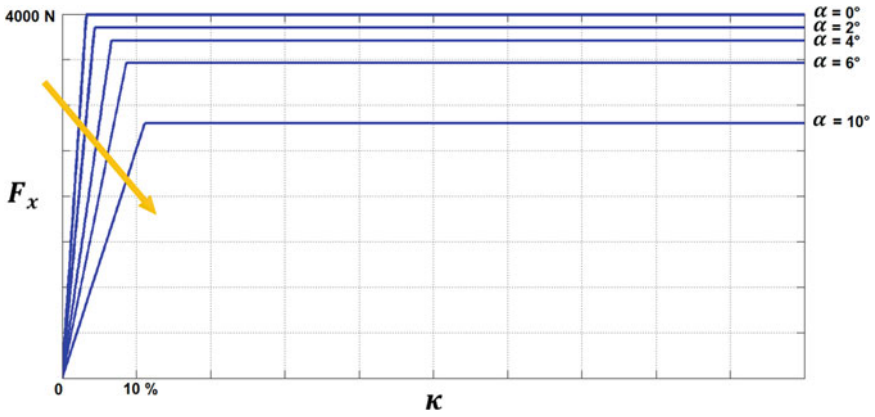


Fig. 30 Variation of the longitudinal force slope with respect to side-slip angle (Linearized Dugoff-based model)

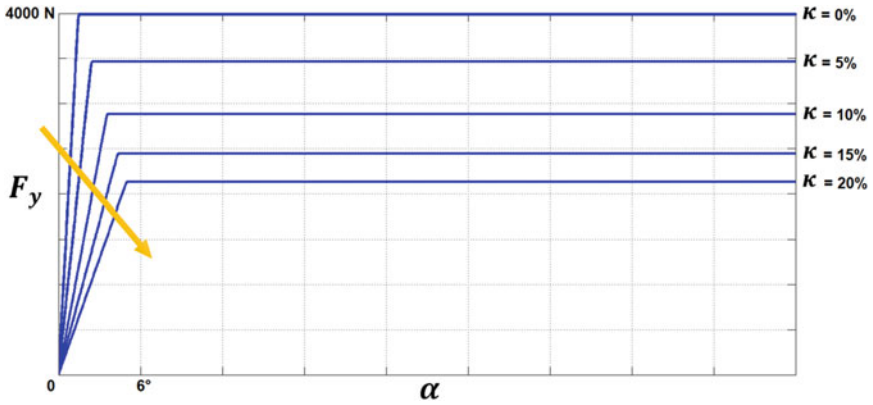


Fig. 31 Variation of the lateral force slope with respect to longitudinal slip (Linearized Dugoff-based model)

2.5.2 Linearized Dugoff-Based Model Evaluation

Note in Fig. 29 that the longitudinal force does not reach the maximum of $\mu F_z = 4000$ N because a part of the tire potential is already consumed by the lateral force. The longitudinal force curve's slope also varies with the side-slip angle because the longitudinal stiffness varies (see Fig. 30).

The same remark holds for the lateral force with respect to the longitudinal slip (Fig. 31).

That is to say that the linearized Dugoff-based model represents well the combined slip behavior in the stable region of the tire. More realistic simulations can be provided using a high-fidelity software as Simcenter Amesim®.

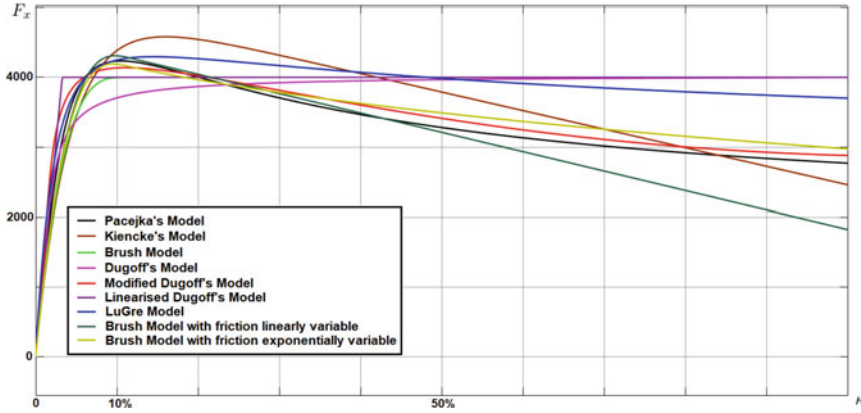


Fig. 32 Comparison of tire models in case of pure longitudinal slip

2.6 Tire Models Comparison

Empirical models' parameters are interpreted in order to meet a physical meaning, using the fact that the multiplication of Pacejka's parameters BCD is equivalent to the stiffness.

2.6.1 Case of Pure Longitudinal Slip

Here we present the simulation of the different models when the tire is only solicited longitudinally. Equations (64)–(66) are then used for Pacejka's model, (79)–(80) for Kiencke's, (85)–(86) for the Brush model, (116), (118)–(119) for Dugoff's, we add (121) for the modified version, we use (134), (136), (140) for the linearized version, and (123)–(125) for LuGre. The other left models lack reliable data to be adapted to the rest of the models. Instead, we add the brush model with a varying friction coefficient to show its impact. Figure 32 gathers the different simulations in the same graph to enable their comparison.

For its well-known accuracy, Pacejka's model can be considered as a reference. We can see that all the models are precise enough in the linear region. As soon as the non-linear zone is reached, the models diverge. While several models exceed 4000 N, the Brush model, Dugoff's model, and its linearized version do not because of the constraints imposed (see Eqs. (86), (118), (140)). Regarding the unstable zone, only Kiencke's model, the LuGre model, the modified Dugoff's model and the Brush models with varying friction coefficient approximate this behaviour. The reason is the variable friction coefficient hypothesis. The approximation gets even better when the friction is considered exponentially variable.

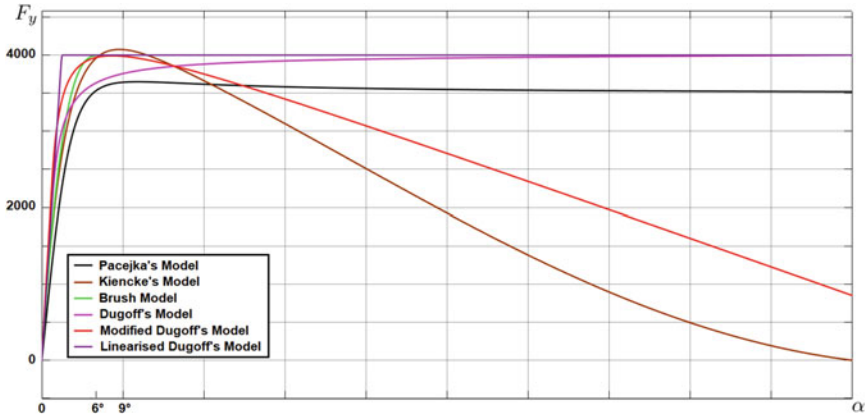


Fig. 33 Comparison of tire models in case of pure side slip

2.6.2 Case of Pure Side Slip

We present here the simulation of different models when the tire is only solicited laterally. Equations (64)–(66) are then used for Pacejka’s model with different parameters (see Pacejka, 2005), (79), (81) for Kiencke’s, (87)–(88) for the Brush model, (117)–(119) for Dugoff’s, we add (122) for the modified version, and we use (138), (139), (141) for the linearized version. Figure 33 gathers the different simulations in the same graph to enable their comparison.

The models here are less accurate with respect to Pacejka’s model. The main reason is that Pacejka takes into account several additional criteria that the other models do not, as carcass flexibility, camber and so on (Pacejka, 2005). Another difference is that Pacejka considers an anisotropic friction. In fact, while the longitudinal force exceeds 4000 N, the lateral force does not even reach it. However, in the linear zone, the models exhibit the same behaviour. In the unstable region, the Brush model, Dugoff’s model, and its linearized version saturate without decreasing because of the constraints imposed (see Eqs. (88), (118), (141)). In contrast, Kiencke’s model and the modified Dugoff’s model decrease in the unstable zone, but fail to describe well the true behaviour of the tire. For more precision, one could think of the Brush model in case of anisotropic friction with exponentially varying friction coefficient.

2.6.3 Case of Combined Slip

The Longitudinal Force: We introduce a constant side slip angle of 4° and we vary the longitudinal slip to evaluate the coupling effects. The weighting functions (67) are then added to Pacejka’s model, Eq. (77) are used for Brach’s model using Pacejka’s equations of pure slip, (79), (80) for Kiencke’s, and (89)–(90) for the Brush model. Same equations as pure longitudinal slip are used for Dugoff’s model, its modified

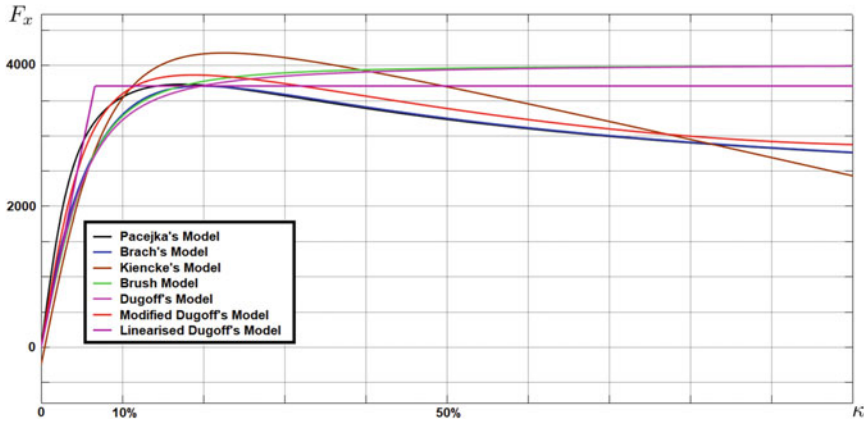


Fig. 34 Comparison of tire models in case of combined slip—the longitudinal force with a side slip of 4°

version, and its linearized version. Figure 34 reassembles the different simulations in the same graph to enable their comparison.

Since a combined slip is simulated, the models' slopes decrease in the linear zone. Here accuracy of the different models is acceptable. As we approach the non-linear zone, errors become more obvious. In the unstable zone, only Brach's model, Kiencke's model, and modified Dugoff's model (all semi-empirical models) illustrate the tire loss of potential phenomenon. Kiencke's model is less accurate whereas Pacejka's curve and Brach's curve overlap. The Brach model and Dugoff's model saturate and reach a maximum of 4000 N which is impossible in practice. The reason is the constant constraints imposed. The linearized Dugoff model solves this problem using dynamic constraints.

The Lateral Force: Here, we introduce a constant longitudinal slip of 10% and we vary the side slip. Same weighting functions are added to Pacejka's model, Eq. (78) are used for Brach's model using Pacejka's equations of pure slip, and (89)–(90) for the Brush model. Same equations than pure side slip are used for Dugoff's model, its modified version, and its linearized version. Figure 35 reassembles the different simulations in the same graph to enable their comparison.

As expected, the lateral force's slope decrease. The difference between models is more obvious in the non-linear zone. Regarding the unstable zone, only Brach's model describes well the tire behaviour with respect to Pacejka's model. Again, the constant constraints used in the Brush model and Dugoff's model are wrong. A dynamic constraint should be used as it is the case in the linearized Dugoff's model.

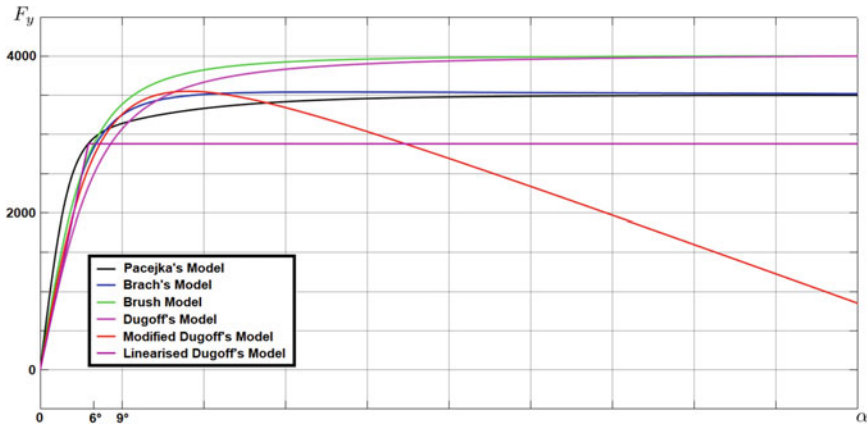


Fig. 35 Comparison of tire models in case of combined slip—the lateral force with longitudinal slip of 10%

2.6.4 Tire Models' Assignment

Tire modelling is essential for vehicle performances improvements, and for the tire performance itself. Four main challenges related to this context can be distinguished: *vehicle dynamics simulation*, *vehicle states estimation*, *vehicle motion control*, and *tire construction*.

Vehicle Dynamics Simulation: This gives an overview of the vehicle global performances off-line. It enables automotive engineers to determine the vehicle weaknesses according to real life scenarios. An exploded view of vehicle subsystems helps then to isolate the problem. This is done by studying the influence of a particular subsystem on the overall vehicle. For a reliable study, vehicle motion should be precisely described. Consequently, tire forces should be as close as possible to experimental data. It is well-known that simulations enable time and cost-saving. Vehicle dynamics simulation improve even chassis and its subsystems' design (Dieter, 2005).

High precision is required to meet experimental data. As simulations are done off-line, tire parameters could be updated manually. Empirical models seem to be a good match for these requirements:

- For high precision, **the Magic Formula of Pacejka** is perhaps the most widely used model. This model was developed to match as close as possible experimental data. Automotive software, e.g. CarSim[®] or AmeSim[®], rely on this model for high fidelity simulations. For pure longitudinal operations or pure lateral performances, one could use Eqs. (64)–(66). For combined slip, weighting functions should be added (67).
- For explicit friction studies, one may favour **Kiencke's model** where a detailed expression of the friction coefficient is provided. Equations (79)–(81) are then used.

- Note that **Brach** was particularly interested in accident reconstruction (Brach & Brach, 2009). Equations (77)–(78) could be used for that matter.

Vehicle Motion Control:

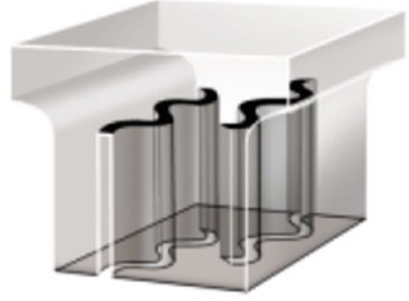
In this chapter, we are interested more in that aspect. Along with the relevant researches in terms of robotic vision and vehicle connectivity, chassis systems control is primary. In automatic control, related dynamics should be modelled, and the relationship between control inputs and desired performances should be formalized. Transfer functions are then deduced for control synthesis. As control synthesis consists of almost inverting the model, tire model should be simple enough. The automation engineer looks most of the time for a linear representation in at least a specific frequency range. This may induce modeling errors and uncertainties. Robust control is then used to ensure plant control despite the uncertainties (Doyle et al., 1989). Consequently, a simpler model is sufficient. Note that there are subsystems designed for a single physical quantity control (Brennan & Alleyne, 2001). Combined slip is then neglected. In contrast, it is important to take account of the combined slip as long as combined maneuvers are concerned, for example braking in a corner.

Simplicity is most of the time translated into linearization. Consequently, the **linearized Dugoff Model** represents an adequate solution for control synthesis. Equations (134)–(141) could then be used to synthesize control commands even in case of combined slip. The simpler linear model that is usually selected (Brennan & Alleyne, 2001) can only be used to synthesize stand-alone subsystems designed for single physical quantity control. The reason is that its parameters C_s and C_α are constant, and therefore, combined slip is not considered.

Vehicle States Estimation: For vehicle control or driver warning, not all the required states are measurable. Certain key parameters should then be estimated to ensure vehicle safe operation. A major difference with respect to vehicle dynamics simulation is that these parameters should be estimated on-line. In this context, the automotive engineer needs a fast computing model that depends on a minimum number of parameters. Moreover, vehicles are operating on unpredictable environments. Parameters should then be updated. To do this, parameters should have a physical meaning so they can be estimated from other measurable parameters through physical equations. A common example is the problem of friction coefficient estimation (Patra & Datta, 2012; Singh & Taheri, 2015). In fact, this conventional parameter depends on the road surface condition that the driver cannot act on. This parameter could be estimated if the tire forces are calculated in advance. As these latter are also not measurable in commercial vehicles due to sensors' cost, vehicle motion sensors are used to estimate them (M'sirdi et al., 2005). Theoretical models are then necessary to face this kind of challenges:

- For fast computation, **Dugoff's model** seems to be a good solution. Minimum parameters are required. And because constant vertical load along a single tire is assumed, relatively simpler equations are used (116)–(119).
- For more precise estimation, one could consider a parabolic vertical load distribution. That is to say choosing the **Brush model**. Several conditions are distinguished in this case:

Fig. 36 Wavy sipes used for water evacuation since the 1930s (Michelin, 2001)



- For pure slip, Eqs. (85)–(88) are used.
- For combined slip and isotropic friction, Eqs. (91)–(94) are preferred.
- For combined slip and anisotropic friction, Eqs. (95)–(99) are used, and Eqs. (104)–(105) are preferred.

Tire Construction: Here, interest is given to the tire's structure. Specific tire performances are studied and improved. For example, in wet soils, indentation phenomenon is corrupted (Michelin, 2001). Water must be evacuated to maintain a decent friction potential. Tread elements geometry is then redesigned to enable water evacuation while keeping optimal operation in dry soils (Fig. 36).

A 2-D model is no longer sufficient, 3D models are often favoured. Ad-hoc Finite Element Methods (FEM) are then used. These non-behavioural methods exceed the scope of this chapter.

2.7 Validation and Relevance of Linearized Tire Models

As we have mentioned earlier, reduced vehicle models are used in the downstream approach. Typically, the bicycle model is preferred for control synthesis in most of the research papers (Bachle et al., 2014; Selby, 2003; Soltani, 2014; Xiong & Yu, 2011). This hypothesis relies on a linear but non coupled model of the tire. Therefore, it has a simpler representation:

$$\begin{cases} F_x = C_s \kappa \\ F_y = C_\alpha \alpha \end{cases} \quad (142)$$

$$\quad \quad \quad (143)$$

The tire behavior is well represented in the linear region, but the model does not take into account the combined slip. This model is more suitable for vehicle stand-alone subsystems. For example, in Xiong and Yu (2011), this simple model is used to develop vehicle longitudinal control logic for a 4WD Hybrid Electric system. In Harada and Harada (1999) and Shuai et al. (2014), the bicycle model is used for

lateral control performance improvement. With the arrival of autonomous vehicles, the virtual pilot must handle multi-objectives control problems. Couplings should be therefore considered, especially at the tire level. For GCC, the linearized Dugoff-based model proposed in this chapter is necessary. To illustrate this, a vehicle global dynamic analysis is described next using Simcenter Amesim[®] platform.

2.7.1 Comparison of Linear Models

To compare both linear models, another requirement should be added. The new linear tire model should be equivalent to the simpler one when there is no combined slip. This requirement enables giving explicit expressions to the different stable operating points κ^* and α^* .

Respecting the previous requirement means:

$$C_s^*(0, \mu, F_z) = C_s \quad (144)$$

According to the expression (134), we find:

$$\kappa^* = \frac{\mu F_z}{8C_s^2} \left[\mu F_z + 4C_s \pm \sqrt{(\mu F_z)^2 + 8\mu F_z C_s} \right] \quad (145)$$

Both results are acceptable because $\mu F_z + 4C_s \gg \sqrt{(\mu F_z)^2 + 8\mu F_z C_s}$. Here, we consider the highest one.

Again, the previous requirement leads to:

$$C_\alpha^*(0, \mu, F_z) = C_\alpha \quad (146)$$

According to the expression (138), we find:

$$\alpha^* = \frac{\mu F_z}{2C_\alpha} \quad (147)$$

The Amesim[®] demo used describes a complete chassis model and provides global dynamics analysis (Fig. 37).

Inputs are steering angle and engine torque. The combined slip could then be observed. The vehicle is a B-segment class car. The model is composed of a chassis model with 15 degrees of freedom with associated subsystems: elastokinematics module, suspension (spring, damper, lower and higher end stop, antiroll bar), aerodynamic module, tire, road, sensors, powertrain unit, braking system and powersteering system. It is a highly nonlinear global model whose purpose is to represent as precise as possible the vehicle behavior. Most importantly, it uses Pacejka's model to calculate tires' efforts. We add supplementary modules that include the new linear Dugoff-based tire model and the classic linear tire model and compare them with The

Magic Formula of Pacejka. Figure 38 illustrates the longitudinal force and Fig. 39 illustrates the lateral force.

When only longitudinal force is requested (acceleration or braking), both linear models are close enough to Pacejka’s model. The lateral force of the classic linear model starts to deviate as soon as the tire is solicited longitudinally and laterally at the same time. Both longitudinal and lateral forces of the classic linear model diverge at severe conditions, for example in a hard brake maneuver while steering or progressive steering at high velocity. In contrast, the new linearized model follows close enough the Magic Formula. When the tire is solicited longitudinally and laterally at the same time, both longitudinal and lateral stiffnesses are updated and dynamic saturations are recalculated. The values of the forces are then lower. This is crucial from a control perspective. Ignoring combined slip may lead to high tire force demand. This could exceed the saturated value and destabilize the vehicle. The new linearized model proposed aims to prevent this scenario.

2.7.2 Further Improvements

So far, all mathematical formulas developed are expressed in steady state conditions. In control synthesis, time response is an important criterion. Subsequently, the transient behavior should be considered. Tire force delay is caused by carcass deflections (Pacejka, 2005; Svendenius, 2003). This is usually represented by a first order lag (Klomp & Lidberg, 2006; Martino, 2005):

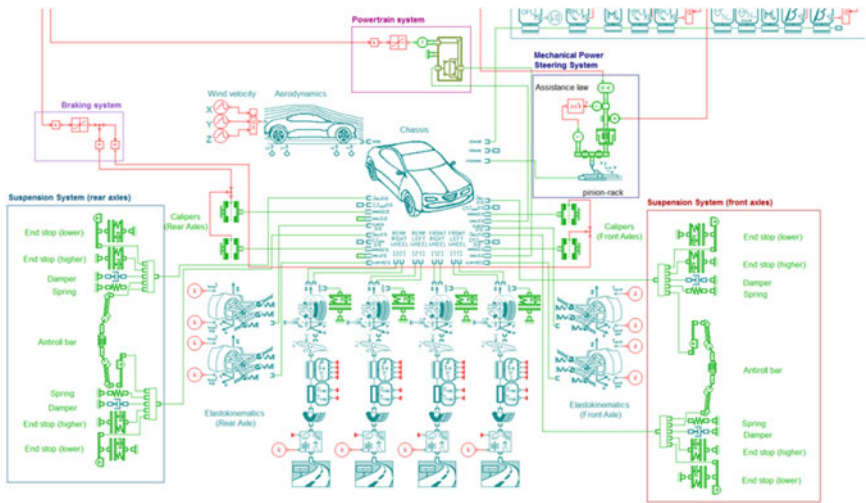


Fig. 37 The chassis model of 15 degrees of freedom in Amesim® environment

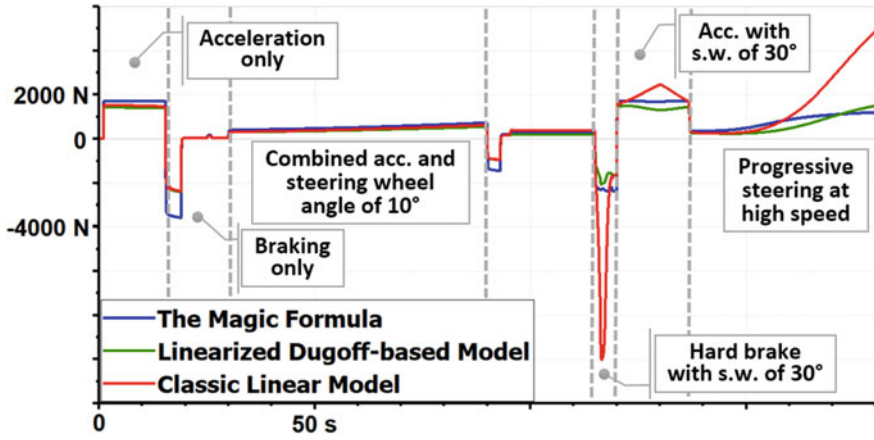


Fig. 38 Amesim® comparison of Pacejka model with linearized models—front left tire longitudinal force

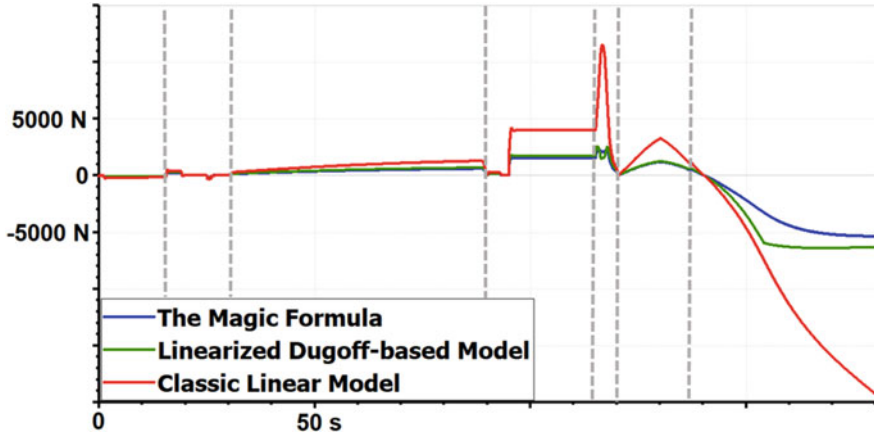


Fig. 39 Amesim® comparison of Pacejka model with linearized models—front left tire lateral force

$$F_i = \frac{1}{1 + \tau_i s} F_i^{ss} \quad (148)$$

i is either x or y , τ_i is the time-constant, s is the Laplace operator, and F_i^{ss} is the steady-state force that have been developed in the previous sections.

The carcass deflections change with the vehicle speed (Pacejka, 2005). The time-constant in Eq. (148) depends then on the vehicle's speed:

$$\tau_i = \frac{3L_i}{V_x} \quad (149)$$

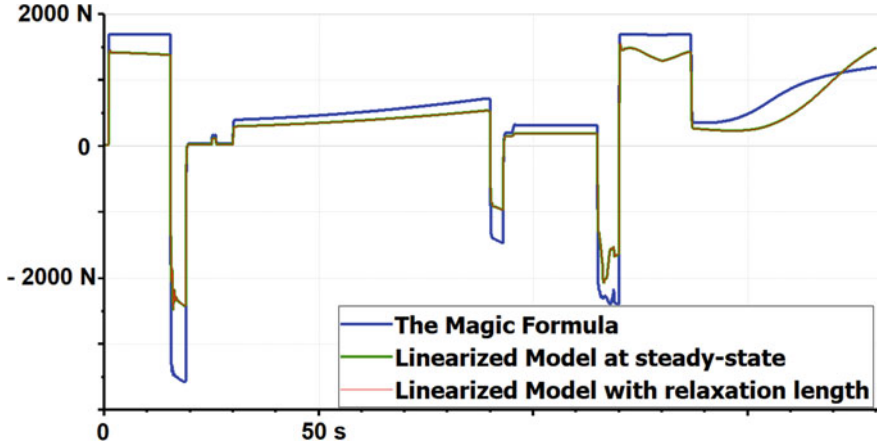


Fig. 40 Amesim[®] comparison of Pacejka model and linearized models with and without transient behavior—front left tire longitudinal force

L_i is defined as the relaxation length and V_x is the vehicle longitudinal speed. The relaxation length represent the distance that tire rolls before the lateral force reaches 63% of its final value (Pacejka, 2005). This does not influence much the shape of the tire response as Fig. 40 shows.

This is mainly due to the low value of the relaxation length (around 0.02 m). In control synthesis, the transient behavior could be ignored to prevent the generation of high order controllers.

2.7.3 Controllability

The linearized model is developed for control synthesis suitability. A state-space representation can be proposed:

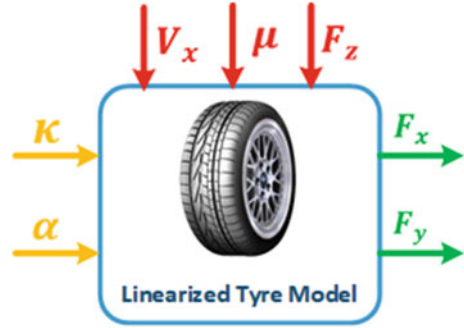
$$\begin{cases} \dot{x}(t) = A(t)x(t) + B(t)u(t) \end{cases} \quad (150)$$

$$\begin{cases} y(t) = C(t)x(t) \end{cases} \quad (151)$$

where:

- $x(t) = \begin{bmatrix} F_x \\ F_y \end{bmatrix}$: is the state vector,
- $y(t) = \begin{bmatrix} F_x \\ F_y \end{bmatrix}$: is the output vector,
- $u(t) = \begin{bmatrix} \kappa \\ \alpha \end{bmatrix}$: is the input vector,

Fig. 41 Linearized tire module with varying-parameters



- $A(t) = \begin{bmatrix} -\frac{1}{\tau_x} & 0 \\ 0 & -\frac{1}{\tau_y} \end{bmatrix}$: is the state matrix,
- $B(t) = \begin{bmatrix} \frac{C_s^*}{\tau_x} & 0 \\ 0 & \frac{C_\alpha^*}{\tau_y} \end{bmatrix}$: is the input matrix,
- $C(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$: is the output matrix.

Without forgetting the constraints (140) and (141).

Note that both state and input matrices contain varying parameters. Although stiffness coefficients depend on input variables, they cannot be split as we need the whole expression to be multiplied by its corresponding input variable. According to the structure of the matrices A , B and C , the system is clearly stable, controllable and observable as long as the varying parameters are positive, which is the case. Finally, this representation is suitable for modular and flexible control architectures. A simple tire module can be added as the Fig. 41 shows where the tire block contains Eqs. (150) and (151) and constraints (140) and (141).

2.7.4 Relevance of the New Tire Model

We recall that this model was developed to face GCC issues. As far as the overall chassis is concerned, coordination is a key aspect. We saw that in the downstream coordination approach, rule-based strategies are generally preferred. Autonomous vehicles require additional subsystems. As the number of subsystems increases, development of such rule-based strategies becomes less obvious. Optimization techniques performed upstream the standalone subsystems present a promising alternative in this situation. For ground vehicles, tires are the sole effectors. The command distribution is mainly constrained by tires' potential and actuators' limits. As far as coupled operations and therefore combined slip is concerned, the potential available for one

direction varies due to the other direction. This new tire model offers the possibility to take into account this phenomenon by varying the tires' stiffness in each direction when the tire is solicited both longitudinally and laterally. In addition, the overall potential may vary if the friction changes or the vertical load varies. For the first cause, this can happen for example if it starts raining and the road becomes wet. For the second cause, the vertical load can vary due to roll dynamics when turning, and pitch dynamics when braking or accelerating, which is more likely to happen. The new tire model depends on both the friction coefficient and the vertical load. The maximum efforts are then updated if μ and F_z are well estimated. To sum up, three reasons can lead to an overestimation of the tire potential and lead to a wrong control distribution: combined slip, friction change or vertical load variation. The new tire model takes into account these three phenomena in order to favour the tires with greater potential when distributing the commands, and optimize the vehicle's overall potential.

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Vehicle Steering and Suspension Kinematics/Compliance and Their Relationship to Vehicle Performance©



Gene Lukianov

Abstract This chapter integrates tire performance data, steering, suspension kinematics/compliance and weight transfer into a unified visualization of vehicle steady state performance. It discusses the tire force and moment convention that is generally used to define the tire contact patch forces and offers an alternative convention that clarifies the visualization of the forces acting upon the tire contact patch for suspension kinematic and compliance design. Wheel end architecture is examined considering different supporting link arrangements and a case is presented to consider rear wheel end caster angle as a significant contributor to vehicle dynamic performance. Vehicle kinematic and compliance design principles and engineering practices are presented for a integrated kinematic and compliance design solution. The consequences of roll center location and migration and inherent compliance characteristics due to kinematic design are discussed. Measured kinematic and compliance test data is presented and discussed. Vehicle weight transfer and weight transfer distribution due to mass properties, wheel rates and roll center locations is presented and overlaid upon tire lateral force vs. slip angle data to reveal tire (axle) pair and vehicle performance considering the grip utilized at each tire contact patch. The overlay of vehicle weight transfer onto tire data offers a fundamental methodology to examine the state and utilization of each tire during a cornering maneuver. Further it provides a basis of logical methodology to optimize corner loads and individual steering angles for fuel consumption efficiency or for maximization of cornering performance.

Keywords Aligning moment • Pneumatic trail • Lateral force • Slip angle • Friction and grip utilization • Caster angle • Suspension kinematics • Suspension compliance • Kinematic and compliance test data • Weight transfer and distribution • Steering ackermann • Steering angle optimization • Fuel consumption • Cornering performance

Vehicle Dynamics is the art and science of engineering wheeled vehicle movement. The specific project goal may be to maximize on-road performance, off-road mobility

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or comfort for general transportation while giving the operator a level of confidence and pleasure operating the vehicle. The science and practice of Vehicle Dynamics requires an understanding of many separate technical areas. Vehicle Dynamicists need to be conversant in the principles and then know how to aggregate them into a complete vehicle. Very subtle sensations and factors are present and must be developed “artistically” by an on-road vehicle development program.

Vehicle Dynamics engineering requires the understanding and application of knowledge regarding multiple vehicle components and subsystems. This session will discuss the first six items; through suspension springing. The other topics will be left for another time and for other publications:

- Tires and Steady State Cornering Performance;
- Suspension Kinematics;
- Suspension Compliance;
- Steering System Kinematics and Compliance;
- Suspension Damping;
- Suspension Springing;
- Brakes;
- Aerodynamics;
- Mass Properties;
- Power Delivery and Distribution;
- Vehicle Body and Frame Structural Stiffness and Deflections;
- Suspension, Powertrain and Vehicle Structural Vibrations;
- Human Factors.

This chapter will present practical Vehicle Dynamics methodologies based upon principles, measurements and physics. It will offer a perspective that clarifies principles presented in theoretical vehicle dynamics texts.

1 Introduction

Vehicle Dynamics involves three separate vehicle motion perspectives. The occupants are located inside the vehicle body which is suspended above the tires which are traveling on a global path. The driver observes both global and local vehicle motions. Additionally, the driver is moving relative to the vehicle body. Different parts of the body, torso, hips, shoulders, head, legs, feet, hands, ears and eyes all sense different inputs and provide feedback to the driver. Vehicle Dynamicists must engineer the vehicle motions and these individual observations into a cohesive whole.

The multitude of factors that comprise Vehicle Dynamic performance overload the measurement and analytical disciplines with data and calculations. Measurements and analysis cannot describe the human opinion of vehicle performance: “I love the way the car steers and handles”. Quantifying the factors is necessary, however combining the factors into a cohesive whole is done by experienced dynamicists with

feeling and passion. Attention to the immeasurable observations and sensations is part of the magic and challenge of Vehicle Dynamics.

This chapter will discuss suspension and steering kinematics and compliance. However to put kinematics and compliance into perspective other aspects of Vehicle Dynamics such as tires, weight transfer and tire utilization need to be discussed. Some prior fundamental knowledge of Vehicle Dynamics and suspension principles is assumed. The entire chapter could repeat definitions and basic knowledge and run out of space to present insights as to how to get it done! There are many texts covering the fundamentals. This document will attempt to offer practical in-depth knowledge.

This chapter is dedicated to Maurice Olley, the father of Vehicle Dynamics and the master whose work and illustrations continued to be used by all subsequent texts. Every Vehicle Dynamicist should study his work.

2 Tire Force Definitions

Vehicle Dynamic Engineering requires the understanding of the forces between the tires and the road surface. The tire contact patches are the only mechanical connections that exist between the vehicle and the road surface and are the primary sources of forces acting on the vehicle to determine its path at lower speeds. Aerodynamic forces that come into play at higher speeds will not be considered as part of this chapter.

In a hypothetical vehicle handling event: The driver initiates drive, brake or cornering forces and then modulates them to maintain the desired path. The vehicle reacts to the generation of control forces by following Newton's Laws of Motion. The vehicle motions can be grouped into global motions describing the vehicle's travel path and local motions; the movement of the vehicle body (sprung mass) relative to the tire patches and the vehicle path. The driver intends to produce the global motions, however the driver feels and observes both the global motions and the superimposed local motions of the sprung mass. Engineering both motions is necessary to produce admired, satisfactory vehicle performance and driver sensations.

Contact patch forces are reacted through a multitude of vehicle components such as tires, wheels, axles, suspension uprights, suspension links, springs and sway bars as well as dampers and suspension snubbers. Visualizing the forces generated between the tire and the road provides an excellent starting point to comprehend the challenges and logic of suspension kinematic and compliance design.

2.1 SAE Tire Convention

Historically, tire contact patch forces have been defined using SAE Convention J670E. This methodology utilizes three orthogonal forces acting upon the geometric center of the tire-ground contact and three moments, each acting around a coordinate

axis, to define the contact patch forces. The tire is conceptualized to be a simple disc contacting the ground, having a variable vertical load, steering angle, camber and pressure.

The force (F_y) generated perpendicular to the vertical/longitudinal (steering) plane is defined as the lateral force. The longitudinal force in the plane of the wheel is defined as (F_x). The vertical force perpendicular to the ground is defined as (F_z). The moment (M_z) generated around the vertical (Z) axis as the tire is steered and lateral force is developed is defined as the aligning moment. The tire needs to be steered more than the intended path in order to generate the expected lateral force. The difference between the steering angle and the actual direction of travel is defined as the slip angle. The moment (M_x) around the longitudinal (X) axis is defined as the overturning moment. The moment (M_y) around the lateral (Y) axis is defined as the rolling resistance moment (Fig. 1).

Using the SAE convention of forces and moments, tire cornering performance is characterized by plotting lateral force and aligning moment as functions of slip angle. A range of vertical forces bracketing curb load and representing weight transfer which occurs during cornering is used to develop multiple plots. Incremental camber angles and tire pressures can be included as additional variables.

The actual tire contact patch forces are not located at the contact patch geometric center and their locations are not fixed. The three moments are used to define the force locations relative to the tire-ground contact origin. The lateral tire force is

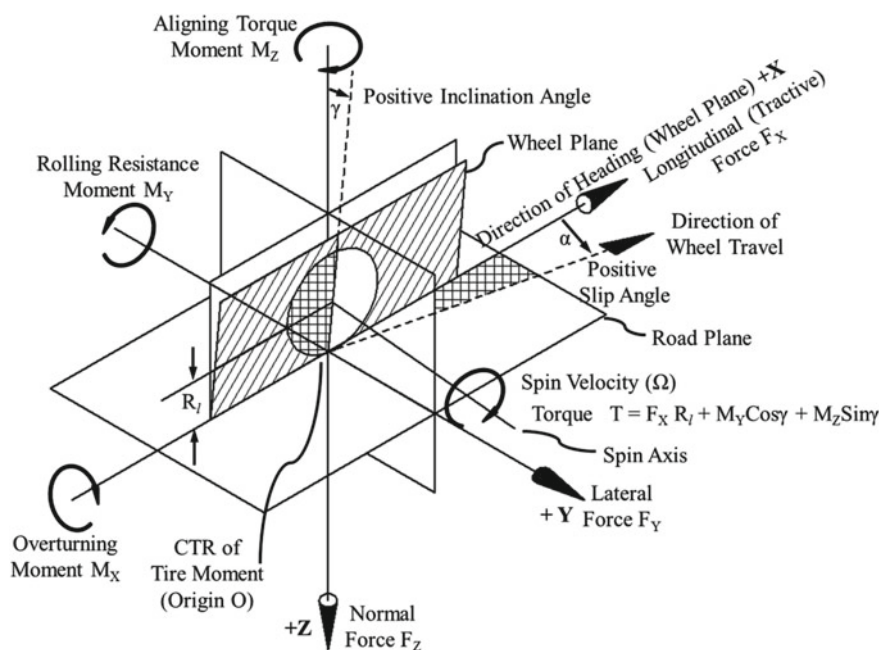


Fig. 1 SAE tire force and moment convention

initially located behind the geometric center of the contact path and moves forward as steering, lateral force and slip angle increase. This longitudinal offset distance is known as the ‘pneumatic trail’. Aligning moment is the moment resulting from the lateral force being multiplied by the pneumatic trail.

Aligning moment increases and then falls, Fig. 2, (even going negative) as steering, lateral force and slip angles increase. The reason is that as lateral force increases, pneumatic trail is diminishing. Since aligning moment is the multiple of rising lateral force and diminishing pneumatic trail, a rising/falling aligning moment characteristic is observed. When pneumatic trail moves in front of the geometric center the aligning moment reverses direction, becomes negative, and continues to progress until the tire is saturated (Figs. 3, 4).

For pneumatic tires used on passenger road vehicles, tire lateral force generated by steering is initially located approximately 35 mm behind the geometric center of the tire contact patch (at nominal curb weight). As the lateral force increases with steering, the pneumatic trail reduces towards zero following a consistent curve shape. As slip angle increases pneumatic trail falls roughly linearly. The pneumatic trail can cross the zero point and become negative at high cornering loads. Pneumatic trail increases with tire vertical loading and increases with lower tire pressures. The characteristic curve shape of the pneumatic trail versus lateral force and slip angle remain similar with changes of load, camber and pressure (Fig. 5).

The other two moments originate for similar reasons. The overturning moment is an artifact of the vertical force location moving laterally depending upon tire camber

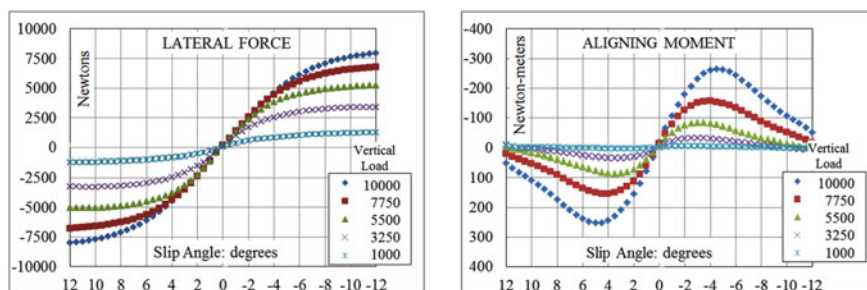


Fig. 2 Tire lateral force and aligning moment versus slip angle

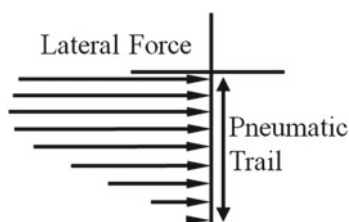


Fig. 3 Lateral force positioned by pneumatic trail

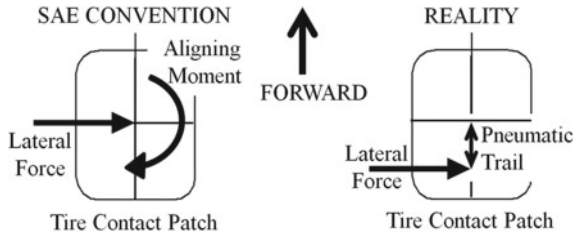


Fig. 4 Aligning moment compared to pneumatic trail

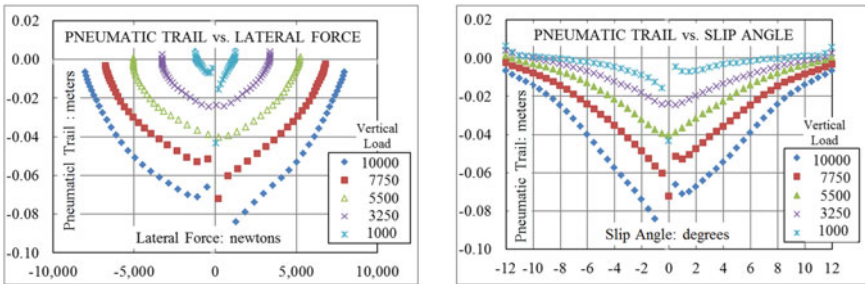


Fig. 5 Pneumatic trail versus lateral force and slip angle

and lateral forces. The rolling resistance moment is an artifact of the vertical force location being slightly ahead of the geometric center under free rolling conditions. The SAE Convention does not account for the longitudinal force moving laterally off center: The lateral location of the longitudinal force is positioned at the tire geometric center.

The SAE methodology is accommodated in analytical suspension calculations. It is also compatible with tire force testing machines that measure forces through load cell arrangements. The conceptualization of practical mechanical suspension solutions is impaired by this depiction using three forces and three moments.

2.2 Tire Contact Patch Dynamics

The tire contact patch shape is not constant. It changes shape depending upon the contact patch forces and the orientation of the tire to the road surface. The contact patch forces move longitudinally and laterally across the contact patch depending upon the operating conditions and the geometric orientation of the tire relative to the ground. The dynamic contact patch between the tire and road surface is difficult to quantify. It is possible however to infer contact patch characteristics from observations of tires in action.

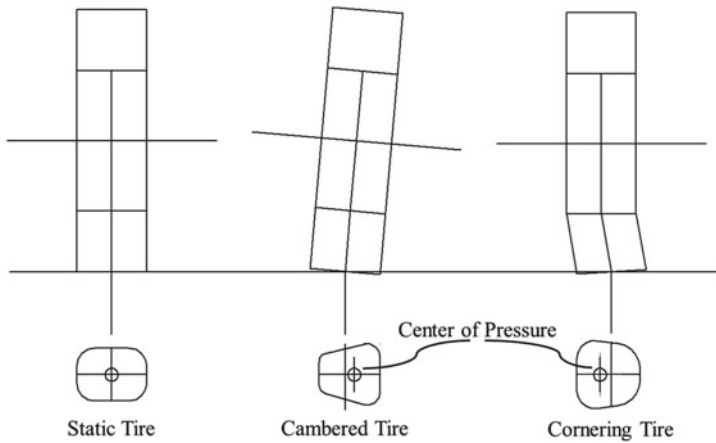


Fig. 6 Tire contact patch deflection

At rest and zero camber the contact patch would be symmetrical laterally and longitudinally. The patch shape can be observed to be roughly oval or rectangular with rounded corners. The tire patch enlarges with higher vertical loads and with lower tire pressure.

The contact path transitions to a rounded trapezoidal shape with increasing tire-road orientation angle and with the increasing lateral force. The lateral location of the vertical force between the tire and the road surface can be expected to move to the center of pressure (Fig. 6).

2.3 Alternative Tire Force Convention

The SAE tire force model can be simplified by eliminating the three moments and replacing them with three orthogonal tire forces each appropriately located as calculated from the moments measured:

Lateral force location. The longitudinal location of the lateral force is established by the pneumatic trail calculated from the aligning torque and lateral force measurement.

Vertical force location. The lateral offset of the vertical force can be calculated considering the overturning moment. The longitudinal location of the vertical force solution is difficult to determine because drive and brake torques mask the rolling resistance moment. From a vehicle dynamics point of view the longitudinal location of the vertical force is not critical. An assumption can be made that the longitudinal location is longitudinally coincident with the tire patch geometric center.

Longitudinal force location. The lateral location of the longitudinal force is important because it generates steering moments. The solution is confounded by the

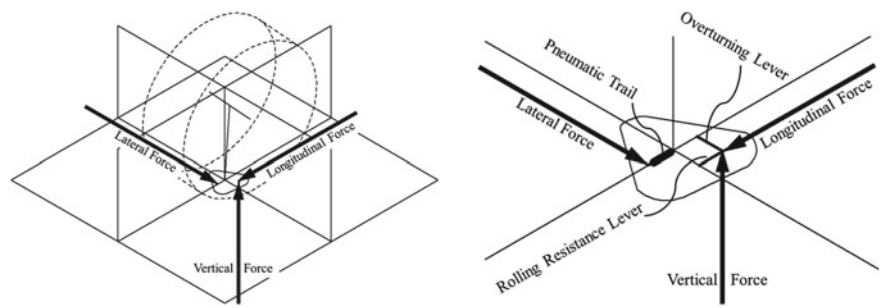


Fig. 7 Tire contact force locations. (free rolling condition)

Table 1 Tire contact force locations and lever equations

Tire contact patch force locations			
	Orientation	Lever arm	Lever equation
Lateral force	Longitudinal	Pneumatic trail	$\frac{\text{Aligning moment}}{\text{Lateral force}}$
Vertical force	Lateral	Overturning lever	$\frac{\text{Overturning moment}}{\text{Vertical force}}$
	Longitudinal	Rolling resistance lever	$\frac{\text{Rolling resistance moment}}{\text{Vertical force}}$
Longitudinal force	Lateral	Overturning lever	$\frac{\text{Overturning moment}}{\text{Vertical force}}$

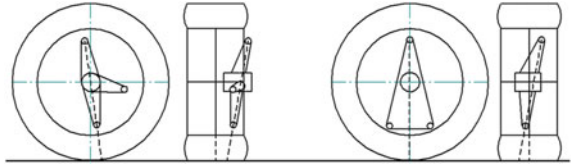
presence of the aligning moment due to the lateral force. An assumption can be made that the lateral location of the longitudinal force is approximately coincident with the tire’s contact patch center of pressure. The center of pressure can be assumed to be coincident with the lateral offset of the vertical force as defined by the overturning moment (Fig. 7, Table 1).

The use of three forces and three moments causes confusion when engineers attempt to visualize wheel end and suspension solutions. Three forces moving predictably relative to the contact patch geometric center can clarify the understanding and simplify the conceptualization of suspension solutions. This methodology simplifies the visualization of suspension architecture solutions. This alternative three force concept will be used throughout this course.

3 Wheel End Architecture

The “wheel end” of an independent suspension can be defined as the sub assembly outboard of the suspension arms. It consists of the tire/wheel/stub axle and suspension upright. It includes the brakes if they are outboard. The wheel end is connected to the vehicle via suspension control arms or links. Both ends of a vehicle have their own wheel end architectures. They are generally symmetrical left to right (Fig. 8).

Fig. 8 Front and rear wheel end architecture



The wheel end is constrained laterally and longitudinally via suspension links. Vertical constraint is provided by some combination of springs, sway bars and suspension snubbers (Fig. 9).

A vehicle wheel end experiences three input forces at the tire patch: (1) Vertical forces due to vehicle weight, weight transfer and road imperfections; (2) lateral forces due to cornering and; (3) longitudinal forces due to acceleration, braking and obstacles.

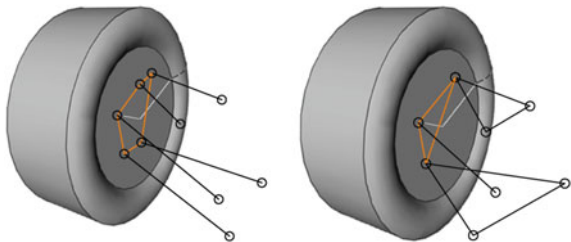
All suspension components deflect under load, some intentionally (e.g. rubber isolators), and some apparent as deflections due to load and strain in the 'rigid' components. The aggregate wheel end kinematics and compliance is the sum of component motions and component deflections.

The magnitude of steering compliance is significantly less than driver steering inputs. Compliance steering values range from hundredths of a degree up to about a degree at maximum load. The values are small but their effects on vehicle dynamic characteristics are significant. All vehicles have a level of steering due to suspension compliance. Race cars with hard joints and road vehicle wheel ends all steer due to compliance.

Front wheel ends have an obvious steering axis defined by the ball joints. Rear wheel ends can be visualized to have a virtual axis around which the wheel end steers.

If the contact patch force is not in line with the steering or virtual steering axis, a moment around the axis is present and steering compliance develops. The location and direction of the lateral and longitudinal contact patch forces relative to the steering axis determine if the wheel end will 'toe in' or 'toe out'. The magnitude of wheel end steering compliance depends upon the length of the moment arms and magnitudes of the input forces, the geometry of the supports providing the reaction forces, and the compliance of the supporting components.

Fig. 9 Wheel end supporting linkage



The direction of steering compliance is critical. If it is an understeering compliance, vehicle dynamic stability is enhanced. If it is an oversteering compliance, stability deteriorates.

3.1 Front Wheel End Architecture

A conventional front suspension wheel end includes upper and lower ball joints and a steering arm joint. The front wheel end architecture has a steering axis that passes through the upper and lower ball joints. It has a caster angle, caster trail, a kingpin angle and a scrub radius describing the steering axis position and orientation relative to the contact patch (Fig. 10).

Front suspension architectures have positive caster placing the steering axis consistently in front of the lateral contact force location thereby producing a reliable understeer moment.

The side view location of the third support point, the outer tie rod end, influences the compliance steer character of a front suspension. The tie rod introduces an apparent stiffness at the end of the steering arm when steering forces are high and deflection is minimized. The apparent stiffness at the tie rod end modifies the location of the steering axis from a purely geometrically determined location to a location defined by geometry and compliance. A biasing axis can be established by defining an axis between the upper ball joint and the outer tie rod location. This biasing axis can increase understeer or contribute to oversteering tendencies depending if it generates an understeering or oversteering moment (Fig. 11).

An outer tie rod location in front and below the axle (low front steer) or in back and above the axle (high rear steer) biases the compliant steering axis forward, increasing understeer. An outer tie rod location behind and below the axle (low rear steer) biases the compliant steering axis backwards, reducing understeer. Compliance

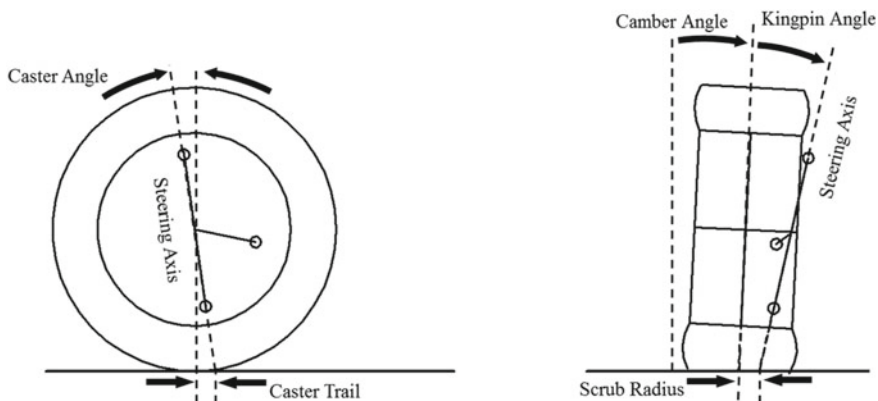


Fig. 10 Front wheel end steering axis definitions

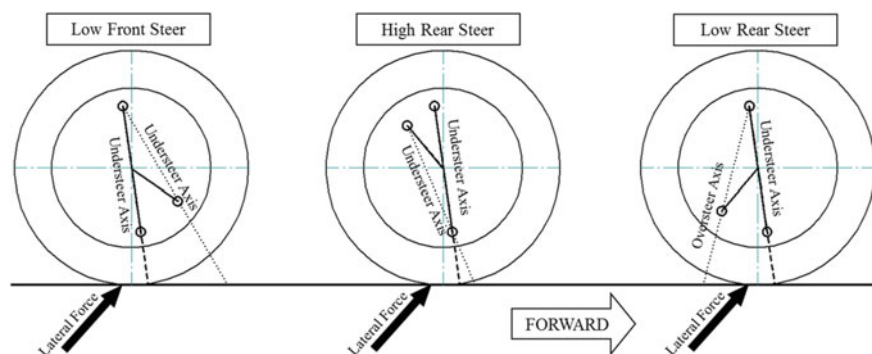


Fig. 11 Front wheel end steering axis bias due to steering arm

testing reveals that a low rear steer architecture sometimes develops compliance oversteer at high steering forces. Special design features such as a rubber isolated cradle are used to counter this architectural oversteer characteristic.

3.2 Rear Wheel End Architecture

The rear suspension of a vehicle generally does not include a steering mechanism but it too has steering compliance due to contact patch forces, wheel end geometry and component deflections. Rear wheel end architecture compliance can be examined using similar methods as used for front wheel end architectures.

Three simple two force cross car links are necessary to laterally constrain a wheel end. Longitudinal support is provided by longitudinal links or triangulation of the lateral links turning them into A-arms. Wheel end architectures with less than three lateral links and two longitudinal links require some links to support multiple forces and moments.

The lateral supports can be visualized as springs allowing deflection at each location. The stiffness of each spring is equal to the total stiffness of that lateral support (Fig. 12).

A steering axis can be visualized in the architecture if two lateral links are stiff and one is soft. The location of the virtual steering axis is bounded by the side view geometrical arrangement of the lateral supports. The location of the virtual steering axis between the bounds is determined by the stiffness of the links. The bounds can be classified as understeering or oversteering contributors depending upon their position relative to the location of lateral contact patch force and the direction of the steering moment generated (Fig. 13).

Virtual steering axis. As the side view geometry is rearranged or the stiffness of the supports is revised the virtual steering axis is relocated and the ground intersection point of the steering axis can be moved in front or behind the lateral contact patch

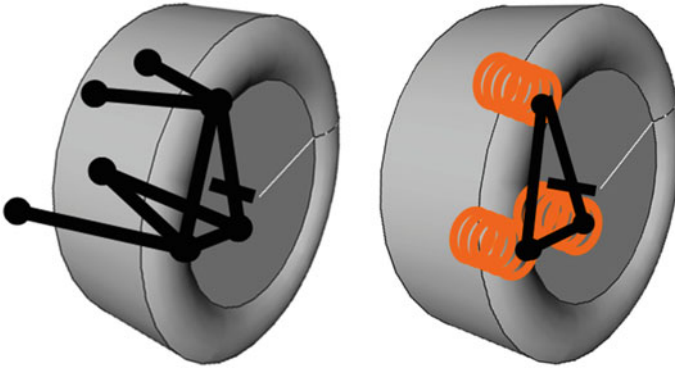


Fig. 12 Rear wheel end lateral compliance model

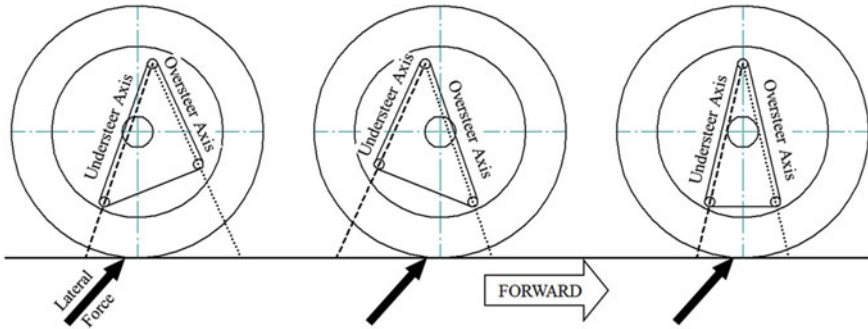


Fig. 13 Rear wheel end virtual steering axis bounds

force location range. The resulting steering compliance can be engineered to be either an understeering or an oversteering compliance. The virtual steering axis will migrate toward a stiffer support and away from a softer support (Fig. 14).

The side view virtual steering axis effectively defines a rear suspension ‘caster’ and ‘caster trail’ (Fig. 15).

The front view location of the virtual steering axis is determined by the positions of the longitudinal supporting forces. The front view also establishes the scrub radius and ‘king pin’ angle (Fig. 16).

A multilink rear suspension with virtual instant centers would need to have the instant centers between pertinent link pairs reduced to three lateral and two longitudinal supports. Once resolved to virtual locations, the bounds of the geometry’s virtual steering axis can be established.

Contact patch forces passing through the virtual steering axis at ground level would not induce any moment around the axis and therefore would not contribute to steering compliance. The intersection of the neutral axis at ground level can be determined by compliance testing a vehicle wheel end at various positions of longitudinal and

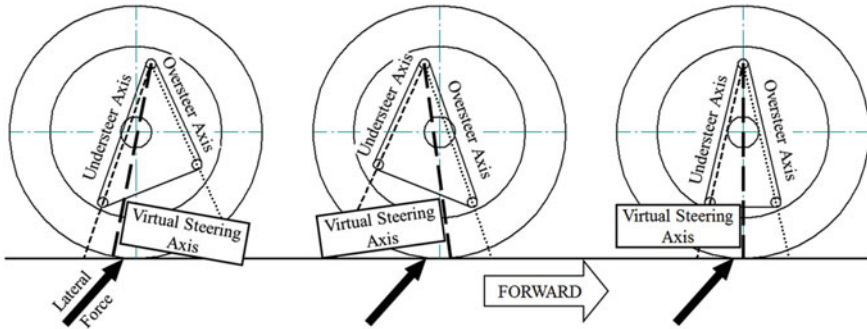
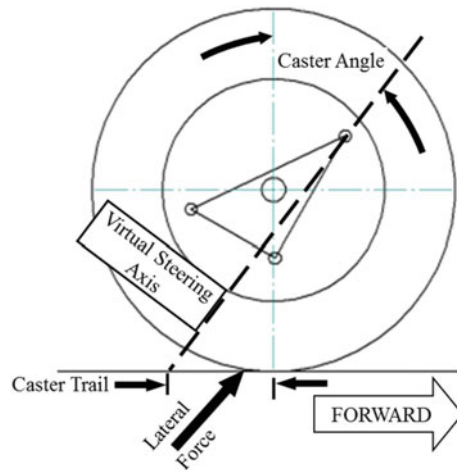


Fig. 14 Rear wheel end virtual steering axis locations

Fig. 15 Rear wheel end negative caster and caster trail



lateral force then plotting the results to determine the location of the virtual steering axis ground intersection point.

Absolute compliance steer values are not established by geometric design, however the direction of steering compliance and the relative magnitude is determined. Compliance steering, as a result of lateral and longitudinal contact patch forces, is a critical factor in acceptable vehicle dynamic behavior (Table 2).

4 Suspension Kinematics

Suspension kinematics establish the geometric motion relationships of the wheel ends to the body and ground as a result of suspension travel, roll, pitch, and steering.

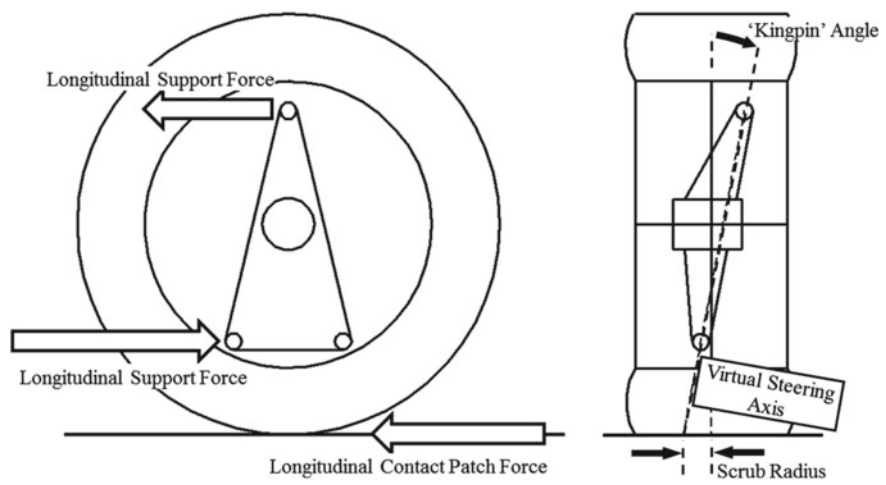


Fig. 16 Rear wheel end ‘kingpin angle’ and scrub radius

Table 2 Steering consequence of rear suspension virtual steer axis location

Rear compliance steer direction			
	Virtual steering axis location relative to force location	Direction of steering compliance: outer, loaded tire	
Lateral force	In front	Oversteer (both tires)	
	Behind	Understeer (both tires)	
Longitudinal force	Inside	RWD under acceleration: understeer	Under braking: oversteer
	Outside	RWD under acceleration: oversteer	Under braking: understeer

The study of suspension kinematics needs to include historic perspectives of other vehicles (competitive comparisons), past personal experience and general vehicle knowledge. Multiple factors and viewpoints need to be considered. There is no perfect solution, but there are excellent solutions. Trade-offs between characteristics are part of the art. The vehicle kinematic solution needs to be one that delivers expected performance with minimal sacrifice or unexpected consequences. Often a factor that was missed or not prioritized appropriately becomes a key reason for unusual behavior or idiosyncrasy in a vehicle’s dynamic behavior. This issue could have been avoided without compromising other characteristics if it was recognized early in the vehicle design process and corrected.

Suspensions are usually variations of four bar linkages. The suspension is assumed to be constrained by non-compliant linkages with the geometric motions determined by linkage movement. Kinematic geometry is not an exact representation

of suspension behavior. Recognition of the assumptions being made, the impact of the assumptions and their relationship to actual behavior is needed.

Suspension kinematics is best studied with simple techniques. Ultimately the suspension is a 3D system; however it is clearer to study the suspension in 2D, from multiple viewpoints, using simple tools and methodologies to gain an understanding of the mechanisms involved. Tools such as pinned and moving cardboard cutouts are a great way to visualize the mechanical action of a suspension. A 2D CAD model makes it easier to visualize instant centers and angles than a 3D CAD model where it's difficult to keep the information clear. Graphical representations are much better for comprehending suspension kinematics and opportunities than numerical tables.

An experienced dynamicist needs to be familiar with the directional performance impact of every suspension characteristic. Will a change contribute to understeer or oversteer? Will the vehicle gain or lose response? Keeping all the factors in mind allows a dynamicist to start organizing a budget of contributors.

Suspension kinematics and compliance need to be examined at a component level and as an assembly to visualize and comprehend the influence of each on wheel end performance. Kinematic and compliance reactions to longitudinal drive and brake forces as well as cornering forces need to be considered. Kinematic engineering makes a number of assumptions:

- The results are valid but not perfectly accurate;
- Instant centers determined are not exact; suspensions with rubber bushings have small shifts in pivot centers due to deflection;
- Components are rigid;
- Roll centers are approximate;
- Roll is assumed symmetrical;
- Scrub radius is approximate;
- Force calculations are approximate;
- Nothing is perfect.

Design decisions regarding wheel end architectures, roll center heights, roll axis elevation and inclination and camber gains need to be made early in the design process, then the front and rear suspension design can proceed. As work progresses points to keep in mind are:

- Roll centers may not be stable. They can move vertically and laterally. Since the roll centers move, the roll axis also moves in space. Lateral motion of the roll centers results in roll axis skewing and unsymmetrical sprung mass motions. Stable roll centers can be achieved with good kinematic design;
- Kinematic roll center solutions are approximate as are 'forced based' roll center solutions. The kinematic solution makes the assumption that roll is symmetrical. Force based solutions make an assumption regarding the allocation of left and right contact patch forces;
- The scrub radius is dynamic. Tire camber and road inclination move the tire center of pressure location off the center of the tire changing the realistic scrub radius lever arm;

- It is efficient to package the wheel end first before connecting it to the vehicle frame via suspension linkages;
- Design activity will be interactive and repetitive until a reasonable solution from all perspectives is accomplished or design time runs out;
- Analysis cannot fully quantify vehicle performance. On road tuning will be necessary to optimize vehicle dynamic performance.

Different suspension architectures naturally have characteristic kinematic and compliance properties. Solid axle, twist beam axle, MacPherson strut and short-long arm architectures all have their inherent unique kinematic and compliance characteristics. Component configurations also influence compliance characteristics.

Suspensions should generally not be over-constrained. Over-constrained suspension linkages require assumptions to be made as to which links change length to allow motion. Over-constrained linkages require flexible isolators with enough deflection to allow link length changes. An over-constrained suspension can result in significant link loads and component stress resulting in unexpected failures. The over-constrained forces are not calculable by analysis of vehicle road loads. The forces can be calculated by considering the suspension motion, determining the link length changes necessary to allow motion and then calculating the link forces based upon the stiffness and length change of each component. The link forces due to over constraint are additive to road loads forces.

Kinematics is best studied graphically from three orthogonal vehicle views: Front View, Side View, and Top View. Each view provides an understanding of a portion of the whole system. The interaction between directions is generally of secondary importance so it is safe to not be concerned with interactions when initially designing a suspension (Fig. 17).

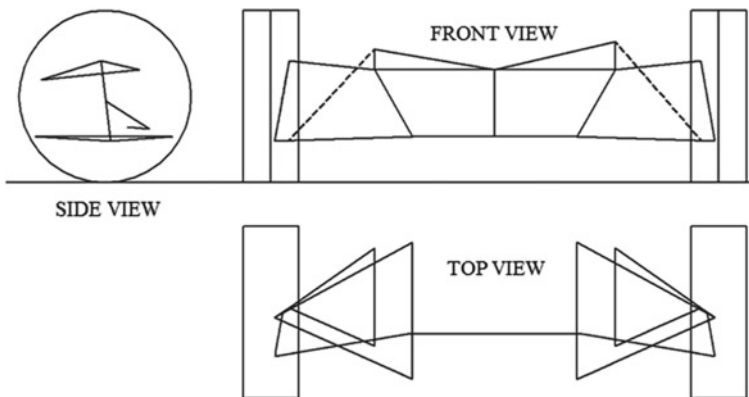


Fig. 17 Suspension kinematic design views

4.1 *Front View Kinematics*

The front view design should be the first kinematic task accomplished. The front view establishes the following suspension characteristics:

- Front view wheel end architecture;
- Steering axis inclination and static scrub radius;
- Brake rotor, caliper and bearing packaging within the wheel;
- Ball joint spacing and positioning;
- Outer tie rod location;
- Static camber;
- Front view packaging of the wheel end, control arms and suspension attachments;
- Front view steering arm, tie rod and steering rack location;
- Control arm instant centers, camber and track change with travel;
- Roll center location and migrations with travel and roll;
- Suspension jacking vector angles;
- Partial solution for toe curves;
- Spring and damper positioning and motion ratio.

The graphical front view clearly reveals the tradeoff relationships between geometry factors such as: roll center height versus track change and aggressiveness of camber change versus track change and roll center height. Suggestions regarding design practices are offered:

- Design for vertically stable roll center relative to the sprung mass (possible), not to ground (impossible);
- Design for laterally stable roll centers in roll;
- Maximized control arm lengths;
- Start with a ratio of 0.62 for the upper control arm length relative to the lower control arm in order to stabilize roll centers;
- Camber change can only compensate for about 35% of body roll before camber change in ride becomes excessive. It is more efficient to reduce roll using sway bars, springs and suspension snubbers;
- The span between upper and lower ball joints is important to minimize camber compliance. Camber stiffness is a function of ball joint span squared;
- Spring and damper effectiveness should be maximized;
- The placement of the steering rack and tie rod length for appropriate bump steer is approximated in the front view. Zero bump steer is rarely an ideal solution. Design in rack vertical position adjustability for corrections and tuning;
- Rear suspensions should be designed the same way as front suspensions. The rear design should consider 'virtual' caster, caster trail, scrub radius and a virtual steer axis to support compliance design (Figs. 18 and 19).

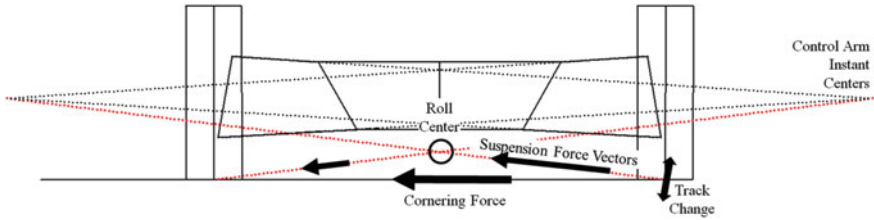


Fig. 18 Suspension front view design factors

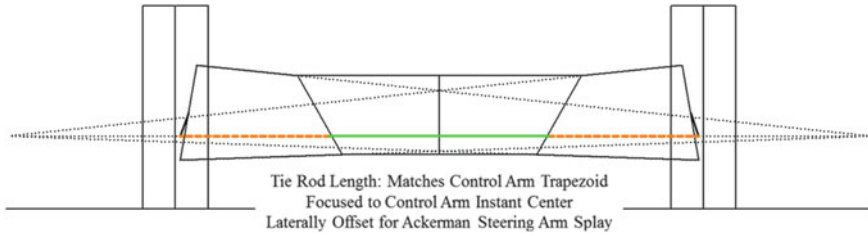


Fig. 19 Front view bump steer solution

4.2 Side View Kinematics

The kinematic side view design of the wheel end is critical to establish lateral force compliance steer characteristics (Fig. 20).

The side view design determines the following design characteristics:

- Side view locations of the lateral links which establish the camber and steering compliance characteristics due to lateral contact patch forces. Side view span between the supports is a significant factor for camber and steering compliance stiffness. Angular stiffness is a function of span squared;
- Design range of caster angle and caster trail prior to refinement due to compliance decisions;
- Side view spindle offset relative to the steering axis;

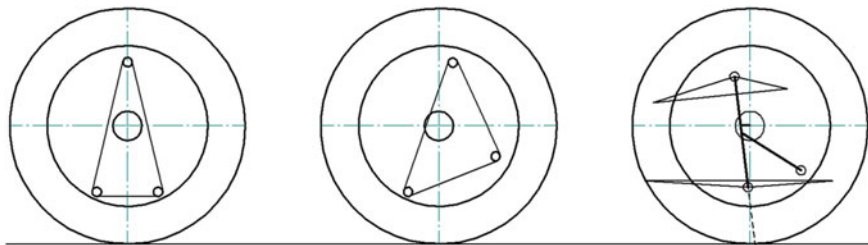


Fig. 20 Wheel end architecture side views

- Steering arm lever length;
- Suspension anti-dive, anti-lift and anti-squat;
- Axle kinematic recession or procession.

Suggestions regarding the side view design are discussed below:

- Steering forces are much easier to comprehend by considering the locations of the longitudinal and lateral forces multiplied by the leverage of each force around the steering axis than summing forces and tire moments. The left and right side Free Body Diagrams (FBD) are different when cornering;
- Side view spindle offset reduces caster trail lateral force leverage while maintaining a high caster angle;
- Caster in unison with front view king pin angle adds negative camber to the outer tire and positive camber to the inside tire as the tires are steered. This advantageous change in camber angle compensates to a degree for body roll.

Procession and recession. Side view inclinations of the control arms determine the longitudinal kinematic wheel end motion when encountering road disturbances. A bump on the road initiating a suspension jounce event has more impact on ride comfort than a rebound event. A suspension that allows the wheel end to move rearward (recession) during jounce motions provides increased ride comfort than a procession geometry (Fig. 21).

Anti-dive, anti-lift and anti-squat factors. During acceleration and deceleration control arm side view inclinations can produce vertical suspension forces which are transmitted to the sprung mass. These ‘anti-forces’ can provide a level of support to the sprung mass reducing sprung mass pitch and suspension deflections as weight transfer occurs. Geometric examinations of side view kinematics provide the percentage of weight transfer that is supported by the ‘anti’ factors.

Brake anti-force vectors (for outboard brakes) originate at the tire contact patch and focus at the suspension side view instant center. For acceleration events the anti-force vectors originate at the axle for independent suspensions and at the tire contact patch for solid axles, focusing at the suspension side view instant center. If acceleration or braking forces are generated at both ends of the vehicle, the longitudinal

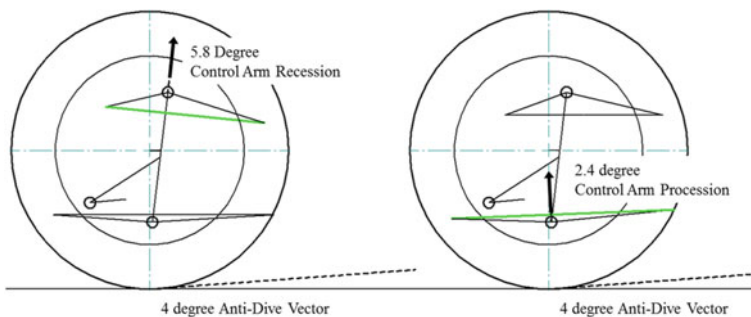


Fig. 21 Control arm recession and procession

forces need to be allocated according to drive and brake system force distributions (Fig. 22).

The front anti-dive percentage for a vehicle with outboard front brakes would be:

$$\text{Front anti dive percent} = 100 \times \frac{(\text{front brake proportion} \times \tan \theta_f)}{\left(\frac{\text{CG height}}{\text{Wheelbase}}\right)} \quad (1)$$

The anti-squat percentage for a rear drive vehicle with an independent suspension would be:

$$\text{Rear anti squat percent} = 100 \times \frac{(\tan \theta_r)}{\left(\frac{\text{CG height}}{\text{Wheelbase}}\right)} \quad (2)$$

Similar equations can be constructed for rear anti-lift while braking and front wheel drive front anti-lift during acceleration.

4.3 Top View Kinematics

Top view kinematic geometry establishes the following characteristics:

- Ackermann steering geometry;
- Geometric control arm proportions;
- Control arm force distribution;
- Control arm and wheel end recession character due to compliance;
- Toe change due to longitudinal forces and component compliance.

Steering geometry. Ackermann geometry is established by projecting the top view steering arm vector to the intersection with the opposite arm. If the intersection is coincident with the rear axle, 100% Ackermann is approximated. An intersection between the front and rear axle implies that Ackermann is greater than 100%. An intersection behind the rear axle implies less than 100% Ackermann. An intersection

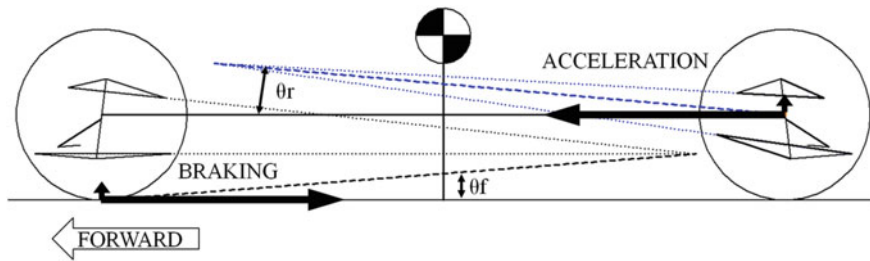


Fig. 22 Suspension longitudinal and 'anti'-force vectors

in front of the front axle implies reverse Ackermann. The steering rack needs to be behind the outer tie rod ends to maintain consistent Ackermann throughout the steering range. Ackermann is discussed in detail within the steering section (Fig. 23).

Control arm forces. Wheel end compliance is an aggregate of local component kinematics, component forces and component stiffness. Local component motion and compliance need to be considered while designing the suspension kinematics.

The control arm top view illustrates the resolution of forces on a control arm. Once the upper and lower ball joint forces are calculated, the reaction forces resulting from a longitudinal force at the ball joint is calculable knowing the length and width of a control arm. The total longitudinal reaction force is calculable, while the division between the two supports depends upon the relative stiffness of the two supports (Fig. 24). The problem can be fully calculated if the control arm is comprised of separate two force links with a ‘virtual’ ball joint. Then the stiffness of the supports is no longer a factor in the force distribution calculation:

$$\sum \mathbf{M}'\mathbf{0}' = \mathbf{0} \quad IF_x \times L = R_{y1} \times W \quad (3)$$

$$\sum F_y = \mathbf{0} \quad R_{y1} + R_{y2} = 0 \quad (4)$$

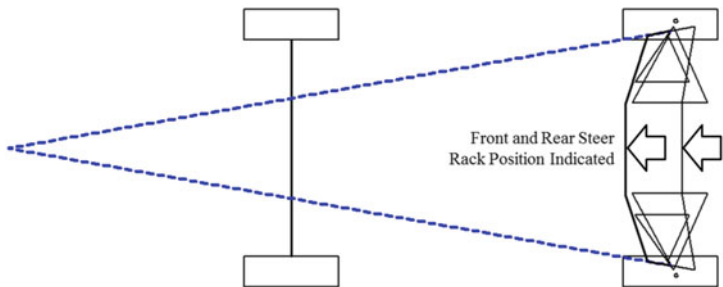
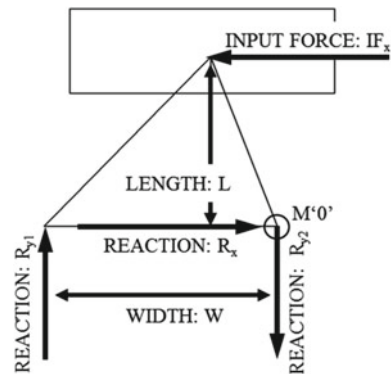


Fig. 23 Ackermann steering solution

Fig. 24 Control arm reaction forces



$$\sum \mathbf{F_x} = \mathbf{0} \quad IF_x = R_x \tag{5}$$

Control arm compliance. A comparison of L-shaped and A-shaped control arms illustrates the kinematic and compliance consequences of component motions. Control arm longitudinal compliance character is bounded by arm geometry, not by bushing stiffness. L-shaped and A-shaped control arm configurations offer different kinematic and compliance possibilities.

The kinematic control arm compliance window can be bounded by considering the possible motions of soft front/hard rear and hard front/soft rear radial rate bushing combinations for an arm experiencing a longitudinal force at the ball joint. The control arm is visualized to rotate around the hard bushing and translating due to longitudinal stiffness (Fig. 25). The ball joint of a lower L-arm will inherently tend to rotate outwards and backwards under braking forces:

The ball joint of a lower A-arm can rotate outwards or inwards depending where the soft and hard bushings are located (Fig. 26).

The control arm compliance motion in concert with steering linkage motion generates a suspension toe change under braking. Front suspension toe out is recognized as a stabilizing (understeer) characteristic during a brake and turning event. Front

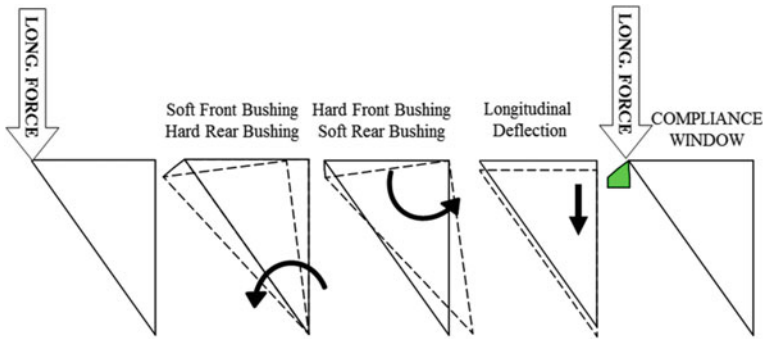


Fig. 25 L-arm deflection bounds and compliance window

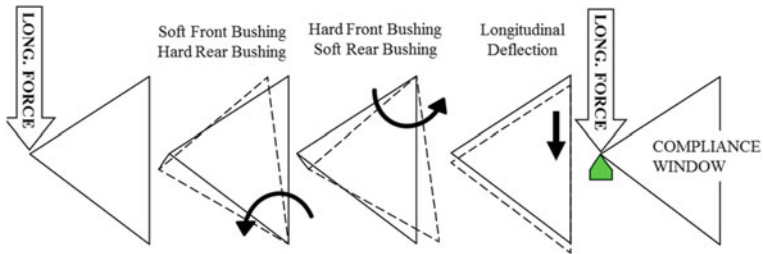
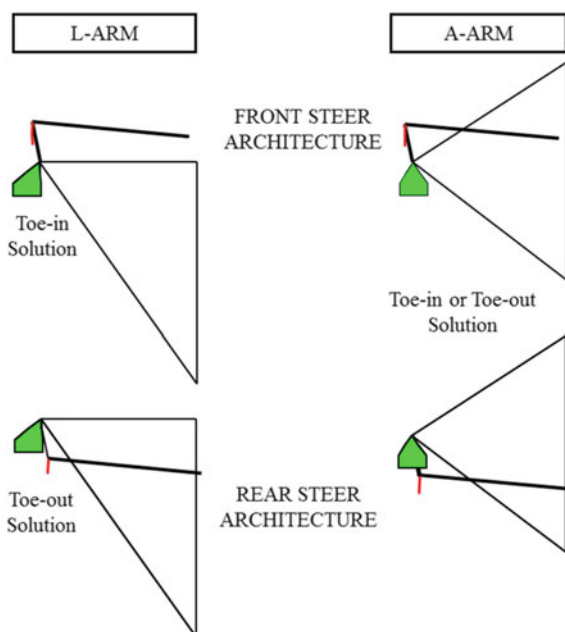


Fig. 26 A-arm deflection bounds and compliance window

Fig. 27 L-arm and A-arm compliance window



suspension toe in under braking is considered an un-stabilizing (oversteer) characteristic. The vehicle dynamic influence of rear suspension toe under braking is opposite the front.

Visualizing the motion of the outer tie rod relative to the ball joint, the L-arm in conjunction with a front steering system produces a toe-in under braking solution. If the steering is a rear steer system the combination will produce a toe-out under braking solution window. The A-arm suspension can produce either a toe-in or a toe-out solution regardless of front or rear steer architecture. The toe direction for the A-arm geometry will depend upon the relative radial stiffness of the front and rear bushings. The compliance tuning opportunities with the A-Arm are broader than with the L-arm (Fig. 27).

The compliance character of the upper control arm and the vertical position of the steering rack between the lower and upper control arms will also contribute to the complete toe compliance solution.

4.4 Suspension Motion Ratio

Suspension motion ratios are used to quantify the effectiveness of a spring or damper within the suspension architecture.

Motion ratio can be defined as spring travel divided by wheel travel or the reciprocal. It is best to make sure that the relationship being discussed is clearly defined.

Often in vehicle dynamics discussions, different terminologies are used by individuals. Attempts to fully normalize definitions are futile, so it's best to assure that all parties understand the properties being discussed. A motion ratio of 1:1 provides a spring and shock absorber deflection equal to the wheel end travel. A motion ratio of 0.5 provides a spring deflection of 50% of wheel end deflection.

Numerous formulas for motion ratio are presented in technical literature. Often these calculations are simplified and inaccurate, poorly representing the mechanism.

A popular method uses the horizontal distance of the lower control arm, the spring lower mount horizontal distance, and the cosine of the spring angle. This equation is not accurate (Fig. 28).

$$\text{Popular Motion Ratio} = \cos(\text{Spring Angle}) \times L1/L2 \tag{6}$$

An accurate kinematic solution is possible if the suspension geometry is diagramed. The front view instant center distance and the perpendicular spring moment to the instant center are used to compare the moments around the instant center (Fig. 29):

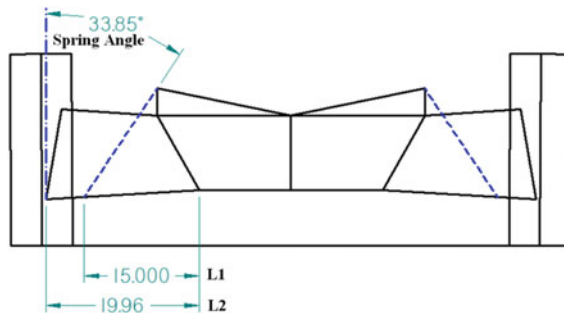


Fig. 28 Popular motion ratio calculation

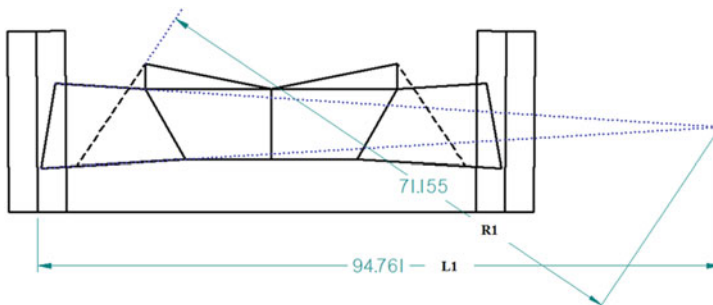


Fig. 29 Method 1: Kinematic motion ratio calculation

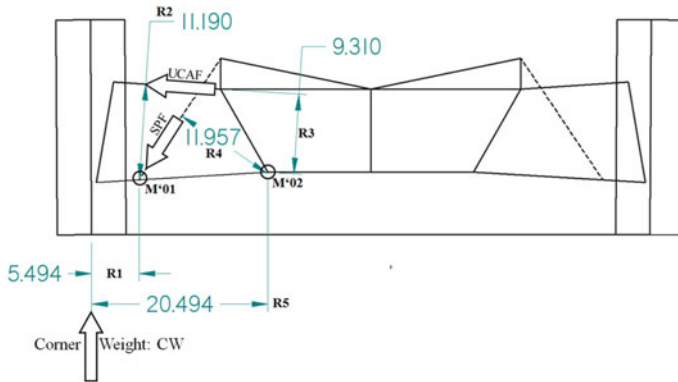


Fig. 30 Method 2: Dimensional equivalent of kinematic motion ratio calculation

$$\text{Motion Ratio Method 1} = R1/L1 \quad (7)$$

An alternative method relies upon the measurement of three perpendicular distances and two horizontal distances. This method considers the moments around the lower spring mount and lower control arm inner pivot to accurately calculate the motion ratio (Fig. 30). It duplicates the prior Method 1 presented:

$$\sum M'01 = 0 \quad \text{Corner weight} \times R1 = UCAF \times R2 \quad (8)$$

$$\sum M'02 = 0 \quad UCAF \times R3 + SPF \times R4 = \text{Corner weight} \times R5 \quad (9)$$

$$\text{Motion Ratio Method 2} = CW/SPF = R4/(R5 - (R1 * R3/R2)) \quad (10)$$

The three methods are compared for accuracy:

- Popular Method: $\text{Cosine}(33.85) * 15/19.96 = 0.62$;
- Method 1: $71.155/94.761 = 0.75$;
- Method 2: $11.957/[20.494 - (5.494 * 9.310/11.190)] = 0.75$.

The popular method is erroneous by seventeen percent in this example.

4.5 Kinematic Measurements

Suspension kinematics (and compliance) characteristics are measured on a full vehicle test bench machine such as the Anthony Best Kinematic and Compliance Test Machine at Morse Measurements of Salisbury, North Carolina (Figs. 31 and 32).



Fig. 31 Anthony Best Kinematic and Compliance measurement machine



Fig. 32 Vehicle mounted on K and C machine

Table 3 Kinematic test matrix

Bounce with sway bars
Bounce without sway bars
Roll with sway bars
Roll without sway bars
Steering kinematics

A kinematic and compliance measurement machine can move and load a vehicle measuring the vehicle suspension motions and tire patch loads while remaining in an indoor laboratory environment. The machine can translate, roll and pitch the sprung mass and introduce forces and moments at each tire patch while measuring the wheel end motions in six degrees of freedom (lateral, longitudinal, vertical, and rotationally around each axis). Such a machine can provide measurements of vehicle suspension characteristics or simulate complex vehicle motion and loading scenarios.

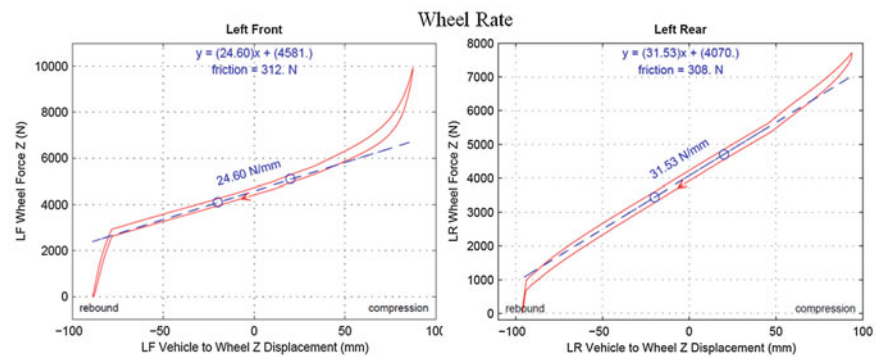
Results for repetitive test cycles are plotted graphically in 2D. A slope is determined across center to determine an on-center coefficient. Test regimes have been standardized; however additional custom combinations can be easily executed and plotted.

Kinematic tests consist of vehicle bounce and roll motion tests. Steering system kinematic tests examine the rotational relationships between the steering wheel and the wheel ends.

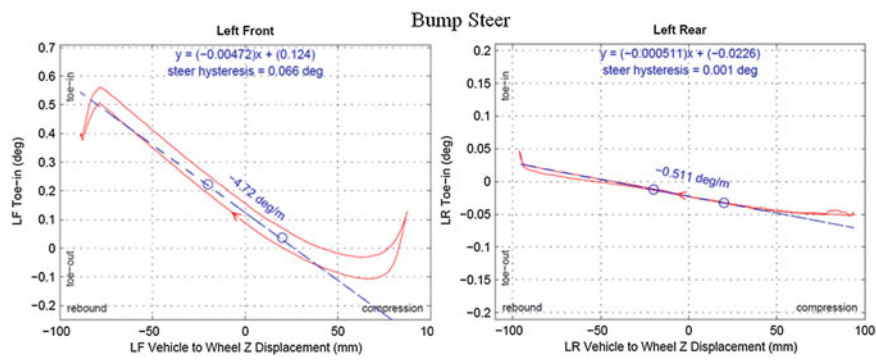
Bounce tests involve moving the vehicle sprung mass vertically and measuring the vertical loads and wheel end motions. Roll tests involve rolling the vehicle around the roll axis using a running measurement of corner loads and adjusting vertical vehicle displacement to maintain the initial axle loads. Both bounce and roll tests are executed with and without sway bars so that the sway bar contribution can be isolated (Table 3).

4.6 Kinematic Test Data

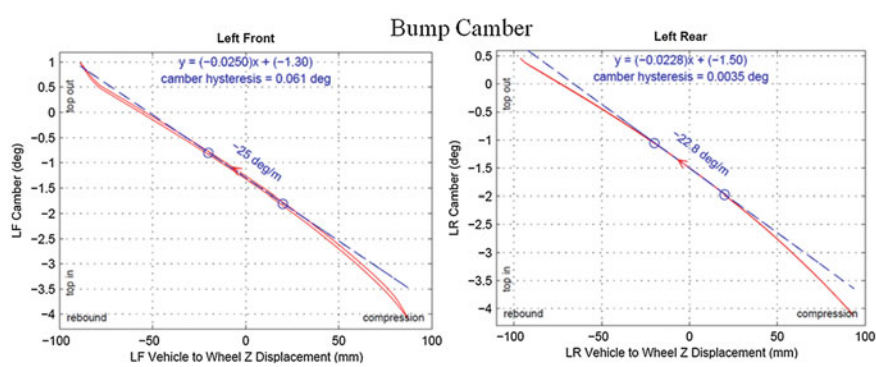
Sample kinematic test data plots with discussion points follow:
Test data courtesy of Morse Measurements.



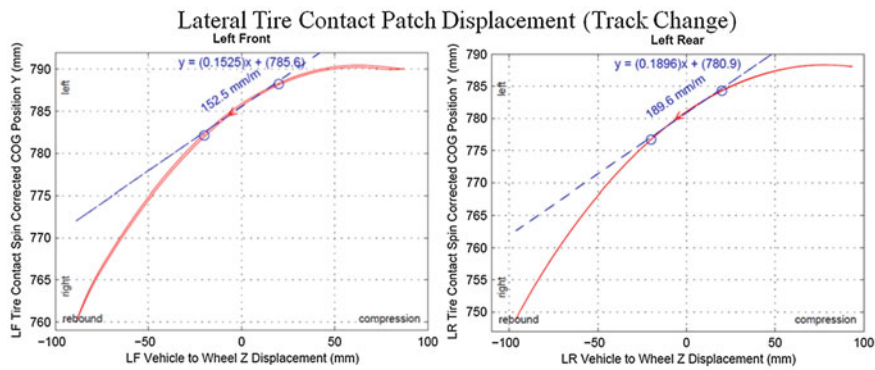
Test Data 1 Front increasing wheel rate in compression due to jounce bumper, rear is more linear



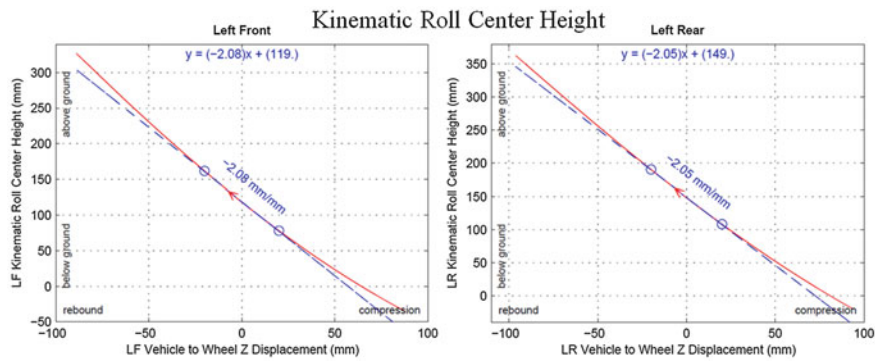
Test Data 2 Front has significant bump steer non-linearity in compression



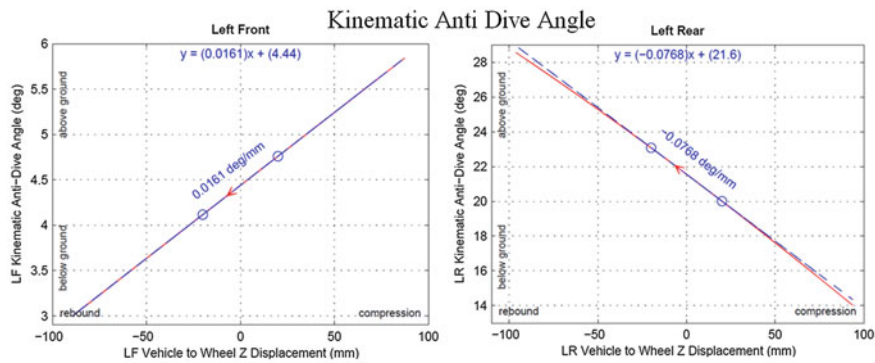
Test Data 3 Increased camber change in compression typical of SLA suspension



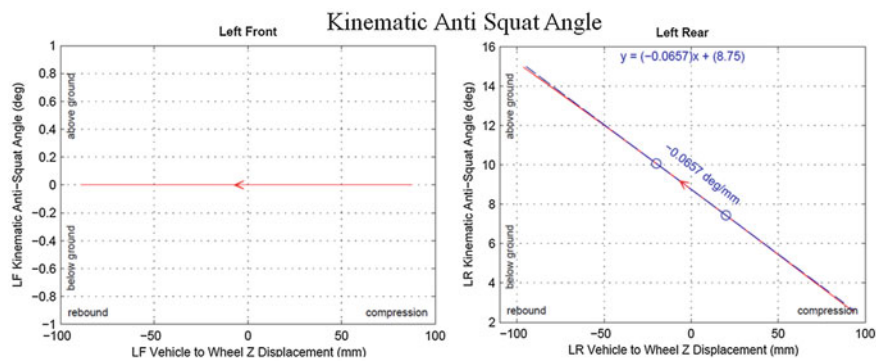
Test Data 4 Decreasing track change in compression reduces loaded tire lateral scuff



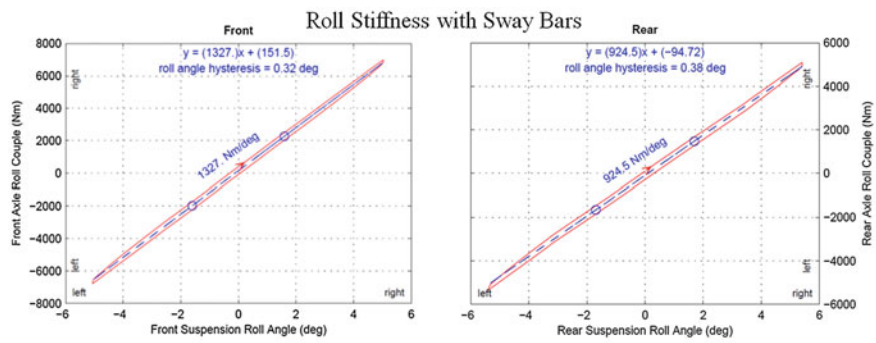
Test Data 5 Significant roll center height changes with suspension displacement



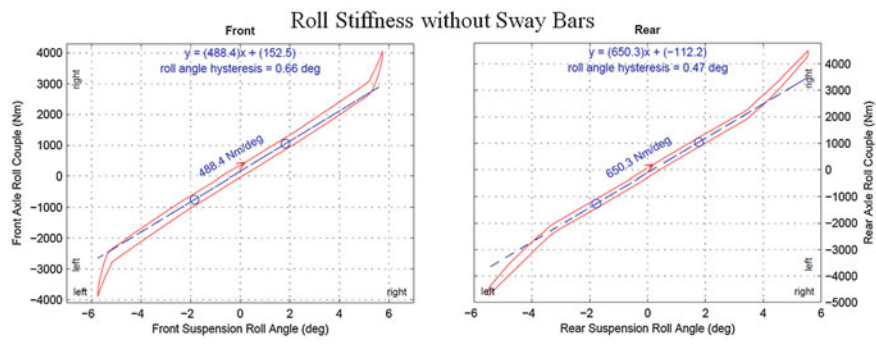
Test Data 6 Rear has significantly more anti-dive than front



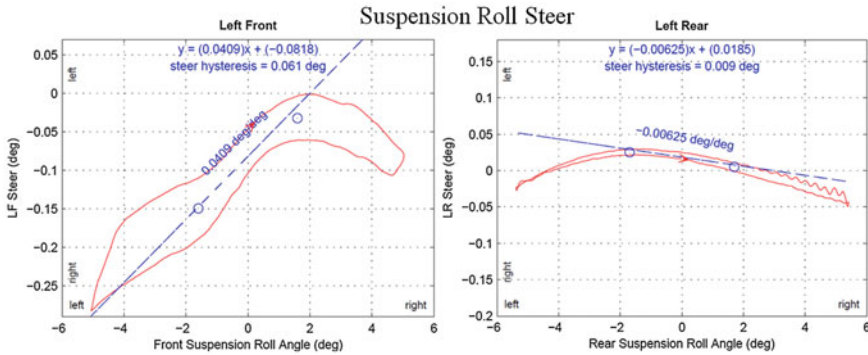
Test Data 7 Vehicle is rear drive, no anti squat calculated for front



Test Data 8 Sway bars provide significant roll stiffness compared to suspension springs alone (Test Data 9).



Test Data 9 Jounce and rebound bumper roll stiffness contribution can be observed



Test Data 10 Reversing roll steer direction in front and rear, front has high hysteresis

5 Suspension Compliance

Suspension compliance involves the design of wheel end deflections resulting from road forces. It does not include suspension deflections due to ride and roll motions.

Compliance occurs as a result of bushing deflection at suspension and chassis joints. It is also a result of 'rigid' suspension component deflections. All mechanical components experience deflections under load. Deflection magnitude depends upon the component geometry, force levels and the stiffness of the component. Generally, suspension supporting components are assumed to be rigid during the design phase. This is not true in actual operation. Often the supporting components are no stiffer than the bushings themselves. A good rule of thumb is that a supporting component should be 5–10 times stiffer than the isolator so that the majority of deflection occurs at the isolator. Defining and measuring stiffness is challenging. Stiffness is the calculated quantity of input force divided by the resulting deflection. Force is relatively easy to define and measure. Deflection measurements depend upon the dimensional frame of reference. The assumptions made as to where and how the component is supported and where measurements start and end significantly influence the measurement. There is no perfect solution.

Suspension compliances are bounded by component geometry. Proper kinematic design allows reasonable compliance values to be achieved with nominal, practical bushing stiffness. If kinematic design is compromised, a suspension will be doomed to be an underperforming architecture regardless of bushing stiffness chosen.

5.1 Compliance Measurements

Vehicle compliance measurements are executed on a test bench such as the Anthony Best K and C Measuring Machine at Morse Measurements that was discussed previously.

The vehicle is generally fixed at a particular suspension height, such as two passenger loading, and all compliance tests are done at that height. Wheel end motion is quantified relative to the vehicle body. Longitudinal force, lateral force or steering moments are applied at the tire patch. Input force and moment directions are either in the same direction or in opposite directions depending upon the test. Wheel end motions are quantified in three directions of translation, plus steering, caster and camber rotations. Vertical force changes are measured simultaneously as horizontal forces are applied. Since the vehicle body is fixed in space, vertical suspension motion is minimized. The principle of super position reasonably applies to K and C measurements.

Familiarity with a vehicle's suspension architecture is necessary so that compliance measurements can be related to the suspension architecture and the compliance contributors can be understood. The L-arm versus A- Arm comparison in the kinematic section illustrates the effect of suspension architecture and component geometry on compliance. A vehicle with upper and lower control arms will have compliance from each arm and the other components sum to total wheel end compliance. Understanding the directions of reaction forces and component compliance provides insights and reveals opportunities.

It is necessary to be able to trace the input forces and moments throughout the suspension to the support points to be able to comprehend the component motions and local compliances (Fig. 33).

Compliance due to longitudinal road surface and braking forces need to be tested separately. For longitudinal road forces, which originate at the tire patch, the unlocked tire is strapped to the test bench loading platen and is allowed to rotate around its axle so that the longitudinal force is resolved through the axle without a wheel end moment. Longitudinal braking forces are applied though the tire patch with the tire locked using the brakes and not strapped to the platen. The brake test method introduces a longitudinal force and a rotational moment due to brake torque into

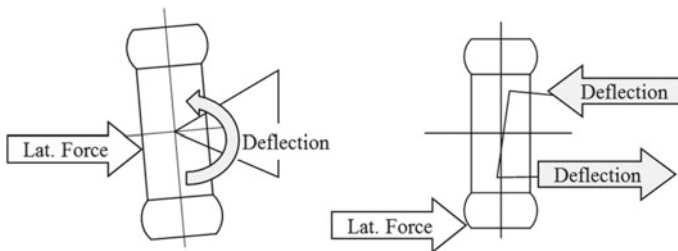


Fig. 33 Wheel end deflections due to lateral force

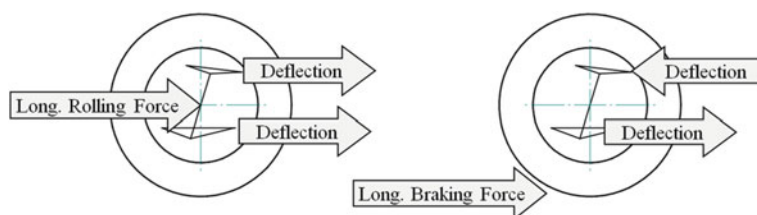


Fig. 34 Wheel end deflections due to longitudinal force

the suspension. The resulting reaction forces and local component deflections are different in direction and magnitude (Fig. 34).

5.2 Steering Compliance

Every vehicle can be visualized to have a steering axis at each suspension corner. The front wheel end steering axis is obvious. The rear wheel end has a virtual steering axis dependent on wheel end geometry and the stiffness of the suspension links. The steering compliance can be either an understeering or an oversteering factor depending upon the direction of the moments around the steering axis. This is a critical factor in suspension design for dynamic performance (Fig. 35).

If the front and rear wheel end steering axes are in front of the lateral force location, the front axle will exhibit compliance understeer while the rear axle will exhibit compliance oversteer. If the rear wheel end is designed such that the virtual steering axis is behind the lateral force location as defined by the pneumatic trail, the rear wheel end would include a consistent understeer contributor. The compliance understeer moment would increase as lateral force increases and moves forward due to pneumatic trail reduction (Fig. 36).

Front wheel end architectures have evolved to have positive caster generating consistent compliance understeer. Independent suspension rear wheel end architectures have not evolved to similarities in architecture. An argument can be made that a rear suspension ought to have a level of negative caster to produce an inherent compliance understeer characteristic similar to the front. A study of rear suspension caster

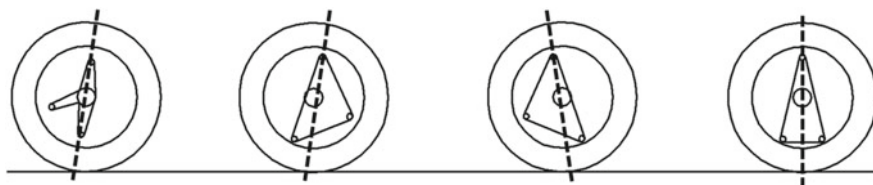


Fig. 35 Wheel end architecture steering axis orientations

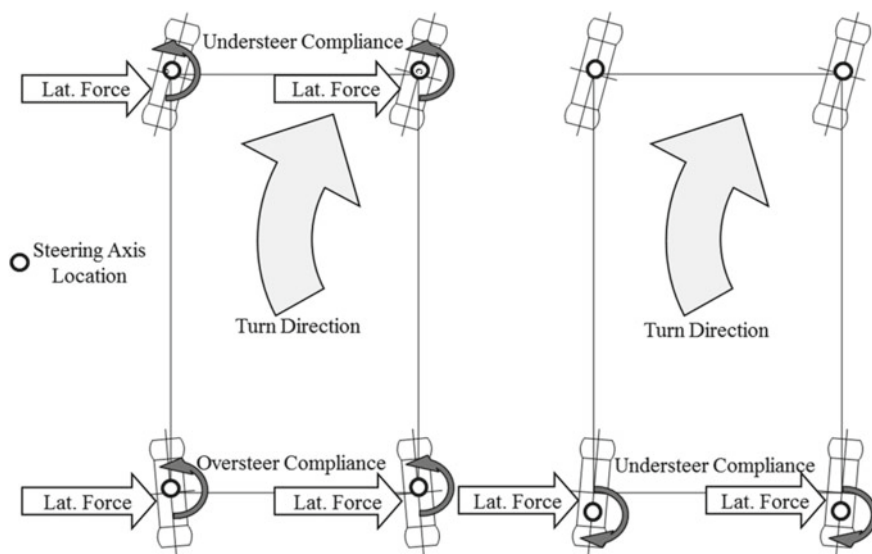


Fig. 36 Compliance steer relative to steering axis location

angles reveals that current production solutions are inconsistent. Vehicle dynamic behavior is significantly different depending upon rear wheel end oversteer or understeer compliance. Even inexperienced, unobservant drivers can feel the difference between the two architectures.

Longitudinal forces generate steering compliance due to the lateral location of the force relative to the virtual steering axis. The lever arm between the longitudinal force and the virtual steering axis can be considered a ‘dynamic’ scrub radius. The dynamic scrub radius can be defined as the lateral distance between the tire’s footprint dynamic center of pressure and the steering axis. The tire footprint center of pressure depends upon the road surface’s orientation to the tire plane and tire deformation due to cornering forces. The center of pressure can shift from one edge of the tire to the other; it is not necessarily in the middle of the tire. Left to right longitudinal force steering compliance is often unsymmetrical because of different road—tire plane orientations, unequal forces and moments. Longitudinal force steering compliance may be countered with control arm and steering linkage kinematics and compliance.

Longitudinal and lateral forces result in translations of the wheel end in addition to camber and steering angle changes. Longitudinal compliance character impacts vehicle ride comfort and harshness. Lateral suspension compliance adds to camber angle compliance summing to an effective ‘footprint compliance’ which is related to steering responsiveness in the front and sloppiness of the rear suspension in transient maneuvers. Suspension footprint compliance is further increased by tire lateral compliance.

5.3 Component Compliance Equation

Component compliance can be divided into the study of supporting components which operate in the linear range and bushing that do not have linear force—deflection characteristics. As the bushing rate increases with load the proportion of compliance between the parts can change significantly. It is always best to consider the stiffness of supporting components.

Isolators are often assumed to be linear in compliance calculations. This first order assumption can be reasonable if the bushings are operating around an unloaded nominal position or the vehicle performance is being analyzed at low G levels. Bushings often have a static load due to vehicle supporting forces, shifting the operating range away from on-center. Such bushings do not operate around zero preload; dynamic forces are superimposed upon the static support forces.

A nonlinear equation is necessary to adequately represent bushings. One equation that is capable of representing bushing non-linearity is:

F_{positive} = C₁d + (C₂e^{d/C₃} - 1) (11)

F_{negative} = -(C₄d + (C₅e^{d/C₆} - 1)) (12)

where, F_{positive or negative} is the direction of force acting on the bushing, d is bushing deflection and C₁₋₆ are bushing equation coefficients. The force—deflection slope must not be zero and the bushing rate should be equal across zero deflection to represent bushing rate continuity between positive and negative directions (Table 4, Fig. 37).

Table 4 Sample compliance equation coefficients for positive force

C ₁	C ₂	C ₃
20,000	50	0.100

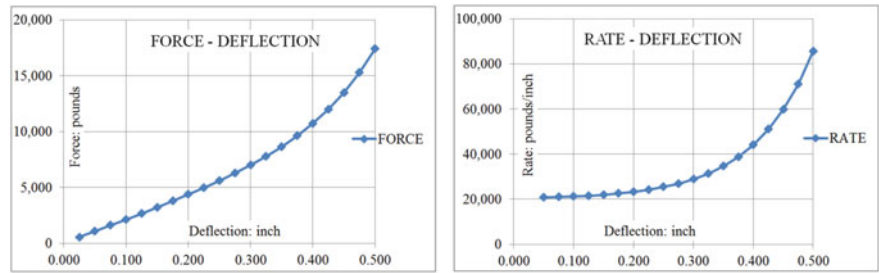


Fig. 37 Sample non-linear bushing characteristics

5.4 Wheel End Compliance

Wheel end steering and camber compliance design potential can be presented graphically considering the side view wheel end geometry. A matrix of camber and steering compliance values using a range of link stiffness combinations is calculated based upon the lateral force being located at a particular pneumatic trail. Such methodology offers an understanding of compliance possibilities given a wheel end geometry and potential bushing rates. A graphical presentation of camber and steering compliance relative to design objectives reveals if the geometry and bushings chosen will satisfy the objectives. The robustness of tuning opportunities is also revealed. A similar approach can be used to examine other design factor combinations (Figs. 38 and 39).

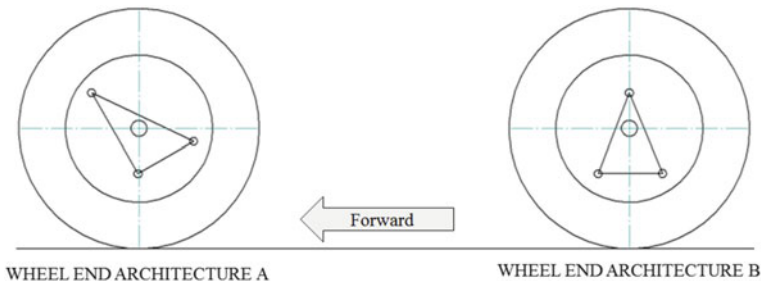


Fig. 38 Wheel end architecture alternatives

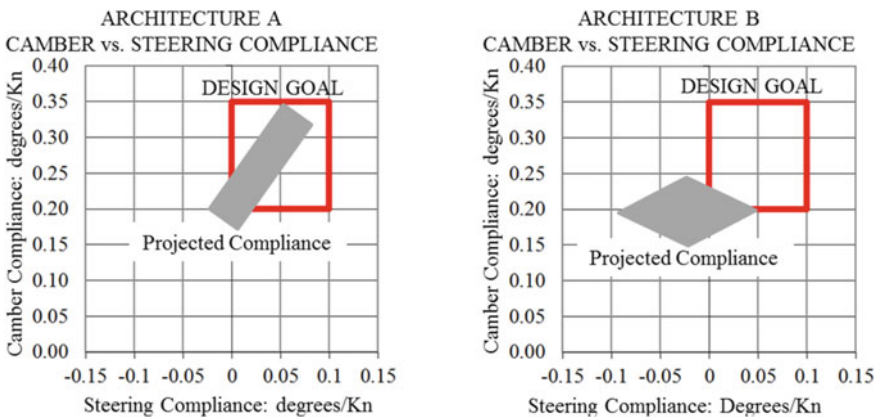


Fig. 39 Alternative wheel end compliance performance versus design goal

5.5 Compliance Testing

Vehicle compliance test have been standardized to some degree. The following chart lists the most common tests (Table 5).

Parallel force compliance tests are approximations of vehicle performance. Opposed force compliance tests are used to eliminate, to a practical degree, compliance due to an isolated cradle or the compliance of the steering system. This allows an assessment of suspension corner contributions alone (Figs. 40, 41, 42).

Table 5 Compliance test matrix

Lateral opposed
Lateral parallel
Lateral parallel, engine off
Lateral parallel, with trail
Aligning torque, opposed
Aligning torque, parallel
Aligning torque, parallel, engine off
Longitudinal, braking and traction
Longitudinal, force at wheel center, brakes off

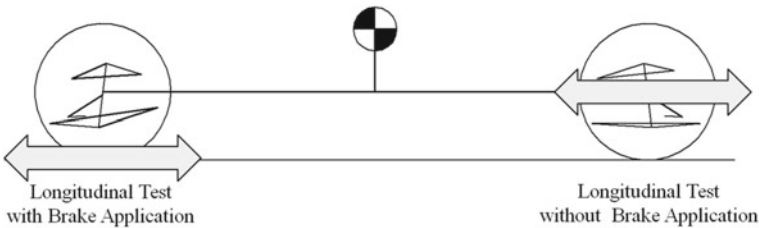


Fig. 40 Longitudinal compliance test scenarios

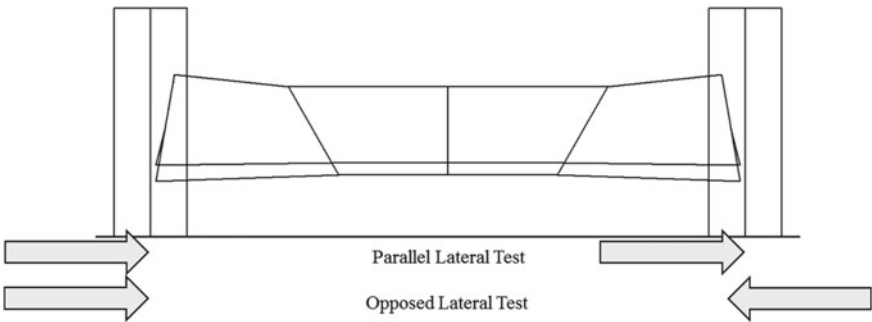
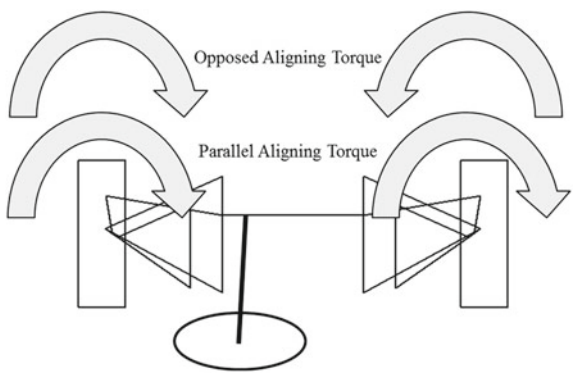


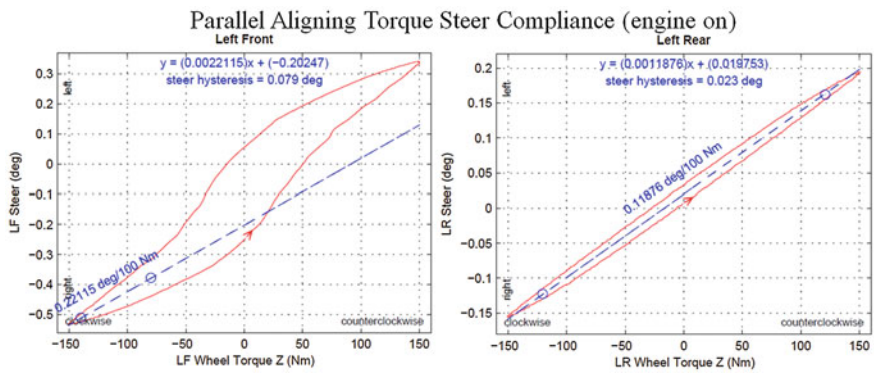
Fig. 41 Lateral compliance test scenarios

Fig. 42 Aligning torque test scenarios

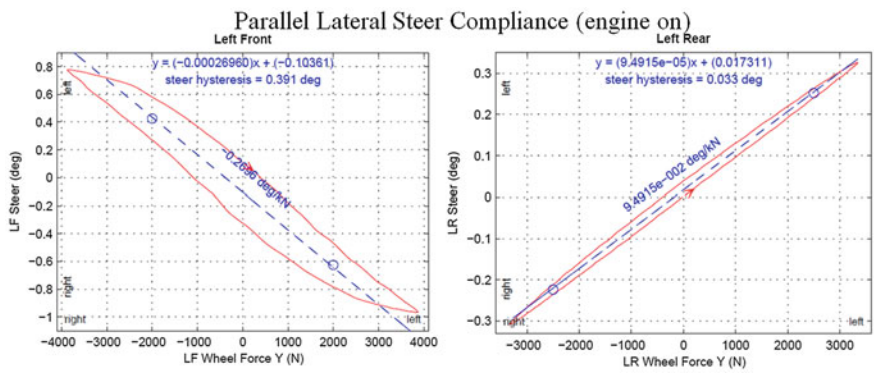


5.6 Compliance Test Data

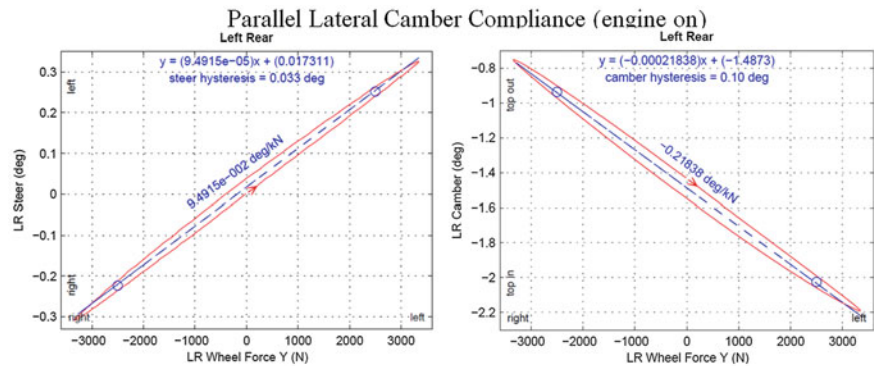
Examples of compliance tests executed by Morse Measurements follow with commentary:



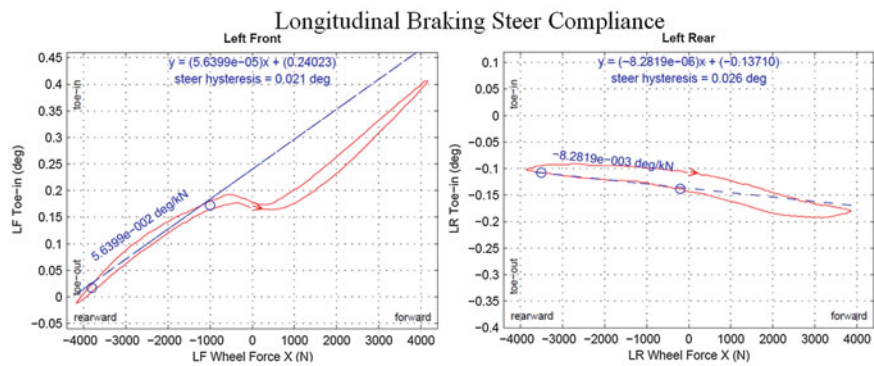
Test Data 11 Hysteresis in front due to power steering assist



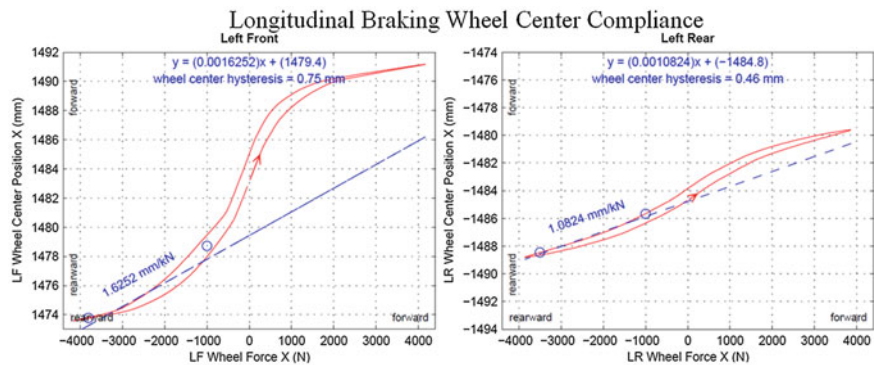
Test Data 12 Hysteresis in front due to power steering assist



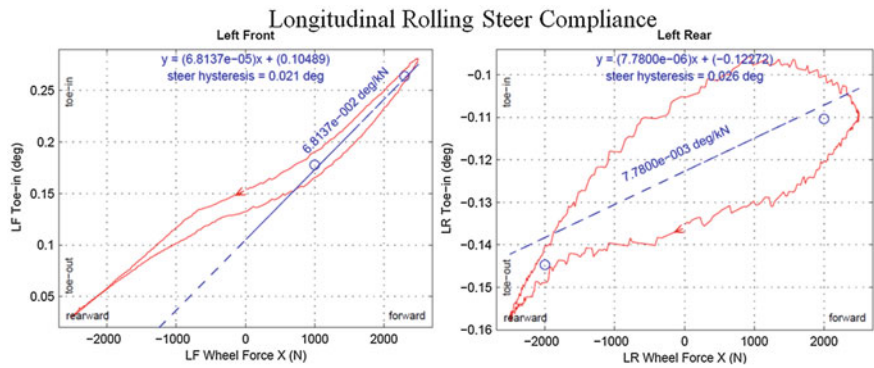
Test Data 13 Front higher compliance and higher hysteresis than rear



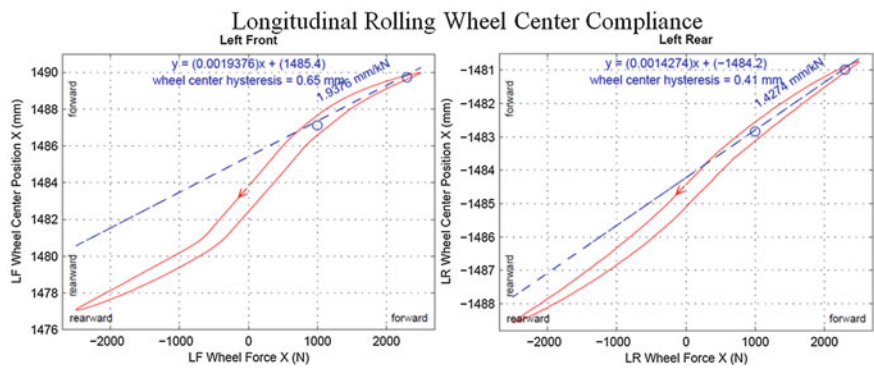
Test Data 14 Non-linear transient in front, with significant toe change



Test Data 15 Increased longitudinal compliance for ride comfort across center



Test Data 16 Rear Y scale is magnified so curve looks odd



Test Data 17 Increased compliance across center for ride comfort

5.7 Understeer Budget

Suspension compliance measurements have a multitude of units which makes it difficult to put the information into perspective. Measurements include units such as degrees/inch, degrees/mm, degrees/degree, degrees/kN and degrees/100Nm. A method to consolidate the different compliance units was introduced by Bundorf and Leffert (1976) *Cornering Compliance Concept for Description of Vehicle Directional Control Properties*, SAE paper 760713. All compliance factors are normalized to common units of degrees per lateral G. This method allows an examination of individual wheel end contributors, the front and rear axle separately, and to quantify the total vehicle with and without tire contributions (Table 6). The understeer budget provides insight into full vehicle dynamics as well as sprung body dynamics which the driver feels as part of the full vehicle experience:

Table 6 Typical understeer budget

Understeer budget (on center, below 0.1 G's)			
	Units	Front	Rear
Vehicle characteristics			
Corner weight	N	3500	2800
Cornering stiffness	N/deg	620	570
Pneumatic trail	m	0.03	0.03
Lateral force steer	Deg/kN	0.05	−0.02
Aligning moment steer	Deg/100 Nm	0.80	−0.12
Roll steer	% deg/deg	1.20	4.87
Roll gradient	deg/G	4.70	4.70
Understeer contributors			
Lateral force steer	deg/G	0.18	−0.6
Aligning moment steer	deg/G	0.84	−0.10
Roll steer	deg/G	0.06	0.23
Total understeer (with out tires)	deg/G	1.07	0.07
Understeer gradient (with out tire)	deg/G	1.14	
Tire contribution	deg/G	5.65	−4.91
Total understeer (with tires)	deg/G	6.72	−4.84
Understeer gradient (with tire)	deg/G	1.88	

Note Individual compliance characteristics can be either understeer or oversteer contributors

5.8 Design and Analytical Tools

Numerous suspension kinematic analysis tools have been developed since many people study kinematics for race performance. Compliance analysis is mostly executed within OEM engineering organizations, often with proprietary tools. The amount of information necessary for compliance calculations is significantly greater than for kinematic analysis. Compliance analysis requires realistic stiffness information regarding each bushing in 6 directions and possibly link stiffness information. This information is usually too difficult to acquire outside of OEM engineering organizations.

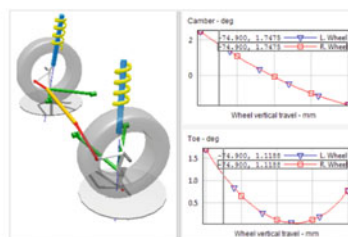
A 2D CAD program is recommended for kinematic analysis. A 2D CAD program can be manipulated to allow motion of the 4-bar linkages to model suspension travel, vehicle body roll or component kinematics. A 2D perspective provides a clear understanding of dimensional aspects and kinematic performance without the high risk of getting confused by 3D models. The author uses Siemens Solid Edge® for 2D suspension kinematic modeling.

For compliance analysis, Mechanical Simulation of Ann Arbor, Michigan offers a SuspensionSim® software package that requires bushing rate inputs for each joint. This software package can be tricked into being a kinematics package if the bushing



SuspensionSim simulates quasi-static Kinematics and Compliance (K&C) tests applied to suspension systems.

SuspensionSim differs in several ways from the vehicle simulation products BikeSim, CarSim, and TruckSim. Rather than using predefined parametric programs with specific multibody models, SuspensionSim uses a multibody solver program that builds models based on user datasets.



Visualize K&C test results with synchronized animation and plotting. Click for the full image.

Fig. 43 SuspensionSim simulation software



World's most established vehicle simulation platform enables accelerated engineering throughout the complete product design process



CarSim delivers the most accurate, detailed, and efficient methods for simulating the performance of passenger vehicles and light-duty trucks. With more than twenty years of real-world validation by automotive engineers, CarSim is universally the preferred tool for analyzing vehicle dynamics, developing active controllers, calculating a car's performance characteristics, and engineering next generation active safety systems.

With manufacturers facing compressed product development cycles, CarSim provides an intuitive set of tools for engineers to quickly evaluate complete vehicles, automobile sub-components, and active controllers in complex, simulated driving environments.



Fig. 44 CarSim vehicle dynamics simulation software

rates are set very stiff in translation and soft in rotation. The software then simulates a kinematic solution as long as the suspension is not over constrained. The software will calculate kinematics and compliance with over-constrained linkages because it calculates linkage length changes as a result of bushing compliance. The accuracy of the kinematic and compliance solutions depends upon the bushings rate values chosen. Mechanical Simulation also produces a full vehicle simulation software package, CARSIM® that is compatible with SuspensionSim (Figs. 43 and 44).

6 Steering Kinematics and Compliance

A conventional vehicle steering system operates the front tires. This system changes the direction of the front tires relative to the vehicle centerline. As a result, the vehicle changes direction. The mechanics of the steering system are discussed in this section. A vehicle's dynamic steering behavior involves more than just the action of the steering system. Some dynamic effects of the steering system are mentioned in the discussions of vehicle dynamic performance.

6.1 Static Steering Forces

Steering forces can be examined separately as static parking forces and dynamic cornering forces. While parking, tires experience a scuffing motion as well as a rolling motion. The parking forces generated by a tire with the brakes applied and wheels locked can be estimated by considering the locked tire contact patch, tire motion, load and friction:

1. Determine corner weight, CW and tire pressure, TP ;
2. Calculate the footprint area, FA ;

$$FA = CW/TP \quad (13)$$

3. Determine the tire contact patch dimensions to equal the footprint area and locate the steering axis within the footprint (Fig. 45);
4. Divide the contact patch into reasonable segments, determine each segment area, SA_i ; then calculate the vertical segment load, VSL_i for each segment;

$$VSL_i = TP \times SA_i \quad (14)$$

5. Determine the radius from each segment centroid to the steering axis ground intersection point, SR_i (Fig. 46);
6. Assume a contact patch coefficient of friction, Mu , (0.9 is a good estimate);

Fig. 45 Contact patch dimensions and steering axis location

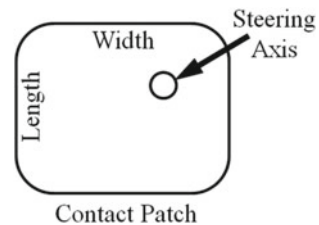
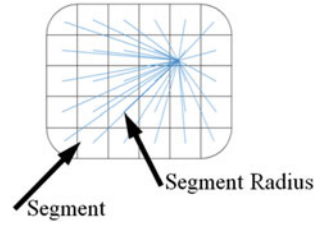


Fig. 46 Contact patch segmentation and radius to steering axis



7. Calculate and sum the moments of the segment friction forces around the steering axis to determine torque to steer one tire, TQ .

$$TQ = \sum (Mu \times VSL_i \times SR_i) \quad (15)$$

6.2 Dynamic Steering Forces

Dynamic steering forces for each tire can be estimated by calculating the moments produced by the lateral and longitudinal forces acting around the steering axis:

1. The sum of pneumatic trail and caster trail is used as the lever arm for the lateral force moment;
2. The dynamic scrub radius is used as the lever arm for the longitudinal force moment.

The front tires may be at different load and geometric conditions due to vertical, cornering and longitudinal forces as well as at different orientations relative to the road surface. The left and right tire moments need to be considered separately and the directions of the moments must be recognized and preserved (Fig. 47).

6.3 Ackermann Steering

Ackermann steering is a horse and buggy solution invented to reduce tire rutting of gravel driveways. The Ackermann principal applies to low G cornering situations; it does not offer a solution at higher G-levels. A reasonable automotive design methodology for quantifying and optimizing steering system geometry at higher G levels has yet to be developed (Fig. 48).

Attempts have been made to optimize front tire steering geometry to maximize vehicle cornering capability. These attempts have brought about parallel steer and reverse Ackermann solutions. Both can provide benefits in high G situations but do not optimize the steering across the operating range and have negative consequences. On road testing rarely supports the optimum theoretical solution.

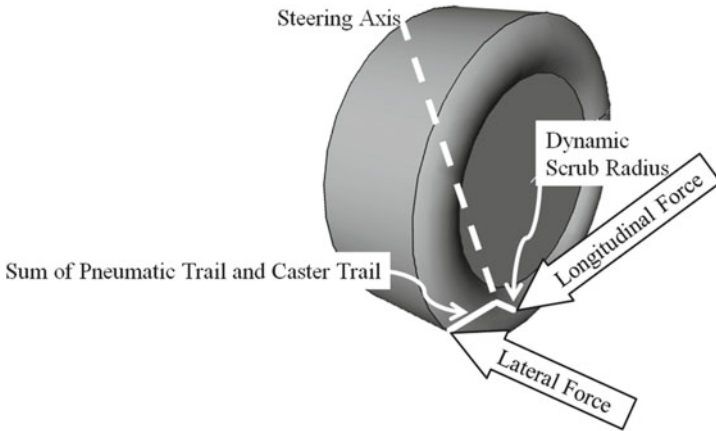
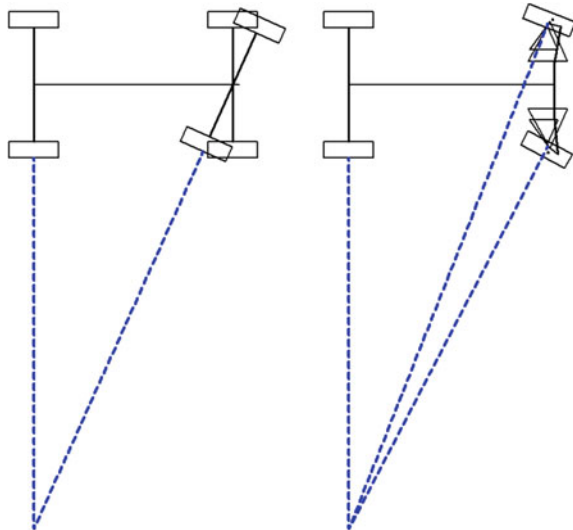


Fig. 47 Dynamic steering contact patch tire force locations

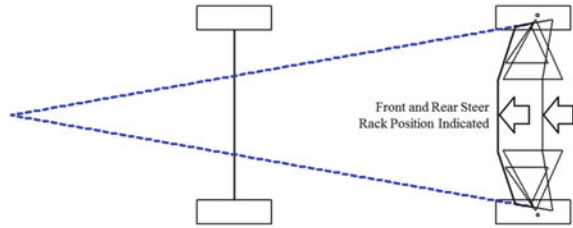
Fig. 48 Horse and buggy versus automotive steering solution



Regardless of the insufficiencies of Ackermann, it is useful for road vehicles which operate at the low end of the G level spectrum. Ackermann does allow more efficient cornering and less tire scuff when driving in cities and parking lots. Ackermann reduces tire squeal and tire wear.

Ackermann solutions are best generated from the top view. The projected steering arm geometry needs to intersect toward the rear of the vehicle. If the intersection aligns with the rear axle, 100% Ackermann is approximated. An intersection in front of the rear axle implies extra Ackermann. An intersection behind the rear axle implies

Fig. 49 Ackermann steering solution



less than 100% Ackermann. Reverse Ackermann is poorly defined mathematically (Fig. 49).

The steering levers need to be splayed to develop Ackermann geometry. The steering rack should be positioned behind the outer tie rod ends so that Ackermann steering properties can be maintained throughout the steering range.

6.4 Steering Kinematic and Compliance Measurements

Some steering subsystem characterization can be accomplished on a K and C machine. The kinematic and compliance tests are adequate to characterize the steering ratio, Ackermann and wheel end compliance. The kinematic and compliance tests are not adequate to characterize the steering rack and its contribution to steering feel. A separate steering rack test bench is necessary to accurately measure steering rack assist, response and friction properties.

The kinematic tests measure the rotational characteristics of the steering system, Ackermann and wheel end geometry, such as king pin inclination and scrub radius. Kinematic steering measurements include an assessment of steering ratio throughout the steering range. Steering ratio oscillation caused by steering column universal joint angles and phasing can be observed.

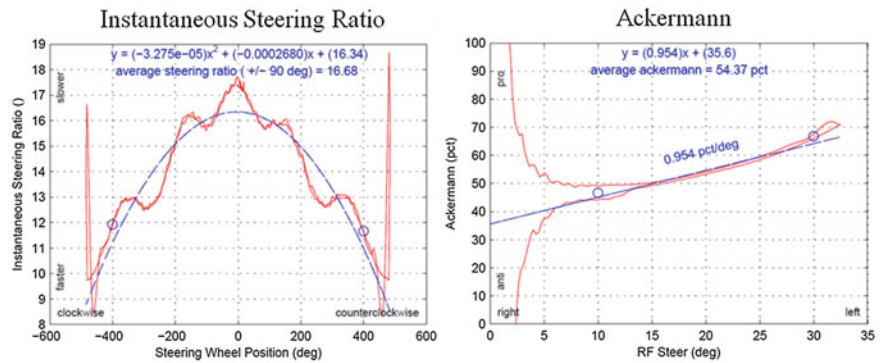
Power steering systems, whether hydraulic or electrical, rely upon a compliant torsion bar member to measure driver steering effort and to determine the assist necessary. Compliance tests are done with engine on and off to quantify the mechanical steering system separately from the assisted system. Engine on parallel aligning torque compliance is evaluated on-center for low G character and past center for higher G character. Engine on parallel force steering compliance is generally observed to be non-linear due to power steering assist tuning (Table 7).

6.5 Steering Kinematic Test Data

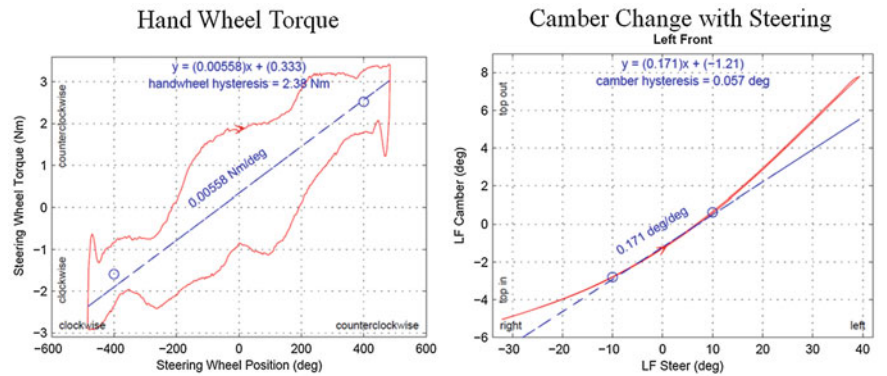
Steering kinematic and compliance data courtesy of Morse Measurements is presented with commentary:

Table 7 Steering kinematic and compliance test matrix

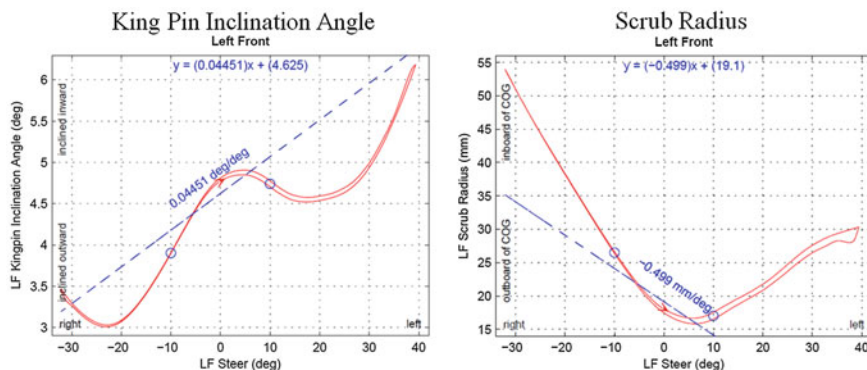
Steering ratio
Ackermann
Handwheel torque
Camber angle
Steering axis geometry
Vertical force



Test Data 18 Variable steering ratio with oscillation due unbalanced steering shaft u-joint angles. Ackermann poorly defined below 5 degrees of steering



Test Data 19 Hand wheel torque data is not refined enough for accurate conclusions



Test Data 20 KPI angle and scrub radius variation may be due to virtual ball joint architecture

7 Weight Transfer with Kinematics and Compliance

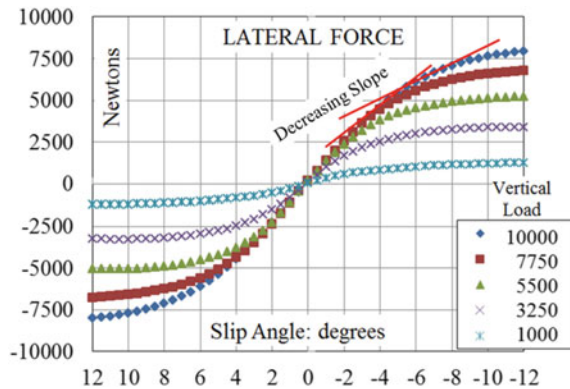
Weight transfer to the outside occurs as a vehicle is cornering. The front to rear weight transfer distribution is critical in order to balance tire behavior and hence vehicle performance for satisfactory dynamics. The better the four tires are utilized, the greater the performance capability.

The fundamental reason for engineering weight transfer distribution is that as the tire's vertical load increases and cornering forces develop, the tire's slip angle increases at a higher rate than the vertical force increase. Consequently, highly loaded tires operate at higher slip angles than lightly loaded tires. If the suspension is tuned so that the front tires support more weight transfer than the rear tires, the front tires develop greater slip angles than the rear tires and an understeering balance is achieved. This inherent characteristic provides a degree of tunable understeer which is used to maintain vehicle stability and dynamic comfort for the driver and occupants (Fig. 50).

7.1 Weight Transfer Allocation

Weight transfer to the outside during cornering is normal for a vehicle with the CG above the ground. Lateral weight transfer is a function of lateral cornering G's, vehicle weight, CG height and vehicle track. The outer tire forces against the ground increase, the inside tire forces against the ground reduce an equal amount. The total vertical force against the ground remains the same (ignoring any aerodynamic effects). During steady state cornering conditions, with four tires on the ground, the front to rear weight transfer proportion can be intentionally modified while the

Fig. 50 Decreasing slope as slip angle increases



left and right side totals and the front and rear totals cannot be changed. As weight transfer is allocated, the chosen outside corner weight increases, the two adjacent corner weights decrease and the diagonal corner weight increases. The weight change at adjacent corners is equal in magnitude but opposite in direction (Table 8).

When a vehicle is cornering on three wheels the lateral weight transfer as well as the front to back distribution is fully determined based upon vehicle CG location, track, wheel base and the accelerations being experienced. It is impossible to allocate weight transfer with suspension design when a vehicle is cornering on three wheels.

The front to rear weight transfer allocation can be adjusted by varying the relative vertical suspension stiffness of the front and rear of the vehicle. This can be done by adjusting spring rates and sway bar stiffness. Weight transfer allocation is also dependent upon suspension kinematics and the wheel-end geometry of the vehicle. Roll center and roll axis concepts are used to explain the mechanics of weight transfer due to suspension kinematics. Front wheel-end steering axis and scrub radius geometry also introduce weight redistributions when steered (Fig. 51).

7.2 Cornering Grip

Cornering forces can be considered as 'grip' which can be defined as the vertical tire force multiplied by the value of 'utilized' friction between the tire and the road surface. An assumption of equal grip levels between the left and right side is appropriate when the tires are equally loaded and lateral cornering force is low (less than 0.1 G). This assumption becomes increasingly inaccurate as lateral Gs build. The left and right side lateral or cornering forces at each corner of the vehicle cannot be determined without knowledge or assumptions of tire performance and grip. Only the total at each end can be calculated directly.

Table 8 Vehicle corner weights with weight transfer and allocation

No weight transfer				Weight transfer equally distributed				Weight transfer 80% allocated to front			
	Left	Right	FRT/RR total		Left	Right	FRT/RR total		Left	Right	FRT/RR total
Front	1000	1000	2000	Front	500	1500	2000	Front	200	1800	2000
Rear	1000	1000	2000	Rear	500	1500	2000	Rear	800	1200	2000
	2000	2000			1000	3000			1000	3000	
	L Total	R Total			L total	R total			L total	R total	

Fig. 51 Lateral weight transfer and reaction forces

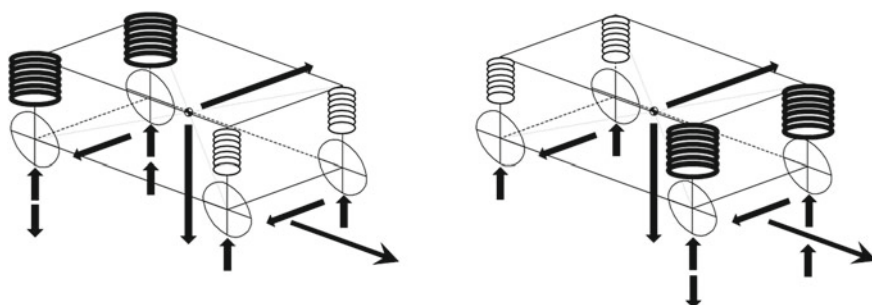
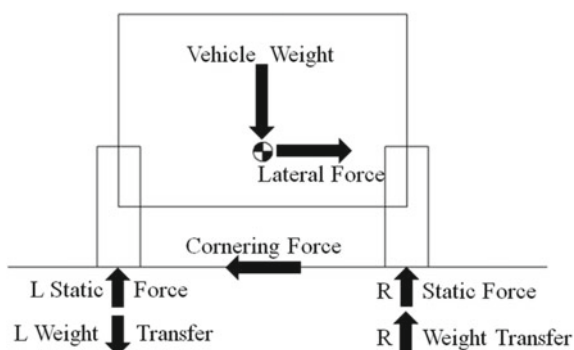


Fig. 52 Impact of suspension stiffness on weight transfer allocation

7.3 Suspension Stiffness

Vertical suspension stiffness provides a mechanism to allocate the weight transfer between the front and rear of the vehicle. If one end is very stiff while the other end is soft, the majority of weight transfer will be at the stiff end. The proportions of front to rear weight transfer are related to the stiffness of the ends relative to each other (Fig. 52).

7.4 Roll Centers and Roll Axis

Vehicle suspensions, as all linkages, have instant centers that define the centers of rotations for the mechanism. A vehicle with an independent suspension has a front view instant center for each corner of the vehicle as well as a collective front view instant center for each vehicle end known as the roll center. These instant centers are not stationary in space and tend to move relative to the vehicle and the ground. This fact is ignored in simplified suspension analysis and errors are introduced resulting in the analysis not being representative (Fig. 53). Knowing where the roll centers

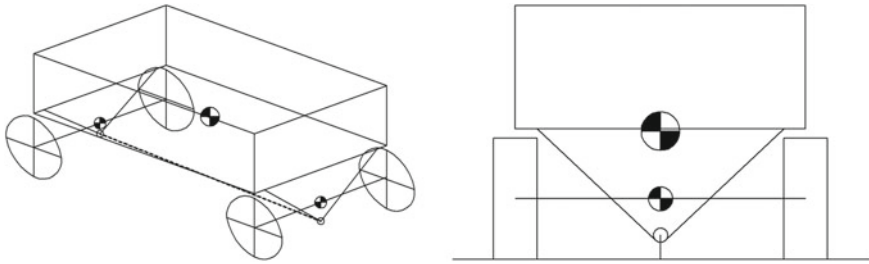


Fig. 53 Roll center model

migrate allows us to determine the state of vehicle weight transfer more accurately. One end of the vehicle can be examined at a time to understand the forces and moments involved in weight transfer and cornering.

The vehicle end model includes a roll center allowing the sprung mass to rotate around the un-sprung mass and to transfer lateral force. The vehicle end has a vertical suspension stiffness associated with each corner. The sway bar in this model is represented as part of the vertical suspension stiffness operating only in roll.

The lateral forces act upon the sprung and un-sprung mass at their CG locations. Assuming a constant cornering G level, the forces exerted on the two masses can be calculated. The sprung mass lateral force reaction is located at the roll center. This reaction force is also an input force into the un-sprung mass. All sprung mass reaction forces can be calculated by considering the sprung mass moment around the roll center. The un-sprung mass reaction forces can be calculated considering the lateral force at the roll center and the moments around ground level.

7.5 Sprung Mass Weight Transfer and Cornering Forces

Sprung mass weight transfer. As lateral forces develop, the sprung mass roll around the roll center is resisted by the left to right difference in vertical force produced by the suspension stiffness and deflection. The resulting body roll and suspension deflection depends upon the total roll stiffness of the vehicle. The amount of weight transferred at each end is equal to the deflection times the total suspension stiffness at that end. Assuming a rigid sprung mass, weight transfer of the sprung mass can be allocated between the front and rear suspension of the vehicle by suspension stiffness decisions. The sprung mass weight must be apportioned between the front and rear based upon the static front to rear weight distribution. Forces that do not change with roll stiffness allocation or roll center heights are indicated as solid arrows (Fig. 54). Hollow arrows indicated forces that change with roll stiffness allocation or roll center height revisions:

Sprung mass moment around the roll center. If the roll center is not at ground level, a moment around the ground develops due to the sprung mass lateral force

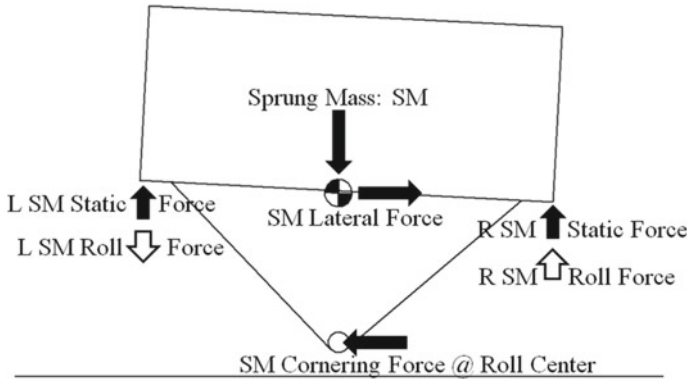


Fig. 54 Sprung mass weight transfer Free Body Diagram

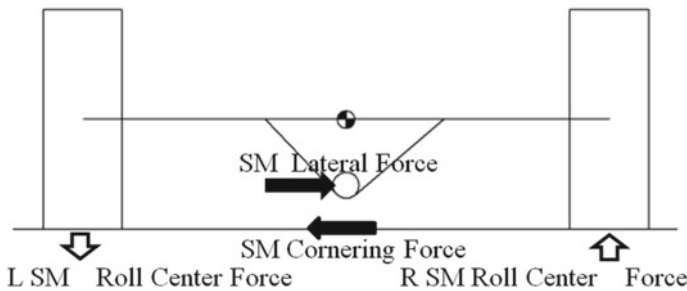


Fig. 55 Partial FBD (spring forces not shown) of sprung mass moment due to roll center elevation

and roll center elevation. This moment is resisted by the pair of vertical forces at the tire patches. These forces do not change as suspension stiffness is allocated between front and rear. They do change with front and rear roll center height; they are not independent of each other. Roll center height revisions effect the roll axis elevation and the moment arm between the sprung mass CG and the roll axis (Fig. 55).

7.6 Total Sprung Mass Weight Transfer

The sprung mass weight transfer is comprised of two sets of forces:

1. The suspension spring and sway bar forces resisting body roll and resulting in suspension deflections. These forces are passed through the un-sprung-mass to the tire contact patches. These forces are allocated between the front and the rear of the vehicle by spring and sway bar stiffness selection;
2. The sprung mass lateral force acting at the roll center at each end of the vehicle. The moments generated by these forces around the ground level are resisted by

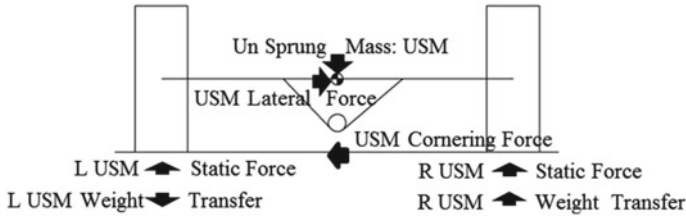


Fig. 56 Un-sprung mass weight transfer FBD

pairs of vertical tire patch forces. These moments and resulting forces are determined by the suspension architecture and mass properties, not by suspension stiffness or weight transfer allocations.

As the roll center heights are modified, the moments of the sprung mass CG around the roll centers and the moments of the sprung mass lateral force around ground level change, as one pair grows, the other pair shrinks.

7.7 *Un-Sprung Mass Cornering Forces*

The un-sprung mass weight transfer forces are fixed by its weight, track and the height of its CG. The forces acting upon the un-sprung mass can be determined with the exception of the lateral force at each tire. The total lateral force at the ground is known but not the left to right distribution unless the individual tire grip is known (Fig. 56).

7.8 *Complete Weight Transfer and Cornering Forces*

Both ends of the vehicle have such Free Body Diagrams and distribute forces and moments in this manner. However, since the sprung mass is assumed rigid, the resistance to body roll can be allocated between the front and rear suspension by adjusting the relative suspension stiffness resulting in a change of vertical force at each vehicle corner.

The complete Free Body Diagram of the forces acting on the front and rear un-sprung mass illustrates the fixed (solid arrows) and allocate-able forces (hollow arrows) (Fig. 57).

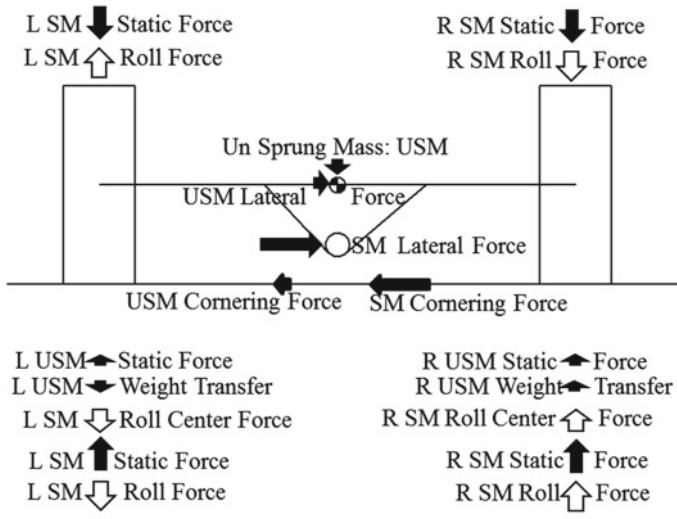


Fig. 57 Complete FBD of un-sprung mass forces and reactions

7.9 Torsional Stiffness

As a vehicle torsional structure deviates from rigid, the sprung mass roll force allocation between the front and rear becomes less effective due to body torsional deflection. When body torsional stiffness is zero, sprung mass roll force allocation becomes impossible. The two ends of the vehicle act independently from a lateral weight transfer point of view and have independent roll angles.

7.10 Weight Transfer Calculation

Weight transfer calculations have been presented in Milliken and Milliken's *Race Car Vehicle Dynamics* and other sources. The Milliken calculation is reproduced here. It is necessary to determine the moment of the sprung mass around the roll axis before calculating the weight transfer at each end of the vehicle. A one G lateral acceleration can be used as a practical assumption for the calculations (Fig. 58).

Calculation of the sprung moment, M_s around the roll axis, Z_R , ft-lbs/radian:

$$M_s = -W_s h_2 (A_y - \Phi) \quad (16)$$

where Φ is the roll angle of the chassis and does not need to be used for small angles. A_y is the lateral acceleration in Gs.

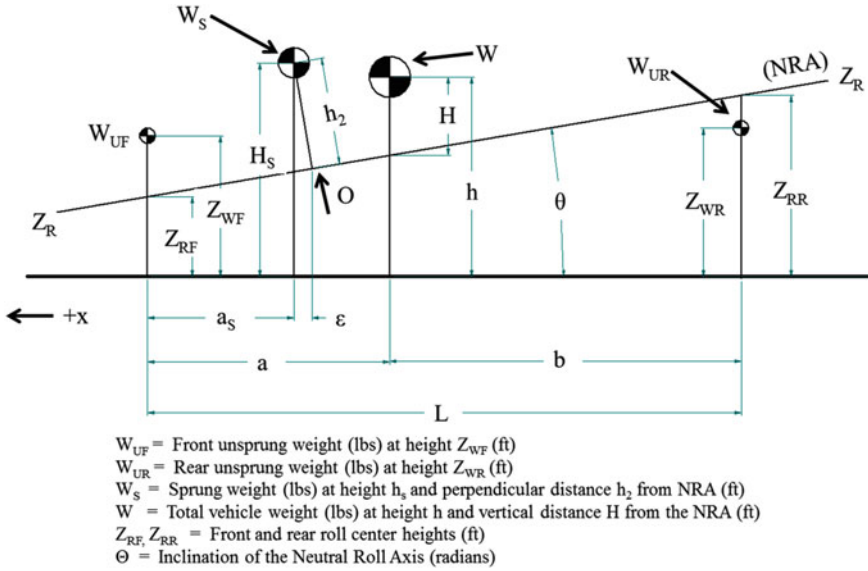


Fig. 58 Vehicle dimensional parameters

The roll sensitivity calculation, Φ/A_Y , is not used in subsequent calculations; it quantifies vehicle roll angle sensitivity to cornering Gs, radians/G:

$$\Phi/A_Y = -W_S h_2 / (K_F + K_R - W_S h_2) \quad (17)$$

where K_F and K_R are the front and rear roll stiffness in ft-lbs/radian.

The front axle weight transfer calculation, $\Delta W_F/A_Y$, provides the increase and decrease of the static corner weight for the front outside and inside corners respectively, in pounds/G:

$$\frac{\Delta W_F}{A_Y} = \left(\frac{W_S}{t_F} \right) \left(\frac{h_2 K_F}{K_F + K_R - W_S h_2} + \frac{Z_{RF}(L - a_s)}{L} \right) + \frac{Z_{WF} W_{UF}}{t_F} \quad (18)$$

where t_F is the front track in feet.

The rear axle weight transfer calculation, $\Delta W_R/A_Y$, provides the increase and decrease of the static corner weight on the rear outside and inside corners respectively in pounds/G:

$$\frac{\Delta W_R}{A_Y} = \frac{W_S}{t_R} \left(\frac{h_2 K_R}{K_F + K_R - W_S h_2} + \frac{Z_{RR} a_s}{L} \right) + Z_{WR} \frac{W_{UR}}{t_R} \quad (19)$$

where t_R is the rear track in feet.

Assumptions in the model:

- Roll centers are fixed and centered laterally;
- Lateral and vertical sprung mass CG motions as a result of roll around the roll axis are ignored;
- The sprung mass is torsionally rigid;
- Jacking forces are not included.

A sample vehicle calculation follows (Tables 9 and 10): Exercising this model at a variety of roll center heights, roll axis elevations and inclinations allows a comparison of vehicle weight transfer and roll characteristics:

Observations regarding weight transfer and roll angle sensitivity to roll center heights:

- Raising the roll center increases weight transfer at that end, reducing the weight transfer at the opposite end;

Table 9 Example vehicle specifications

Sample vehicle				
Variable	Description	Units	Values	Notes
Ay	Lateral acceleration	Gs	1.00	* Adjust with roll axis change
Ws	Weight sprung mass	Pounds	1700	
h2	Perp height of sprung weight above roll axis	Feet	1.833*	
Zrf	Front roll center height	Feet	0.167	
Zrr	Rear roll center height	Feet	0.333	
Kf	Front roll stiffness	Ft-lbs/radian	37,500	
Kr	Rear roll stiffness	Ft-lbs/radian	30,000	
FVt	Front vehicle track	Feet	5,000	
RVt	Rear vehicle track	Feet	5,000	
Wuf	Front unsprung weight	Pounds	150	
Zwf	Frt unsprung wt CG height	Feet	1.042	
Wur	Rear unsprung weight	Pounds	150	
Zwr	Rear unsprung wt CG height	Feet	1.042	
hs	Vert distance sprung weight off ground	Feet	2.000	
W	Total vehicle weight	Pounds	2000	
l	Wheelbase	Feet	8.333	
a	Distance vehicle CG behind front axle	Feet	4.096	Not required for calculation
as	Distance sprung mass CG behind front axle	Feet	4.083	
h	Vehicle CG height	Feet	1.700	Not required for calculation

Table 10 Effect of roll center height and roll axis elevation on weight transfer

Roll center and roll axis sensitivity													
Roll center height				Perp Ht of sprung weight above roll axis	Roll angle	Lateral weight transfer: lbs							
inch Zrf				inch Zrr	inch h2	deg	Total	Front ΔWf	Rear ΔWr	% Frt	% Rear		
Roll axis inclination				0	4	22.02	2.77	771	394	377	51.1	48.9	
				2	2	22.00	2.77	773	423	349	54.8	45.2	
				4	0	21.94	2.77	773	452	321	58.4	41.6	
Roll axis elevation				0	0	24.00	3.04	779	429	349	55.1	44.9	
				2	2	22.00	2.77	773	423	349	54.8	45.2	
				4	4	20.00	2.51	767	418	350	54.4	45.6	

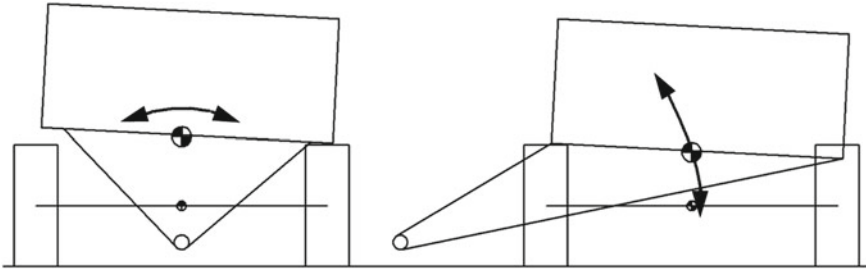


Fig. 59 Sprung mass motion due to central and offset roll center location

- Lowering the roll center reduces the weight transfer at that end and increases the weight transfer at the opposite end;
- Roll axis elevation reversely impacts the sprung mass roll angle;
- Total weight transfer stays the same regardless of roll center or roll axis locations.

7.11 Lateral Roll Center Location

Lateral roll center location does not change the weight transfer at the tires. Lateral roll center location does affect the Free Body Diagrams, the balance of forces, moments and resulting motions and vertical deflections of the sprung mass (Fig. 59).

7.12 Roll Axis Inclination

Sprung mass motion due to roll axis inclination generates a perceived steering response and yaw sensation to the driver. As a roll axis is sloped forward the rolling sprung mass has a component of motion that yaws the sprung mass ‘out’ of the turn. As the roll axis is sloped backwards the sprung mass motion has a component of motion that yaws the sprung mass ‘into’ the turn. These components of motion are observed by the driver sitting inside the sprung mass as understeering, oversteering and stability sensations.

This component of sprung mass motion is described in “*Chassis Design Principles and Analysis*” edited by William and Douglas Milliken, which presents the vehicle dynamics investigations performed by Maurice Olley (Fig. 60). Olley’s unpublished sketch is reproduced directly.

A cardboard box representing the sprung mass with a rod representing the roll centers and roll axis clearly demonstrates the effect of an inclined roll axis on the sprung mass. The device can be reoriented to demonstrate a forward and rearward roll axis inclination. The yaw component of the box relative to the roll axis can be

“If the slope of the roll axis is θ (rad) and the roll angle is Φ (rad.), the yaw angle of the median plane, and of the four wheels is:”

$$\Psi = \Phi \theta \quad *$$

* Excerpt from: Chassis Design Principles and Analysis
Based upon previously unpublished technical notes
By Maurice Olley

If the roll axis is lower in front, the body Yaws to the OUTSIDE of the TURN; an “understeer sensation”

If the roll axis is lower in the rear, the body Yaws to the INSIDE of the TURN; an “oversteer sensation”

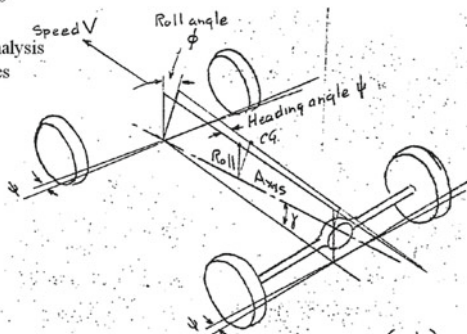
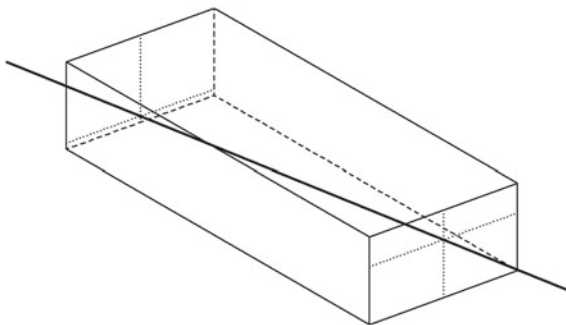


Fig. 60 Maurice Olley's sloped roll axis notes, commentary by the author

Fig. 61 Sloped roll axis box model



observed as the box is rolled around the stationary roll axis simulating a cornering event (Fig. 61).

7.13 Kinematic Stability

If roll centers are not stable in space, the vehicle roll axis is not stable. If the roll axis is not stable, calculations of lateral weight transfer keeping roll centers constant introduce errors. A shifting roll axis changes the motion of the sprung mass relative to the vehicle path as a maneuver proceeds. Unexpected and un-natural sprung mass motions can confound driver sensations and lead to a lack of confidence in vehicle performance.

A stable geometry keeps the roll centers in the middle of the vehicle and at a constant distance relative to the sprung mass as it moves vertically in ride. The roll center is not constant to the ground during suspension travel.

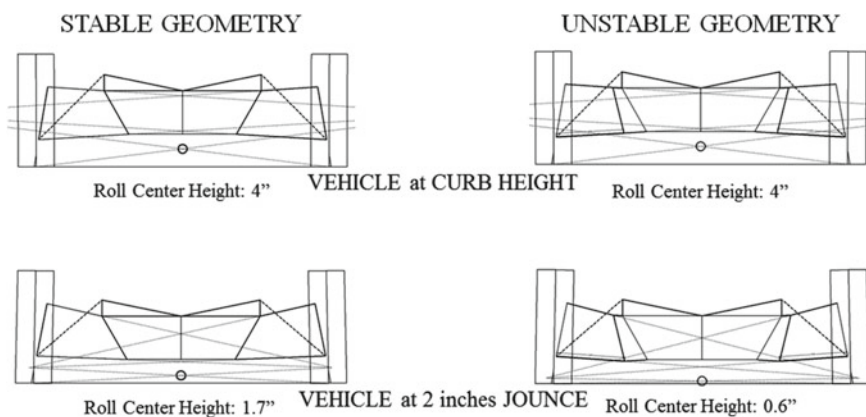


Fig. 62 Stable and unstable roll center geometry at curb and jounce

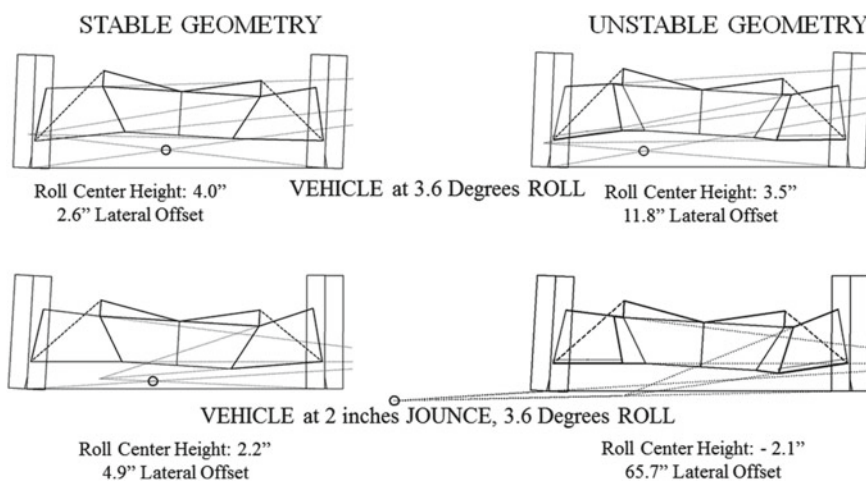


Fig. 63 Stable and unstable roll center geometry in roll and roll/jounce

Graphical suspension models provide insights into a stable and unstable roll center location (Figs. 62 and 63).

Long control arms and an upper arm length of approximately 0.62 relative to the lower arm are important factors in designing stable roll center locations for short-long arm suspensions.

7.14 Anti-Forces

Anti-dive, anti-squat, anti-lift and jacking forces. Vehicle sprung mass attitude and suspension deflections are dependent upon the stiffness of springs, sway bars, suspension travel limiters and temporary longitudinal and lateral ‘anti’ forces.

The temporary forces act as a vehicle experiences lateral and longitudinal accelerations, supporting a portion of the sprung mass vertical load, reducing the forces acting through the springs and sway bars. These temporary forces are passed through the suspension links directly into the vehicle sprung mass bypassing the springs and sway bars. These temporary forces are known as anti-squat, anti-dive, anti-lift, and jacking forces (Fig. 64). They can be quantified applying methodologies similar to the side view kinematic calculations used to determine kinematic ‘anti’ percentages presented in Sect. 4.2.

The front suspension anti-dive force calculation for a vehicle with outboard brakes:

$$\text{Front Anti Dive Force} = \text{Vehicle weight} \times G_{\text{Level}} \times \text{front brake proportion} \times \tan\theta_f \quad (20)$$

The rear suspension anti-squat force calculation for a rear drive vehicle with independent rear suspension:

$$\text{Rear Anti Squat Force} = \text{Vehicle weight} \times G_{\text{Level}} \times \tan\theta_r \quad (21)$$

If the kinematics are such that the resultant of the longitudinal force and the vertical component slopes downwards, the direction of the anti-force reverses and it acts to increase suspension and sprung mass deflections. It becomes a ‘pro’ force.

Jacking forces. Lateral jacking forces can be examined in a similar fashion. The left to right distribution of lateral contact patch force at a vehicle axle must be known or assumed. Once the lateral force distribution decision is made, the inner and outer jacking force magnitudes can be calculated individually using the suspension front view instant centers. Given the roll center is above ground, the outer jacking force will provide support to the sprung mass resulting in reduced forces through the

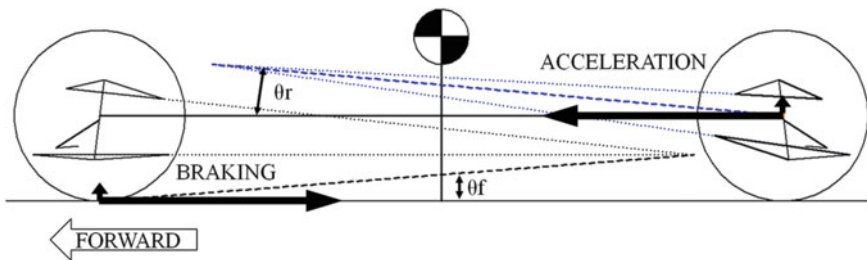


Fig. 64 Longitudinal forces, vertical anti-dive and anti-squat force vectors

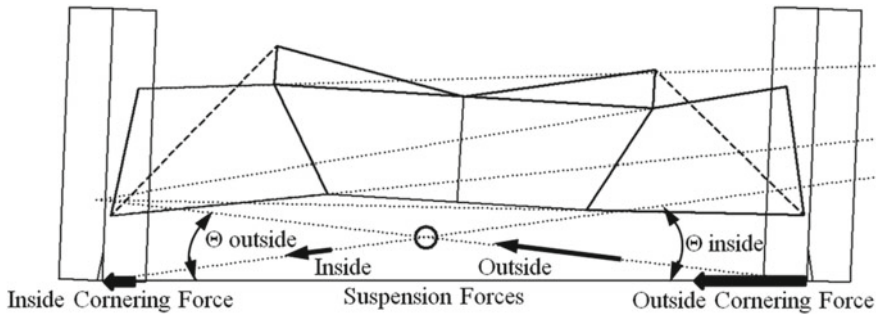


Fig. 65 Cornering force and suspension force vectors

suspension spring and sway bar. The inner jacking force will act to pull the inside down. The inner and outer jacking forces are not symmetrical. Jacking forces can introduce unequal forces into the sprung mass resulting in shifts in sprung mass attitude, suspension instant centers and roll center locations (Fig. 65).

The vertical, jacking, component of the suspension forces at each end of the vehicle can be quantified as:

$$\text{Outside Jacking Force} = \text{Outside wheel cornering force} \times \tan\Theta_{\text{outside}} \quad (22)$$

$$\text{Inside Jacking Force} = -\text{Inside wheel cornering force} \times \tan\Theta_{\text{inside}} \quad (23)$$

The vertical, jacking, components of the suspension forces can be calculated using the individual tire vertical forces and an appropriate grip, or utilized friction level (Sects. 7.2 and 8.4) at each tire:

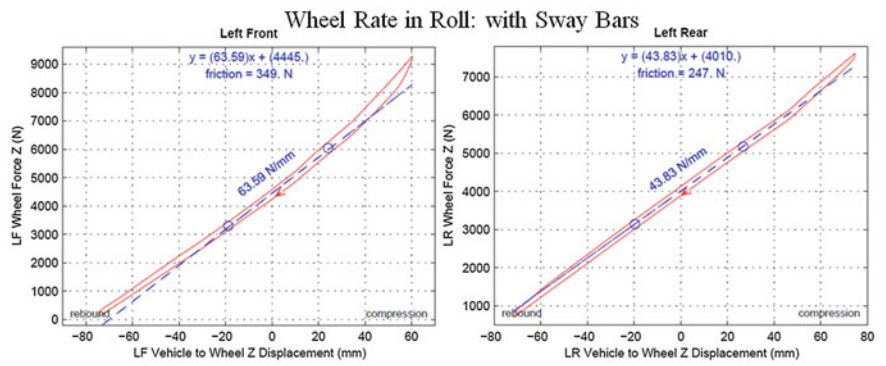
$$\begin{aligned} \text{Outside Jacking Force} &= \text{Outside wheel vertical force} \\ &\times \text{Outside tire grip} \times \tan\Theta_{\text{outside}} \end{aligned} \quad (24)$$

$$\begin{aligned} \text{Inside Jacking Force} &= -\text{Inside wheel vertical force} \\ &\times \text{inside tire grip} \times \tan\Theta_{\text{inside}} \end{aligned} \quad (25)$$

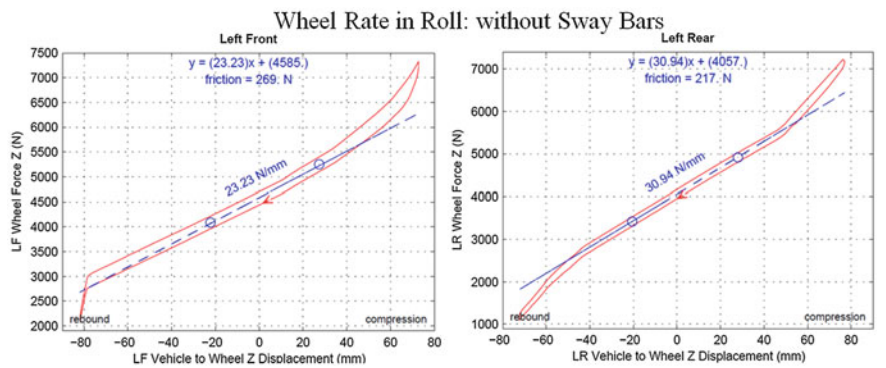
7.15 Sway Bar Systems

Sway bar subsystems can be highly non-linear and need to be quantified separately from the vehicle to measure their performance adequately. Predictive equations are generally optimistic due to the fact that they do not account for the compliance and nonlinearity of the support bushings and the mechanical effects of the vertical link

connections. It is easier to consider the suspension rate contribution of the sway bar system instead of the roll stiffness contribution. The sway bar suspension rate contribution can be added directly to the wheel rate from the suspension spring using identical units. Roll calculations can then proceed from the wheel rate calculations and vehicle track.



Test Data 21 Front wheel rate is 175% greater; rear wheel rate is 42% greater than without sway bars



Test Data 22 Jounce and rebound bumper effects are clearly visible without sway bars

7.16 Dampers

Dampers are intentionally unsymmetrical and highly non-linear. Damper characteristics are developed subjectively because science and analysis has yet to fully explain

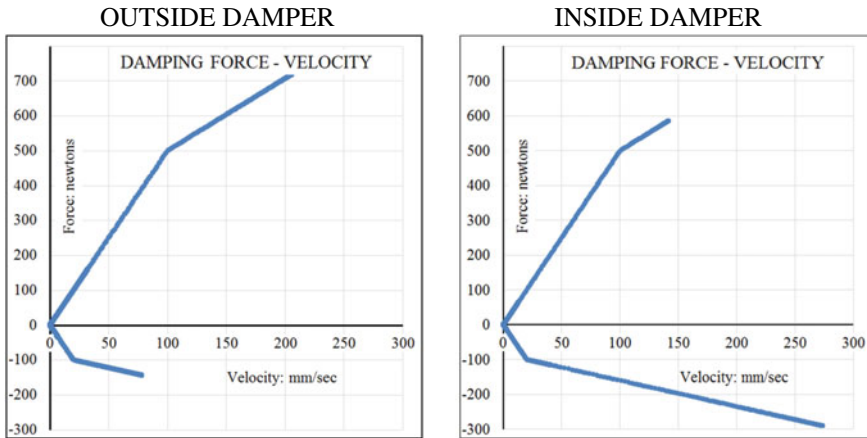


Fig. 66 Damper force–velocity profile

the basis of excellent vehicle performance. Dampers provide unsymmetrical, non-linear damping as well as temporarily influence tire patch loading, vehicle ride height and roll center locations during transient events.

Classical ‘percentage of critical damping’ calculations do not quantify non-linear damping performance adequately. The slope of the force–velocity plot in classical damping analysis does not quantify non-linear damping. The initial slope, the Y-intercept of the high speed slope and the area under the Force -Velocity curve are important factors in developing damping and energy absorption. Impulse mechanics; force–time and mass-velocity, also need to be considered. The more damping developed early results in less damping being needed later to control a disturbance event. Since there is no adequate non-linear damping quantification method, a visual comparison of system performance to an equivalent linear model can provide some insights as to percentage of critical damping.

A time based analysis of a vehicle spring mass system with an unsymmetrical, non-linear damper illustrates the differences in damping forces, velocities and displacements that occur during a vehicle roll transient (Figs. 66 and 67).

Unsymmetrical damper forces result in unsymmetrical tire loading and unsymmetrical suspension displacements during a transient event. The unsymmetrical displacements result in roll center and suspension kinematic conditions not predicted during kinematic studies and weight transfer calculations.

Dampers are major contributors to vehicle transient performance and can be contributors to steady state performance. Classical steady state analysis assumes that the wheel ends do not oscillate during a steady state event. The wheel ends do oscillate at wheel hop frequency which is superimposed upon the steady state condition. Wheel hop oscillations are caused by road surface, tire and wheel rotational imperfections and other physical factors in the suspension. The result is that

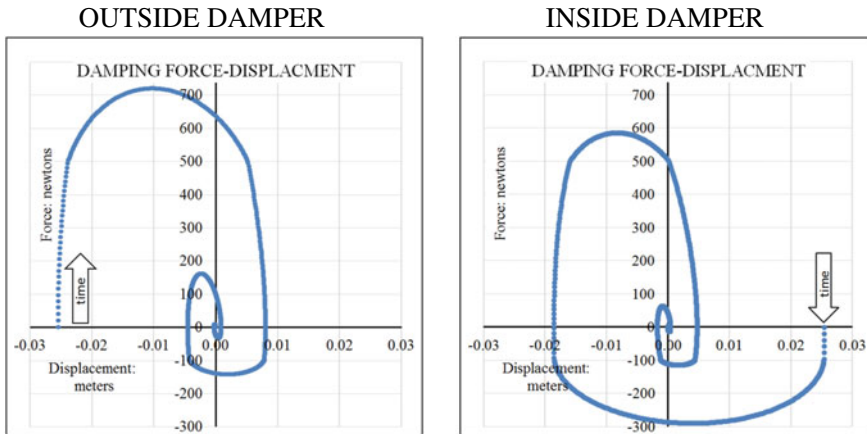


Fig. 67 Damper displacement–time history

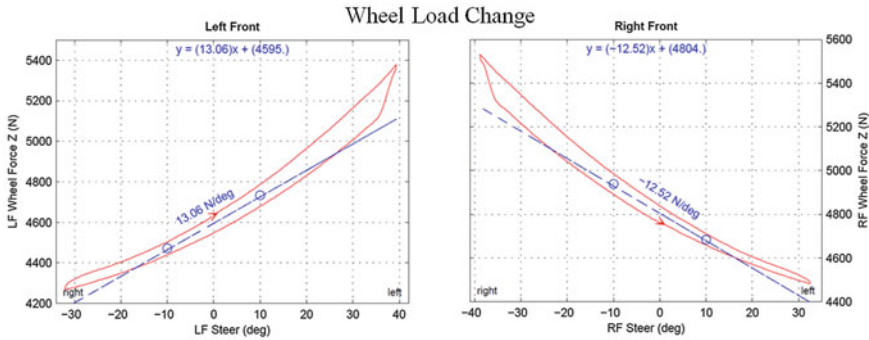
the dampers are functioning and developing unbalanced jounce and rebound forces effecting steady state tire loading and vehicle dynamics.

7.17 *Travel Limiters*

Suspension travel limiters such as jounce and rebound bumpers are highly non-linear force and rate components. They effect suspension stiffness under dynamic ride and roll situations. The effects are not symmetrical. The bushing modeling equation in Sect. 5.3 is adequate for representing jounce and rebound bumpers.

7.18 *Steering Weight Redistribution*

If the tire contact patch was not against the ground when steered, it would move in a three dimensional arc as determined by the king pin inclination angle and scrub radius while biased by caster in the side view. When a vehicle is sitting on level ground, this vertical motion is absorbed by vehicle height and suspension deflections. The front wheel end geometry introduces corner weight redistributions as it is steered. One front corner gains static weight while the adjacent corners lose static weight. The static weight changes in the front produce opposite weight changes at the rear. This weight redistribution is additive to other weight transfer factors.



Test Data 23 Wheel force change must be equal and opposite at all adjacent vehicle corners

7.19 Vehicle Inertias

Vehicle inertias are not factors in steady state operation calculations. However sprung mass roll, pitch and yaw inertias and the total vehicle yaw inertia, which includes the wheel end masses, come into play during transient maneuvers.

7.20 Component and Vehicle Development

Compliance and kinematic factors result in ‘micro steers’; small wheel end toe changes at all four corners and lateral motions of the contact patch due to camber deflections and wheel end lateral translations. These effects happen at different rates resulting in ‘micro-phasing’ of the effects during a vehicle transient event. The effects are subtle compared to the global cornering event. They are difficult to quantify analytically, difficult to measure in vehicle performance testing but are critical in developing a proper dynamic feel of the vehicle. Vehicle testing methodology and analytics have not caught up to subjective quantification of these factors in dynamic situations. The best method to acquire understanding is by testing components and simple K and C measurements to quantify characteristics, then experimenting with an actual vehicle to gain a sense of the dynamic consequences. Component differences that are not discernable in K and C testing and analysis can be quite apparent in subjective dynamic vehicle assessments. The component differences shrouded by global K and C vehicle measurements can be clearly observed when tested as subassemblies or components.

8 Handling and Tire Utilization

8.1 Steady State and Transient Handling

Vehicle steady state handling analysis does not involve time factors or vehicle inertias. Steady state handling can be analyzed as a static state of the vehicle in a corner. Vehicle transients such as corner entry, corner departure or lane changes are more complex. Tires do not generate cornering force immediately. It takes a certain distance for the tire to reach a fully deflected and stable loaded state. This characteristic is referred to as relaxation length. The time to achieve stable cornering forces depends upon the vehicle velocity. Kinematic and compliance factors also develop at different rates influencing transient vehicle dynamics.

8.2 Understeer Budget

Vehicle sublimit modeling is generally satisfactory when the weight transfer, kinematic and compliance factors are assumed to be linear coefficients as measured around the nominal vehicle suspension setting. The understeer budget assumes these coefficients are linear.

Aligning moment poses a problem for modeling since a linear coefficient is a poor representation. Aligning moment rises and then falls as lateral force and slip angle develop. Replacing aligning moment with a pneumatic trail coefficient addresses this problem as the pneumatic trail change with slip angle is a reasonably linear function (Fig. 68).

Conventional understeer budgets include separate factors for lateral force steer and aligning moment steer. This approach is valid on-center (the aligning moment coefficient is calculated based upon the rising aligning moment) however it is not accurate at higher G levels when the aligning moment reverses slope and ultimately becomes negative.

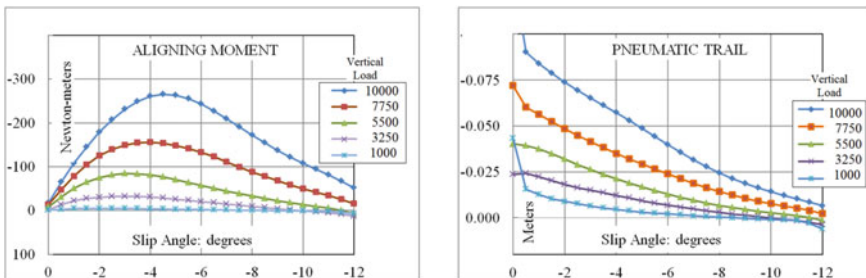


Fig. 68 Aligning moment and pneumatic trail versus slip angle

Replacing the lateral force steer and aligning moment factors with a single factor that recognizes the rising lateral force and the falling pneumatic trail together provides a clearer understanding of steering compliance due to lateral force generation (Tables 11 and 12).

Rear suspension compliance has a direct effect on transient handling and subjective vehicle dynamics assessments (Tables 13 and 14). Changing rear suspension compliance oversteer to understeer results in a significant change in total vehicle understeer as illustrated in the comparisons below:

Rear suspension compliance character is especially important in transients because the steering and the cornering forces develop at different times. Front cornering force develops first as the front tires are turned to initiate the maneuver. The rear of the vehicle reacts to the front input. Given a steering input at the front axle, the vehicle experiences a lateral translation at the CG and a rotation around the CG due to vehicle yaw inertia. The proportion of translation and rotation depends upon the compliance of the rear axle and weight transfer that drives rear tire slip angles. If the rear compliance is stiff, the vehicle will tend to translate more than yaw. If rear compliance is soft, the vehicle will tend to yaw more than translate at the initiation of the event. The sensations to the observers (the driver and passengers) are distinctly different. The yaw and translation amplitudes and the ‘micro-phasing’ of these events contribute to the driver’s sensations and observations.

Table 11 Understeer budget with lateral force and aligning moment compliance

Understeer budget			
Independent front/solid rear, + understeer effect, – oversteer effect	Front deg/G	Rear deg/G	Source/comments
Weight distribution effect; (tire connering stiffness)	7.2	–6.6	Tire properties, weight, roll stiffness distrib
Aligning torque on rigid body	0.1	–0.1	Very minor effect
Suspension roll camber	1.2	0.0	No rear axle camber change
Suspension roll steer	0.5	–0.5	Rear roll oversteer
Lateral force deflection steer compliance	0.2	0.2	Minimal rear compliance
Aligning torque deflection steer compliance	1.3	0.1	Minimal rear compliance
Sum total cornering compliance	10.5	–6.9	
Total understeer = 10.5 – 6.9 deg/G	3.6		deg/G

Table 12 Understeer budget using lateral force and pneumatic trail

Understeer budget		Combined Lat. Force and aligning moment compliance	
Independent front/solid rear; + understeer effect, – oversteer effect	Front deg/G	Rear deg/G	Source/comments
Weight distribution effect; (tire cornering stiffness)	7.2	–6.6	Tire properties, weight, roll stiffness distrib
Suspension roll camber	1.2	0.0	No rear axle camber change
Suspension roll steer	0.5	–0.5	Rear roll oversteer
Cornering Force/pneumatic trail compliance	1.5	0.3	Minimal rear compliance
Sum total cornering compliances	10.4	–6.8	
Total understeer = 10.5 – 6.7 deg/G	3.6		deg/G

Table 13 Understeer budget with rear compliance oversteer

Understeer budget		Combined lateral force and aligning moment compliance	
Independent front/solid rear; + understeer effect, – oversteer effect	Front deg/G	Rear deg/G	Source/comments
Weight distribution. effect; (tire cornering stiffness)	7.2	–6.6	Tire properties, weight, roll stiffness distribution
Suspension roll camber	1.2	–1.2	Rear camber change
Suspension roll steer	0.5	0.0	Rear neutral
Cornering force/pneumatic trail compliance	1.5	–1.0	Rear compliance oversteer
Sum total cornering compliances	10.4	–8.8	
Total understeer = 10.4 – 8.8 deg/G	1.6		deg/G

8.3 Tire Forces

Tire test bench data is presented in a coordinate system aligned parallel to the axle, perpendicular to ground and parallel to the wheel plane. Lateral force measurement is aligned to the axle not the direction of travel. The steering angle is greater than the slip angle for an understeering vehicle. The direction of travel is defined by the steering angle minus the slip angle.

Measured lateral force is not the same as the cornering force experienced by a vehicle. The cornering force is perpendicular to the tangential direction of circular motion and the slip angle not to the wheel plane. For small steering angles this fact

Table 14 Understeer budget with rear compliance understeer

Understeer budget		Combined lateral force and aligning moment compliance	
Independent front/solid rear; + understeer effect, – oversteer effect	Front deg/G	Rear deg/G	Source/comments
Weight distribution effect; (tire cornering stiffness)	7.2	–6.6	Tire properties, weight, roll stiffness distribution
Suspension roll camber	1.2	–1.2	Rear camber change
Suspension roll steer	0.5	0.0	Rear neutral
Cornering force/pneumatic trail compliance	1.5	1.0	Rear compliance understeer
Sum total cornering compliances	10.4	–6.8	
Total understeer = 10.4 – 6.8 deg/G	3.6		deg/G

can be ignored. As slip angle increases the error increases. Presenting cornering force as the lateral force multiplied by the cosine of slip angle improves the representation of cornering forces at high slip angles (Fig. 69).

Retardation force becomes significant to vehicle performance as cornering force increases: The retardation force increases as the sine of the slip angle. For small angles, doubling the slip angle doubles the retardation force. Longitudinal retardation force can be observed in vehicle skid pad tests. As a vehicle operates on a skip pad, significant power must be added as lateral G's and speed increases. For the same G level, a tire operating at a higher slip angle requires more power than a tire operating at a lower slip angle. As slip angle increases, retardation force can approach 20% of lateral force at 10 degrees slip angle (Fig. 70).

Fig. 69 Horizontal tire contact patch force orientations and locations

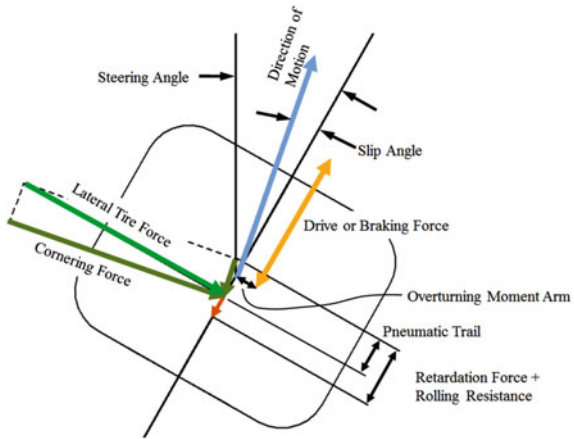
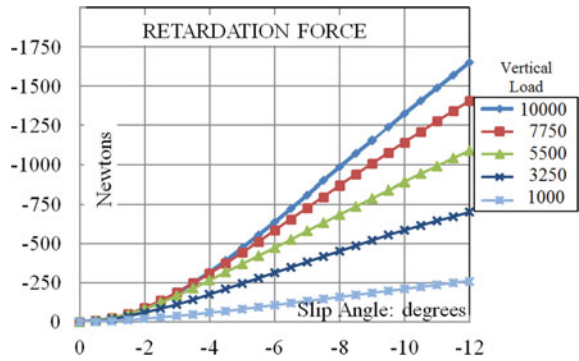


Fig. 70 Longitudinal retardation tire contact patch force



8.4 Lines of Constant Grip

Tire data is normally presented as slip angle on the X-axis and lateral force on the Y axis. Lines of constant lateral grip can be superimposed on this plot. Grip is defined as lateral force (or cornering force) divided by vertical force. Grip is a representation of the tire lateral traction at a particular state. It can also be considered a value of ‘utilized mu’ or ‘utilized friction’ (Fig. 71).

Lateral Grip = *Lateral force(or cornering force)/normal force* (26)

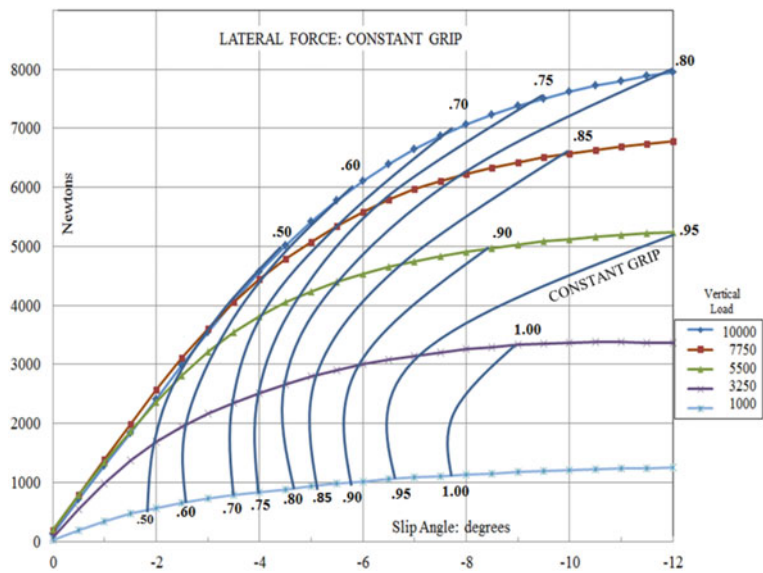


Fig. 71 Lines of constant grip overlaid on tire lateral force versus slip angle plot

As normal force (tire vertical load) increases, additional slip angle is necessary to develop the same grip. The highest grip or maximum friction is available at low normal forces.

8.5 Vehicle State Examination

A vehicle in a steady state corner operates at a constant lateral G-level (ignoring slight differences in turn radii). The front and rear cornering grip levels equal the cornering G's being developed.

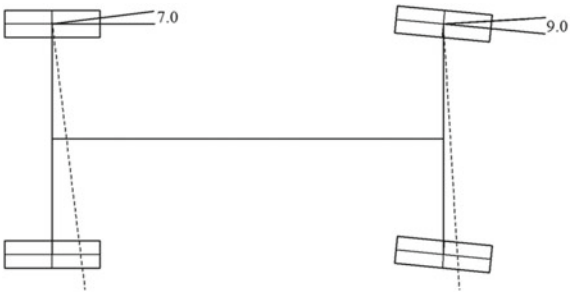
A hypothetical vehicle with weight transfer and steering geometry can be superimposed upon the tire lateral force versus slip angle plot in order to examine tire performance at each vehicle corner (Table 15).

The vehicle is modeled operating in a 300 foot diameter corner at low speed and at 0.8 G. Outer steering angle at 0 G is determined to be 3.59°. At 0.8 G the outer front tire is estimated to operate at 9.0° slip angle and the outer rear tire is estimated to operate at 7° slip angle. The understeer coefficient is estimated to be 2.5° per G (Fig. 72).

Table 15 Vehicle specifications

Hypothetical vehicle
115" Wheel base, 68" track
Vehicle weight: 4968 lbs, 54% front, 50% left
Sprung weight: 4468 lbs
Total roll stiffness: 113,706 ft-lbs/radien, 63.6% front
Unsprung weight: 250 lbs per end
Roll center heights: 2" front, 4" rear
Vehicle CG height: 24.8 in.
Sprung mass CG height: 26 in.
Parallel front axle steer geometry (no Ackermann)
Solid rear axle (no rear wheel toe change)

Fig. 72 Hypothetical vehicle slip angles and radius of rotation



Orienting the inner tire centers of rotation to the vehicle's point of rotation, the inner tire slip angles would be slightly greater than the outer tire slip angles. The difference between inner and outer slip angles is minimal. For this analysis the inner and outer slip angles are assumed to be identical. This vehicle can be considered to have parallel steer in the front and a solid axle in the rear (Table 16).

Weight transfer and tire cornering states. Equations equating the sum of individual tire cornering forces at each axle to the lateral G force at that axle are written for incremental g-levels. The weight transfer calculations provide the vertical loads at each corner; however the individual tire grip levels are not known and are treated as variables.

The weight transfer and tire cornering state equation for the front axle of the vehicle during cornering:

$$\begin{aligned} &\textbf{Cornering GLevel} \times \textbf{Front Weight} \\ &= (\textit{outer front corner vertical force} \times \textit{outer front tire grip}) \\ &+ (\textit{inner front corner vertical force} \times \textit{inner front tire grip}) \end{aligned} \quad (27)$$

The equivalent equation for the rear axle of the vehicle during cornering:

$$\begin{aligned} &\textbf{Cornering GLevel} \times \textbf{Rear Weight} \\ &= (\textit{outer rear corner vertical force} \times \textit{outer rear tire grip}) \\ &+ (\textit{inner rear corner vertical force} \times \textit{inner rear tire grip}) \end{aligned} \quad (28)$$

These calculations are repeated in increments of 0.1 G from 0.5 to 0.8G for the sample vehicle (Table 17).

Steady state and tire performance overlay. The tire load states at each G-level are superimposed upon the tire plot, locating each solution where:

- The sum of inner and outer tire lateral force equals the cornering force at each vehicle end, with the tires at calculated vertical loads and the same slip angle (Fig. 73).

This produces a graph of the vehicle and tire state where four conditions are satisfied:

- The vehicle's weight transfer calculations are preserved;
- The four tires are operating at their calculated vertical loads;

Table 16 Vehicle steering and slip angles

Tire angles		Rear	Front
Base Ackermann Steering	Outer	0.00	3.59
	Inner	0.00	3.73
Slip angles @ 0.8 G	Outer	7.00	9.00
	Inner	7.25	9.26

Table 17 Vehicle corner weights with weight transfer and tire grip equations

Cornering G-level	Corner weight				Tire pair equations
		Left	Right		
0.5 G	Front	8545	3368	Front equation	$8545 (\text{MuFout}) + 3368(\text{MuFin}) = 0.5(11914) = 5957 \text{ N}$
	Rear	6801	3383	Rear equation	$6801 (\text{MuFout}) + 3383(\text{MuFin}) = 0.5(10184) = 5092 \text{ N}$
0.6 G	Front	9063	2851	Front equation	$9063 (\text{MuFout}) + 2851(\text{MuFin}) = 0.6(11914) = 7148 \text{ N}$
	Rear	7142	3042	Rear equation	$7142 (\text{MuFout}) + 3042(\text{MuFin}) = 0.6(10184) = 6110 \text{ N}$
0.7 G	Front	9581	2333	Front equation	$9581 (\text{MuFout}) + 2333(\text{MuFin}) = 0.7(11914) = 8340 \text{ N}$
	Rear	7484	2700	Rear equation	$7484 (\text{MuFout}) + 2700(\text{MuFin}) = 0.7(10184) = 7128 \text{ N}$
0.8 G	Front	10,098	1815	Front equation	$10098 (\text{MuFout}) + 1815(\text{MuFin}) = 0.8(11914) = 9531 \text{ N}$
	Rear	7826	2358	Rear equation	$7826 (\text{MuFout}) + 2358(\text{MuFin}) = 0.8(10184) = 8147 \text{ N}$

- The inner and outer slip angles are equal as established by the vehicle's kinematics;
- The sum of tire lateral force at each end equals the vehicle cornering force at each end.

Steady state and tire grip. Lines of constant tire grip superimposed on the tire and vehicle state plot reveal the grip at each individual tire (Fig. 74).

This plot presents the performance state of each tire as the vehicle is cornering. It reveals that the four tires are not contributing equally to vehicle cornering grip. Considering the lines of constant grip, the outside tires operate at lower grip levels while the inner tires operate at higher grip levels than the vehicle as a whole (Table 18). Each of the four tires operate at different grip levels:

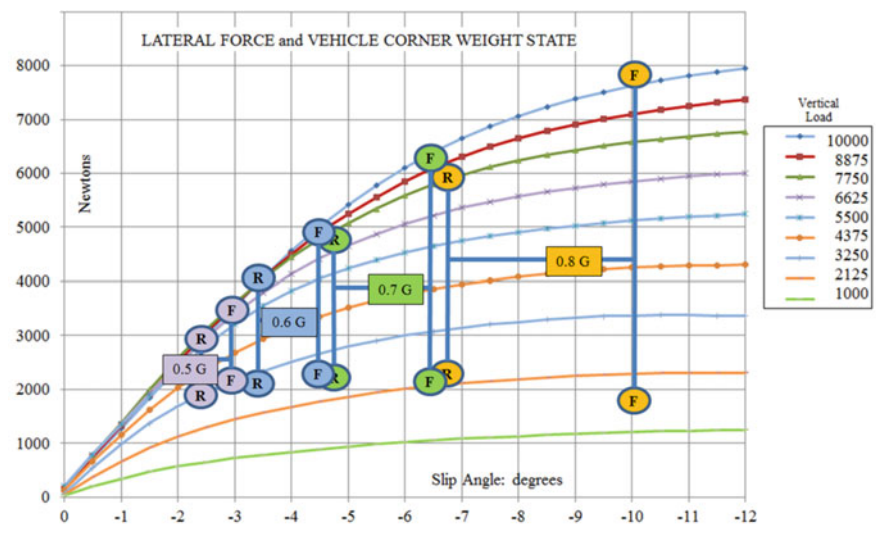


Fig. 73 Vehicle G-level, weight transfer and tire states overlaid on tire force versus slip angle plot

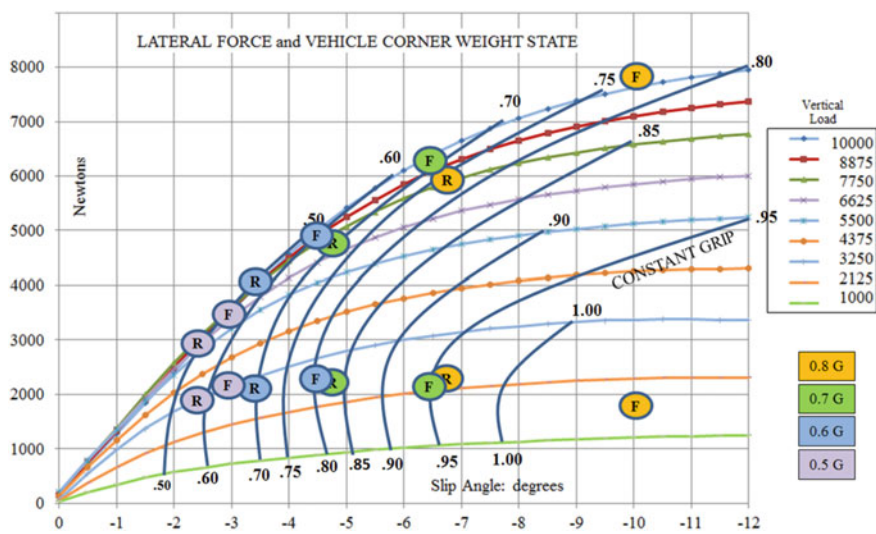


Fig. 74 Vehicle weight transfer and tire states, lines of constant grip overlaid on tire lateral force versus slip angle plot

8.6 Weight Transfer, Tire Force and Slip Angle Progression

The progression of the tire and vehicle state as G levels and slip angles increase can be plotted. As G levels and slip angles increase, the outer tires progress across lines

Table 18 Individual tire grip

		Outer grip	Inner grip
0.5 G	Front	0.48	0.65
	Rear	0.49	0.58
0.6 G	Front	0.57	0.80
	Rear	0.58	0.70
0.7 G	Front	0.68	0.95
	Rear	0.70	0.83
0.8 G	Front	0.77	1.04
	Rear	0.77	0.97

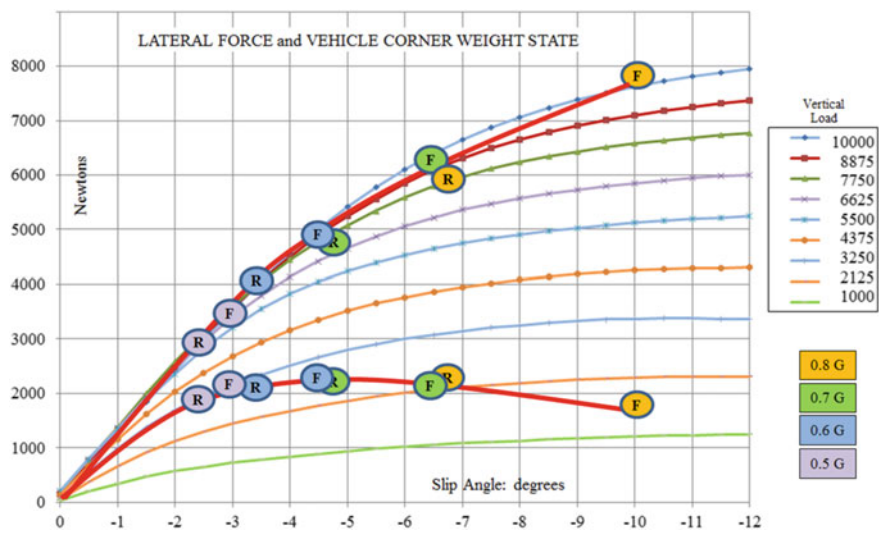


Fig. 75 Vehicle weight transfer and tire state progression across tire lateral force versus slip angle plot

of increasing vertical load. The inner tires progress across lines of decreasing vertical load resulting in a slope reversal. Scanning up and down these lines illustrates the tires’ state as a corner is entered, maintained and exited. This plot does not consider vehicle roll or tire camber sensitivity. The plotting methodology could be refined to include both as well as other tire performance variables (Fig. 75).

8.7 Vehicle Design Opportunities

The consequences of current vehicle steering architectures can be quantified by considering tire utilization concepts where the tires states are examined for equality

of grip, minimization of retardation force or maximization of tire performance. The combined tire performance and vehicle state plot provides insights into possible new vehicle design and performance solutions:

- The plots reveal that the outer tires are ‘under-utilized’ while the inner tires are ‘over-utilized’ considering the vehicle cornering G level and tire grip;
- The methodology provides an examination of tire slip angles and tire grip while cornering and can lead to logical vehicle solutions incorporating independent steering at each vehicle corner;
- Maximum cornering grip and vehicle lateral G’s can be deduced if the four tires were allowed to steer independently;
- Fuel economy could be improved by minimizing total retardation force if the four tire steering and slip angles were determined independently.

The methodology concepts presented can lead to new vehicle architecture and design opportunities:

- Logical tire performance based solutions for ‘programmed Ackermann’ and four wheel steer solutions;
- Potential for maximizing fuel economy or performance with reduced tire size and weight.

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Tyre Mechanics and Thermal Effects on Tyre Behaviour



Andrea Genovese and Francesco Timpone

Abstract This chapter deals with tyre mechanics and it has a particular focus on thermal effects on its dynamical behaviour. In the first part the typical tyre structure is introduced together with the tyre mechanical/dynamical behaviour according to a classical approach, so recalling the main kinematic and dynamic quantities involved in tyre pure and combined interactions. The core of this chapter is the description of a physical-analytical tyre thermal model able to determine the thermal status in each part of the tyre useful for vehicle dynamics modelling and driving simulations in order to take into account thermal effects on tyre interactions and consequently on vehicle dynamical behaviour. Successively also the tyre wear modelling is faced, after a brief introduction to the different models available in literature some considerations are reported concerning the thermal effects on wear.

1 Introduction to the Tyre

The wheel is considered one of the most important inventions, because it found use in a wide range of applications, as well as transportation vehicles, construction equipment and internal parts of machineries (Mark et al., 2013). Nowadays, there are tyres for several categories based on vehicle's applications: tyres for racing and passenger vehicles, tyres for light and heavy trucks and finally tyre for special use, as agricultural, military or heavy industry. Depending on their application, different constructive composite solutions are adopted employing significant quantities of canvas, steel wire and fabric reinforcements.

Regardless of the design or application, all tyres must fulfil some fundamental tasks, as to provide load-carrying capacity, to ensure dampening effect, to

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properly transmit driving and braking torques, providing cornering force and generating minimum noise and minimum vibrations (Guiggiani, 2014; Pacejka, 2012; Rajamani, 2012). The intrinsic structure dampening characteristics, elastic properties of rubber and unique deformability and recovery capacity make the tyre the only device that satisfies all of the above-mentioned tasks.

2 Tyre Structure

The tyre is an under pressure, inextensible and deformable casing, made of composite material of polymeric matrix (vulcanized rubber and additives such as carbon black and silica) with reinforcement plies of steel or synthetic fibres (Mark, 2007; Mark et al., 2013). It has the shape of a solid of revolution (toroidal ring) with the function to bear statically and dynamically a given load, to transmit to the ground the longitudinal and lateral forces necessary to motion, to ensure the directionality of the vehicle enabling the steering and the insertion of it in the driver's desired trajectory, to perform both traction and braking force by means of adherence with the road surface, improve passenger comfort contributing to the vehicle suspension. The materials of modern pneumatic tyres are synthetic rubber, natural rubber, fabric and wire, along with carbon black and other chemical compounds. All these numerous components are built up on a drum and then cured in a press under heat and pressure. Heat facilitates a polymerization reaction that cross-links rubber monomers to create long elastic molecules. These polymers create the elastic quality that permits the tyre to be compressed in the area where the tyre contacts the road surface and spring back to its original shape under high frequency cycles.

Currently, the most common tyre in automotive industry is the tubeless type, lacking of inner tube. The inner side of the tyre is constituted by a special rubber coating, highly airtight, called liner. The parts constituting a tubeless tyre are the following (Fig. 1):

- Tread, a thick layer of rubber compound, commonly styrene-butadiene (SBR) copolymers (Farroni et al., 2014b), which provides the interface between the tyre and the road, and whose surface is crossed by a “pattern” able to ensure a good grip in both wet and dry conditions, as well as good characteristics of silent running. The pattern is composed by a particular arrangement of full (ribs) and void (grooves) volumes; the latter are designed to channel the water out and away ensuring a “dry” contact between rubber and soil.
- Shoulder, the area including the tread lateral extreme edge and the beginning of the sidewall. The shoulder rubber is the thickest so that the design must allow for the easy diffusion of heat generated within the tyre while driving. That's why the shoulder section plays a crucial role in a good heat dispersal.
- Sidewall, the part between the shoulder and the bead bundle. It consists of a thin flexible rubber layer, intended to protect the carcass plies against side impacts, such as the sidewalls edges. The sidewall helps to make the tyre rigid and improves

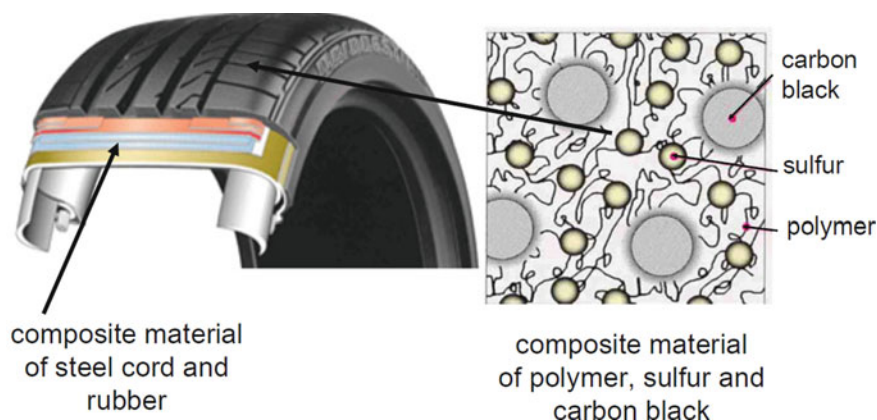


Fig. 1 Tyre structure (Nakajima & Nakajima, 2019a)

the ride quality; generally speaking, a high sidewall tyre is more comfortable than a low one as it is more flexible and hence able to better absorb the road unevennesses. A tyre's type, size, structure, pattern, manufacturing company, product name and various characters are indicated here.

- **Bead**, a tyre/rim coupling element; the core of a tyre's bead is made up of several individually rubber-coated steel wires formed into a hoop, that is then provided with a rubber apex. The bead function is to ensure that the tyre must be slightly tightened around the rim, in order to prevent relative rotations or, in the case of a sudden drop in inflation pressure, the tyre will not fall off the rim. This coupling is particularly critical, considering that all the actions exchanged between tyre and road are transmitted to the vehicle via the bead/rim contact surface that has a very limited extension and therefore is subjected to high stress levels.
- **Inner liner**, a thin layer of specially formulated compound, placed on the inner surface of tubeless tyres, that resists air diffusion and improves air retention by lowering permeation outwards through the tyre. Generally made of a synthetic rubber called butyl, or of a rubber of the polyisoprene variety, the inner liner maintains the air inside the tyre.
- **Belt**, a strong reinforcement found between the tread and the carcass in a radial or belted bias tyre. It consists in a restrict expansion of carcass plies, whose functions enclosure a stabilizing the tread area and a providing impact resistance. Varying the belt width and angles affects vehicle ride and handling characteristics. Alternate belt constructions with materials other than steel is also often utilised.
- **Carcass**, the most important framework of a tyre, constitutes its resistant structure, mainly acting in normal interaction. It is composed of one or more layers of plies placed under the tread (or under belt plies in radial tyres). Every single ply consists in series of mutually parallel cords of very durable and at the same time flexible material, surrounded by the vulcanized rubber compound. Each carcass

ply extends from bead to bead, wrapping and transferring on them the stresses generated by inflation pressure and interaction forces.

Body plies arrangement gives the name to the structure of the tyre: nowadays the most commonly employed structure is the radial one, in which the carcass is composed of one or more plies disposed through the meridian planes of the toroid. The radial carcass is made more stable by the belt annular reinforcing structure which runs below the tread.

The belt plies have very small inclination angles, between 5° and 10° and absorb the shear stresses which arise in working conditions. The advantages of radial are mainly due to the fact that the tread is independent from the carcass; consequently, the tangential stresses (absorbed by the belt plies) are decoupled from normal ones (absorbed by radial carcass plies), while in bias tyres all stresses decompose according to the crown angles. Furthermore, the radial arrangement reduces fatigue and wear: being the sidewall thinner, the material is less subjected to hysteresis, occurring for shear between plies and generating heat to be dissipated. Radial tyres are characterized by better grip, more stability and a higher braking efficiency, acting as safer than conventional ones; they also allow greater energetic saving (increased durability and fuel economy) and a greater level of comfort (more shock absorption) (Allouis et al., 2016). The other type of structure, still used in some applications for trucks, trailers and farm implements, is called diagonal or bias because of the plies cross on the carcass being disposed according to proper inclinations respect to equatorial plane called “crown angles”. Note that in these tyres there are not belt plies; therefore, it is quite simple to recognize a radial tyre, being the sidewall plies fewer than the tread ones, for the presence in the latter of belt reinforcements.

3 Mechanics of Tyres

The tyre is fundamental in vehicle dynamics and, therefore, many automotive companies spend a lot of time on developing the tyre structure and improving its behaviour within the contact with the road.

Hence, the tyres have to fulfil several functions, such as supporting the weight of the vehicle, cushioning the vehicle over surface irregularities, providing sufficient traction for driving and braking and, further, providing adequate steering control and direction stability. For these and other reasons, the study of tyre mechanics is essential for comprehending the vehicle performance.

Reference system. In order to describe the phenomena involved in the interaction with the road, together with the tyre behaviour and the forces and moments arising during the vehicle motion, it is necessary to define an axis system that serves as a reference for the definition of several parameters. One of the commonly used axis systems recommended by ISO855 standard is shown in Fig. 2.

First, the basic components playing an active role in tyre mechanics should be defined: the rim, which is the outer edge of a wheel holding the tyre and is assumed as

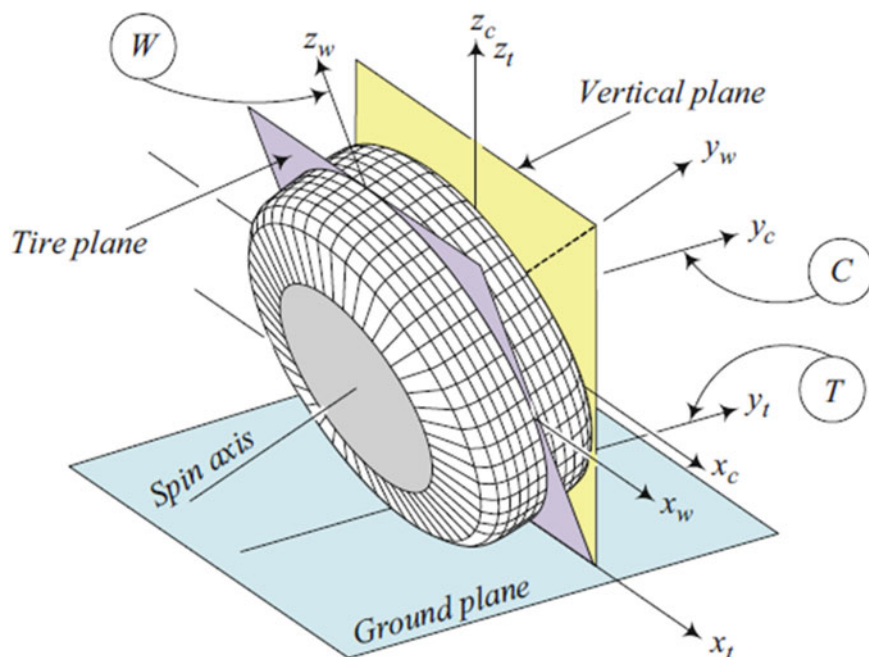


Fig. 2 Tyre reference system (Jazar, 2019)

a rigid body, the flexible carcass of the inflated tyre and the contact patch between the tyre and the road. For simplicity, the road is assumed to have a hard and flat surface, like a geometric plane. This is a good model for any road with high quality asphalt paving, since the texture of the road surface is not relevant for the definition of rim kinematics. Once assumed the road and the rim as rigid bodies, the origin of the axis system is set on the centre of the tyre contact area. The x -axis is the intersection of equatorial wheel plane and the ground plane with a positive direction forward; z -axis is perpendicular to the ground plane, which has the positive direction upward; y -axis on the ground plane and its direction is chosen so that the axis system satisfies the right-hand rule (Jazar, 2019).

There are three forces and three moments acting on the tyre from the ground: tractive force (or longitudinal force) F_x is the component in the x direction of the resultant force exerted on the tyre by the road, lateral force F_y is the component in the y direction, and normal force F_z is the component in the z direction. “Overturning moment” M_x is the moment about the x axis exerted on the tyre by the road, “rolling resistance” moment M_y is the moment about the y axis, and “aligning torque” M_z is the moment about the z axis. With this axis system, many performance parameters of the tyre can be conveniently defined; for instance, the longitudinal shift of the centre of normal pressure is determined by the ratio of the rolling resistance moment to the normal load, the lateral shift of the centre of normal pressure is defined by the ratio of

the overturning moment to the normal load and the integration of longitudinal shear stresses over the entire contact patch represents the tractive or braking force.

There are two important angles associated with a rolling tyre: “slip angle” and “camber angle”: slip angle α is the angle from the x axis to the velocity vector of the centre of tyre contact, about the z axis; camber angle γ is the angle formed between the xy plane and the wheel plane. Lateral interaction forces at the tyre/ground interface are a function of both the slip angle and the camber angle. In the following, brief notes about lateral and longitudinal tyre dynamics will be provided; for more detailed analyses the consultation of wide available bibliography is suggested (Connelly & Huston, 1994; Dashora, 1994; De Castro et al., 2012).

Tyre kinematics. Let Q be a point on the rim axis y_c (see Fig. 3). The position of the rim with respect to the flat road depends only on the height h of the point Q and on the camber angle γ . The latter is the angle between the rim axis and the road plane. The rim, being a rigid body, has a defined angular velocity Ω . Therefore, the velocity of any point P of the space moving together with the rim is given by the well-known equation:

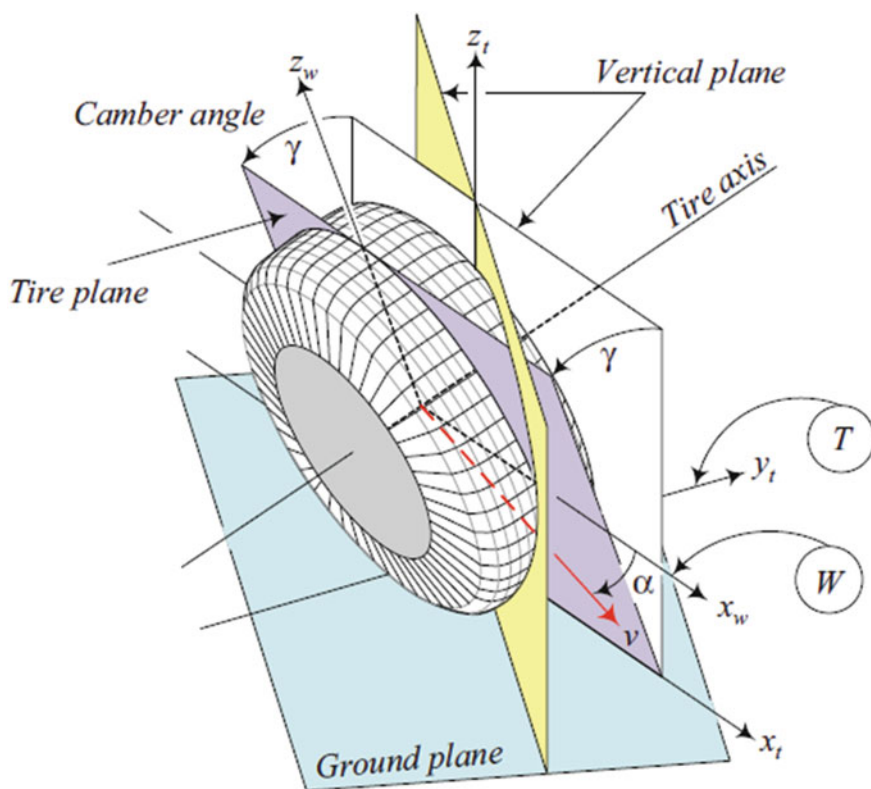


Fig. 3 Tyre reference system planes (Jazar, 2019)

$$V_P = V_Q + \Omega \times \overline{QP} \quad (1)$$

where V_Q is the velocity of the point Q and $\overline{QP}$ is the vector connecting Q to P. The three components of V_Q together with the Ω ones are the six parameters that completely determine the rim velocity field. The angle between the velocity V_P corresponding to the centre of contact patch, which is parallel to the flat road, and the x axis of the reference system is called slip angle α . The latter is fundamental in the lateral interaction between the tyre and road. In order to describe the wheel motion, it is common to evaluate the following vectorial magnitude, also called slip:

$$s = \frac{V - \Omega \times R}{V_x} \quad (2)$$

where V is the wheel centre velocity, which is parallel to the flat road, Ω is the angular velocity of the wheel, R is the pure rolling radius and V_x is the x axis component of the wheel centre velocity. The quantity $\Omega \times R$ is defined pure rolling velocity and matches with the wheel centre velocity as soon as the tyre works in pure rolling conditions. Distinguishing the slip components along x and y axis, we can define the following magnitudes:

$$\begin{cases} s_x = \frac{V_x - \Omega \times R}{V_x} \\ s_y = \frac{V_y}{V_x} = \tan \alpha \approx \alpha \end{cases} \quad (3)$$

The first quantity is called longitudinal slip, whereas lateral slip is the second one. About the longitudinal slip, it is possible to differentiate the following cases:

- Wheel working in pure rolling condition: there are no differences between the wheel centre and each rim point velocity
- Wheel working in global slip condition (traction phase): the tyre rotates around the wheel axis, but the vehicle does not move forward
- Wheel working in global locking condition (braking phase): the tyre behaves as a rigid body during the vehicle-braking phase.

However, taking into account what happens in the contact patch, the tyre tread usually works in pseudo-slippage condition. Actually, the tread of a tyre is deformable, whereas its belt is not stretchable. Consequently, for example, when a vehicle brakes, the road surface pulls the contact patch backwards, but only the tread is distorted. The tread blocks recline, causing a relative movement between the bottom of the rubber block, in contact with the road surface, and the belt. This is the shear phase (or pseudo-slippage), which occurs at the leading edge of the contact patch (Calabrese et al., 2013; Nakajima & Nakajima, 2019b).

As the rubber tread block gets closer to the trailing edge of the contact patch, the stress increases and the rubber block, whilst remaining sheared, goes into effective slippage condition with the road surface. This means that a mismatch in the velocity

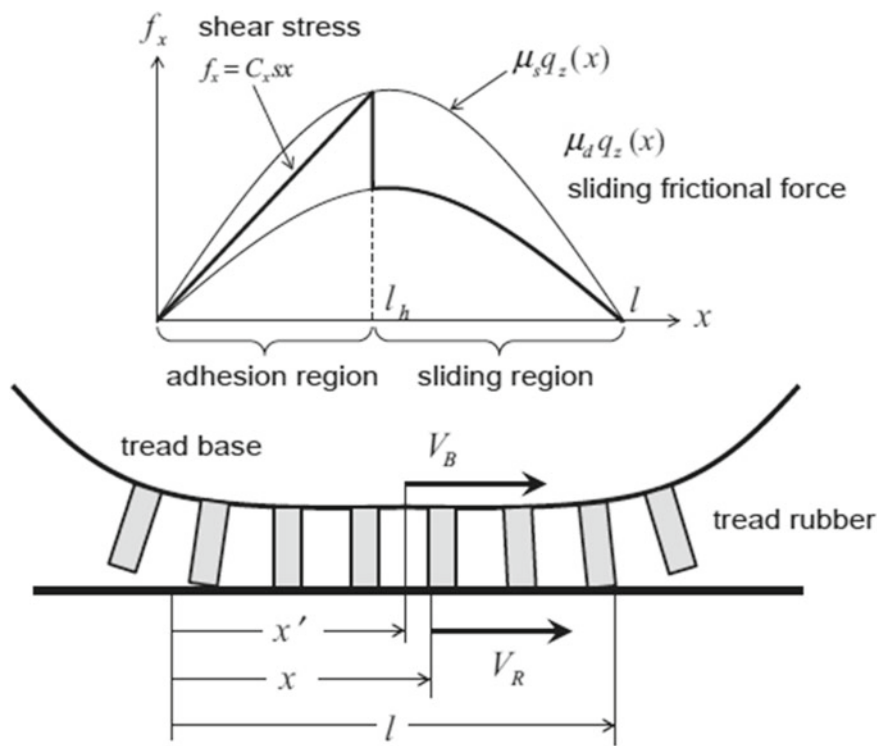


Fig. 4 Tyre slippage condition (Nakajima & Nakajima, 2019b)

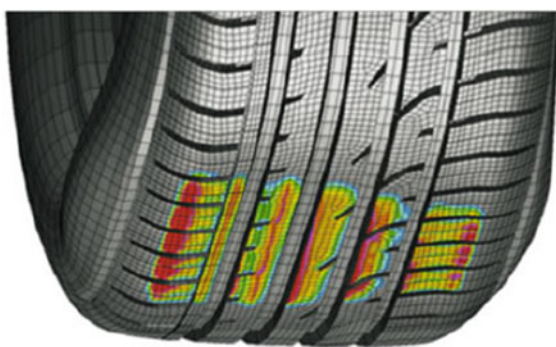
value occurs between the points of the tread in contact with the road and the road surface ones as depicted in Fig. 4.

Mechanisms involved in tyre–road interaction. As previously anticipated, the tread is made of rubber compound, which is an elastomeric material to which the tyres owe a large part of their grip capacity. Grip implies contact between two surfaces: one is the tyre surface and the other is the road surface. The contact patch is the region where the tyre is in contact with the road (see Fig. 4). The patch shape and the position with respect to the reference system depend on the tyre operating conditions: forward speed, load condition, wheel alignment, internal pressure, road roughness, tyre temperature and wear level.

- Road roughness and molecular adhesion. Regarding the road characteristics, grip depends on the type of road surface, the road roughness and wet or dry conditions. More precisely, grip comes from road roughness effects and molecular adhesion. These are mechanisms involved in the relative slippage between the elastomer and the road surface. The first mechanism is about the frequency excitation of the material by the road texture. Indeed, the rubber is distorted when it slips over the rough acnes, whose size varies from 1 cm to 1 μm . This mechanism is also known as

indentation phenomenon, which emphasizes the penetration of the road asperities into tyre tread rubber (Genovese et al., 2020b; Arricale et al., 2020). The second mechanism takes place in a microscale and is amplified by slippage. Therefore, in both tyre–road phenomena, the viscoelastic properties of the tread rubber and its hysteresis, which are characterized by destructive and non-destructive techniques (Carputo et al., 2020; Farroni et al., 2021; Genovese & Pastore, 2021), as well as ultrasound (Lionetto & Maffezzoli, 2009) and optical (Martorelli et al., 2020) methods, play an important role. In the road roughness effects analysis, two main features of the road geometry must be examined considering the tyre grip: the macro and micro roughness. The macroroughness is the name given to the road surface texture when the distance between two adjacent acnes is between 100 microns and 10 mm (Fig. 5). This roughness level contributes to indentation: the tread block strikes against the rough spot and deforms, but, by a hysteresis effect, it does not immediately revert to its initial shape on the other side of the rough spot. This asymmetrical deformation generates a force field that counteract the slippage. The microroughness, instead of macro one, refers to the road surface texture, when the distance between two consecutive rough spots is less than 100 microns. This is related to the surface roughness of the aggregates and sands within the road composition. Since the road roughness effects on the tyre tread arise from small bumps on road spots, the corresponding phenomena occur within a range of stress frequencies between 100 Hz and 1 MHz. The molecular adhesion, instead of roughness effects, outcomes from molecular interactions occurring at the tyre tread–road interface and involves the Van der Waals bonding. Indeed, the rubber molecular chains follow a cycle of stretching and braking, which generates friction phenomena between internal particles in a specific material volume (viscoelastic behaviour). The stress cycle provides three phases: the first phase consists in creating bonds between the tread and the road; in the second one, the molecular chains are stretched and a friction force is generated due to viscoelastic behaviour that opposes the skidding phenomenon; in the last phase, the bond breaks and forms again farther on (Genovese et al., 2019, 2020a, c). It is clear that the essential condition for adhesion to be operative is a direct contact between the tread rubber and the road surface (i.e. when the road is clean and dry). Molecular

Fig. 5 Tyre–road interaction in simulation



adhesion phenomenon usually occurs in a frequency range between 1 MHz and 1 GHz.

- Tyre compound. The viscoelastic behaviour of the tyre tread rubber leads some phenomena involved in the interaction with the road (Carputo et al., 2021; Genovese et al., 2020b). Actually, the consideration of viscoelasticity is important for an accurate stress–strain analysis of the tyre structure, as well as the computation of energy loss for tyre rolling resistance, which is principally the total effect of the hysteretic losses that occurs as a tyre rolls on the road. For this reason, a part of the mechanical energy is converted to heat as a result of the rolling resistance phenomenon: effectively, the tyre consumes a portion of the power transmitted to the wheels, thus leaving less energy available for moving the vehicle forward. Therefore, the rolling resistance plays an important part in increasing the vehicle fuel consumption (Batchelor, 2000; Calabrese et al., 2013). The rolling resistance could be also identified as the effort required for the tyre to keep rolling. It depends on the tyre employed, the nature of the surface on which it rolls and the operating conditions, such as inflation pressure, load and speed. As already mentioned, rolling resistance is largely a result of energy dissipation within the volume of the tyre. Indeed, hysteretic losses amount for about 80–95% of the total rolling resistance and most research efforts, which are related to reducing rolling resistance, always focus on this aspect. This dissipation occurs as consequence of the tyre materials, as soon as they are cyclically deformed during rolling. The rolling resistance arises because the normal pressure $p(x)$ in the leading half of the contact patch is higher than that in the trailing half. This pressure field asymmetry is clearly due to the hysteresis in the tyre, as said before. Therefore, the vertical resultant F_z of the pressure distribution is offset towards the front of the contact patch. However, the tread rubber exhibits a viscoelastic behaviour together with suitable flexibility and the tyre is designed so that it safeguards these properties when is in service. In this way, both flexibility and hysteresis are helpful to friction, which produces grip thanks to the road roughness effects and the molecular adhesion. Unfortunately, at the same time the viscoelastic behaviour of tyre tread is responsible of the rolling resistance generated in a frequency range of 10–150 Hz. For these reasons, the tyre design is constantly developing in order to find a compromise between the viscoelastic behaviour effects. The viscoelastic behaviour is an advantage in terms of tyre grip with the road, but it generates the rolling resistance phenomenon due to the hysteresis and energy loss. Actually, as a loaded tyre completes a revolution, it deforms within the contact patch area and dissipates a part of the energy stored. To reduce the rolling resistance, it is necessary to minimize the energy loss due to the cycle deformation of the tread rubber. This means selecting less hysteretic polymers in the tyre composition or increasing the distances between reinforcing fillers and other raw materials, such as sulphurs and oils. However, the implementation of both solutions it is quite difficult because minimizing tyre energy loss means compromise the tread grip and resistance to wear. The main effort is to find the best trade-off solution for the overall tyre performance in order to achieve a good grip, high wear resistance and

low rolling resistance in any tyre working conditions (Batchelor, 2000; Calabrese et al., 2013).

4 Tyre Role in Vehicle Dynamics

The tyre plays a fundamental role in the vehicle dynamics which in its turn is subjected to three different types of force fields: the gravitational, the aerodynamic and the tyre–road interaction ones.

The interaction forces field refers to the phenomena occurring in the contact patch between the tyre and the road. This field is due to the application of a torque—driving or braking—around the wheel axis and a force applied on quantities are transmitted to the road thanks to the tyre contact patch. This force–torque system at a given point of the contact patch is statically equivalent to any set of forces or distributed load. Therefore, regardless of the degree of roughness of the road, the distributed normal and tangential loads in the contact patch yield a resultant force F and a resultant torque vector M :

$$\begin{aligned} F &= F_x i + F_y j + F_z k \\ M &= M_x i + M_y j + M_z k \end{aligned} \quad (4)$$

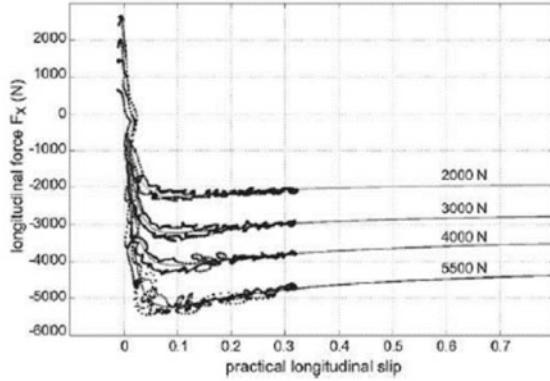
The resultant couple M is simply the moment about the point O , but any other point could be selected. The traditional components of the magnitudes in equation set (4) have been already defined and are summarized in the following: F_x is the longitudinal force, F_y is the lateral force; F_z is the vertical load (or normal force); M_x is the overtuning moment, M_y is the rolling resistance moment and M_z is the self-aligning torque (Allouis et al., 2016).

Thanks to the experimental tests carried out on tyres and to physical-analytic models, it is possible to determine the tyre–road interaction forces laws. These expressions state that the vertical load depends on tyre deformations, whereas the longitudinal and lateral forces depend on the corresponding slip factors, longitudinal and lateral, respectively. In the following paragraphs, the longitudinal and lateral load will be briefly described; for more details about the tyre–road interaction, see the suggested references (Allouis et al., 2016; Batchelor, 2000).

Pure longitudinal interaction. One of the tyre testing aims is the full identification of the functions able to describe the relationships between the motion and the position of the rim and the force and moment exchanged with the road in the contact patch. It is meaningful to perform experimental tests for the so-called pure slip conditions. It means setting that the longitudinal and lateral forces depend only on the corresponding slip factors and on the vertical load, whereas the self-aligning moment on the vertical load and lateral slip factor.

To comprehend the phenomena involved in the contact patch during the longitudinal interaction, we should take into account a vehicle, which is going to brake. If no sliding takes place on the contact patch, the relationship between the longitudinal

Fig. 6 Longitudinal interaction



force F_x and the longitudinal slip s_x can be considered as linear:

$$F_x = C_x s_x \quad (5)$$

where s_x can be in traction phase, $s_{x,T}$ or in braking phase, $s_{x,B}$. C_x is the tyre longitudinal stiffness, often called braking stiffness:

$$C_x = \frac{\partial F_x}{\partial s_x} |_{s_x} \quad (6)$$

In Fig. 6, the typical behaviour of the longitudinal force F_x as a function of the practical longitudinal slip under braking conditions, for several values of the vertical load F_z , is shown.

It is important to point out that the longitudinal forces decrease as a linear function of the slippage ratio in a small range of longitudinal slip ratio s_x . In this case, the longitudinal tyre stiffness definition (Eq. 6) is correct and the tyre blocks work in adherence. Moreover, the vertical load influences the longitudinal force F_x : the latter grows less than proportionally with respect to the vertical one. Hence, the global longitudinal friction coefficient μ_x can be defined as the ratio between the peak value of the longitudinal force and the corresponding vertical load.

Pure lateral interaction. When a tyre is not subject to any force perpendicular to the wheel plane, it will move along this last; if a side force F_s is applied to a wheel, a lateral force will be developed at the contact patch, and the tyre will move along a path at an angle equal to the slip angle α with the wheel plane, mainly due to the lateral elasticity of the tyre, as shown in Fig. 7. The lateral force developed at the tyre/ground contact patch is usually called cornering force $F_{y\alpha}$ when the camber angle of the wheel is zero; the relationship between the cornering force and the slip angle is of fundamental importance to the directional control and stability of road vehicles. When the tyre is moving at a uniform speed, the side force F_s applied at the

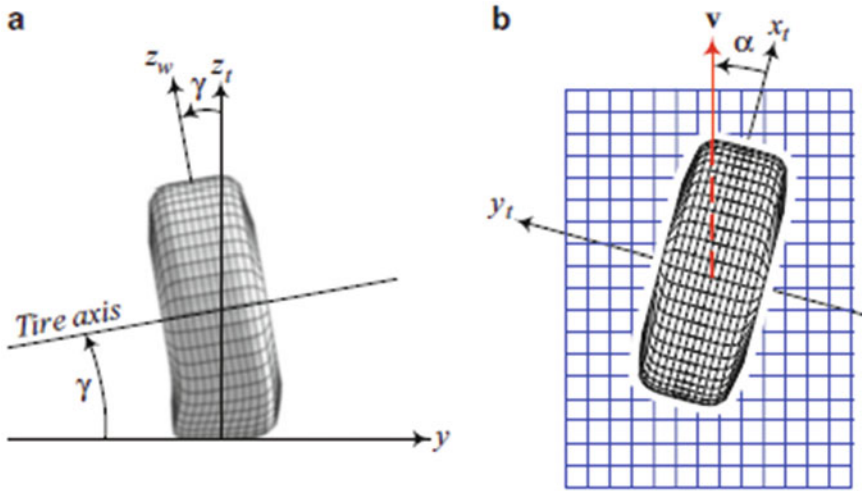


Fig. 7 Lateral interaction physics: **a** camber angle, **b** sideslip angle (Jazar, 2019)

wheel centre and the cornering force $F_{y\alpha}$ developed in the ground plane are usually not collinear: at small slip angles, the cornering force in the ground plane is normally behind the applied side force, giving rise to a torque which tends to align the wheel plane with the direction of motion. This torque is called the “aligning” or “self-aligning torque”, and it is one of the restoring moments which help the steered tyre return to the original position after performing a curving manoeuvre. The distance between the side force and the cornering force is called the “pneumatic trail”, and the product of the cornering force and the pneumatic trail determines the self-aligning torque.

To properly approach a vehicle within a turn, the driver has to act on the steering wheel. Every vehicle taking a turn is subjected to a side force, F_c , which tends to force it out of its curve. To keep vehicle on the path, in each tyre–road contact area a centripetal force must arise, F_y , which globally stabilizes the side force.

The relationship between the cornering force and the slip angle is of fundamental importance to the vehicle handling and stability of road. Typical plots of the cornering force as function of the slip angle show that for angles below a certain range, the lateral force is approximately proportional to the slip values. Beyond them, the cornering force increases at a lower rate with an increment of the slip angle and reaches its maximum value as soon as the tyre begins sliding laterally.

It is clear from the diagram in Fig. 8, that for low slip angle values the lateral force increases linearly. Therefore, the relationship between the friction force and the corresponding kinematic parameter can be expressed as follows:

$$C_{y,\alpha} = \left. \frac{\partial F_{y\alpha}}{\partial \alpha} \right|_{\alpha=0} \quad (7)$$

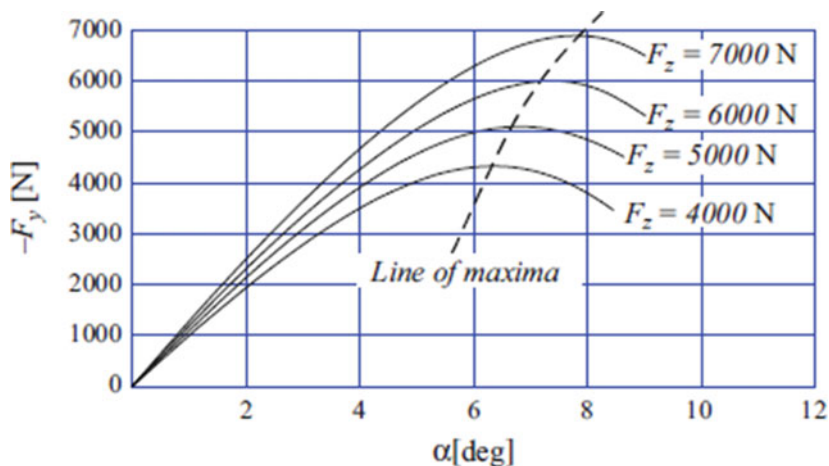


Fig. 8 Lateral interaction: Lateral force of a sample tyre for different normal loads (Jazar, 2019)

where $C_{y,\alpha}$ is known as cornering stiffness. This quantity indicates the slope of the curve at the origin of the coordinate axis system. The cornering stiffness generally increases with the load, but the rate of increase declines as load increases (see Fig. 9). High performance vehicles on a dry road will exhibit their maximum cornering ability using large tyres operating at relatively light loads. Inflation pressure usually has a moderate effect on the cornering properties of a tyre, but in general, cornering stiffness increases with an increase of the inflation pressure. However, the relationship between the cornering force and the normal load is non-linear; it means that the transfer of load from the inside to the outside tyre during a turning manoeuvre will

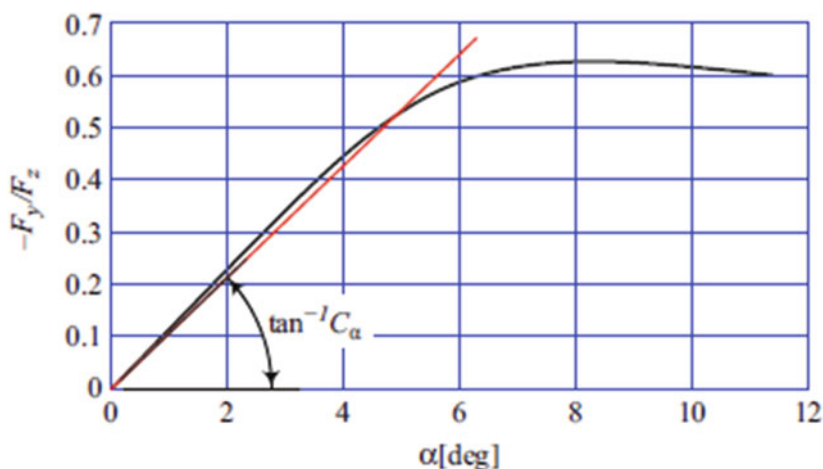
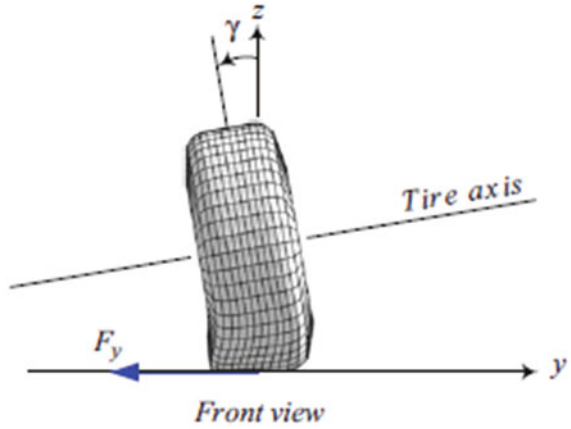


Fig. 9 Lateral stiffness (Jazar, 2019)

Fig. 10 Cambered tyre behaviour (Jazar, 2019)



reduce the total cornering force that a pair of tyres can perform, making so possible to act on the under/over steering behaviour of the whole vehicle modifying the value of the roll stiffness, able to manage the load transfers (Connolly & Huston, 1994; Farroni et al., 2017).

Camber thrust. Camber causes a lateral force usually referred to as “camber thrust” F_{yy} , and the development of this thrust may be explained in the following way: a free-rolling tyre with a camber angle would revolve about point O, as shown in Fig. 10; however, the cambered tyre in a vehicle is constrained to move in a straight line, developing therefore a lateral force in the direction of the camber in the ground plane. It has been shown, that the camber thrust is approximately one fifth the value of the cornering force obtained from an equivalent slip angle for a bias-ply tyre and somewhat less for a radial-ply tyre.

To provide a measure for comparing the camber characteristics of different tyres, a parameter called “camber stiffness” is often used; it is defined as the derivative of the camber thrust with respect to the camber angle evaluated at zero camber angle:

$$C_{y,\gamma} = \frac{\partial F_{yy}}{\partial \gamma} |_{\gamma = 0} \quad (8)$$

Similarly to the cornering stiffness, normal load and inflation pressure have an influence on the camber stiffness. It has been calculated that for truck tyres, the value of the camber stiffness is approximately one tenth to one fifth of that of the cornering stiffness under similar operating conditions.

The total lateral force of a cambered tyre operating at a certain slip angle is the sum of the cornering force $F_{y\alpha}$ and the camber thrust F_{yy} :

$$F_y = F_{y\alpha} \mp F_{yy} \quad (9)$$

If the cornering force and the camber thrust are in the same direction, the positive sign should be used in the above equation. For small slip and camber angles, the relationship between the cornering force and the slip angle and the one between the camber thrust and the camber angle are essentially linear; the total lateral force of a cambered tyre at a slip angle can, therefore, be determined by:

$$F_y = C_{y,\alpha}\alpha \mp C_{y,\gamma}\gamma \quad (10)$$

As discussed previously, the lateral forces due to slip angle and camber angle produce a torque, but the component due to slip angle, however, is usually much greater and mainly responsible of the aligning torque acting on tyres in ordinary driving conditions.

Interaction between tangential forces. In the discussion about the cornering behaviour of tyres, the effect of the longitudinal force has not been considered. However, quite often both the side force and the longitudinal force are present, such as braking in a turn. In general, tractive (or braking) effort will reduce the cornering force that can be generated for a given slip angle; the cornering force decreases gradually with an increase of the tractive or braking effort. At low values of tractive (or braking) effort, the decrease in the cornering force is mainly caused by the reduction of the cornering stiffness of the tyre. A further increase of the tractive (or braking) force results in a pronounced decrease of the cornering force for a given slip angle.

This is due to the mobilization of the available local adhesion by the tractive (or braking) effort, which reduces the amount of adhesion available in the lateral direction. It is interesting to point out that if an envelope around each family of

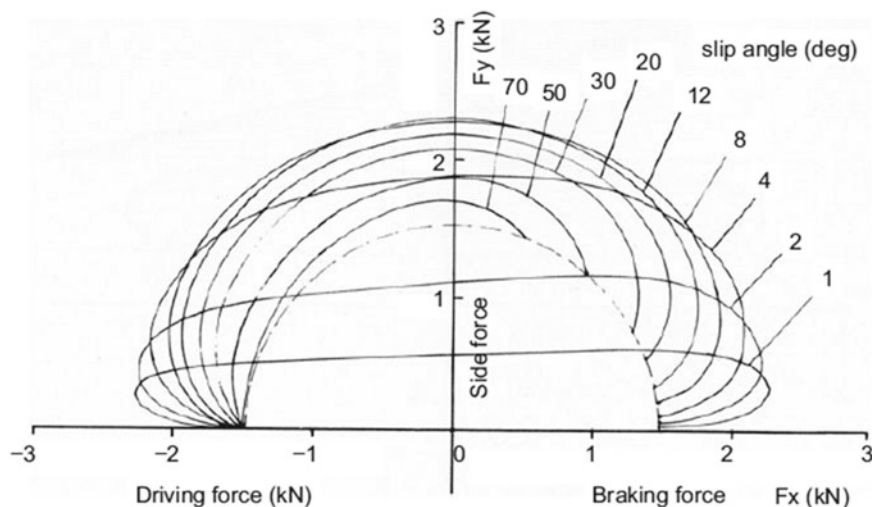
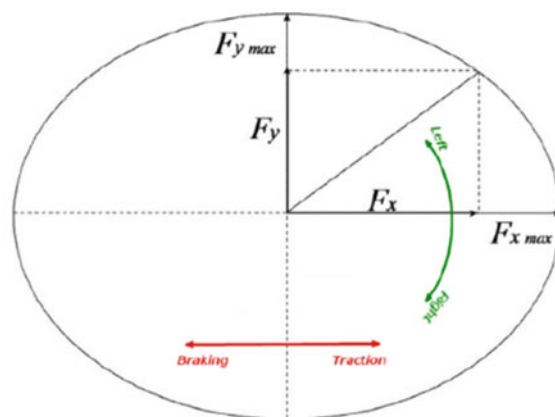


Fig. 11 Effect of longitudinal force on the cornering characteristics (Nakajima & Nakajima, 2019c)

Fig. 12 Friction ellipse

curves of Fig. 11 is drawn, a curve approximately semi-elliptical in shape may be obtained. This enveloping curve is often referred to as the friction ellipse.

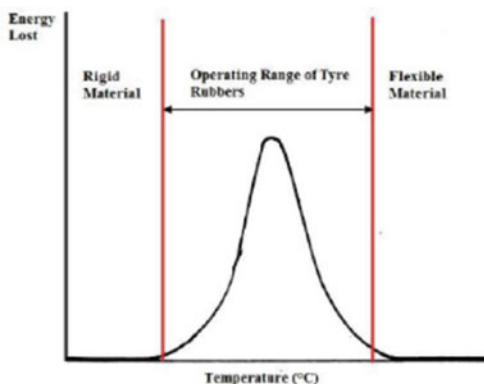
The friction ellipse concept is based on the assumption that the tyre may slide on the ground in any direction if the resultant of the longitudinal force (either tractive or braking) and lateral (cornering) force reaches the maximum value defined by the coefficient of friction and by the normal load on the tyre. However, the longitudinal and lateral force components may not exceed their respective maximum values $F_{x, max}$ and $F_{y, max}$, as shown in Fig. 12. $F_{x, max}$ and $F_{y, max}$ can be identified from measured tyre data and constitute respectively the major and minor axis of the friction ellipse.

5 Tyre Working Conditions Effects

The vehicle usually operates under a range of different external conditions, so the passenger or race tyres are subjected to varying load, pressure, speed and temperature. All these conditions influence not only the rolling resistance, but also the tyre tread response in transient phenomena during the interaction of the road (Batchelor, 2000; Calabrese et al., 2013).

Temperature effect on rubber behaviour. The working temperature of a tyre usually depends on several aspects, as well as the type of the tyre itself, the way the vehicle is driven and the environmental temperature. Typically, the internal temperature of a standard passenger tyre lies between 20 and 60 °C. Clearly, the tyre temperature increases as soon as the external one is higher. In the usual working conditions, the amount of energy dissipated by the tyre tread compound, which exhibits a viscoelastic behaviour, when subjected to a cyclic load, decreases as the temperature increases, as shown in Fig. 13. The suitable temperature working range matches with the maximum energy lost area due to viscoelastic behaviour so that friction coefficients are high. Very low temperatures turn the tyre tread behaviour

Fig. 13 Temperature effect on viscoelastic compound dissipation

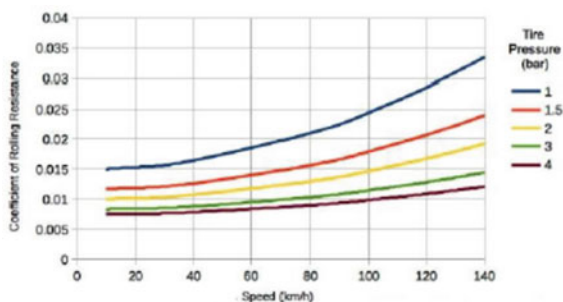


into rigid one. This means that in braking transient conditions, the tyre is not able to compensate the difference between the wheel centre and the rolling speed $\Omega \times R$ and the speed of the vehicle, because the tread compound is less deformable, and the vehicle's wheels may be locked. Contrariwise, in driving conditions, the wheels go round and the vehicle does not move forward at the same time (the longitudinal slip ratio is infinite).

In cornering conditions, instead, the vehicle goes straight rather than taking the bend, if the tyre rubber behaves as a rigid material. Actually, the slip angle indicates how the tyre is deformed in order to let the vehicle rightly corner. This means that the velocity vector changes its direction slower than tyre one due to viscoelastic behaviour of the tread rubber.

Pressure effects on tyre rolling resistance. Each vehicle tyre is subjected to an applied vertical load F_z , which influences the longitudinal and lateral loads in the tyre–road contact patch. Particularly, the relationship between the longitudinal/lateral and vertical load is non-linear due to contact patch saturation phenomenon along the x and y axis according to the reference system adopted. Further, it is found that the rolling resistance increases nearly proportionally to the applied load due to the increased levels of bending and shearing that take place. Although the rolling resistance is nearly a linear function of the normal load, the energy lost increment due to higher loads generates a tyre temperature rising. The latter outcomes in a lower hysteretic loss coefficient for tyre tread elastomer material. Therefore, the rolling resistance coefficient often decreases with an increasing load. The inner pressure variations usually affect the global deformation of the tyre, as well as the vertical load. Particularly, a decrease of the pressure produces a rapidly increase of the rolling resistance, because it reduces the deformation of tyre blocks in the contact patch. This also generates an increase of bending and shear stresses of tyre tread. For passenger car tyres, the inflation pressure is not usually high enough to be a meaningful source of the total energy loss. Because of these phenomena occurring at different temperatures, the variation of the rolling resistance as function of the temperature, and thus of the heat radial generative term within the tyre structure, is not linear; clearly, it is lower

Fig. 14 Pressure on influence on rolling resistance



when the external temperature is higher: between 10 and 40 °C, an increase of 1 °C corresponds to a reduction in rolling resistance of about the 0.6% under normal load conditions. In addition to the pressure, the rolling resistance also depends on vehicle speed. Principally, as soon as the wheel centre velocity rises, the frequency of deformation of tyres increases, causing an increment of the tyre tread rubber loss factor. At the same time, a higher heat generation occurs: so, the tyre temperature rises, whereas the tyre loss factor decreases. The combined influence of the above effects gives rise to a typical increment of the rolling resistance with vehicle speed, but in some cases, it may decrease due to temperature effects on the tyre tread behaviour and the loss factor. Anyway, the variations of rolling resistance as function of the speed are generally much smaller than the pressure and temperature ones, as clearly plotted in Fig. 14.

6 Tyre Thermal Modelling in Vehicle Dynamics and Driving Simulations

The tyre characterization under the thermal point of view is particularly important to understand the tyre thermodynamic behaviour. The tyre stiffness and transient behaviour are deeply linked to thermal effects concerning deep tyre layers. Furthermore, the tyre tread bulk temperature, usually not measurable online but providable by means of a specifically developed tyre thermal model, is a crucial additional data channel to evaluate the tyre optimal thermal range and therefore to optimize the tyre performance in different ambient conditions and upon diverse track pavements.

Nowadays everyone playing a role in automotive sector is looking for the optimal solution in order to model and to understand the tyres behaviour in both experimental and simulation environments. The ability to predict the interior temperature distribution, and thus the grip behaviour of the tyre, is fundamental in terms of the vehicle handling improvement and of the asset optimization according to highly variable outdoor testing conditions.

As an example, since 2009, the test limitations imposed by race organizers in F1 have seen the participating teams speeding up the development of driving simulators

to help drivers and technicians to prepare for upcoming events devising the vehicle set-up (Shim & Margolis, 2005). Detailed circuit characteristics, such as their geometry or bumps and kerbs shape and location, are directly linked to the downforce levels, braking duty and tyre loads reachable under the defined set-up conditions. The teams generally run a large number of laps in the simulator, programmed with a variety of fuel loads and grip levels, to ensure as many scenarios as possible to be covered and to analyse as many race outcomes as possible (Farroni et al., 2018).

During the last years, engineering advances in sensor technologies allowed to introduce sensing elements even inside the wheel. The easy accessibility to innovative high-tech and reliable sensors, useful for a wide range of objectives, allowed the number of telemetry channels to increase significantly. The bigger volume of data collected during both tests and races to be processed met the availability of the reliable and predictive physical models, able to tune the vehicle and its components for optimum performance.

Vehicle optimal setup is deeply linked with the correct compound working conditions, achievable with a properly optimized suspension layout in terms of load map and of suitable wheel alignment geometry. In order to optimize the compound characteristics in all the vehicle-working conditions and therefore the grip, two directly opposing influences should be taken into account: the frequency of contact with the track road, corollary of the car's speed, and the prevailing working tyre temperature. The tuning of completely heterogeneous parameters set, as tyre pressure, tyre internal structure, suspension geometry and the vehicle characteristics, is achievable only if all the above component subsystems are properly modelled within the simulation environment.

A crucial parameter for tyre structure thermodynamic characterization is the thermal diffusivity, defined as the ratio between the thermal conductivity and the product of the specific heat and density. This parameter is a function of temperature and it varies with different compound compositions, especially exalted in the competition tyres. An original non-destructive technique to evaluate the thermal diffusivity with a specific model, called TRT-Lab, has been developed by University of Naples (Allouis et al., 2016). A significant benefit of using this innovative procedure is the fact that TRT-Lab and the Thermo Racing Tyre (TRT) models (described in the followings) (Allouis et al., 2016; Farroni et al., 2014a) share the same structure, differentiating only in the boundary conditions. The thermal characteristics identified by means of the cited procedure are then employable within the thermodynamic model of the tyre, able to predict the temperature gradients with a high level of accuracy within the entire tyre structure. Starting from the TRT model outputs in terms of temperatures and of local thermal exchanges, physical considerations particularly suitable to distinguish the optimal thermal range of each compound and the variation of tyre dynamic behaviour due to the thermal effects, can be carried out.

The introduced TRT is a physical-analytical tyre thermal model, based on the Fourier's law of heat transfer applied to a three-dimensional domain. A series of innovative upgrades has been applied to the original version in order to take in account all the thermal phenomena actively concurring in the tyre's working life, as the sidewalls presence, the exhaust gases impact at the rear axle and the inhomogeneous

distribution of local variables (pressure, stress and sliding velocity) within the contact patch.

In the years, new implementations to the already existing TRT thermo-dynamic model have been applied with specific purposes: to add new structure elements and, thus, to enhance and optimize the physical response, but not at the expense of the real-time simulation. The so realized TRT-evo model includes the presence of the sidewalls, actively involved in the convective heat exchanges respectively with the external airflow and the inner gas fluid located inside the inflation chamber.

The effect of the blown exhaust gases or of the heating brake thermal impact upon the outer tyre layer, which is directly linked to the tyre temperature distribution, is also taken into account. This effect depends on the particular aerodynamic solutions adopted by the car designers and it can represent a key factor for the management of tyres in terms of optimization of the grip/temperature relationship.

To contemplate an inhomogeneous distribution of local variables inside the contact patch (for example, stress and deformations due to the camber thrust or due to the influence of inflation pressure upon the contact patch shape), the outer tyre surface is meshed as a pattern of ribs.

TRT-evo—an evolution of Thermo Racing Tyre model. Starting from TRT tyre thermo-dynamical model, a new physical thermal model has been developed (Farroni et al., 2019a). In addition to the temperature distributions within the singular tyre layers, the influence of a large number of parameters can be evaluated; among them: wheel alignments (as camber or toe), inflation pressure, different track and weather conditions, and the influence of the vehicle settings.

To exploit the entire amount of grip available on the tyre–road interface in order to preserve the highest level of handling performance preventing the tyre from sliding, more and more physical phenomena regarding the vehicle and its subcomponents have to be taken into account within the integrated vehicle control systems (Trächtler, 2004; Xiong et al., 2012). Tyre characteristics in terms of braking and cornering stiffness and maximum amount of grip at various wheel operating conditions is a function of a wide range of variables, among which the tyre compound core and sidewall temperatures, which modify the inner chamber air pressure (Genna, 1996; Sackett, 1990), and the tyre mechanical structure characteristics (Umeno, 1998), as shown in Fig. 15. Differences in terms of interaction characteristics, in which a considerable stiffness decrease due to a higher tyre temperature is clearly appreciable, are shown both for longitudinal and lateral tyre interaction curves. The data represented has been non mentioned due to confidentiality agreements with the companies involved in the published researches.

The knowledge of the optimal temperature range of a generic tyre compound is particularly significant in the motorsport, where tenth of a second per lap makes difference, as represented in Fig. 16. Thanks to the availability of the tread core and bulk temperature, not directly measurable but provided by the TRT model, it is possible to obtain interesting correlations, because internal temperature offers the best results concerning the correlation between grip and temperature. It can be attributed to the fact that the external surface temperature varies with fast dynamics,

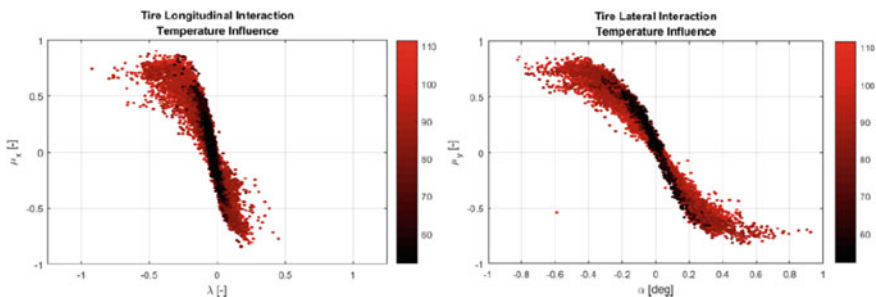


Fig. 15 Temperature influence on tyre interaction characteristics curves

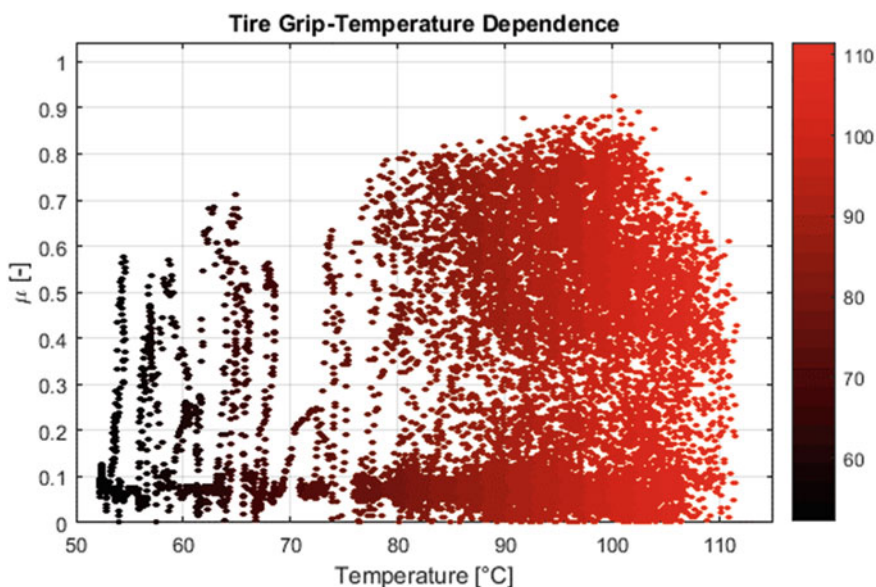


Fig. 16 The tyre in analysis shows an optimum thermal range between 95 and 103 °C, where the grip reaches highest values

which does not allow to modify in such short time the whole tread characteristics. Therefore, the tyre frictional behaviour can be more properly linked to the tread bulk layer to analyse and to optimize the compound mechanical characteristics starting from its thermal behaviour, which can be evaluated by means of a specific tyre thermal model.

Structure and working principles of the physical thermal model. In the original version of Thermo Racing Tyre model, only the tread and the layers underlying it were taken into account within the tyre thermo-mechanical system. Indeed, the tyre structure was lacking of sidewalls, and consequently able to reproduce the

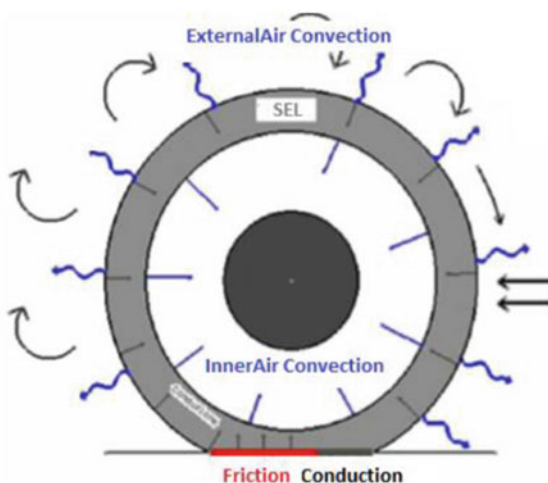
temperature distribution only in a portion of the system in contact with road, disregarding important structural elements influencing the entire interaction phenomenon. However, thanks to the availability of the bulk temperature, provided by means of the TRT model, it was possible to obtain useful correlations, from which, for example, the compound optimal thermal range could be identified. Consequently, the overall vehicle frictional behaviour, involving the tyre compound mechanical properties, could be optimized starting from the polymer thermal characteristics, which can be appropriately evaluated by means of an indoor test procedure and, then, a specifically developed TRT-Lab model (Allouis et al., 2016).

The TRT model takes into account the following thermo-dynamic phenomena, as represented in Fig. 17:

- heat generation due to tyre–road tangential interaction, known as Friction Power
- heat generation due to effect of tyre cyclic deformation during the rolling, known as SEL (Strain Energy Loss)
- heat exchange with the external environment due to thermal conduction between the tread and the road
- heat exchange with the external environment due to convection of the tread layer with the outside air
- heat exchange with the external environment due to convection of the innerliner layer with the inner air
- heat conduction between the tyre layers due to the temperature gradient.

Since one of the key objectives of the TRT-evo is represented by the possibility to run in real-time, the computational cost, connected to a more complex model to manage, has been decreased by simplifying the mesh of the previous model version and, thus, by reducing the state vector length so making it suitable for real time analyses.

Fig. 17 Thermal exchanges within the original TRT model



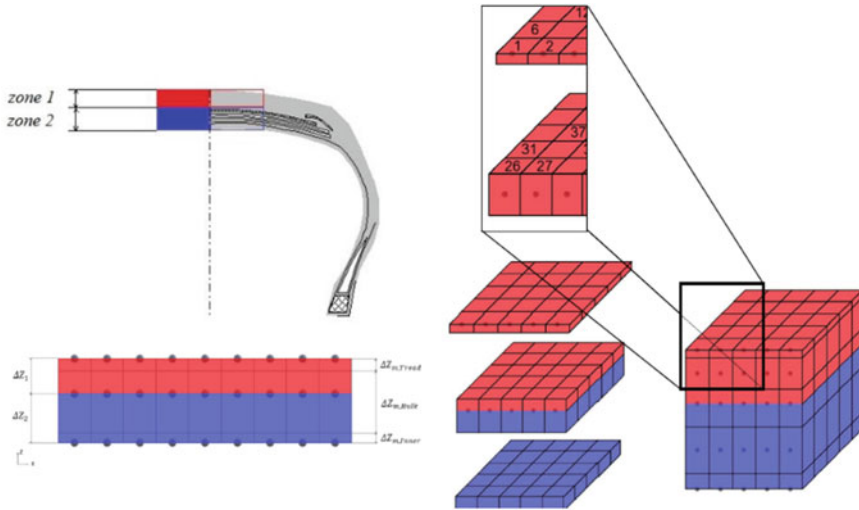


Fig. 18 TRT material bi-zone discretization of the tyre tread-belt structure

The implementations applied to the already existing TRT thermo-dynamic model for the innovative TRT-Lab have a key purpose: to add new structure elements and, thus, to enhance and optimize the physical response, but not at the expense of real-time simulation.

The least computationally expensive solution to take into account the tyre tread high inhomogeneity has resulted in the adoption of a thermal equivalent dual-zone material configuration along the radial direction, as represented in Fig. 18:

- tyre tread constituted only by rubber compound, accountable for the generation of tangential interaction forces within the contact patch extension
- tyre carcass highly composite structure, including belt, body plies and inner liner, determining the carcass shape and contact patch shape.

With special regard to the tyre intensive thermal properties, the zones of homogeneous material have been characterized by the following set of physical parameters: density ρ , specific heat c_v and thermal conductivity k . The thermal diffusivity of the track pavement has been preliminarily analysed for different tracks to accurately model the heat exchanges dealing with the tyre–road conduction term.

Simplified tyre structure

The computational cost, connected with the number of the state variables and therefore equations considered, can be reasonably reduced substituting the node grid configuration of the tyre internal layers with a singular node configuration, as represented in Fig. 18. Consequently, the number of nodes can be reduced from n to $(n/m + m)$ (where m is the number of internal layers), considering a singular rib configuration, assigning the same thermal intensive properties (density ρ , specific

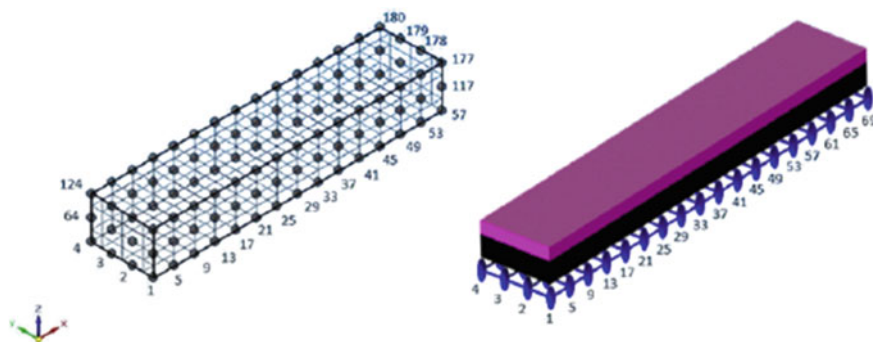


Fig. 19 Simplified mesh adopted for real-time simulation purpose

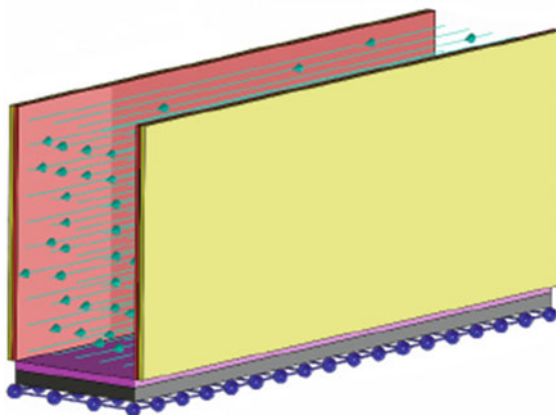
heat c_v and thermal conductivity k). The reason for which it is possible to simplify the discretization, condensing the tyre entire internal grid layers to singular nodes, is that their thermal inertias, even within a very fast vehicle dynamics, is too ponderous to induce a significant temperature circumferential gradient. The tread external layer is still modelled by means of a grid of nodes, whereas a singular node configuration has been adopted for the internal tyre layers: tread bulk (intermediate parallelepiped in black) and belt-innerliner (in magenta) in Fig. 19.

Sidewalls implementation within tyre structure

The TRT-evo model consequent gain in terms of performance execution allowed to include the sidewalls within the tyre physical structure without undermining the simulation real-time requirement. Due to a lower computational model burden, a further discretization along the circumferential direction could be possible within the external layer to reproduce more accurately the particularly fast thermal dynamics of the tyre surface.

During rolling, tyres are subject to periodic deformations, which induce time dependent load history on the tyre system. As a consequence of both elastic and viscous mechanical characteristics and because of a low thermal diffusivity, the temperatures tend to arise inside the tyre structure. The modelling of tyre sidewalls, actively involved in the convective heat exchanges with inner and outer air fluxes and subject to fatigue induced by tyre–road interaction, is evidently necessary to understand the thermal aspects relating to optimal operation purpose and long service life (Hernández-Olivares et al., 2007; Steyn & Haw, 2005). Including the sidewalls within the tyre structure allows to get a more physical representation of the whole wheel chamber, providing an interesting instrument to study and understand how the inner gas dynamics can influence the thermo-mechanical behaviour of the tyre. Indeed, even if the above elements do not take directly part in the tyre–road interaction, a considerable fraction of Strain Energy Loss (SEL) is dissipated inside the sidewalls because of large deformations. As shown in Fig. 20, the implemented sidewalls have been discretized, each by means of at least two nodes. Since the sidewall geometry

Fig. 20 Discretization of the tyre sidewalls



is particularly irregular within the meridian reference plane, the sidewall equivalent dimensions have been calculated starting from the tyre CAD. The sidewall nodes, called external (in yellow) and internal (in red), are respectively in contact with the external air flux and with the inner gas contained inside the wheel chamber.

Aero/Exhaust blow gas/Brakes implementation

The vehicle rear end is able to create downforce by means of three different contributions: diffusers, rear wings and proper deflection of the exhaust gases. The optimal solution would allow linking these three areas together to create the best amount of downforce with least drag. The engine exhaust gases, besides being used for downforce purposes, can be also used to provide further heat in case of need for the achievement of the optimal working temperature, around the tyres. The thermal contribution coming from the exhaust gases, directed to the rear of the car, deeply depends on the aerodynamic geometry adopted and it can be specifically designed to invest different areas of the tyres.

If improperly canalized, the additional thermal sources risk to damage the tyre because of too high thermal stress levels induced. So, every solution adopted has to be studied in detail in order to properly handle the supplementary heat flow, whose correct employment may represent a key factor for the management of the tyres in terms of optimization of the grip/temperature relationship during particular operating phases (De Castro et al., 2012; Review). Starting from these considerations, it is undeniable the importance that a study, able to take into account the extremely severe thermal conditions due to localized hot gas flows, could have for racing teams. The phenomena have been modelled inside the TRT-evo model, allowing to take into consideration an external localized thermal source.

Multi-Rib implementation

One of the main limits of the TRT was definitely the fact that it was based on global quantities, which are not able to describe the inhomogeneous distribution of local

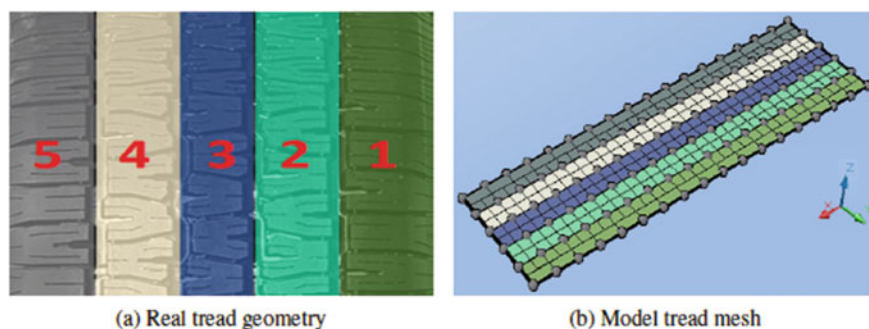


Fig. 21 The tyre tread external mesh: 5 rib configuration

variables inside the contact patch, for example stresses and deformations due to camber thrust. The modelling of local ribs within the TRT-evo tyre structure allows to study the effects linked with the most common tyre configurations, like non-zero camber angles or under/over inflation conditions. At the present stage, the model includes a certain number of ribs, as represented in Fig. 21. To determine the amount of friction power per each rib, it becomes necessary to provide different local forces and sliding velocities, evaluated for example by means of specific contact models. Moreover, per each rib TRT-evo provides the possibility of diversifying both the thermal power terms and the material mechanical/thermal properties. This allows to evaluate the temperature distribution within the whole tyre: inside the contact patch taking into account the distribution due to camber thrust or under/over inflation tyre condition, and regarding the part of tread in contact with the external airflow influenced by a specific aerodynamic solution. Furthermore, each rib is characterized by specific thermal properties in order to model more tread compounds along the width direction and compound local degradation due to wear, also depending on temperature.

Mathematical thermal model. The TRT-evo thermodynamic tyre model is based on the use of the diffusion equation of Fourier applied to a three-dimensional domain.

The complexity of transient thermo-dynamic phenomena under study and the degree of accuracy required imply the dependence of thermodynamic quantities, and in particular of the thermal diffusivity terms, on the temperature. Furthermore, the non-homogeneity of tyre has made it necessary to consider the variation of the above parameter also along the thickness.

The TRT-evo is a physical-analytical model based on the Fourier's law conceived in the Euler reference system. Thus, the tyre is considered motionless and the boundary conditions, linked with external environment thermal exchanges, vary during the simulation. The road is represented by a geometric homogeneous even plane, in contact with the tyre portion corresponding to the contact patch under a particular load condition. The remaining tyre surface is involved in the convective heat exchange with the external air, whose entity is determined by means of specific

CFD analysis, on whose basis interpolative analytical functions are identified to guarantee the real-time attitude. The road T_r and the air T_{air} temperatures concern the boundary conditions of the model, taking into account the track weather conditions.

Therefore, the Fourier equation takes the following formulation applied to a considered three-dimensional heterogeneous domain (Kreith et al., 2011; Patankar, 1980):

$$\frac{\partial T}{\partial t} = \frac{\dot{q}_G}{\rho c_v} \cdot \left(\frac{\partial^2 k(z, T) \cdot T}{\partial x^2} + \frac{\partial^2 k(z, T) \cdot T}{\partial y^2} + \frac{\partial^2 k(z, T) \cdot T}{\partial z^2} \right) \quad (11)$$

Writing the power balance equations for each elementary mass associated to each node, in relation to its position within the tyre structure, the matrix equation of the entire model can be obtained in the typical state form (Minkowycz et al., 2000):

$$\begin{pmatrix} \frac{\partial T_1}{\partial t} \\ \frac{\partial T_2}{\partial t} \\ \vdots \\ \frac{\partial T_n}{\partial t} \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} + \frac{1}{\rho C_v} \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \cdot \begin{pmatrix} T_1 \\ T_2 \\ \vdots \\ T_n \end{pmatrix} \quad (12)$$

Each i -th node, whose temperature is determined in real-time, is represented by a parallelepiped volume V_i , equal to:

- external node grid

$$V_i = \Delta x \Delta y \Delta z_{m,i} \quad (13)$$

- internal mononode configuration

$$V_j = LW \Delta z_{m,j} \quad (14)$$

in which Δx and Δy are respectively the dimensions along longitudinal-circumferential and transversal-wide directions, while the quantity $\Delta z_{m,i}$ represents the thickness of a node mass associated to i -th rib of the external node grid and the quantity $\Delta z_{m,j}$ is the tyre tread thickness of j -th layer.

Remembering that the tangential conductive exchanges of the internal tyre nodes have been neglected due to their singleton mesh configuration and therefore the radial conduction has been considered in Eq. 11.

Moreover, each node has a mass expressed as follows:

$$m_j = C_\rho V_j \quad (15)$$

where the dimensionless C_ρ coefficient depends on its position in the grid.

Writing the balance equations for each generic node needs the modelling of heat generation and of heat exchanges with the external environment. The heat exchange between the tyre and the external environment, remembering that the radiation mechanism is neglected by hypothesis, it can be classified as follows:

Heat exchange with the road

The thermal conductive exchange between the tread and the asphalt has been modelled taking into account the particular shape and extension of the contact patch, provided by means of the RCH-tire model (Farroni et al. 2019b) or thanks to the results of specific test procedures. The term for such exchanges, for the generic i -th node will be equal to:

$$\frac{k_{t,r}}{\Delta z_{m,j}} (T_r - T_i) A_{CP,i} \quad (16)$$

where:

- $k_{t,r}$ is the equivalent conduction coefficient between the node i , with thermal diffusivity characteristics at temperature T_i , and the road, expressed as:

$$k_{t,r} = \frac{k_r k_i}{k_r + k_i} \quad (17)$$

- T_r is the track temperature [K]
- T_i is the temperature of node i [K]
- $A_{CP,i}$ is the contact patch area portion upon i -th node
- $\Delta z_{m,j}$ is the thickness of tyre tread external layer.

Heat exchange with the outside/inside air

The whole mechanism of the heat transfer between a generic surface and a moving fluid at different temperatures is described by natural and forced convection equations. Therefore, the heat exchange with the outside air is modelled by the mechanism of forced convection, occurring when there is relative motion between the car and the air, and by natural convection, when such motion is absent. Supposing the tyre invested by the air similarly to a cylinder invested transversely from an air flux, the forced convection coefficient is provided by the Newton's law of cooling formulation:

$$h_c (T_{fluid} - T_i) (A_i - A_{CP,i}) \quad (18)$$

where:

- h_c is the convection coefficient between the node i and the fluid, given by:

$$h_c = \frac{k_{air}}{L} \left[k_h \left(\frac{v_x L}{v_{air}} \right) \right] \quad (19)$$

- k_{air} is air conductivity, function of the average tread surface temperature and the air temperature [W/m/K]
- v_x is considered to be equal to forward speed of the vehicle (air speed is supposed to be zero) [m/s]
- v_{air} is the kinematic viscosity of air [m²/s], empirically obtained by means of:

$$v_{air} = (5 \cdot 10^{-10} T_{t,air}^2 - 10^{-6} T_{t,air}^2 + 0.0008 T_{t,air} + 0.135.) \cdot 10^{-4} \quad (20)$$

- $\Delta Z_{m,inner}$ is the thickness of the tyre inner surface layer
- L is the characteristic length of the heat transfer surface [m]
- $T_{t,air}$ is the air temperature at the boundary tyre layer [K]
- k_h is a tuning parameter [—].

The values of h_c evaluated with the above formulation are in a good agreement to the results obtained by means of CFD simulations, once a tuning parameter k_h has been properly evaluated.

The determination of the convection coefficient h_c is based on the classical approach of the dimensionless analysis. Indeed, the natural convection coefficient h_c is given by the following formulation:

$$h_c = \frac{k_{air} Nu}{L} \quad (21)$$

in which, considering the specific case:

- Nu is the Nusselt number, representing the ratio of convective to conductive heat transfer across to the boundary [—]

$$Nu = 0.53 Gr^{0.25} Pr^{0.25} \quad (22)$$

- Gr is the Grashof number, which approximates the ratio of the buoyancy to viscous force acting on a fluid [—]
- Pr is the Prandtl number, defined as the ratio of momentum diffusivity to thermal diffusivity [—].

The innerliner convection exchange with the inner gas function of turbulence level within the inner air chamber, evaluated through specific CFD calculations.

Friction Power—FP

Concerning the tyre, the heat is generated in two different ways: for friction phenomena arising at the interface with the asphalt and because of stress-deformation cycles to which the entire mass is subjected during the exercise (SEL).

The first heat generation is connected with the thermal power produced at tyre–road interface because of interaction; in particular, it is due to the tangential stresses that, in the sliding zone of the contact patch, do work dissipated in heat. Friction power can be associated directly to the nodes involved in the contact with the ground, and it is calculated as referred to global values of force and sliding velocity, assumed to be equal in the whole contact patch:

$$FP = F_x v_{sx} + F_y v_{sy} \quad (23)$$

where:

- F_x is the longitudinal interaction force [N]
- F_y is the lateral interaction force [N]
- v_{sx} is the longitudinal sliding velocity [m/s]
- v_{sy} is the lateral sliding velocity [m/s].

where a part of this thermal power is transferred to the tyre and the remaining to the asphalt.

In the original TRT version, the contribution of each tyre node towards the generation of the Friction Power, and therefore the knowledge local interaction forces and velocities, was not known. Indeed, the heat generated by means of friction power mechanism transferred to the tyre was equally distributed to all the nodes in contact with the ground.

The TRT-evo model, thanks to rib implementation and to RCH-tire model outputs (Farroni et al., 2019b), in terms of local pressures and velocities, allows to obtain uneven heat generation distribution and therefore local stresses maps within the contact patch, particularly important for studying wear mechanisms.

Strain Energy Loss—SEL

The energy dissipated by the tyre, $\dot{q}_{SEL}$, because of cyclic deformations is due to a superposition of several phenomena: intra-ply friction, friction inside singular plies, non-linear viscoelastic behaviour of all rubbery components, etc.

During the rolling, the entire tyre is subjected to the cyclic deformations with a frequency corresponding to the tyre rotational speed. During the motion, portions of tyre, entering in sequence in the contact area, are subjected to deformations, which cause energy loss and then heat dissipation.

In the TRT-evo model, the amount of heat generated by deformation (SEL) is estimated through experimental tests carried out deforming cyclically the tyre in three directions (radial, longitudinal and lateral). Estimated energies do not exactly coincide with the ones dissipated in the actual operative conditions, as the deformation mechanism is different; it is however possible to identify a correlation between them on the basis of coefficients estimated from real data telemetry and from the specifically developed dynamic analysis involving RCH-tire model.

At the current stage, the empirical SEL formulation is a function of the following parameters and it deeply depends on the tyre characteristics:

$$SEL = f(F_z; F_x; F_y; \omega; \gamma; p_{in,air}) \quad (24)$$

where:

- F_z is the vertical interaction force [N]
- ω is the air pressure within the wheel chamber [rad/s]
- γ is the wheel alignment camber angle [rad]
- $p_{in,air}$ is the air pressure within the wheel chamber [N/m²].

Contact patch area calculation

The size and the shape of contact area are obtained by means of specifically developed test procedures, based on the pressure sensitive films or employing the MBD/FEM models, as the RCH-tire model (Farroni et al., 2019b).

Since the characterization procedures are linked to static application of vertical load F_z in different conditions of internal pressure p_{in} , air and wheel alignment configuration in terms of camber γ and toe angle ψ of the vertical load F_z , the contact patch configuration concerning its shape and extension refer to static load conditions.

However, the instantaneous dynamic contact patch extension and shape can be rather different, because of particular transient conditions of wheel loading, centrifugal effect on the rolling tyre and viscoelastic tyre intrinsic characteristics. The preliminary tuning of the RCH-tire parameters to fit both static and dynamic experimental data, respectively represented by vertical rim lowering and contact patch extension and shape in quasi-static conditions and strain energy loss cycle areas in different load and frequency conditions due to viscous properties of the tyre structure, can constitute a valid instrument to implement the contact patch dynamic characteristics within the TRT-evo model.

At the current stage, TRT-evo is able to adopt real contact patch areas from both the experimental activities or the MBD/FEM model outputs, as a function of the following data inputs:

$$A_{CP} = f(F_z; p_{in,air}; \gamma) \quad (25)$$

Constitutive Equations

The power balance equations, on heat transfers, are written for each elementary mass associated to each node, and differ in relation to their position inside the grid.

Indicating the heat conductivity of the tyre compound and belt-innerliner layers respectively with k_1 and with k_2 , an example of balance equations formulation is represented in Fig. 22.

In particular, two different situations are described for the same node 2:

- Node 2 is supposed in contact with road (Fig. 22a). It is possible to notice the term identifying the cooling with the road, characterized by the presence of the

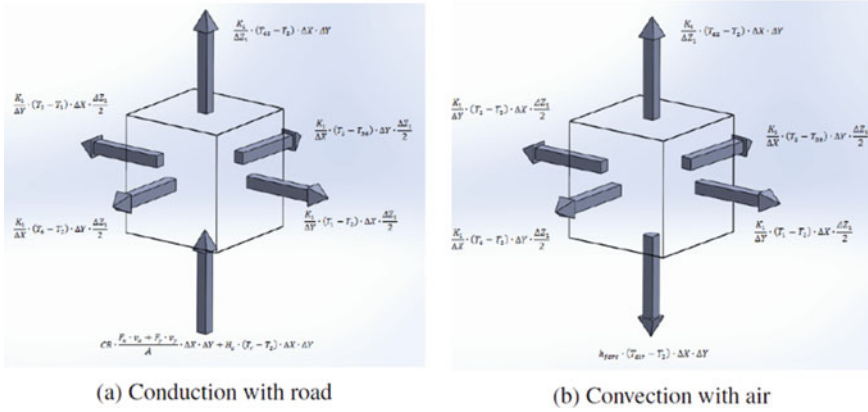


Fig. 22 Balance equations of tread external nodes

coefficient $H_c = \frac{k_{t,r}}{\Delta Z_{m,i}}$, and the one connected to the Friction Power generative term.

- Node 2 is supposed in contact with air (Fig. 22b). Note the absence of generative term and the presence of convective term identifying the exchange with the outside air, characterized by h_c coefficient.

In the model, the tyre is considered motionless and the boundary conditions rotating around it to take into account the fact that elements belonging to the surface layer will be affected alternatively by the boundary conditions corresponding to the contact with the road and to the forced convective exchange with the external air.

Finally, the matrix equation at the basis of the model is Eq. 12, in which a_{ij} is the generic coefficient, relative to the energy balance equation of the node i , that multiplies the j -th node temperature, while b_i is the generic coefficient not multiplying nodes temperatures.

The input data set, the TRT-evo model requires are summarized in Table 1.

Table 1 TRT-evo model inputs

Physical quantity	Description	Unit
F_x	Vertical interaction force	[N]
F_y	Longitudinal interaction force	[N]
F_z	Lateral interaction force	[N]
v_x	Forward speed at the wheel hub centre	[m/s]
v_{sx}	Longitudinal sliding speed	[m/s]
v_{sy}	Lateral sliding speed	[m/s]
ω	Wheel angular velocity	[rad/s]
γ	Camber angle	[rad]

The structural characteristics, the thermal properties of the tyre and the thermal conductivity of the track are also required.

Some of these data result from the telemetry measurements available for different tracks and are preliminarily analysed in order to check their reliability; others, such as in particular the ones related to structural and thermal characteristics of the tyre, are estimated on the basis of measurements and tests conducted on the tyres.

In addition to surface, bulk and inner liner temperature distributions, the model also provides the thermal flows involving the tyre, such as the flow due to the external air cooling, the one due to the cooling with the road, the one with the inflation air as well as the flows due to friction, hysteresis and exchanges between the different layers.

7 Tyre Wear Modelling

Tyre wear is a topic of particular interest both for road safety enhancement and for vehicle performance optimization. In the first case, the reduction of the braking and directional capabilities due to the thinning of the tyre tread layer is one of the main responsible of the ineffective water drainage on wet soils and in general of the tyre-related accidents. As concerns handling performances, in particular in the racing field, wear reduces the maximum available friction coefficient value (Fig. 23) provided by tyres, that can implicate a considerable decrease in the driving forces at

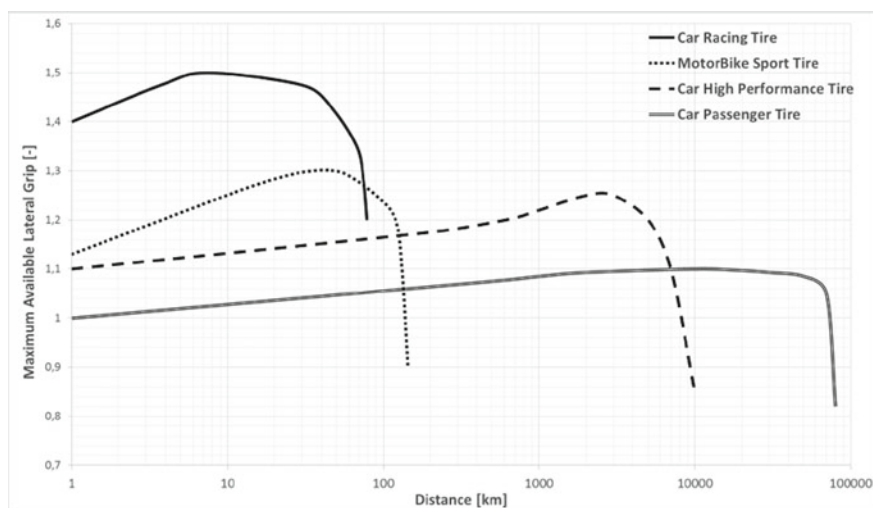


Fig. 23 Percentage of tyres of tyre-related crash vehicles in each category of tyre tread depth (*Data Source NMVCCS 2005–2007*)

the ground, a way out of the optimal trajectory and a consequential increase in the lap-time.

The first analytical models describing wear phenomena between two bodies in mutual contact are linked to the names of the German mathematician and geometer Theodor Reye and of the English engineer John F. Archard, who, in different periods, described sliding wear basing it on the theory of asperity contact (Archard, 1953). The Archard equation was developed later than the Reye's hypothesis, though both came to the same physical conclusions, that the volume of the removed debris due to wear is proportional to the work done by friction abrasive forces arising at the contact interface.

Early experimental studies concerning tyre wear were carried out by Schallamach and Turner (1960), who stated wear increases as the square of the slip, while at constant slip, it increases with stiffness and decreases with elastic hysteresis of the wheel. Saibel and Tsai (1973) and Shepherd (1986) focused on specific wear phenomena proposing local stress models able to predict sliding and abrasion in the contact patch, linking wear phenomena to them. Sueoka et al. (1997) dedicated to a different kind of phenomenon, polygonal wear, caused by the first natural vibration mode of the tyre in the vertical direction, occurring when tyre pressure is inadequate or suspension defines incorrect wheel angles. Le Maître et al. (1998) carried out a wide outdoor experimental campaign, presenting tyre wear performance as a statistical distribution based on the usage severity of each tested vehicle.

In the new century, with the increase of tyre performances and of the attention to road safety theme, many authors developed wear models: Braghin et al. (2006) predicted local wear distribution combining a mathematical model of the tyre with an experimentally determined local friction and wear law. Huang et al. (2015), basing on Mohsenimanesh and Ward's studies (2010) developed a recent theory able to estimate the tread mass removal by means of a three-dimensional pressure and sliding local prediction.

In the following an innovative approach is proposed, not considering tyre wear as a consequence of working dynamics (Li et al. 2012a), or as a phenomenon to be studied as influenced by external factors (Li et al. 2012b) anymore, but as a starting unavoidable condition whose effect on tyre thermal behaviour and on the overall tyre and vehicle frictional performance can be analysed with an innovative approach.

The main objective of this chapter is to provide the entire instruments and knowledge of physical phenomena useful to understand and to provide valid instruments to develop a reliable wear mechanism formulation on the basis of data collected from the models.

Wear is due to many causes and involves three major consequences: loss of performance, loss of security and environmental pollution. Depending on the context, the focus on the consequence can be moved: in motorsport tyre's fundamental role is to guarantee high performance as long as it is possible, in road vehicles safety has a decisive importance compared to the performance target, in big densely-populated cities the pollution due to tyre dust (small rubber fragments that detach from the tyre with dimensions in the order of microns) can be of considerable importance.

According to Le Maître et al. (1998), five important sources causing wear can be identified, and each of them may induce different wear mechanisms and therefore rate variations:

- **Routes and driving style.** Driving style and routes are an input for the generation of the tyre/road interaction forces, which are directly linked to the distributions of stress and strain within the contact patch. The variations in terms of wear rate can be interpreted through different acceleration levels imposed by the driver to the vehicle. Indeed, professional drivers would be able to reach higher accelerations than a common driver, archiving an increasing wear at the same route.
- **Ground.** Contact patch main phenomena like friction, adhesion and abrasion highly depend on the roughness parameters and the type of ground.
- **Season.** The main season parameters that exert a decisive influence on the tyre wear mechanism are the ambient temperature and humidity, on which directly depend tyre temperature and thus thermo-mechanical fatigue of tyre structure and tread compound.
- **Vehicle.** The average life of a tyre may vary depending on the vehicle characteristics like weight, suspensions and steering geometry, implicating diverse strain levels within the tyre composite structure due to vehicle balance.
- **Tyre.** The tyre itself has also a major effect on wear phenomenon. Since the tyre is deeply integrated system, the most important parameters acting on tyre wear may be identified among the stiffness and geometry parameters, working pressure of inflation, and tread and composite material structure characteristics.

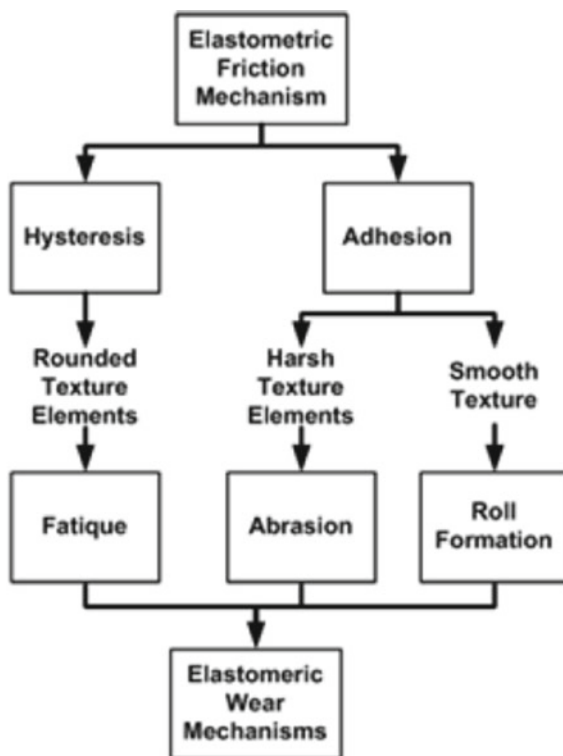
Wear and friction. The rolling tyre can be seen as a system, whose tread rubber elements are undergoing with prescribed motion laws within the contact patch with road, and this elementary macroscopic modelling of tyre frictional behaviour provides a basic understanding of rubber friction and tyre mechanics simultaneously.

The various mechanisms of friction and wear occur in different combinations when a tyre is subjected to braking, driving, free rolling and cornering manoeuvres. For example, a free rolling tyre exhibits the fatigue mechanism for the repeated and cyclic loading and unloading of discrete road asperities.

Under wet conditions with severe braking, the same tyre will experience locked wheel sliding on the particular road surface under investigation. Due to the presence of water, the forces of adhesion between tyre and road are very weak (to be neglected); on the other hand, hysteresis friction term and wear abrasion term will induce a more severe form of the abrasive wear mechanism. In contrast, locked wheel sliding of a tyre on a dry harsh texture will make a severe and perhaps irreversible abrasive wear resulting in local overheating and the production of rubber wear fragments. A schematic diagram of deeply interconnected friction and wear mechanisms within rubber-like materials is represented in Fig. 24 to better understand the underlying complex phenomena in analysis.

As said, there are two principal components of rubber friction: adhesion and hysteresis (Genovese et al., 2019). Depending on the surface, kinematic-dynamic working and temperature conditions, diverse adhesion and hysteresis friction terms

Fig. 24 Schematic diagram of friction and wear mechanism



contribute to the total grip amount. On a perfectly smooth texture, adhesion will contribute to the wear phenomenon, where wear local tear irregularities should be less present on the tread surface. On the other hand, on harsh textures, the adhesion mechanism arises abrasive or cutting wear, where the asperities of the road texture and the continuous deformation due to tyre rolling may lead to phenomena of hysteresis and fatigue wear. The fatigue mechanism is relatively mild in intensity but continuous, meanwhile the abrasive wear phenomenon is severe and usually of short duration.

Moreover, the link between friction and wear actually is bidirectional, the wear can be seen as a consequence of friction, but the same friction has a component resulting from wear.

Adhesion

The adhesion term of friction derives from the molecular links between rubber and the underlying surface. Therefore, the adhesion forces are closely related to the contact surface and compound viscoelastic characteristics. If the two surfaces were perfectly smooth, the effective contact area would be the same as the observed one, but the real surfaces are rough on molecular scale and the contact is thus limited to the highest peaks of the two surfaces. Therefore, the real contact area depends on the surface

profiles, on the proprieties of the materials, on the contact pressure, on temperature and on tyre operating kinematic conditions.

Deformation and hysteresis

Hysteresis is a material bulk phenomenon. A movement of the tyre compound on the real road texture results in deformation of the rubber due to penetration within surface asperities. Since the rubber material is viscoelastic, the deformation generates the hysteresis loop, whose amplitude deeply depends not only on dynamic and kinematic load conditions but also on excitation frequency and material temperature.

If the rubber recovers the deformation slowly due to a high viscoelastic rubber characteristic in particular operating conditions, the compound indentation would only affect the peaks of surface irregularities, not reaching the road roughness valleys. In addition, the influence of this effect should be taken into account on the generation of the effective contact area and the pressure distribution, which will finally result in diverse interaction force generation values.

According to (Lorenz et al., 2013), the friction term due the deformation is strictly dependent on the vertical load. To evaluate the hysteretic grip term ensuring to neglect the adhesive friction term, the surface should be lubricated. A qualitative trend of different quantities, among them of friction force due the deformation F_{def} and the quantity C_{def} as functions of vertical load and penetration level, defined as:

$$C_{def} = \frac{F_{def}}{F_v} \quad (26)$$

where F_v is the vertical load.

Figure 25 depicts the trend of C_{def} and F_{def} as a function of the vertical load.

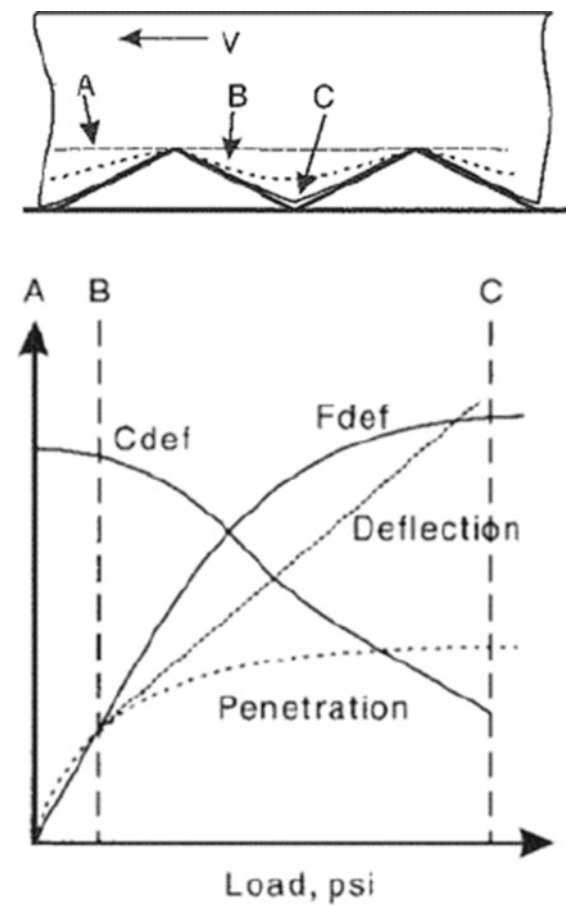
The level of penetration A represents very light load on the rubber, where F_{def} grows almost linearly with the load and C_{def} remains constant due to the penetration of the asperities in the rubber. In these conditions, the effective contact area increases proportionally and this condition is valid for very small values of vertical load.

As the load increases, the relationship between load and the friction force is no longer comparable to the linear behaviour. The force still grows with the load but no longer linearly until the force curve changes its slope and the coefficient of friction begins to decrease (between B and C). The rubber starts to saturate the available space between the road asperities with the result that the deformation involves all the rubber and not only the surface. The trend of F_{def} is saturated and does not increase with the vertical load, bringing down the value of C_{def} .

Tearing

As the deformation forces and sliding speed goes up, local stress can exceed the tensile strength of the rubber (function in its turn of compound temperature and excitation frequency in those operating conditions). When the polymer bonds and

Fig. 25 Schematic diagram of friction and wear mechanism



cross-links are stressed to failure, the material cannot recover completely the deformation and this causes tearing. Tearing phenomenon absorbs energy, resulting in additional friction force. Indeed, wear phenomenon is the ultimate result of tearing.

When rubber moves on a hard-smooth surface, no true sliding occurs. The only relative motion is in the waves, where contact between the rubber and the counter face is temporarily lost. Then, the contact area is crossed by waves of detachment.

The rubber adheres to the smooth surface at the leading edge. Then, waves of detachments move across the rubber surface. The energy required for the separation of the rubber from the surface is much higher than the energy gained in the previous attachment, due the adhesion forces. Because of the material viscoelastic proprieties, the rubber absorbs some energy requiring more force to move the slider.

In conclusion, the material deformation is a key element in rubber friction generated during sliding on both smooth and rough surfaces. For this reason four chapters

have been dedicated to properly describe the tyre starting from its structural characteristics to be able to evaluate local pressure and velocity distribution within the contact patch, the compound viscoelastic characteristics modifying its properties as function of frequency and temperature, a thermal model able to evaluate the temperatures of different tyre layers and the thermal exchanges within of tyre composite structure and finally the road roughness properties.

Wear irregularity. Tyre wear can be roughly divided into two categories: regular wear and irregular wear. Regular tyre wear shows an even wear on the external surface of the tread. On the other hand, irregular wear results in local spots on the tyre external surface, where wear phenomenon progresses faster than other spots. These local worn spots generally produce extra tyre noise and vibration, which reduce the tyre performance in terms of handling and comfort. Tyre wear is generally a combination of regular and irregular wear phenomena.

If it is difficult to predict the phenomena concerning the regular wear, it is even harder to estimate the irregular wear, since a complete suitable stress–strain deformation distribution should be estimated instant by instant and pavement local irregularities and the probable presence of tiny rubber and dust particles should be also taken into account.

Abrasion pattern and graining

The graining is a fatigue phenomenon of the tyres due to the tyre–road relative surface motion. When the rubber slides against the road texture, the abrasion creates rubber lumps on the tread surface, reducing the amount of available grip. These lumps then break off, leaving small craters on the external tread surface. One of the most common reasons for the graining of the front tyres is an unbalanced setup which can produce an excess of under-steer and the consequent sliding. Another factor is the low environmental temperature, since the tread surface cools on long straights and leads to a sudden decrease of the loss factor and thus to different viscoelastic response capacity, where tread compound becomes more brittle.

The result of the graining is the abrasion of the tyre surface. The orientation of the abrasion pattern indicates the direction of sliding relative movement between tyre and road.

In most cases, the side graining on the front tyres can be noticed, although on some circuits may appear longitudinally on the rear tyre. This happens when there is the need of many traction phases with great amount of sliding. Sliding on smooth slopes does not always produce abrasion. Abrasion severity depends on the shape of uneven road surfaces rather than on their size. Laboratory experiments have shown that the abrasion is less if the scrolling takes place in an inert atmosphere. Thus, a possible solution to reduce the extent of this phenomenon can be to insert antioxidants in the rubber composition. Some examples of abrasion patterns are illustrated in Fig. 26. Sliding in random directions does not produce abrasion patterns; in fact, the latter one is the consequence of continuous slip in the same direction.

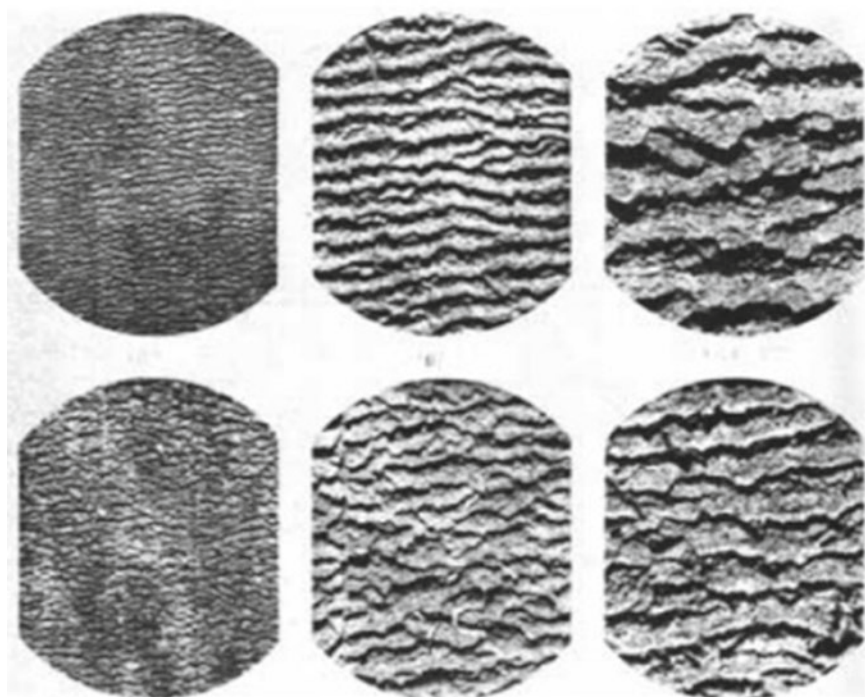


Fig. 26 Abrasion patterns

This phenomenon occurs usually when the track is not rubberised and when the tyre is too soft or adherent or when the tyre is very stressed before it arrives at the working temperature.

Blistering

Lack of the vertical load, problems concerning the vehicle setup or overly aggressive driving style can lead to blistering phenomenon. This phenomenon occurs when a portion of the rubber at the interface between the belt and the tread overheats abnormally causing small bubbles of air within the tread compound which implicate a rubber detachment.

These rubber detachments can have different surfaces depending on the thermal and mechanical stress level to which the tyre has been subjected. Some real hole will be visible on the external tyre surface, which can be often very deep, even reaching the tread–carcass interface. The tyre overheating can occur due to completely different phenomena, as tyre working conditions, inflation pressure, vehicle setup, ambient and track conditions and finally the driving style. Overheating can also be caused by a very dirty track with little grip where the tyres tend to slip, causing abnormal local heating of the tyre tread.

Working conditions dependence. The tyre/road interaction force depends on the structure of the tyres and their adherence is strongly influenced by the temperature. As seen, wear is mostly influenced by the friction force, therefore, it is indirectly related to the temperature. Tyre wear is also influenced by the road surface, the sliding speed and tyre's material characteristics.

Sliding speed

Using a specific rubber polymer sliding on smooth glass and a silicon-carbide sandpaper the major contributions to the friction and wear can be distinguished as a function of another key variable: the sliding speed.

In Fig. 27 the trend of friction coefficient C_f is represented for the same rubber specimen of different pavement textures: glass, silicon carbide and dusted silicon carbide. The glass surface performs as a perfectly smooth surface where the only frictional forces that rubber exchange during sliding should be due to the adhesion phenomenon. Siliconcarbide sandpaper perform as a rough surface where both adhesion and hysteresis contributes are present, meanwhile on dusted silicon-carbide sandpaper only the deformation friction contribute should be present, as the finely powered magnesia should eliminate adhesion term preventing mutual contact between rubber and pavement texture. The vertical scale is represented by the coefficient of friction C_f and the horizontal one is the logarithm of sliding speed. The curves are characterized by different peaks at diverse sliding speeds so that, once viscoelastic compound properties are fixed and temperature stabilized, there will be an optimum working range for each road texture, expressed by means of sliding velocity and thus of excitation frequency.

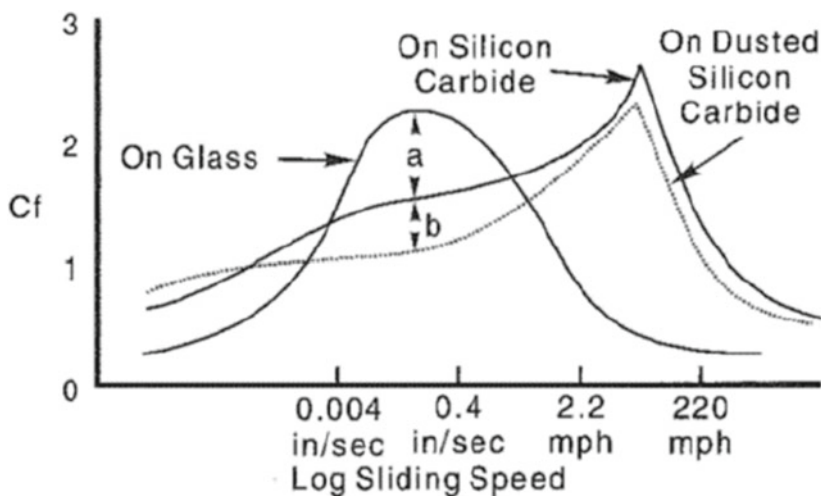


Fig. 27 Speed sensitivity of friction components

The distance a between the rubber/glass and rubber/silicon-carbide curves represent the difference in terms of friction coefficient concerning the rubber grip respectively on smooth and rough surfaces. This indicates that the adhesive contribute of friction force dominates on hysteretic term because the latter one takes into account of the reduced real contact area on the rough pavement. The distance b is the maximum distance between dusted and undusted rubber/silicon-carbide curves, where the difference in terms of friction coefficient represents the adhesive term on the rough surface. It has to be highlighted that since glass and siliconcarbide are materials with different molecular properties, the sliding speed in correspondence of b is different compared to the velocity value corresponding to the maximum friction coefficient on the smooth glass surface.

In contrast with metal friction mechanism, both the adhesive and the hysteretic friction terms for rubber-like materials are viscoelastic. The adhesion peak occurs at a sliding speed of about 1 cm/s which is a typical value of slip speed in the contact patch of a rolling tyre. The second or hysteresis peak occurs at about 150–200 km/h and corresponds to sliding velocities in the contact patch for high speed panic locked braking wheel. As already said, the curves are obtained for a fixed temperature, as each curve with its peaks will move to higher frequencies (or sliding speeds) as the temperature raises.

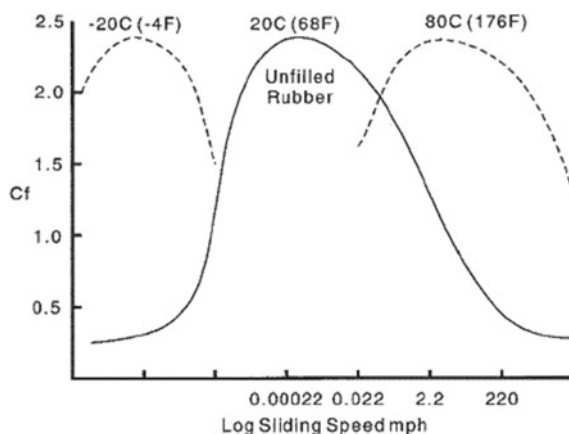
From a tyre design viewpoint, the curves obtained should be used to optimize the tyre behaviour at the extreme limits of driving behaviour: slow speed rolling and high-speed sliding, trying to magnify the amount of available grip in all tyre working conditions.

The characteristic peaks of rubber friction are similar to the characteristics of wear or abrasion processes, since both friction and wear are viscoelastic.

Temperature and frequency effects

Figure 28 represents the trend of the generic coefficient of friction as a function of sliding speed of a generic rubber over a glass surface for three diverse temperature

Fig. 28 Temperature sensitivity of friction components



conditions. The two extreme temperatures are the glass transition temperature and the temperature for which the coefficient of friction is maximum, depending on the type of rubber in analysis. Below the glass transition temperature polymeric materials become rigid and brittle assuming easy tendency to shatter. With higher temperatures values polymers have elastic proprieties and the ability to undergo plastic deformation without encountering fractures.

The direct dependence of wear on temperature is highlighted by scientific evidences (Li et al., 2012b). Despite the direct and indirect experimentally demonstrated dependence of the wear on temperature, the wear models in literature try to predict tyre wear considering almost exclusively the dependence on friction force and sliding speed, because the main problem is to measure or simulate the tyre temperature and the thermal exchanges in the various operating conditions.

Asperities geometry influence

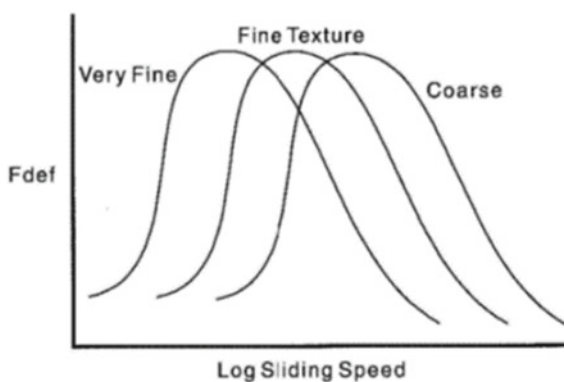
Road asperities have a significant role for the friction forces due to the hysteretic deformation. As said before, analysing the friction force behaviour against sliding speed for different surface textures, for each texture (Fig. 29) there is an optimum slide speed at which a fixed rubber reaches the maximum friction coefficient.

Contact stress or, in other words, contact force per unit area, deeply depends on the applied load value and the shape of the asperities.

In addition to the statistical roughness parameters, it is also important to consider the shape of asperities. As an example, in literature the theoretical texture shape sensitivity is available considering cylindrical and pyramidal asperities. For pyramidal asperities, the deformation forces will be all inclined forming the same angle with respect to the sliding direction not depending on the rubber penetration level. On the other hand, for cylindrical asperities, the angle deeply depends on the penetration level of the rubber. In particular, it increases from 0° up to maximum 90° .

The curve stemming from pyramidal asperities presents an unchanging slope. By contrast, the slope of the curve deriving from the cylinder asperities increases with the penetration.

Fig. 29 Surface texture effect on rubber friction



Wear modelling. Conventionally, the wear is experimentally measured. Otherwise, the tyre wear can be predicted by the tyre vibration and the modal analysis (Huang et al., 2013; Sueoka et al., 1997). Recently, the tyre wear calculation method through FE analysis is a growing trend (Dorsch et al., 2002; Põdra & Andersson, 1999).

In literature, there are several wear models based on scientific tests and direct observation of the wear phenomenon. The most representative wear models have been implemented and compared.

In addition, the cited studies also focus on the modelling of a tyre model to simulate the behaviour of the tyre in different working conditions. Objectives of their analyses are those models which, through mathematical formulations, could be able to describe the instantaneous or global wear phenomenon of a tyre as a function of several physical characteristics. Many other models available in the literature seem to be based on the same models, adopting therefore similar formulations. For this reason, six main wear models have been analysed in more detail.

Wear formulation—Reye

In 1860 Reye proposed a theory to explain the consumption of a solid body when it slides with friction on a rough surface (Reye, 1860).

Reye formulated his wear model starting from a parallelepiped of length $2h$, height l and a unitary thickness. The parallelepiped lying on a rigid plane is pushed against this plane by a uniform pressure p . The parallelepiped is made of an elastic, homogeneous and isotropic material.

If the block is stationary, it remains in equilibrium due to the pressure p exerted by the plane where it lays. Applying a uniform tangential stress q , the block remains in equilibrium until q reaches the value $f \cdot p$, with f represents the friction factor, where the block begins to slide.

According Reye's theory, the tangential stresses that will be exerted by the supporting base along the lower face of the block, produce the deterioration of a thin layer of material just adjacent to the base.

This layer, called the "worn region", has constant width l_w proportional to the tangential stress. The Reye's wear hypothesis asserts that the volume of material V [m^3] removed in each interval of time due to wear phenomenon is proportional to the work done by the friction forces W_f [J] in the same time interval:

$$V = K_{reye} W_f \quad (27)$$

Wear formulation—Archard

On the basis of Reye's hypothesis, a British engineer John F. Archard has developed the equation that takes its name (Archard, 1953; Archard & Hirst, 1956). The Archard's wear formulation is a simple model describing the abrasive wear and it is based on the theory of asperity contact.

Archard has formulated his law considering two circular surfaces of diameter $2a$, in relative sliding. At zero time (Fig. 30a), the surfaces form a circular contact area. In a second time, after a sliding distance y , the relative position of the slides is shown in

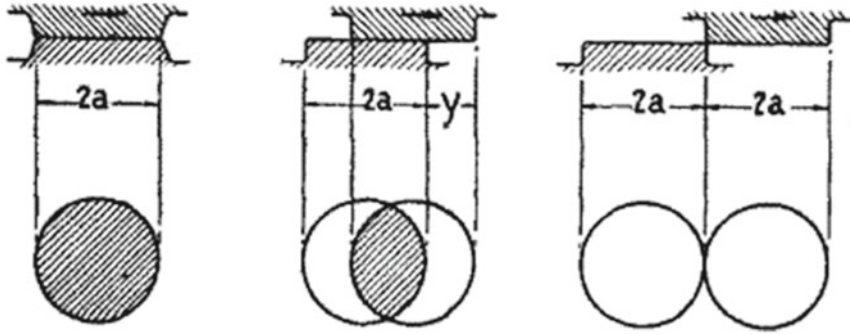


Fig. 30 Representation of single contact in sliding surfaces. **a** Maximum contact area of radius a . **b** After sliding through distance y . **c** After sliding through a distance $2a$; contact area just reduced to zero

Fig. 30b. After sliding a distance $2a$, the contact area under consideration decreased to zero in Fig. 30c. Therefore, the wear phenomenon may be expressed as a function of the load through the following expression:

$$V = \frac{K_{archard} PL}{H} \quad (28)$$

where:

- V : total volume of wear [m^3]
- $K_{archard}$: dimensionless constant [—]
- P : normal load [N]
- L : sliding distance [m]
- H : the hardness of the softest material between the surfaces in mutual contact.

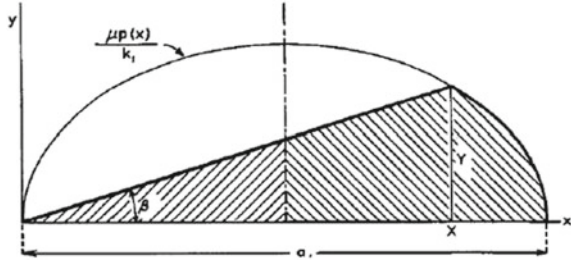
Archard assumes that, since not every contact results in a wear particle, $K_{archard}$ may be regarded as a probability factor that depends on the combination of the materials used.

Wear formulation—Schallamach

Schallamach was the first to focus specifically on tyre wear rather than on rubber wear. He assumed proportionality between abrasion and frictional energy dissipation (Schallamach & Turner, 1960), describing the deformation of the tyre rubber within the contact patch. Calculating the wear amount as function of slip and dynamic operating conditions of a tyre, Schallamach comes to the formulation of his law of wear:

$$A = \frac{\gamma \mu}{v} \int_x^a w \cdot p(x) dx \quad (29)$$

Fig. 31 Tyre/road interaction force to vary the pressure within the contact patch



where:

- A : abrasion per unit distance travelled
- γ : abrasion per unit energy dissipation
- v : travelling velocity
- a : length of area of contact
- x : longitudinal coordinate of point in the contact patch
- w : sliding velocity
- $p(x)$: pressure distribution per unit length of contact area.

By substituting into the footprint contact pressure distribution law (Fig. 31) and by integrating, the following expression of abrasion coefficient A is obtained:

$$A = \gamma \cdot s \cdot F \quad (30)$$

where s is the slip velocity and F the friction force in the contact area. Even if it comes from a different reasoning, Schallamach comes to the same formulation of Reye. This confirms what it is previously said, namely, many models in the literature are based on previously developed models.

Wear formulation—Shepherd

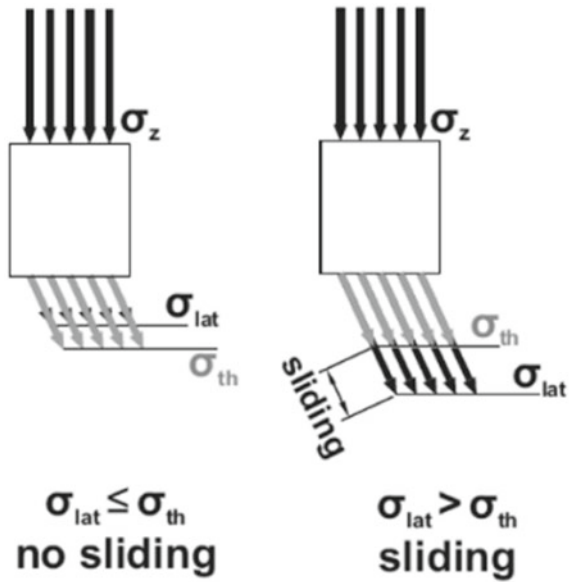
Starting on Schallamach wear model, Shepherd has formulated his model. Tread abrasive wear is supposed to be directly proportional to the contact area and inversely proportional to the block shear stiffness (Shepherd, 1986).

Shepherd considers sliding to appear when the lateral stress σ_{lat} is larger than the threshold stress σ_0 , as reported in Fig. 32. The rubber is considered not to slide when the lateral stress is lower than the threshold stress. In the latter case, Shepherd neglect the abrasive wear at all. The threshold stress is calculated by:

$$\sigma_{th} = \mu_0 \cdot \sigma_z \quad (31)$$

where μ_0 is a constant coefficient of friction and σ_z is the local pressure. The difference between the lateral stress and the threshold stress is assumed to stand only in the sliding stress. Shepherd formulated the abrasive wear as follows:

Fig. 32 Sliding according to Sheperd's wear model



$$\begin{cases} V = \frac{A}{K}(\sigma_{lat} - \sigma_{th}); & \text{if } \sigma_{lat} > \sigma_{th} \\ V = 0; & \text{if } \sigma_{lat} \leq \sigma_{th} \end{cases} \quad (32)$$

where A is the contact area, $k_{shepherd}$ is the shear block stiffness and B is the wear factor.

Wear formulation—Sueoka

Sueoka is known for his research on wear pattern formations at contact with rotating systems, that has adapted his research to automotive tyres (Sueoka et al., 1997).

Sueoka has analytically investigated the polygonal wear of car and truck tyres at a constant forward velocity. He assumed the polygonal wear is caused by vertical force variation as a result of the first vertical natural mode of the tyre belt. The tyre is approximated by a rigid ring model and the tyre wear is approximated by a wear model with a time delay. The rigid ring model consists of a single degree of freedom system of the tyre belt in the vertical direction.

Sueoka has based his wear model on Shepherd's abrasive wear model, where the contact patch of the tyre is approximated by a contact point, where the stresses distribution is transformed into global forces. Sueoka wear model formulation is the following:

$$U(T) = U(t - T) + v_{sueoka} \beta P^{n_{sueoka}}(t) \quad (33)$$

where:

- U : wear quantity [m]
- t : time [s]
- T : time delay [s]
- v_{sueoka} : wear factor [m/N]
- $P(t)$: normal force [N]
- n_{sueoka} : dimensionless wear exponent [–].

The wear quantity at time t consists of the wear of the previous revolution $U(t - T)$ and wear increment at t . The time delay is the time of one tyre revolution. The wear increment is considered to be caused by sliding in the lateral direction. Sueoka linked the lateral sliding to the lateral force acting at the contact area. Coefficient β is assumed to be a sort of friction coefficient, which depends on the toe angle α and the camber angle γ .

$$\beta = |C_\alpha \alpha + C_\gamma \gamma| \quad (34)$$

where C_α and C_γ are respectively cornering stiffness and camber stiffness factors.

Wear formulation—Braghin

Braghin proposed a mathematical model of the tyre with an experimentally determined local friction and wear law (Braghin et al., 2006). His model is derived from the observation of experimental tests carried out in laboratory conditions. Tests were carried out on two different testing machines. The imposed slip angle testing machine and the imposed rotational slippage testing machine have undergone wear tests in order to derive the wear law.

In order to validate the model Braghin also collected the data from outdoor tests that were carried out with two passenger cars on a public road. Each car was equipped with a different set of tyres: one car was equipped with four all-season tyres (A tyres) and the other car was equipped with four winter tyres (B tyres). The two rubber specimens for two testing machines have completely different geometry. To compare the wear specimens results it is necessary to normalize both the frictional power and the mass loss. The frictional power is scaled with the contact area while the mass loss is divided by the travelled distance multiplied for the specimen's transversal dimension (covered area). It results in the following wear model:

$$\tilde{m} = k_{1,braghin} \tilde{w}^{\tilde{k}_{2,braghin}} \quad (35)$$

where:

- $\tilde{m}$: mass loss per unit covered area
- $\tilde{w}$: frictional power per unit contact area
- $k_{1,braghin}, k_{2,braghin}$: wear constants of the rubber compound function of temperature and surface roughness.

Wear formulation—UNINA

The wear model developed in this discussion originates as an evolution of the models in the literature with special attention to the influence that the tyre temperature has on wear phenomenon.

The TRT-evo model is an analytical-physical thermal tyre model able to estimate the tyre temperature distributions even of the deepest levels, usually not measurable online, in completely different ambient and operating conditions.

In order to formulate the first wear law, five different track datasets, provided by motorsport partner, have been processed by means of the TRT-evo model to obtain temperatures instant by instant to be taken into account within the wear Unina formulation. Therefore, being the vehicle equipped with four tyres of the same compound characteristics, 20 different wear datasets are virtually available for the same compound.

Analysing the available data, the following dependencies have been studied:

- Friction Power. Therefore, the dependence reported in Fig. 33, has been formulated as:

$$W_{FP} = a_{Unina} FP^{b_{Unina}} = a_{Unina} \left(\frac{|F_x v_{sx}| + |F_y v_{sy}|}{A_{CP}} \right) \quad (36)$$

where W_{FP} is the wear rate, FP is the friction power and coefficients a_{Unina} , b_{Unina} are coefficients taking into account the boundary conditions in terms, not yet

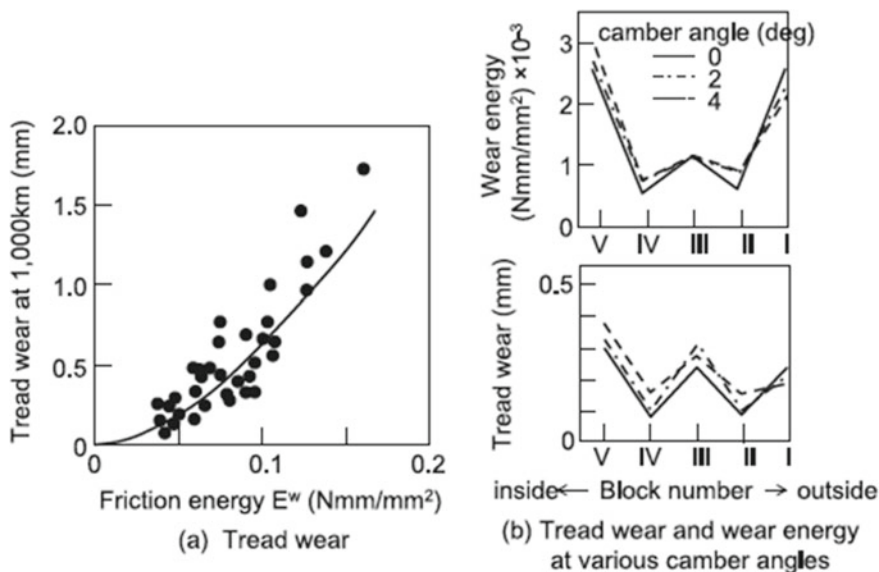


Fig. 33 Friction power influence: wear energy and tread wear (Jazar, 2019)

considered, as road texture and pressure/velocity distributions within the contact patch.

- Temperature. Considering the tyre temperature distribution the mean temperature T^* obtained as weighted between the tyre external instantaneous surface $T_{surface,CP}$ within the contact patch and the tyre T_{bulk} temperature has been implemented to take into account the dependence on wear phenomenon. The wear rate due to temperature has been supposed linearly increasing with temperature formulated by means of the following expression:

$$K_T = K1_T + K2_T T^* \quad (37)$$

where $K1_T$ and $K2_T$ are both positive coefficients.

Therefore, the complete first Unina wear formulation is the following:

$$V = W_{FP} K_T \quad (38)$$

where V represents the wear, expressed in $[m^3]$.

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Torque Vectoring Control for Enhancing Vehicle Safety and Energy Efficiency



Basilio Lenzo

Abstract Torque vectoring control is one of the most interesting techniques applicable to electric vehicles with multiple motors. Essentially it is the possibility to allocate desired amounts of torque to each motor. With an uneven allocation of torque between left and right sides of the vehicle, a direct yaw moment can be generated and exploited to enhance the vehicle handling behaviour. This allows to enhance vehicle safety, stability and cornering performance. Significant energy efficiency benefits are also achievable, either as a main priority or as a secondary objective when the main target is the vehicle handling behaviour. This Chapter explains the underlying principles of each element of a torque vectoring control framework: reference generator, high level controller, and low level controller. Notable applications are also presented and discussed in detail.

1 Introduction

In general, torque vectoring can be defined as a feature that allows to control torque at individual wheel level. The most frequent reason for employing torque vectoring is the improvement of vehicle longitudinal and/or lateral dynamics (Esmailzadeh et al., 2003; Lenzo et al., 2019b)—hence affecting vehicle safety, stability and cornering performance. This is normally achieved by allocating different amounts of torque to the left side and right side of a vehicle, resulting in the generation of a yaw moment at vehicle level—it is not uncommon to see the concepts of direct yaw moment control (DYC) and torque vectoring used interchangeably, even if actually the former is achievable through a specific use of the latter. Torque vectoring can also be used to

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enhance energy efficiency in electric vehicles (Sforza et al., 2019; De Filippis et al., 2016, 2021).

Based on the above definition, in some sense torque vectoring is already implemented in modern passenger cars. In fact, these are equipped with Anti-lock Braking System (ABS) and Electronic Stability Control (ESC), that control friction braking torques on individual wheels when safety-critical conditions are detected. However, this individual wheel torque control is limited to braking torques, i.e. a standard passenger car cannot control individual traction torques. Cars are normally equipped with a single source of power (very often that is an internal combustion engine) and an open differential, which implies that the torque is evenly distributed between left and right side of a driven axle (Guiggiani, 2018).

A way to achieve torque vectoring for traction torques is the use of appropriate mechanical systems, including limited slip differentials or more complex systems known as torque vectoring systems (TVS, Piyabongkarn et al., 2010). Despite being effective, often these approaches have limitations including the impossibility to allocate more torque on the faster wheel of the axle (in case of limited slip differentials) and/or the inability to achieve a left-right torque bias in certain driving conditions, such as off-throttle situations.

Torque vectoring can be achieved, by definition, in electric vehicles with multiple motors. In this case the constraints of mechanical systems are overcome, i.e., it is possible to allocate more torque to the faster wheel of an axle, and it is possible to achieve a left-right torque bias even when the overall vehicle torque demand is zero. Clearly, there are still intrinsic constraints, for example the amount of left-right torque bias is limited by the torque characteristics of the available motors and the vehicle layout. Figure 1 shows a schematic of a vehicle layout featuring four on-board motors, while other layouts are investigated, e.g., in De Novellis et al. (2014a) and Tuncay et al. (2011).

Consistently with the focus of this book on future trends, this Chapter explores torque vectoring techniques for electric vehicles with multiple motors. Here, interestingly, torque vectoring may be continuously active to provide consistent vehicle dynamics benefits, which is an important difference with respect to traditional control systems such as ABS and ESC.

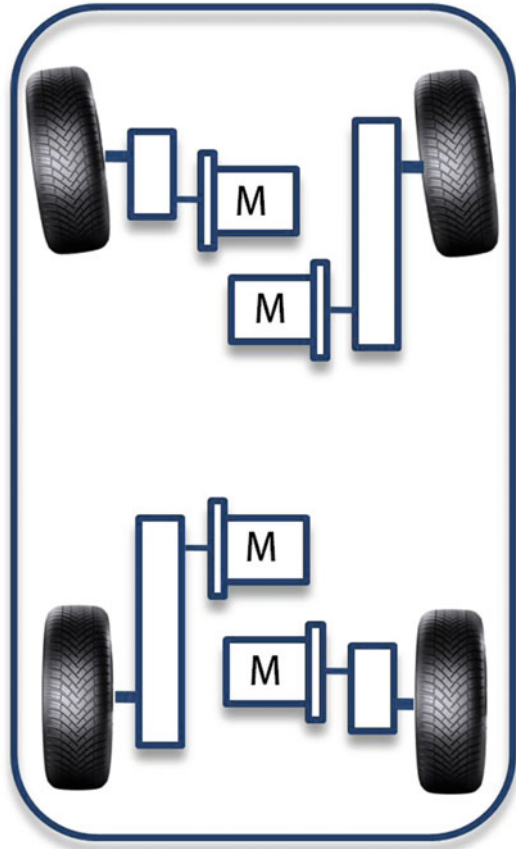
2 Torque Vectoring Control Framework

The majority of the literature agree that a torque vectoring control framework is composed by three main blocks (Fig. 2):

Reference generator

The reference generator (Sect. 3) combines information on measured/estimated vehicle states and on the driver input, thereby generating reference values for relevant vehicle quantities. Normally these are a reference yaw rate, r_{ref} , and in some cases

Fig. 1 Schematic of vehicle layout with four independent motors. In this example the on-board configuration requires appropriate gear transmissions to reach the wheels, as opposed to an in-wheel motor configuration



a reference sideslip angle, β_{ref} . Such reference(s), to be followed by the vehicle through torque vectoring, are given as input to the high level controller.

High level controller

The high level controller (Sect. 4) takes the output target quantities from the reference generator, as well as the driver input (e.g. accelerator/brake pedal positions) and vehicle states (e.g. vehicle velocity). Such inputs are elaborated to compute the desired total torque T_{tot} and yaw moment M_z to be applied at the vehicle level.

Low level controller

The task of the low level controller (Sect. 5) is to map the desired total torque and yaw moment into values of torque demand for each motor. Considering a typical electric vehicle layout featuring four motors, there are potentially four degrees of freedom (DOF), i.e. the four values of torque that can be assigned, T_{ij} , where $i = 1, 2$ respectively for front and rear, $j = 1, 2$ respectively for left and right. Since T_{tot} and M_z are imposed by the high level controller, practically there are only two available

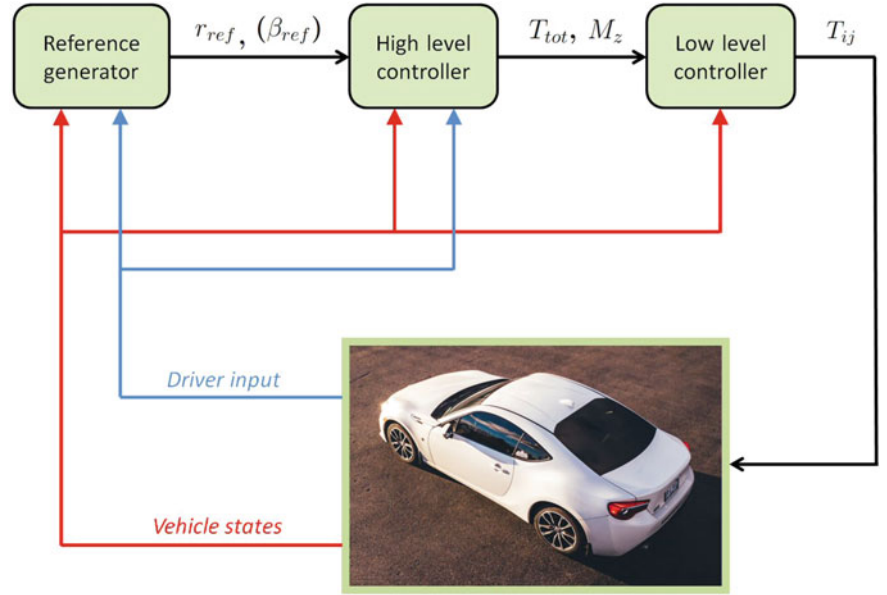


Fig. 2 Typical layout of a torque vectoring control framework

DOF. In other words, the desired total torque and yaw moment can be achieved with virtually ∞^2 combinations of the four motor torques. Relevant vehicle states (e.g. vehicle velocity), along with physical constraints (e.g. nominal motor torque), may be used to determine the most appropriate combination.

The underlying principles of the three blocks, as well as some notable applications, are covered in the remainder. Where relevant, the Adapted ISO sign convention (Pacejka, 2006) is adopted.

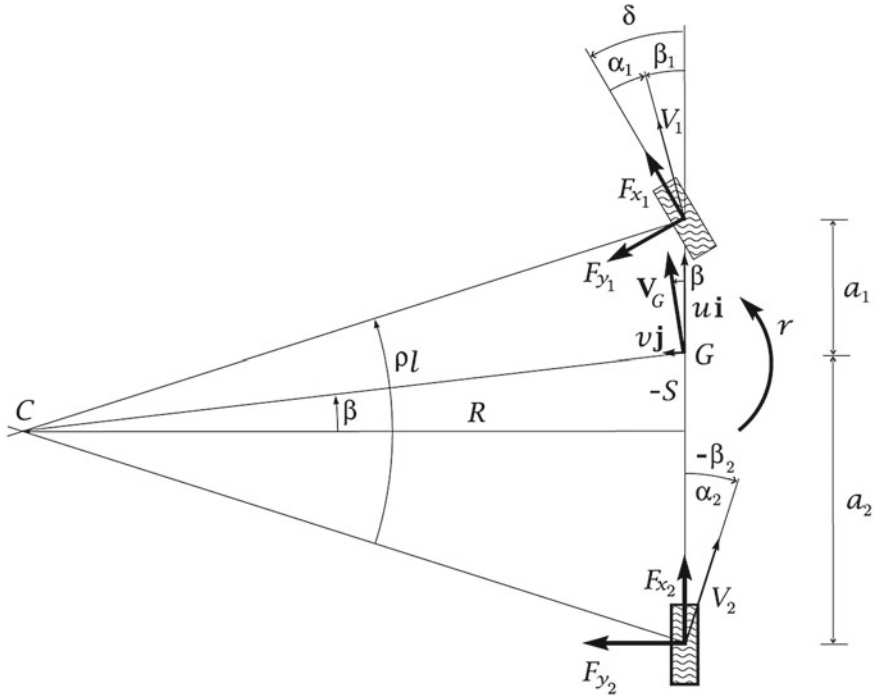


Fig. 3 Single-track vehicle model (adapted from Guiggiani, 2018)

3 Reference Generator: r_{ref} , (β_{ref})

In light of the above, this Section first focuses on the generation of a reference yaw rate. The concurrent control of yaw rate and sideslip angle is dealt with in Sect. 3.3.

3.1 Fundamentals on the Design of r_{ref}

The reference yaw rate is defined based on a simple principle, that is the redesign of the vehicle cornering response. This concept is elaborated hereinafter.

As well-known, for a single-track vehicle model with no rear wheel steering (Fig. 3) the congruence relationships (Lenzo & De Castro, 2019) are:

$$\alpha_1 = \delta - \beta_1 \quad (1)$$

$$\alpha_2 = -\beta_2 \quad (2)$$

where α_1 and α_2 are respectively the apparent slip angles of the front and rear axle, δ is the wheel steer angle, β_1 and β_2 are respectively the sideslip angle at the front and rear axle centre. Owing to the classical hypothesis of car assumed as a rigid body in planar motion, and small sideslip angles,

$$\beta_1 = \arctan \frac{v + ra_1}{u} \approx \frac{v + ra_1}{u} \approx \beta + \frac{ra_1}{u} \quad (3)$$

$$\beta_2 = \arctan \frac{v - ra_2}{u} \approx \frac{v - ra_2}{u} \approx \beta - \frac{ra_2}{u} \quad (4)$$

where u and v are respectively the longitudinal and lateral components of the velocity of the vehicle centre of mass, a_1 and a_2 are respectively the vehicle front and rear semi-wheelbases, r is the vehicle yaw rate (often also indicated with $\dot{\psi}$), $\beta = \arctan (v/u)$ is the vehicle sideslip angle (also known as vehicle slip angle). Therefore:

$$\alpha_1 \approx \delta - \beta - \frac{ra_1}{u} \quad (5)$$

$$\alpha_2 \approx -\beta + \frac{ra_2}{u} \quad (6)$$

Subtracting 6 from 5:

$$\delta = \frac{r(a_1 + a_2)}{u} + \alpha_1 - \alpha_2 \quad (7)$$

which can be rewritten as

$$\delta = \frac{l}{R} + \alpha_1 - \alpha_2 \quad (8)$$

where $l = a_1 + a_2$ is the vehicle wheelbase and $R = u/r$ is, formally, the lateral coordinate of the vehicle instant centre (point C in Fig. 3), which is often confused with the vehicle radius of curvature.¹ Although instant centre and centre of curvature do not coincide in general (Guiggiani, 2018), they do in some cases, including steady-state conditions. Using a linear tyre model, the expressions of α_1 and α_2 can be replaced by the constitutive equations:

$$\alpha_1 = \frac{F_{y1}}{C_1} \quad (9)$$

$$\alpha_2 = \frac{F_{y2}}{C_2} \quad (10)$$

where F_{y1} and F_{y2} are respectively the front and rear axle lateral forces, and C_1 and C_2 are respectively the front and rear axle cornering stiffness values.

¹ Also, sometimes R is confused with the distance between instant centre and centre of mass $\frac{\sqrt{u^2 + v^2}}{r}$ (or simply CG in Fig. 3), this is fine for small values of v .

The equilibrium equations of the single-track model read:

$$F_{y_1} + F_{y_2} = ma_y \quad (11)$$

$$F_{y_1}a_1 - F_{y_2}a_2 = J_z\dot{r} \quad (12)$$

where m is the vehicle mass, J_z the vehicle moment of inertia with respect to a vertical axis through the centre of mass, a_y the vehicle lateral acceleration.

In steady-state conditions $\dot{r} = 0$ and $a_y = \dot{v} + ur = ur$, so from 11 and 12, F_{y_1} and F_{y_2} can be obtained as:

$$F_{y_1} = \frac{ma_y a_2}{l} \quad (13)$$

$$F_{y_2} = \frac{ma_y a_1}{l} \quad (14)$$

Finally, by replacing this result in 8 together with 9 and 10:

$$\delta = \frac{l}{R} + \frac{m(C_2 a_2 - C_1 a_1)}{l C_1 C_2} a_y \quad (15)$$

which is often written as:

$$\delta = \frac{l}{R} + K l a_y \quad (16)$$

where K is the understeer coefficient, or understeer gradient:

$$K = \frac{m(C_2 a_2 - C_1 a_1)}{l^2 C_1 C_2} \quad (17)$$

Note that the definition of K given above corresponds to the “stability factor” in Genta (1997) and differs from the more classical definition, K_{us} , by a factor l :

$$K_{us} = \frac{m(C_2 a_2 - C_1 a_1)}{l C_1 C_2} \quad (18)$$

which would produce

$$\delta = \frac{l}{R} + K_{us} a_y \quad (19)$$

instead of 16. A very interesting approach to the definition of understeer is given in Guiggiani (2018), which coincides with 17 in case of linear tyres. The interested reader may also look at Bucchi and Frenzo (2016); Lenzo et al. (2019a) for another definition of the understeer coefficient.

Remarkably, 16 and 19 can be written in the form:

$$\delta = \delta_{kin} + \delta_{dyn} \quad (20)$$

where δ_{kin} is denoted as kinematic wheel steer angle (also known as Ackermann angle):

$$\delta_{kin} = \frac{l}{R} \quad (21)$$

and δ_{dyn} is denoted as dynamic wheel steer angle:

$$\delta_{dyn} = K_{us}a_y = Kl a_y \quad (22)$$

Note that similar definitions could be provided in terms of the steering wheel angle δ_{sw} :

$$\delta_{sw} = \delta_{sw,kin} + \delta_{sw,dyn} \quad (23)$$

where $\delta_{sw,kin}$ is the kinematic steering wheel angle and $\delta_{sw,dyn}$ is the dynamic steering wheel angle. The relationship between δ_{sw} and δ can be expressed by a suitable map (an example of map is shown in Lenzo et al. (2019a)) or, when appropriate, it can be simplified as a factor called steering ratio, S_r : $\delta_{sw} = S_r \delta$.²

The relationship between $\delta_{sw,dyn}$ (or δ_{dyn}) and a_y is known as the vehicle cornering response, or vehicle understeer characteristic, which is the design target. Equation 16 can be rewritten as:

$$\delta = \frac{rl}{u} + Klur \quad (24)$$

which, solving for r , provides a formulation for the reference yaw rate:

$$r_{ref} = \frac{u}{l(Ku^2 + 1)} \delta \quad (25)$$

This approach to design the vehicle cornering response is used by many authors, that simply use the understeer gradient as a tuning parameter (Siampis et al., 2016; Esmailzadeh et al., 2003; De Novellis et al., 2013; Koehler et al., 2015; Wong et al., 2016; Vignati et al., 2016; Ding et al., 2017; Parra et al., 2018), although with slight differences such as the use of:

- different symbols for the same quantities;
- different definitions of understeer gradient, very often K_{us} , sometimes K ;
- the total vehicle velocity V instead of u as they are nearly the same for small sideslip angles ($u = V \cos \beta$);
- the steering wheel angle δ_{sw} instead of the wheel steer angle δ .

² S_r may be roughly between 12 and 20.

In any case the understeer gradient is chosen to be positive. That implies an understeering behaviour, which in a standard vehicle is preferred to oversteer for safety reasons. Finally, it should be considered that normally:

- the reference yaw rate obtained via 25 is saturated, for example Ding et al. (2017) and Rajamani (2011) suggest:

$$|r_{ref,MAX}| = 0.85 \frac{\mu g}{V} \quad (26)$$

where μ is the estimated available tyre-road friction coefficient, and g is the gravitational acceleration;

- a first order lag is applied to r_{ref} to provide a smoothing action, i.e., in the Laplace domain the actual reference yaw rate is:

$$r_{ref,actual}(s) = \frac{r_{ref}}{\tau_1 s + 1} \quad (27)$$

where s is the Laplace operator and τ_1 a suitable time constant.

A more comprehensive approach to the definition of the desired understeer characteristic is described in the following section.

3.2 Design of the Full Vehicle Cornering Response and Driving Modes

The method shown in 25 implies a simple linear relationship between dynamic steering angle and lateral acceleration. However this is only an approximation of a real vehicle behaviour, accurate only for low values of a_y , as shown e.g. in Fig. 4. It would then seem natural to go beyond the mere design of a tuning coefficient, by actually designing the whole understeer characteristic of the vehicle—so as to replicate a realistic shape. Although in principle any profile might be chosen, the designer should aim for a cornering response with clear benefits with respect to the baseline vehicle but not excessively far from it, to avoid unnatural vehicle behaviour. It should also be considered that too large deviations from the behaviour of the baseline vehicle may require a too severe control action which might not be practically achievable by the torque vectoring controller, due to physical limits on the maximum M_z .

Figure 4 shows a typical understeer characteristic and a potential characteristic achievable by torque vectoring. Desirable targets of the design of the understeer characteristic include: (i) increase of the steering responsiveness of the vehicle; (ii) extension of the region of linear cornering response; (iii) increase of the maximum lateral acceleration. In particular, De Novellis et al. (2014a) propose a piecewise function as the relationship between dynamic steering wheel angle and lateral acceleration³:

³ Again the sideslip angle is assumed small so that $V \approx u$.

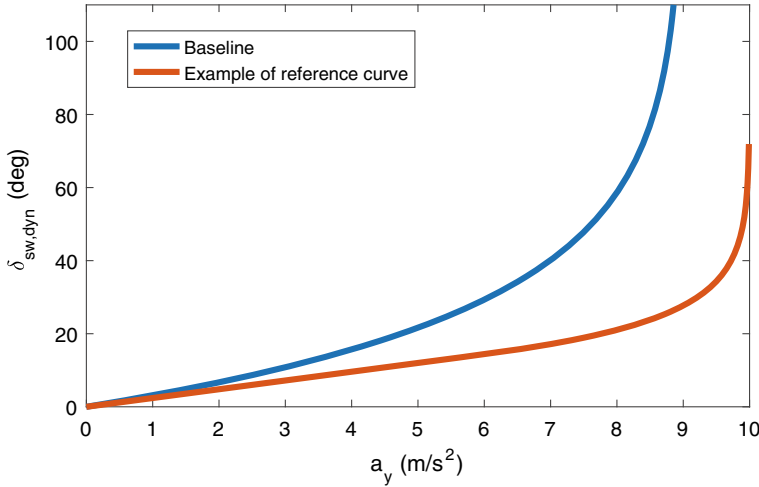


Fig. 4 Examples of vehicle cornering response: baseline curve and a potential reference curve with reduced understeer gradient, extended linear region (up until $a_y^* = 7 \text{ m/s}^2$), increased maximum lateral acceleration ($a_{y,MAX} = 10 \text{ m/s}^2$)

$$a_y = \begin{cases} \frac{1}{K_{us}\delta_{sw,dyn}} & \text{if } \delta_{sw,dyn} \leq a_y^* K_{us} \\ a_{y,MAX} + (a_y^* - a_{y,MAX}) e^{\frac{K_{us}a_y^* - \delta_{sw,dyn}}{(a_{y,MAX} - a_y^*)K_{us}}} & \text{if } \delta_{sw,dyn} > a_y^* K_{us} \end{cases} \quad (28)$$

which addresses the aforementioned desirable targets by allowing to tune three parameters:

- K_{us} , already defined, represents the initial slope of the curve. For instance, a value of K_{us} lower than for the baseline vehicle results in a more reactive vehicle (closer to a neutral behaviour).
- a_y^* is the lateral acceleration limit of the linear region.
- $a_{y,MAX}$ is the maximum achievable lateral acceleration.

The maximum lateral acceleration $a_{y,MAX}$ depends on the estimated available tyre-road friction coefficient, μ , and on the longitudinal acceleration, a_x .⁴ Practically, $a_{y,MAX}$ can be computed either from a g-g diagram considering semi-axes $\approx \mu g$ (where g is the gravitational acceleration) or through the friction ellipse approach (Genta, 1997) based on the knowledge of a_x and the maximum achievable force obtained through a tyre model (which needs a reasonable estimate of μ), as discussed in Mangia et al. (2021). It is worth to mention that $a_{y,MAX}$ is typically limited to $\approx 9 - 10 \text{ m/s}^2$ in normal conditions (dry asphalt), but it might be larger in specific cases, e.g. high quality tarmac, special tyres, etc.

⁴ The effect of a_x on the cornering response can be seen in De Novellis et al. (2015a), Figs. 6 and 7.

An intuitive explanation of the possibility to increase the maximum lateral acceleration is given as follows. By considering again the equilibrium equations of the single-track model 11 and 12, this time accounting for the presence of a direct yaw moment M_z :

$$F_{y1} + F_{y2} = ma_y \quad (29)$$

$$F_{y1}a_1 - F_{y2}a_2 + M_z = J_z \dot{r} \quad (30)$$

By solving for F_{y1} and F_{y2} in steady-state conditions:

$$F_{y1} = \frac{ma_y a_2}{l} - \frac{M_z}{l} \quad (31)$$

$$F_{y2} = \frac{ma_y a_1}{l} + \frac{M_z}{l} \quad (32)$$

As discussed earlier, all vehicles are normally understeering vehicles, thus the front axle tends to saturate before the rear axle does. If the yaw moment M_z has the same sign as the yaw rate, which is denoted as “destabilising yaw moment” (for example, if a_y is positive, i.e. the vehicle turns left, and M_z is positive), i.e. counterclockwise, then according to 31 and 32 the front axle lateral force will be less than for the baseline vehicle (for which $M_z = 0$), while the rear axle will experience a larger force. This way it is possible to exploit more of the rear axle force availability, delaying the saturation of the front axle. Of course, M_z should not be excessive. Note that the effect of the combined (longitudinal-lateral) interaction should also be considered, since M_z is obtained through longitudinal forces.

The reference yaw rate, r_{ref} , can be obtained as an offline look-up table based on the following procedure:

- for given values of a_y , a_x , μ and V , calculate $\delta_{sw,dyn}$ by inverting 28, i.e.

$$\delta_{sw,dyn} = \begin{cases} K_{us}a_y & \text{if } a_y \leq a_y^* \\ K_{us}a_y - K_{us}(a_{y,MAX} - a_y^*) \ln \frac{(a_{y,MAX} - a_y)}{(a_{y,MAX} - a_y^*)} & \text{if } a_y > a_y^* \end{cases} \quad (33)$$

- compute $\delta_{sw,kin}$ based on l , $R \approx V^2/a_y$ and the steering map of the vehicle
- compute $\delta_{sw} = \delta_{sw,kin} + \delta_{sw,dyn}$
- associate the above values of δ_{sw} , V , a_x to the steady-state relationship $r_{ref} = a_y/V$

Finally, the obtained dataset is inverted so as to provide the reference yaw rate as a function of measurable parameters, i.e. steering wheel angle, vehicle speed, longitudinal acceleration. An example is shown in Fig. 5. In practical applications, a first order lag is applied, as per 27.

The idea of designing the whole vehicle cornering response can be further extended to the design of not just one, but multiple understeer characteristics, denoted

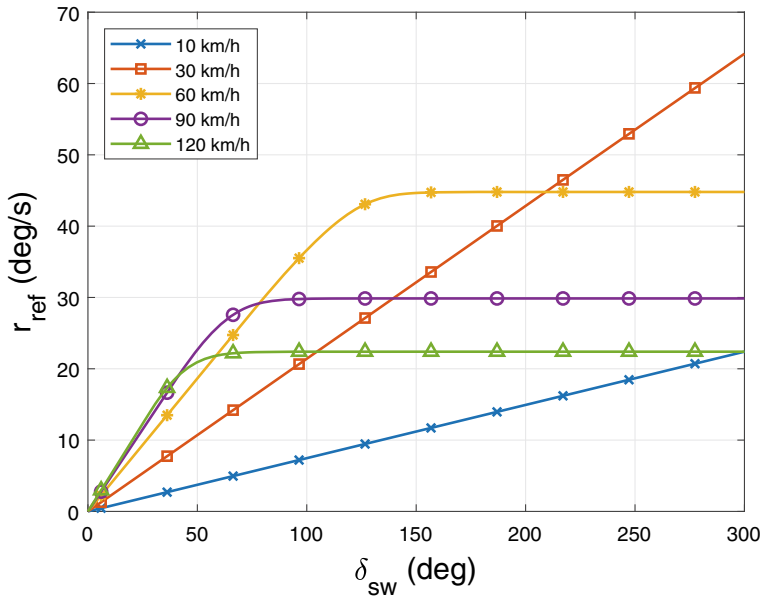


Fig. 5 Example of the reference yaw rate map as a function of the steering wheel angle for different values of vehicle speed, $a_x = 0$ (from Mangia et al., 2021)

as driving modes (De Novellis et al., 2015a). The driver may select a specific driving mode to adapt the vehicle behaviour according to specific driving conditions and/or driver needs or preferences, e.g. improving the fun-to-drive, adapting to low friction conditions, etc.

3.3 Concurrent Yaw Rate and Sideslip Angle Control

As discussed, the vehicle sideslip angle may also be taken into account, along with the vehicle yaw rate, in the reference generator. The main rationale for controlling the sideslip angle is related to vehicle stability:

- Yaw rate control normally requires a good estimate of the available friction coefficient (see 26 and 33). When the estimate is inaccurate,⁵ r_{ref} may not be compatible with the available friction conditions, thus the vehicle may end up in safety-critical conditions, such as drifting or incipient loss of vehicle control, e.g. spin.
- The estimation of the available tyre-road friction coefficient is a very challenging research problem (Guo et al., 2018). However, the estimation of β is in general easier (Cheli et al., 2007; Chindamo et al., 2018).

⁵ For instance, the yaw rate controller is designed for dry conditions, $\mu \approx 0.8 - 1$, but the road is wet. Or, even in dry conditions, μ happens to be overestimated.

- β is rather small in standard driving conditions, and it tends to assume relatively large values in safety-critical conditions. Hence, ensuring that β is limited through sideslip angle control is an effective way for enhancing vehicle stability.

Before describing some of the available solutions, an important concept should be pointed out and explained in detail. While it would be desirable to independently control both yaw rate and sideslip angle with a suitable yaw moment, actually that is not possible. The single-track vehicle model presented earlier (Eqs. 29, 30, 9, 10, 5, 6) can be rewritten, using $a_y = \dot{v} + ur \approx \dot{\beta}u + ur \approx \dot{\beta}V + Vr$, in the following state-space representation with state $x = [\beta \ r]^T$:

$$\dot{x} = Ax + B_i M_z + B_d \delta \quad (34)$$

where the dynamic matrix is

$$A = \begin{bmatrix} \frac{Y_\beta}{mV} & \frac{Y_r}{mV} - 1 \\ \frac{N_\beta}{J_z} & \frac{N_r}{J_z} \end{bmatrix} \quad (35)$$

the input matrix is

$$B_i = \begin{bmatrix} 0 \\ \frac{1}{J_z} \end{bmatrix} \quad (36)$$

the disturbance input matrix is

$$B_d = \begin{bmatrix} \frac{C_1}{mV} \\ \frac{C_1 a_1}{J_z} \end{bmatrix} \quad (37)$$

and the stability derivatives Y_β , Y_r , N_β and N_r are (Genta, 1997):

$$\begin{aligned} Y_\beta &= -(C_1 + C_2) \\ Y_r &= -\frac{(a_1 C_1 - a_2 C_2)}{V} \\ N_\beta &= -(a_1 C_1 - a_2 C_2) \\ N_r &= -\frac{(a_1^2 C_1 + a_2^2 C_2)}{V} \end{aligned} \quad (38)$$

From a control perspective, in 34 the yaw moment is the only input, while there is no control on the steering angle which is managed by the driver, so δ is for-

mally a disturbance. The system can be shown to be controllable⁶—more precisely, state-controllable. However, as discussed in Skogestad and Postlethwaite (2007), this property should better be denoted as “pointwise controllability” in the sense that although the system can possibly reach any desired configuration $[\beta_{ref} \ r_{ref}]^T$, there is no guarantee that the system can hold such state values for a certain amount of time (from a very small amount of time to potentially ∞). To be able to control β and r independently, the system needs to be “functionally controllable” (Skogestad & Postlethwaite, 2007) which here reduces to requiring at least two control inputs. This point is also raised by Kaiser (2015) stating that, to control both yaw rate and sideslip angle, the yaw moment is not sufficient on its own and an additional control variable is needed, such as an active steering system.

With this in mind, many authors have proposed interesting techniques to design M_z by appropriately mediating between the two control targets while guaranteeing safety and stability for the vehicle. A standard approach is to define a reference sideslip angle, β_{ref} , and integrate it together with the reference yaw rate in the high level controller, as discussed in Sect. 4.

Different ways have been proposed to define β_{ref} . For instance, Siampis et al. (2016) choose β_{ref} as the sideslip angle obtained by a single-track model in steady-state conditions:

$$\beta_{ref} = \frac{C_2 l a_2 - a_1 m V^2}{C_2 l (l + K_{us} V^2)} \delta \quad (39)$$

which can be obtained from the equilibrium equations 11 and 12 by inserting 9, 10, 5, 6, assuming $\dot{r} = \dot{v} = 0$, then solving one of the equilibrium equations for r and substituting into the other equilibrium equation, and finally solving for $\beta(\delta)$. Note that the dependency on C_1 is embedded in K_{us} . An explicit version, without the use of K_{us} , is presented in Rajamani (2011).

As discussed above, the main rationale for controlling β is to ensure it does not exceed some safety-critical threshold, to ensure vehicle stability. For this reason, some authors simply choose $\beta_{ref} = 0$, however that does not necessarily ensure a maximum safety margin: β is the sideslip angle at the vehicle centre of mass, where even in case of slip-free conditions there is a non-zero kinematic contribution $\beta_{kin} \approx \frac{a_2}{l} \delta$ (Lenzo et al., 2017b).

On the same wavelength—i.e., bounding β —is the β_{ref} formula proposed in Vignati et al. (2016), which bounds the sideslip angle within $\pm \beta_{th}$, where β_{th} is a threshold representing the maximum acceptable magnitude of β :

$$\beta_{ref} = \beta_{th} \tanh\left(\frac{\beta}{\beta_{th}}\right) \quad (40)$$

where β is the current value of sideslip angle.

⁶ Unless $mV^2 + C_1 a_1 - C_2 a_2 = 0$.

Since, as discussed, controlling r and β at the same time with M_z is challenging, it is clear that imposing both references at the same time implies that, in general, neither of them will be followed exactly—the relative priority may be defined, e.g., by appropriately weighting a yaw rate-based yaw moment contribution and a sideslip angle-based yaw moment contribution, or through the elements of matrix Q in the LQR cost function 56. Hence, even in standard driving conditions with small β , the yaw rate tracking performance (handling requirement) would be compromised without significant stability benefits. It would then seem reasonable to develop a control that accounts for β only when its magnitude is of concern, i.e., beyond a certain threshold. As a result, the handling requirement would not be sacrificed unless stability becomes a priority. This idea may be exploited in two different ways, as detailed below.

In Tota et al. (2018),

$$\beta_{ref} = \begin{cases} \beta & \text{if } |\beta| < \beta_{th} \\ \beta_{th} \text{sgn}(\beta) & \text{if } |\beta| \geq \beta_{th} \end{cases} \quad (41)$$

The interpretation of 41 is that, when the value of β is not of concern, the reference sideslip angle is imposed as the same as the current sideslip angle, i.e. no sideslip-related control action is required because there is purposely no difference between β_{ref} and β . So, the yaw moment is only used to track r_{ref} . Another relevant aspect of this approach is that when $\beta_{ref} \neq \beta$, i.e. an intervention on the sideslip angle is required, the reference yaw rate is corrected with a contribution proportional to the yaw rate variation provoked by the sideslip-related yaw moment contribution.⁷

The solution presented in Lenzo et al. (2021), instead, does not need to define a β_{ref} , yet it proposes a correction of the reference yaw rate based on the actual value of sideslip angle:

$$r_{ref} = r_h - F(r_h - r_s) \quad (42)$$

where r_h is the reference yaw rate calculated on the basis on handling requirements, e.g. through 25 or the procedure described in Sect. 3.2. The stability yaw rate, r_s , represents an achievable yaw rate in the current tyre-road friction conditions:

$$r_s = \begin{cases} r_h & \text{if } |r_h| < \left(\frac{a_y - \text{sgn}(a_y) \Delta a_y}{V} \right) \\ \left| \frac{a_y - \text{sgn}(a_y) \Delta a_y}{V} \right| \text{sgn}(r_h) & \text{if } |r_h| \geq \left(\frac{a_y - \text{sgn}(a_y) \Delta a_y}{V} \right) \end{cases} \quad (43)$$

The factor F is defined as:

$$F = \begin{cases} 0 & \text{if } |\beta| < \beta_{act} \\ k_1 \frac{|\beta| - \beta_{act}}{\beta_{th} - \beta_{act}} & \text{if } \beta_{act} \leq |\beta| \leq \beta_{th} \\ k_2 & \text{if } |\beta| > \beta_{th} \end{cases} \quad (44)$$

⁷ See Table 1 in Tota et al. (2018).

and k_1 , k_2 and Δa_y are positive tuning parameters. The parameters β_{act} and β_{th} represent respectively an activation threshold and, again, an upper threshold. If $|\beta| < \beta_{act}$, there is no sideslip-based correction in 42.

The key aspect of this solution is that the reference generator produces a single output, r_{ref} , and it accounts for β -related requirements, too. This benefits the design of the high level controller, which is not required to mediate between conflicting targets. Although a mediation between yaw rate and sideslip angle requirements is inevitable (because the system is functionally uncontrollable), here such mediation is achieved at the level of the reference generator, allowing for a more “physical” design approach. It is also worth to mention that a similar control principle was adopted in Zanchetta et al. (2019) for the concurrent control of yaw rate and hitch angle for an articulated vehicle (a car towing a trailer).

Finally, Gasmi et al. (2019) present an interesting review on how to possibly choose β_{th} —some authors propose a formula depending on μ (Rajamani, 2011), others propose a formula depending on V (Kiencke & Nielsen, 2000). A sensitivity analysis on β_{act} and β_{th} is also provided in Lenzo et al. (2021).

4 High Level Controller: T_{tot} , M_z

This section briefly describes methods to determine T_{tot} , then it extensively discusses how to elaborate M_z . The final part of this section presents experimental results obtained on a vehicle prototype within the European Project iCOMPOSE (Lenzo et al., 2019b) and further practical remarks.

A general aspect relevant for both T_{tot} and M_z is the understanding of feedforward and feedback controllers. Given a reference value for the generic variable of interest (e.g. the vehicle speed in case of T_{tot} , or the yaw rate in case of M_z):

- A *feedforward control action* is calculated based on a mathematical model of the system to be controlled, regardless of the actual value of the variable of interest.
- A *feedback control action* is calculated based on the error, i.e. the difference between reference value and actual value of the variable of interest.

Often a combination of feedforward and feedback actions is used, to blend their benefits. The reason is that the feedforward contribution helps to provide a fast response of the system (the control action is generated straightaway, hence no need to wait for the feedback action to build up⁸), while the feedback contribution allows to compensate for unmodeled effects and disturbances—both practically unavoidable—thus ensuring a precise tracking action. While feedforward controllers are rarely used alone, it is not uncommon to use feedback controllers only. For a much more complete discussion on this topic, the interested reader is referred to Isermann (2012) and De Novellis et al. (2015b).

⁸ For instance, e.g., Manning and Crolla (2007) suggest that the feedforward contribution “tends to reduce the time lag between steer angle input and vehicle reaction”.

4.1 Calculation of T_{tot}

Traditionally, T_{tot} is calculated (Fig. 2) based on accelerator and brake pedal positions (driver input) and vehicle speed (vehicle state). First, the desired total traction/braking force for the vehicle, F_{tot} , can be computed. Then, the desired total torque can be calculated based on F_{tot} , the wheel dynamics (depending on the wheel angular acceleration and on the wheel moment of inertia) and the wheel radius (which in general is not constant, Vantsevich et al., 2005, 2017). In a simplified scenario, $T_{tot} = F_{tot}R_w$, where the wheel dynamics is neglected and R_w is the average wheel radius, assumed constant.

To compute F_{tot} , the accelerator and brake pedal positions can be interpreted to define a value of desired acceleration, a_{des} . Then, an option is to calculate F_{tot} based on a simplified vehicle equilibrium equation:

$$F_{tot} - F_r = ma_{des} \quad (45)$$

which essentially states that F_{tot} needs to generate a_{des} while: (i) overcoming the resistance contributions, F_r , during traction; (ii) accounting for the favourable action of F_r , during braking. F_r includes contributions such as aerodynamic drag and rolling resistance, which can be modelled rather easily with reasonable approximation (Genta, 1997).

Alternatively, a more precise characterisation of F_r can be obtained with coast down tests (Preda et al., 2010; White & Korst, 1972), and it can be embedded in a generic drivability map that produces F_{tot} based on vehicle speed and accelerator/brake pedal positions, as in Lenzo et al. (2017b).

Clearly, both 45 and the drivability map are feedforward approaches. However in some cases, e.g. when performing standardised manoeuvres or simply during highway driving, reaching and maintaining a certain speed may be necessary. That can be easily achieved through a feedback approach, e.g. a PID (Proportional-Integral-Derivative) controller based on the difference between reference and current value of vehicle speed.⁹

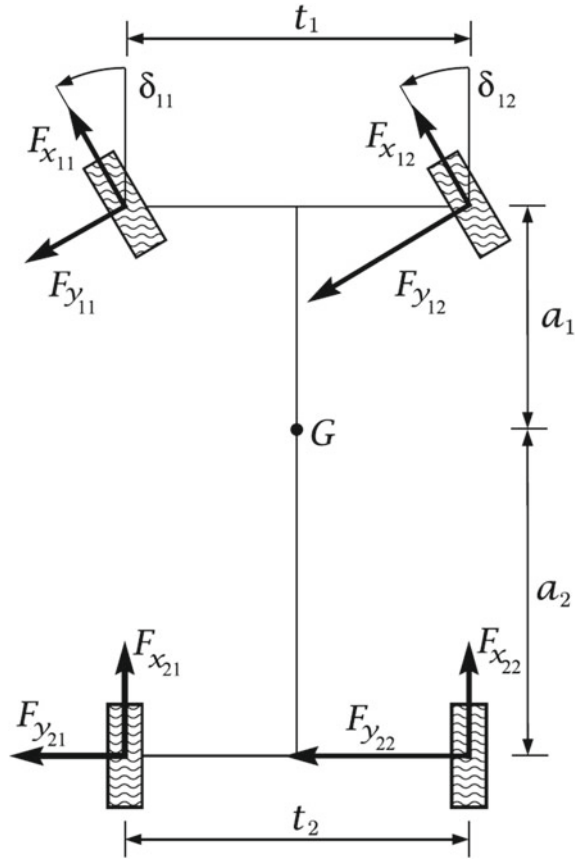
4.2 Calculation of M_z

To calculate M_z it is customary to use either a feedback controller or a combination of a feedforward and a feedback controller. To understand the practical implications of each contribution, it is useful to look at the equilibrium equations of a classic double-track vehicle model (Fig. 6):

$$ma_x = F_{x11} \cos \delta_{11} + F_{x12} \cos \delta_{12} + F_{x21} + F_{x22} - F_{y11} \sin \delta_{11} - F_{y12} \sin \delta_{12} - F_r \quad (46)$$

⁹ In other words, a simple “cruise control” logic.

Fig. 6 Double track vehicle model (adapted from Guiggiani, 2018)



$$ma_y = F_{y_{11}} \cos \delta_{11} + F_{y_{12}} \cos \delta_{12} + F_{y_{21}} + F_{y_{22}} + F_{x_{11}} \sin \delta_{11} + F_{x_{12}} \sin \delta_{12} \quad (47)$$

$$\begin{aligned} J_z \dot{r} = & (F_{y_{11}} \cos \delta_{11} + F_{y_{12}} \cos \delta_{12} + F_{x_{11}} \sin \delta_{11} + F_{x_{12}} \sin \delta_{12}) a_1 + \\ & -(F_{y_{21}} + F_{y_{22}}) a_2 + (F_{x_{12}} \cos \delta_{12} - F_{x_{11}} \cos \delta_{11}) \frac{t_1}{2} + \\ & +(F_{x_{22}} - F_{x_{21}}) \frac{t_2}{2} + (F_{y_{11}} \sin \delta_{11} - F_{y_{12}} \sin \delta_{12}) \frac{t_1}{2} \end{aligned} \quad (48)$$

where $F_{x_{ij}}$ and $F_{y_{ij}}$ are, respectively, the longitudinal and lateral tyre-road forces for wheel ij , t_i the vehicle tracks, δ_{1j} the wheel steer angles, $i = 1, 2$ respectively for front and rear axle, and $j = 1, 2$ respectively for left and right side.

The yaw equilibrium equation, 48, can be rewritten as:

$$J_z \dot{r} = N_f + N_y + N_d \quad (49)$$

where:

$$N_f = F_{x11} \sin \delta_{11} a_1 + F_{x12} \sin \delta_{12} a_1 \quad (50)$$

$$N_y = (F_{y11} \cos \delta_{11} + F_{y12} \cos \delta_{12}) a_1 - (F_{y21} + F_{y22}) a_2 + (F_{y11} \sin \delta_{11} - F_{y12} \sin \delta_{12}) \frac{t_1}{2} \quad (51)$$

$$N_d = (F_{x12} \cos \delta_{12} - F_{x11} \cos \delta_{11}) \frac{t_1}{2} + (F_{x22} - F_{x21}) \frac{t_2}{2} \quad (52)$$

Based on the idea of torque vectoring presented at the beginning of this Chapter, the variables that can actually be controlled to generate a yaw moment are the longitudinal forces, F_{xij} . So, $M_z = N_f + N_d$, where $N_d \gg N_f$ because in normal driving conditions $\delta_{ij} \ll 1$.

On the other hand, it is quite difficult to model the behaviour of N_y because there is no direct control on the lateral forces, F_{yij} . These depend on the tyre behaviour, that is challenging to characterise accurately, especially in real time (Acosta & Kanarachos, 2018; Guo et al., 2018). Furthermore, it should be noted that M_z itself influences the lateral forces (e.g. see 31, 32) and, as a consequence, the vertical loads (see paragraph 2.4 in Frendo et al. (2007)) which in turn affect the lateral forces. These aspects further justify why a feedforward control alone is not a suggested option for direct yaw moment control, yet it should be used together with a feedback contribution.

With that in mind, it is still of interest to derive an expression for a feedforward yaw moment, $M_{z,FF}$. For simplicity, here this is done for a single-track vehicle model with linear tyres.¹⁰ Starting from the equilibrium equations 29 and 30, considering $\beta \ll 1$ and quasi-steady-state conditions ($\dot{\beta} = \dot{r} = 0$, but it might be $\dot{V} \neq 0$):

$$F_{y1} + F_{y2} = ma_y = m(\dot{v} + ur) \approx m(\dot{V}\beta + V\dot{\beta} + Vr) = m(\dot{V}\beta + Vr) \quad (53)$$

$$F_{y1} a_1 - F_{y2} a_2 + M_{z,FF} = J_z \dot{r} = 0 \quad (54)$$

By using the congruence equations 1 and 2, as well as the constitutive equations 9 and 10, setting $r = r_{ref}$ and rearranging for $M_{z,FF}$:

¹⁰ In principle the double-track model above could also be used, with potentially any tyre model—however in case of strong nonlinearities and/or if the tyre model accounts for combined interactions, for computational reasons it would be more appropriate to derive a $M_{z,FF}$ map, as in De Novellis et al. (2013).

$$M_{z,FF} = -\frac{(-C_1 a_1 + C_2 a_2)(C_1 \delta V - r_{ref}(C_1 a_1 - C_2 a_2 + mV^2))}{V(m\dot{V} + C_1 + C_2)} + \frac{C_1(a_1^2 + a_2^2)}{V} r_{ref} - C_1 a_1 \delta \quad (55)$$

It is worth to remark that this value of $M_{z,FF}$ would bring the reference yaw rate to r_{ref} (with no need of feedback controller) if, ideally, the system behaves exactly according to the equations used above. Finally, because it was obtained in quasi-steady-state conditions, the above expression for $M_{z,FF}$ might provoke undesired yaw rate oscillations and overshoots in extreme transients. Such issue can be addressed by adding a dynamic feedforward contribution that modifies the transient response of the system, which can be designed in the Laplace domain as shown in De Novellis et al. (2015b).

For the feedback contribution, a widely used approach is the implementation of a PID controller (Astrom & Murray, 2010) based on the error between reference and actual value of the controlled variable, e.g. the yaw rate. Practically, when an integral contribution is present, suitable reset conditions—e.g. depending on a_y (or the error) being within a predetermined threshold or not—and an anti-windup correction should be included (De Novellis et al., 2015b). The PID parameters may be tuned in many ways, including the Ziegler-Nichols method, the Linear Quadratic Regulator (LQR) (He et al., 2000) etc. In the LQR case, the idea is to minimise the following performance index:

$$J = \int_0^\infty (x^T Q x + M_z R M_z) dt \quad (56)$$

which needs a linear model of the system rearranged in the form

$$\dot{x} = Ax + B_i M_z + U \quad (57)$$

where x is a vector containing the error and its derivative/integral as appropriate (e.g., the error and its integral if a PI scheme is used), A is the dynamic matrix, B_i is the input matrix, and U is everything else (e.g. steering angle contribution as in 34, modelled disturbances). For example, Vignati et al. (2016) use a single-track vehicle model with linear tyres and a Proportional-only controller for yaw rate and sideslip angle, for which $x = [\beta_{ref} - \beta \quad r_{ref} - r]^T$ applies. The matrices $Q \geq 0$ and $R > 0$ penalise excessive error and excessive control action, respectively. The optimal control action, M_z^{opt} , can be computed as¹¹:

$$M_z^{opt} = -R^{-1} B_i^T P x \quad (58)$$

¹¹ The interested reader might find interesting to know how to obtain this result from, e.g., Stanford-University (2020).

where $P = P^T \geq 0$ is the solution of the following Algebraic Riccati Equation (ARE):^{12, 13}

$$A^T P + PA - PB_i R^{-1} B_i^T P + Q = 0 \quad (59)$$

Further interesting approaches for the feedback controller are H- ∞ control (Lu et al., 2016), Sliding Mode Control (SMC) and its variants such as Second Order Sliding Mode (SOSM), Integral Sliding Mode (ISMC) (Goggia et al., 2015; Chen et al., 2018), explicit or implicit Model Predictive Control (MPC) (Zhou & Liu, 2009; Guo et al., 2020). In Parra et al. (2018) M_z is calculated through a fuzzy logic approach based on sideslip angle error, yaw rate error and yaw rate error derivative. A comparison among four feedback control techniques, including PID and SOSM, is presented in De Novellis et al. (2014c).

It is also possible to combine multiple feedback action schemes. For instance Tota et al. (2018) combine a PI controller with an ISMC controller. The PI controller gains are obtained through a LQR according to Eqs. 56 and 57, with $x = [\beta_{ref} - \beta \quad r_{ref} - r \quad \eta]^T = [e_\beta \quad e_r \quad \eta]^T$, and $\dot{\eta} = e_r$. The ISMC (based on a suitable sliding variable¹⁴ accounting for both e_β and e_r) is used to estimate the system perturbation, so the nominal controller (PI) is continuously enhanced by the ISMC perturbation compensator.

4.3 Experimental Results and Further Remarks

This section present a selection of relevant experimental results obtained within the European Project iCOMPOSE (Lenzo et al., 2019b). The vehicle demonstrator was a Range Rover Evoque prototype equipped with four identical on-board electric powertrains (Figs. 7 and 8).

Based on the approach described in Sect. 3.2, Fig. 9 shows:

- The experimental cornering response of the vehicle obtained via skid-pad tests with the baseline configuration (no yaw moment) and with the controlled vehicle using a driving mode denoted as “Sport mode”—because it implies increased steering responsiveness (less understeer gradient than the baseline configuration). As per design, the region of linear vehicle operation is extended and the maximum lateral acceleration is increased.
- The yaw rate measured during step steer tests (100 km/h, 100°) for the baseline vehicle and the “Sport mode” controlled vehicle, highlighting that even if the

¹² Note that this is not a Differential Riccati Equation (DRE) because of the infinite horizon used in 56 (otherwise the right-hand-side of this equation would be $-\dot{P}$ instead of 0).

¹³ Because the single-track vehicle model assumes constant vehicle speed, a gain scheduling approach is often implemented.

¹⁴ The interested reader is also referred to Utkin and Shi (1996), Utkin et al. (2017).

controller is designed in quasi-steady-state conditions,¹⁵ it brings benefits in transients, too. Specifically, it is possible to observe a considerable reduction of the yaw rate oscillations and of the settling time.

Figure 10 depicts the measured sideslip angle during step steer manoeuvres in which the vehicle implemented the concurrent yaw rate and sideslip controller from Tota et al. (2018), based on PI+ISMC. Here, the sideslip thresholds are intentionally set to large values so as to achieve a controlled drift condition with the desired value of sideslip angle. Similar experimental results are discussed in Lenzo et al. (2021).

Before presenting results of the concurrent yaw rate and sideslip angle control (through Eqs. 42, 43, 44), it is interesting to note that the whole discussion presented in Sect. 3.3 implicitly assumed that the controller uses the classically defined sideslip angle, i.e. the angle between the vehicle longitudinal axis and the velocity vector at the centre of mass (CM) of the vehicle (this was also used in Sect. 3.1: $\beta = \arctan v/u$). Despite this is very common in the literature, it is also natural to wonder what the effect of a difference reference point would be on the controller. This was studied in Lenzo et al. (2017b), which tested the controller using, in 44, values of sideslip angle at different points of the vehicle specifically the front axle, the centre of mass and the rear axle. The PI+ISMC controller discussed above was used as high level controller.

The resulting sideslip angle at the centre of mass for a slalom manoeuvre on a slippery road is shown in Fig. 11, while Fig. 12 shows two relevant frames captured at the test facility. While the baseline vehicle (no yaw moment) produces large values

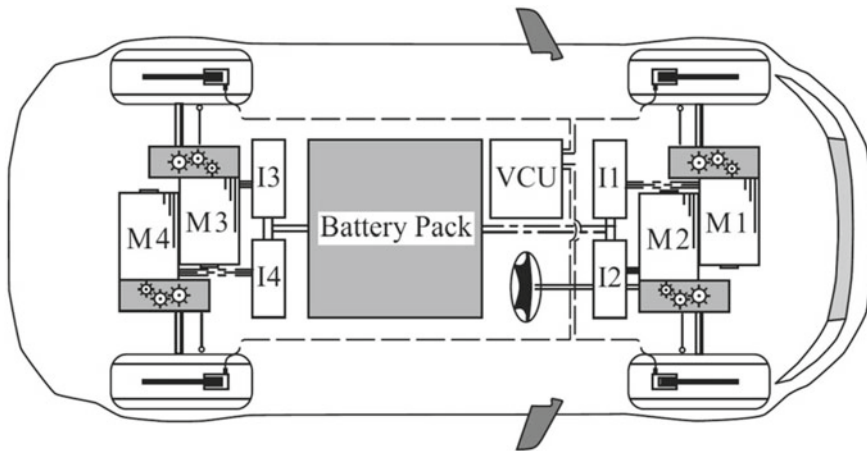


Fig. 7 Schematic of the architecture of the Range Rover Evoque demonstrator. M1-M4 = switched reluctance motors; I1-I4 = inverters; VCU = vehicle control unit (reproduced from Lenzo et al., 2019b)

¹⁵ The reference yaw rate is designed based on the steady-state relationship $r_{ref} = a_y/V$, but in general $a_x \neq 0$ is also allowed.



Fig. 8 The Range Rover Evoque during an obstacle avoidance manoeuvre (ISO 3888-2)

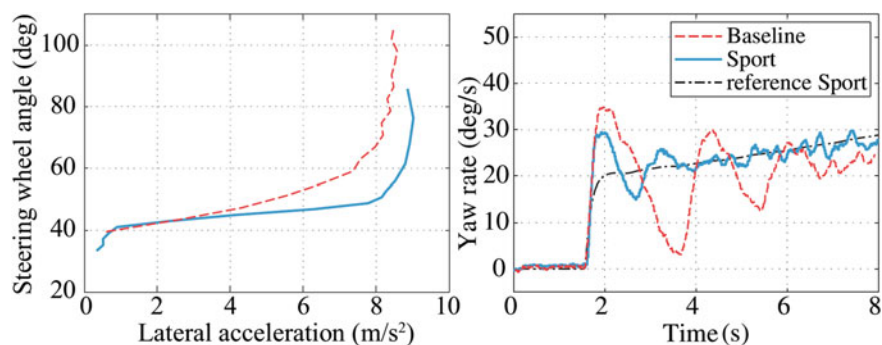


Fig. 9 Results of the design of the understeer characteristics: (left) experimental cornering response; (right) yaw rate response (and reference value) during step steer tests (reproduced from Gruber et al., 2016)

of β , the controlled vehicle based only on handling requirements for standard road conditions is even worse, eventually spinning shortly after 8 s (Fig. 11). On the other hand, when the sideslip angle is accounted for in the definition of the reference yaw rate, the controller provides a significantly smoother, more consistent and safer vehicle behaviour.¹⁶ Interestingly, Fig. 11 shows that when the sideslip angle at the

¹⁶ It should also be noted that compared to a conventional ESC, the torque vectoring controller interventions can be seamlessly and continuously generated, without necessarily decreasing the vehicle speed.

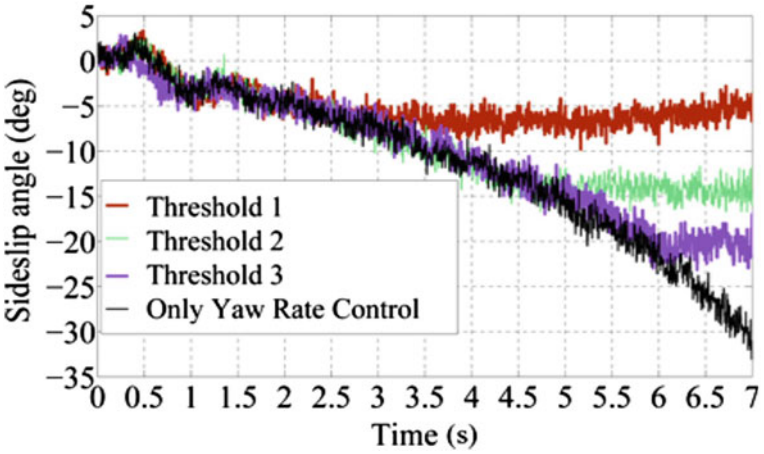


Fig. 10 Vehicle sideslip angle control during step steer tests (reproduced from Tota et al., 2018)

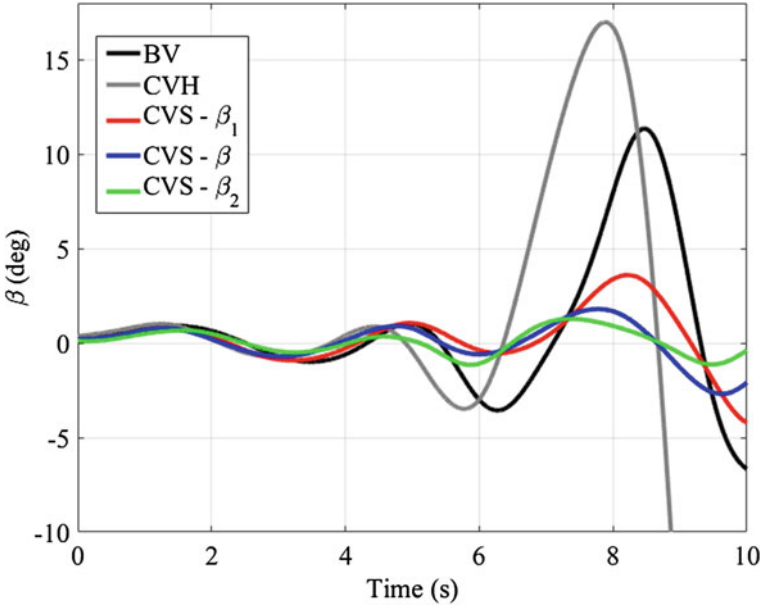


Fig. 11 Time history of the vehicle sideslip angle during a slalom on a slippery road, for different vehicle configurations: Baseline vehicle (BV), controlled vehicle solely based on the handling yaw rate (CVH), controlled vehicle with concurrent yaw rate and sideslip angle control, for three positions of the sideslip angle used in the controller formulation (CVS) (reproduced from Lenzo et al., 2017b)



Fig. 12 A slalom test on a slippery road: (left) without controller, the driver needs to countersteer to keep the vehicle on track; (right) with controller, providing a much smoother behaviour (reproduced from Lenzo et al., 2017b)

rear axle, β_2 , is used in 44, the sideslip angle remains within a tighter range than when using either β or β_1 .¹⁷ The reason of this effect is discussed hereinafter.

The sideslip angle at a generic point P, β_P , can be defined as the angle between the vehicle longitudinal axis and the velocity vector at P or, equivalently, the arc tangent of the ratio between lateral and longitudinal components of the velocity of P. These components can be easily computed through fundamental rigid body kinematic formulas such as Rivals theorem, as shown below. Figure 13 shows a schematic including velocities (V) and sideslip angles (β) at the vehicle front axle, centre of mass (CM), rear axle and a generic point P (note that in the x-y reference system chosen, x_P is positive and y_P is negative). Once known u and v , hence the sideslip angle at the centre of mass, $\beta = \arctan v/u$,¹⁸ then the sideslip angle at the generic point P is:

$$\beta_P = \arctan \frac{v + rx_P}{u - ry_P} \quad (60)$$

which reduces to 3 or 4 when P coincides respectively with the centre of the front axle or the rear axle. A further relevant point is that in general there are two contributions to sideslip angle: a kinematic contribution and a dynamic contribution. For the generic point P, it is

$$\beta_P = \beta_{P,kin} + \beta_{P,dyn} \quad (61)$$

¹⁷ Figure 11 plots β , but the same trend was found also for the other locations, as discussed in Lenzo et al. (2017b).

¹⁸ In iCOMPOSE a dedicated sideslip angle sensor was used, to ensure a reliable measurement of u and v in any condition, so as to be able to fully focus on the design of the controllers.

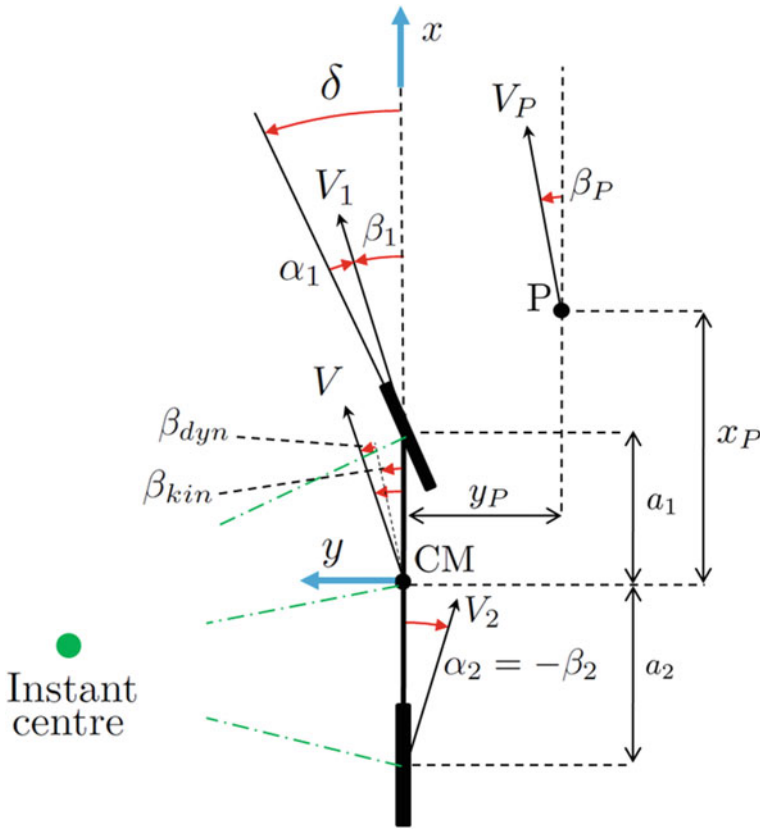


Fig. 13 Vehicle schematic showing relevant quantities at: front axle, centre of mass (CM), rear axle, generic point P

The kinematic contribution, $\beta_{P,kin}$, is by definition related to the vehicle kinematics, in particular to the steering angle and some geometrical quantities. The dynamic contribution, $\beta_{P,dyn}$, is instead related to slip angles. The kinematic and dynamic sideslip contribution at the centre of mass, respectively β_{kin} and β_{dyn} , are shown in Fig. 13. From a control design point of view, it would not make sense to try and correct a kinematic contribution because this is, by definition, an intrinsic property of the system.¹⁹ The ideal target of the control action is the dynamic contribution only, which is consistent with the idea of limiting tyre slips. Provided that there is no rear steering capability, the kinematic sideslip angle contributions at front axle, centre of mass and rear axle can be approximated respectively as δ , $\delta a_2/l$, 0.²⁰ As a

¹⁹ Another example would be a parking manoeuvre, which entails high steering angles, hence high kinematic sideslip angles, but low tyre slips, hence small dynamic sideslip angles.

²⁰ That is, the wheel steer angle divided by the vehicle wheelbase and multiplied by the distance between the rear axle and the point of interest.

result, $\beta_2 = \beta_{2,dyn}$, implying that controlling β_2 in 44 implies that only the dynamic sideslip angle contribution is corrected by the controller. This is also consistent with the fact that the rear axle is responsible for stability in cornering. This explains the results of Fig. 11.

Another important aspect of experimental testing (and simulation) is the objective assessment of performance of the developed controllers. To do so, it is customary to adopt appropriate performance indices. In general these can be defined depending on the specific application. Here, some of the most common performance indices are reported:

- the root mean square error (RMSE) of the magnitude of the yaw rate error, $RMSE_r$, which assesses the yaw rate tracking performance:

$$RMSE_r = \sqrt{\frac{1}{t_f - t_i} \int_{t_i}^{t_f} (r_{ref}(t) - r(t))^2 dt} \quad (62)$$

where t_i and t_f are, respectively, the initial and final time of the relevant part of the recorded dataset (e.g. initial and final time of a specific manoeuvre);

- the root mean square error of the magnitude of the sideslip angle error, $RMSE_\beta$, which assesses the sideslip angle tracking performance²¹:

$$RMSE_\beta = \sqrt{\frac{1}{t_f - t_i} \int_{t_i}^{t_f} (\beta_{ref}(t) - \beta(t))^2 dt} \quad (63)$$

- the normalised integral of the magnitude of the control action, $IACA$, which assesses the amount of yaw moment effort:

$$IACA = \frac{1}{t_f - t_i} \int_{t_i}^{t_f} |M_z(t)| dt \quad (64)$$

Further performance indices for the concurrent control of yaw rate and sideslip angle are discussed in Lenzo et al. (2017b).

5 Low Level Controller: T_{ij}

This section elaborates the mathematical relationships between the outputs of the high level controller (T_{tot} and M_z) and T_{ij} . Then, specific criteria on how to achieve an optimal distribution of the wheel torque are discussed. The final part of this section presents experimental results, obtained on the same vehicle prototype seen in Sect. 4.3, and further practical remarks.

²¹ Obviously this index makes sense only if β_{ref} is defined. As discussed, concurrent yaw rate and sideslip control is possible even without defining β_{ref} , by integrating the sideslip angle in the reference yaw rate formulation, as shown in 42.

5.1 Relationships Among T_{tot} , M_z and T_{ij}

The role of the low level controller is to determine T_{ij} that satisfy the following general expressions, which can be obtained through a basic free body diagram of the vehicle²² (Fig. 6):

$$\sum_{i,j} F_{ij} \cos \delta_{ij} = F_{tot} \quad (65)$$

$$\sum_{i,j} (-1)^j F_{xij} \frac{t_i}{2} \cos \delta_{ij} - \sum_{i,j} (-1)^i F_{xij} \frac{a_i}{2} \sin \delta_{ij} = M_z \quad (66)$$

By neglecting the wheel dynamics and assuming that the wheel radius is approximately the same for all wheels, $F_{xij} = T_{ij}/R_w$, 65 and 66 reduce to:

$$\sum_{i,j} T_{ij} \cos \delta_{ij} = T_{tot} \quad (67)$$

$$\sum_{i,j} (-1)^j \frac{T_{ij}}{R_w} \frac{t_i}{2} \cos \delta_{ij} - \sum_{i,j} (-1)^i \frac{T_{ij}}{R_w} \frac{a_i}{2} \sin \delta_{ij} = M_z \quad (68)$$

Finally, because in normal driving conditions $\delta_{ij} \ll 1$, 67 and 68 become:

$$\sum_{i,j} T_{ij} = T_{tot} \quad (69)$$

$$\sum_{i,j} (-1)^j \frac{T_{ij}}{R_w} \frac{t_i}{2} = M_z \quad (70)$$

For a vehicle with two motors (e.g. front left and front right), this is a system of two equations and two unknowns, which is completely determined. If the vehicle features more than two actuators, there are more unknowns than equations, meaning additional DOF can be conveniently exploited. For a typical vehicle configuration with four electric motors, 69 and 70 become²³

$$T_{11} + T_{12} + T_{21} + T_{22} = T_{tot} \quad (71)$$

$$- \left(\frac{T_{11}}{R_w} \frac{t_1}{2} + \frac{T_{21}}{R_w} \frac{t_2}{2} \right) + \left(\frac{T_{12}}{R_w} \frac{t_1}{2} + \frac{T_{22}}{R_w} \frac{t_2}{2} \right) = M_z \quad (72)$$

²² Note that lateral force contributions, F_{yij} do not appear in these equations because they cannot be controlled, as extensively discussed in Sect. 4.2.

²³ Note that 68 is equivalent to $N_d + N_f = M_z$, where N_d and N_f are given in 52 and 50.

By introducing the reasonable assumption that front and rear track widths are the same, $t_1 = t_2 = t$, 72 can be rewritten as:

$$-(T_{11} + T_{21}) + (T_{12} + T_{22}) = M_z \frac{2R_w}{t} \quad (73)$$

where it is now possible to isolate the torque demand for each side of the vehicle: $T_L = T_{11} + T_{21}$ for the left side, $T_R = T_{12} + T_{22}$ for the right side. So, 71 and 73 can be rewritten as:

$$T_L + T_R = T_{tot} \quad (74)$$

$$-T_L + T_R = M_z \frac{2R_w}{t} \quad (75)$$

Solving 74 and 75 for T_L and T_R :

$$T_L = \frac{T_{tot}}{2} - \frac{M_z R_w}{t} = \frac{T_{tot}}{2} - \Delta T_{RL} \quad (76)$$

$$T_R = \frac{T_{tot}}{2} + \frac{M_z R_w}{t} = \frac{T_{tot}}{2} + \Delta T_{RL} \quad (77)$$

where the torque bias between right and left side is defined as

$$\Delta T_{RL} = \frac{-T_L + T_R}{2} = \frac{M_z R_w}{t} \quad (78)$$

As a result, the desired total torque and yaw moment demand at vehicle level are equivalent to two independent values of torque demand, one per each side of the vehicle (76 and 77). Since the overall torque demand for each side is determined, the two available DOF can simply be seen as torque distribution factors between front and rear motor, one per each side. Hence, the four wheel torque demands, T_{ij} , can be written in the form:

$$\begin{cases} T_{11} = \left(\frac{T_{tot}}{2} - \Delta T_{RL} \right) \sigma_L \\ T_{21} = \left(\frac{T_{tot}}{2} - \Delta T_{RL} \right) (1 - \sigma_L) \\ T_{12} = \left(\frac{T_{tot}}{2} + \Delta T_{RL} \right) \sigma_R \\ T_{22} = \left(\frac{T_{tot}}{2} + \Delta T_{RL} \right) (1 - \sigma_R) \end{cases} \quad (79)$$

where σ_L and σ_R are front-to-total torque ratios, respectively for left and right vehicle side.

Based on the independence of the vehicle sides, the following section will discuss how to optimally choose the front-to-total torque ratio, σ , for a generic vehicle side.

5.2 Computation of σ

In principle, any optimisation criterion may be used in the low level controller to determine σ . Typically, energy efficiency criteria are adopted. It is then important to appreciate what the sources of power loss are and how to handle them.

The study Pennycott et al. (2015) extensively discusses the main sources of power loss for electric vehicles. While power losses due to aerodynamic drag and rolling resistance are of significance, they depend on parameters such as the vehicle speed, so they cannot be influenced by σ . Power losses caused by drivetrains (electric motors, inverters, transmissions if present) and tyre slips, instead, are affected by wheel torque distribution.

The tyre slip power losses can be divided in longitudinal slip power losses and lateral slip power losses, and their expressions are respectively

$$P_{loss,x,ij} = F_{xij} v_{slip,x,ij} \quad (80)$$

and

$$P_{loss,y,ij} = F_{yij} v_{slip,y,ij} \quad (81)$$

where $v_{slip,x,ij}$ and $v_{slip,y,ij}$ are, respectively, the longitudinal and the lateral slip speed at wheel ij . Some studies choose to focus on the minimisation of tyre slip power losses by selecting σ based on the estimated vertical load on each wheel, with the idea to allocate higher torque demands where there is more vertical load, i.e. more availability of grip. In doing so, many authors assume a linear relationship between available grip and vertical load (Parra et al., 2018), that is only an approximation and may be inaccurate (Guiggiani, 2018). In any case, tyre slip power losses are often less significant than drivetrain power losses, unless the vehicle experiences high lateral accelerations (Pennycott et al., 2015). Other studies have shown that in some cases, depending on the type of electric motor installed, optimising slip losses might be slightly better than optimising drivetrain power losses (De Novellis et al., 2013, 2014b). However, relying on tyre slips may not actually be viable, because tyre slips are not directly measurable and their estimation is still challenging (Guo et al., 2018). This has motivated many studies on methods to minimise the power losses occurring in the drivetrains.

Drivetrain power losses have shown to be fairly well represented by third order polynomial functions of speed and torque demand (Lin & Xu, 2015; Mahmoudi et al., 2015; Lenzo et al., 2017a). Specifically Mahmoudi et al. (2015) propose:

$$P_{loss,ij}(T_{ij}, \omega_{ij}) = \sum_{m,n} k_{mn} \left(\frac{T_{ij}}{T_0} \right)^m \left(\frac{\omega_{ij}}{\omega_0} \right)^n P_0 \quad m, n = 1, 2, 3 \quad (82)$$

where ω_{ij} is the angular speed of motor ij and T_0 , ω_0 and P_0 are normalising factors. Instead, Lenzo et al. (2017a) interpolate the drivetrain power losses with a cubic polynomial function of the sole torque demand:

$$P_{loss,ij}(T_{ij}, V) = a_{ij}(V)T_{ij}^3 + b_{ij}(V)T_{ij}^2 + c_{ij}(V)T_{ij} + d_{ij}(V) \quad (83)$$

with the coefficients a_{ij} , b_{ij} , c_{ij} and d_{ij} depending on the vehicle speed, V . The physical interpretation of d_{ij} is that it represents losses independent of T_{ij} , such as those due to rolling resistance.

The contributions Dizqah et al. (2016), Lenzo et al. (2017a) propose fast parametric torque allocation strategies—denoted as control allocation (CA) strategies—based on the experimental assessment of the drivetrain power losses. Both studies make the hypothesis (verified experimentally) that $P_{loss,ij}$ is a strictly monotonically increasing function of T_{ij} , and that $P_{loss,ij}$ has an inflection point, i.e. a transition between a concave shape and a convex shape, for a positive value of T_{ij} . Applying these requirements to 83 implies $a_{ij}, c_{ij}, d_{ij} > 0$, $b_{ij} < 0$, and $b_{ij}^2 < 3a_{ij}c_{ij}$.²⁴

In Dizqah et al. (2016) and in part of Lenzo et al. (2017a) a vehicle layout with four identical electric motors is considered (thus $a_{ij} = a$, $b_{ij} = b$, $c_{ij} = c$ and $d_{ij} = d$ in 83), and a further assumption is introduced, i.e., that the optimal wheel torque distribution is either:

- Single-Axle (SA): the whole side torque is allocated to a single wheel, i.e. either on the front wheel ($\sigma = 1$) or the rear wheel ($\sigma = 0$)—note that if the motors are identical, $\sigma = 0$ and $\sigma = 1$ yield the same power losses for a vehicle side
- Even-Distribution (ED): the side torque is evenly distributed between front and rear wheel ($\sigma = 0.5$)

On this basis, a “switching torque” $T_{sw}(V)$ is defined for each vehicle speed V as the side torque demand for which the SA and ED solutions are equivalent. Formally²⁵:

$$P_{loss,1j}(T_{sw}, V) + P_{loss,2j}(0, V) = P_{loss,1j}(T_{sw}/2, V) + P_{loss,2j}(T_{sw}/2, V) \quad (84)$$

To calculate $T_{sw}(V)$, Lenzo et al. (2017a) propose two options: Hybrid Control Allocation (H-CA), where $T_{sw}(V)$ is calculated by interpolation of the experimental power loss curves; Explicit Control Allocation (E-CA), where $T_{sw}(V)$ is obtained through the analytical form 83, which combined with 84 gives

²⁴ These can be easily obtained imposing that the first order derivative of $P_{loss,ij}$ with respect to T_{ij} must be positive, and that the second order derivative is zero for a positive value of T_{ij} .

²⁵ Note that the left-hand side can be either $P_{loss,1j}(T_{sw}, V) + P_{loss,2j}(0, V)$ or $P_{loss,1j}(0, V) + P_{loss,2j}(T_{sw}, V)$ since, as discussed, $\sigma = 0$ and $\sigma = 1$ yield the same power losses for a vehicle side.

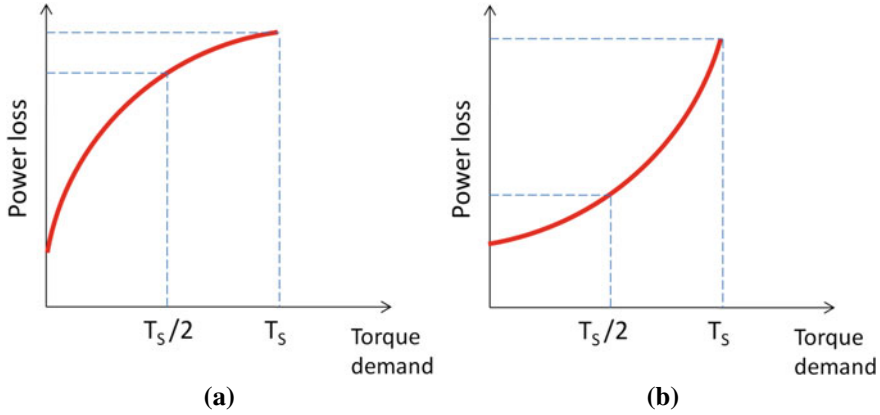


Fig. 14 Power loss functions of a single drivetrain for a given vehicle speed and side torque demand T_S . In case **a** $P_{loss}(T_S) < 2P_{loss}(T_S/2)$. In case **b** $P_{loss}(T_S) > 2P_{loss}(T_S/2)$

$$T_{sw}(V) = -\frac{2}{3} \frac{b}{a} \quad (85)$$

For both H-CA and E-CA the optimal solution is $\sigma = 1$ (or $\sigma = 0$ ²⁶) if the side torque demand is lower than $T_{sw}(V)$, and $\sigma = 0.5$ when the side torque demand is greater than $T_{sw}(V)$. To give a more intuitive explanation, it is convenient to analyse the two extreme cases shown in Fig. 14:

- if the concavity is always negative, the optimal solution is to distribute the torque on one wheel only, SA, $\sigma = 0$ or $\sigma = 1$, as evidenced in Fig. 14a—in this case $T_{sw}(V) = \infty$;
- if the concavity is always positive, the optimal solution is the even distribution of the torque between front and rear motor, ED, $\sigma = 0.5$, as evidenced in Fig. 14b—in this case $T_{sw}(V) = 0$.

As discussed earlier, this approach assumed a-priori that the optimal torque distribution strategy is only either SA or ED (provided that the motors are all the same). It would be natural to wonder whether values of σ other than 0, 0.5 and 1 might be better. This was studied in Lenzo et al. (2017a) which propose the Implicit Control Allocation (I-CA). The I-CA basically assigns a specific value of σ for any combination of side torque demand and vehicle speed.

In terms of practical implementation on the vehicle, H-CA, E-CA and I-CA can easily be run in real time on hardware with low computational processing power, with the switching torque values (for H-CA and E-CA) or the σ map (for I-CA) simply stored as a look-up table in the controller.

²⁶ While $\sigma = 0$ and $\sigma = 1$ are equivalent in terms of power losses, $\sigma = 1$ is preferred for safety reasons since, from a vehicle dynamics point of view, understeer is better than oversteer.

Finally, it is worth to mention that Lenzo et al. (2017a):

- propose algorithms for the optimal distribution of wheel torque in case motors are not all the same;
- show that longitudinal and lateral load transfers, respectively due to longitudinal and lateral acceleration, only affect the term d_{ij} of the power losses but not the optimal front-to-total torque ratio, assuming that the rolling resistance depends linearly on the vertical load.

5.3 An Alternative for M_z : The Energy Efficiency Mode

As thoroughly discussed, M_z is an output of the high level controller which, in turn, uses the reference generator signals that are based on a desired driving mode. In the majority of the literature, driving modes are designed around appropriate vehicle handling requirements, which is the main priority. As a lower priority, the criterion of energy efficiency is addressed in the low level controller.

Another possibility, less explored in the literature, is to consider energy efficiency as the only target: in fact, M_z could be chosen based on energy requirements, too. Some recent works look at the design of understeer characteristics with the aim of improving energy efficiency (Lenzo et al., 2016; De Filippis et al., 2018). In De Filippis et al. (2018) a rule-based approach is proposed considering losses due to motor, inverter, transmission and longitudinal slip, along with the hypothesis of 4 identical and independent motors characterised by power losses expressed with the cubic polynomial function 83. For example, for low-medium values of longitudinal force (specifically, if $T_{tot} < 9/5T_{sw}$), the drivetrain power losses are minimised by a yaw moment that allocates the whole torque demand on either side of the vehicle. Actually, to minimise tyre slip power losses, too, the overall torque demand should be on the external side of the vehicle.²⁷ Another interesting consequence of the analysis in De Filippis et al. (2018) is that the optimal M_z is zero if the switching torque is zero, i.e. if the drivetrains are characterised by a convex shape for the whole range of torque demand.

In light of this, Mangia et al. (2021) propose an integrated torque vectoring framework including an “Energy efficiency” mode inspired to De Filippis et al. (2018), along with traditional “handling” driving modes such as those described in Sect. 3.2:

- When an “handling” driving mode is selected, M_z is imposed by the reference generator and the high level controller, and the low level controller manages σ_L and σ_R based on energy efficiency criteria.
- When the Energy efficiency mode is selected, the reference generator is bypassed and M_z , σ_L and σ_R are selected with the only target of minimising energy consumption.

²⁷ It is worth to note that, hypothetically, assigning more than the overall torque demand on the external vehicle side would imply a negative (regenerative) torque on the inner side, which is far from optimal (Lenzo et al., 2017a).

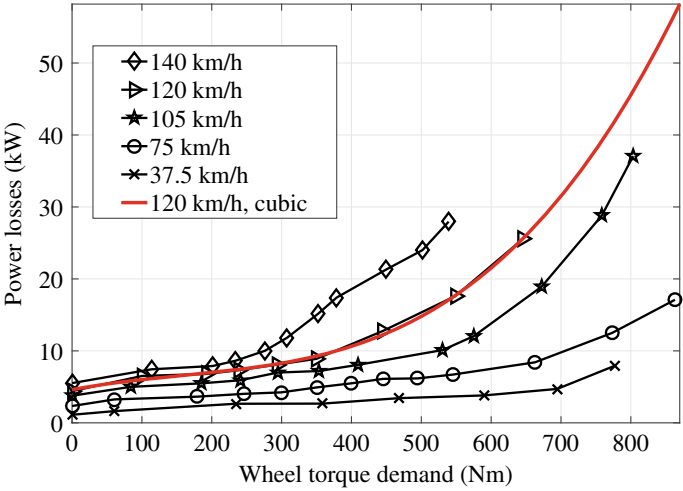


Fig. 15 Power losses of a single drivetrain for different vehicle speeds, and cubic interpolant for $V = 120$ km/h



Fig. 16 The experimental setup on the rolling road facility (reproduced from Lenzo et al., 2019b)

5.4 Experimental Results and Further Remarks

This section present a selection of relevant experimental results obtained within the European Project iCOMPOSE (Lenzo et al., 2019b), with the vehicle demonstrator seen in Sect. 4.3.

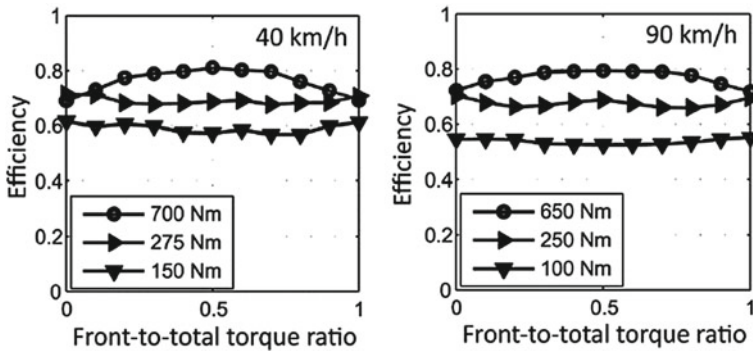


Fig. 17 Measured drivetrain efficiencies as a function of σ for different side torques and vehicle speeds of 40 and 90 km/h

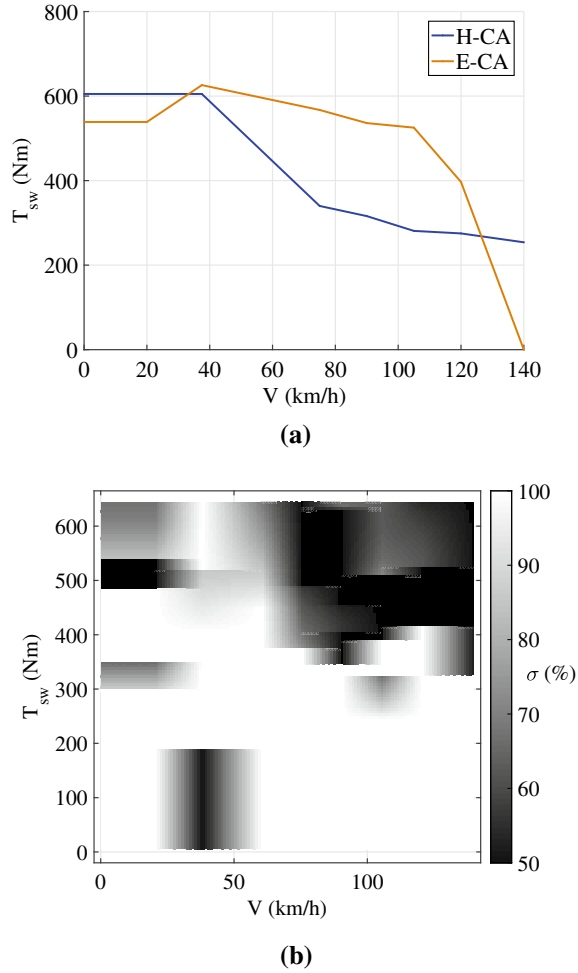
Figure 15 shows the power losses of a single drivetrain as a function of the torque demand for different vehicle speeds, obtained through extensive experimental tests on a MAHA rolling road facility (Fig. 16). To demonstrate the effectiveness of the function 83, Fig. 15 also shows the interpolating function for a sample vehicle speed.

Figure 17 shows the measured drivetrain efficiency (i.e., the ratio between mechanical power output, measured at the MAHA rollers, and electrical power input) of the drivetrain for two selected speeds and various torque demands: as discussed earlier, for low torque demands $\sigma = 0$ or $\sigma = 1$ are better than $\sigma = 0.5$, and vice-versa for sufficiently high torque demands. The switching torque, T_{sw} , is depicted in Fig. 18a which shows the results of the calculations via piecewise interpolation (H-CA) and via the analytical expression 85 (E-CA). Figure 18b shows the I-CA map: interestingly, the blue curve of Fig. 18a overlaps rather well with the transition zone between white and dark grey of Fig. 18b. This is not a surprise since the idea of the H-CA and E-CA strategy (i.e., use $\sigma = 1$ below T_{sw} and $\sigma = 0.5$ above T_{sw}) is an approximation of the I-CA.

The benefit of H-CA, E-CA and I-CA was assessed through the measurement of the energy input required for several driving cycles including, e.g., New European Driving Cycle (NEDC), Artemis Road, etc. Results²⁸ showed that generally H-CA, E-CA and I-CA outperform SA and ED by between 2 and 4%. While the I-CA is in principle better than H-CA and E-CA because it uses the whole range $0 \leq \sigma \leq 1$, experiments showed that not only the energy consumption improvements of the I-CA were hardly significant, but that practically the continuous variation of the torque distribution between front and rear axle provokes undesired vehicle vibrations which seriously hinder drivability and comfort. Between H-CA and E-CA, the former is recommended as it provides slightly better results than the E-CA, simply because of the more accurate approximation of the power losses and the switching torque. Additionally, despite these CA strategies are not explicitly designed for cornering

²⁸ Details are in Lenzo et al. (2017a).

Fig. 18 Elaboration of the power losses shown in Fig. 15: **a** switching torque using the H-CA and the E-CA; **b** σ map for the I-CA



conditions, skid-pad tests were executed for lateral accelerations between 2 and 8 m/s^2 , and the H-CA brought experimentally measured energy savings of up to 5% with respect to the SA and ED strategies.

Regarding the energy consumption benefits achieved through the design of the reference cornering response of the vehicle, several experimental skidpad tests were conducted. As a result, it was found that for the case-study vehicle the optimal cornering response in terms of energy efficiency is close to the neutral steering condition, and can reduce the measured inverter input power by even more than 10% (Fig. 19).

It should also be noted that the driving style has a large impact on power consumption, too: energy efficiency optimisation algorithms cannot be much effective if associated with an aggressive driving style.

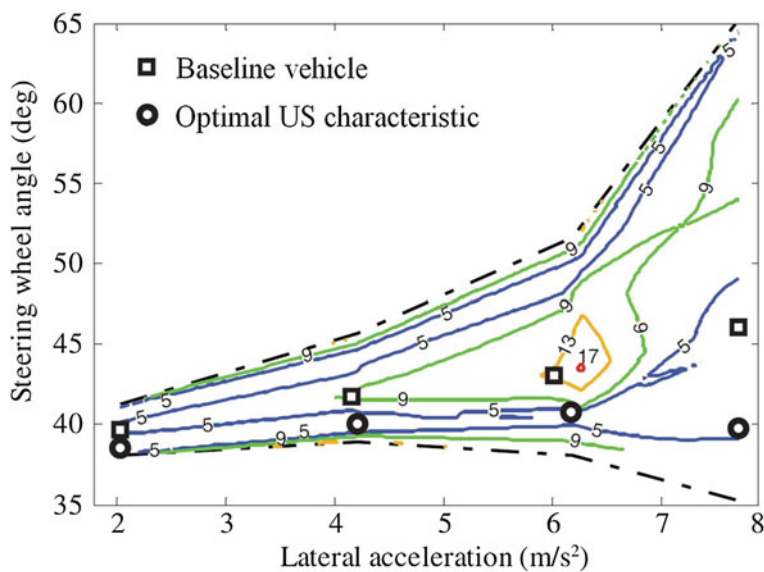


Fig. 19 Understeer diagram with indication of the relative power input increase (in percentage) with respect to the optimal understeer characteristic (reproduced from Lenzo et al., 2019b)

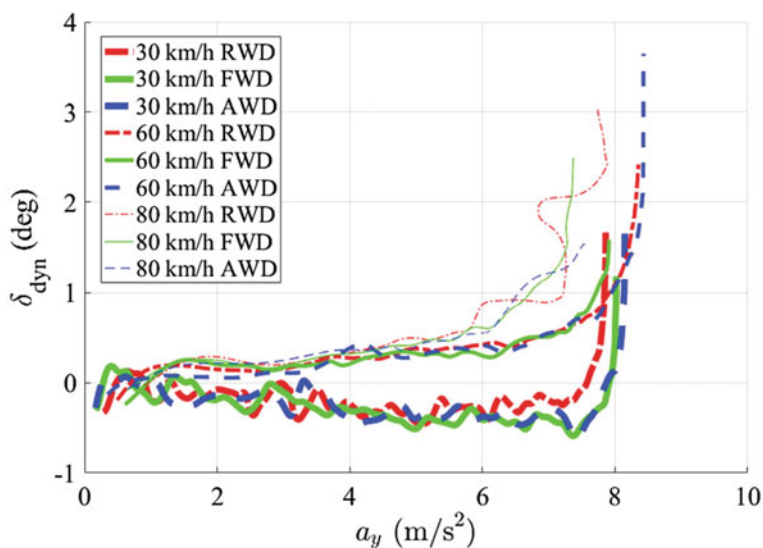


Fig. 20 Dynamic steering angle as a function of the lateral acceleration (reproduced from Lenzo et al., 2019b)

An important practical remark for onboard motor configurations is that potential fast modulations of the motor torque during transients might lead to drivability and possibly yaw stability problems. To solve this issue, De Novellis et al. (2015b) developed a specific anti-jerk control function, called Active Vibration Controller (AVC). Essentially its effect corresponds to the one of a virtual damper between the wheel to the electric drivetrain. As a consequence, the AVC is based on the difference (reported to the same shaft) between motor speed and wheel speed:

$$T_{ij,AVC} = T_{ij} - c_{AVC}(\dot{\omega}_{ij} - \dot{\omega}_{w,ij}/\tau) \quad (86)$$

where $T_{ij,AVC}$ is the torque demand actually sent to the wheel by the AVC, ω_{ij} is the angular speed of motor ij , $\omega_{w,ij}$ is the angular speed of wheel ij , c_{AVC} is a tuning factor, τ is the overall gear ratio.

A further interesting aspect of the vehicle demonstrator used in iCOMPOSE is that it allowed to emulate architectures such as Front Wheel Drive (FWD), Rear Wheel Drive (RWD) and All Wheel Drive (AWD) by simply changing the vehicle control settings. This inspired a study (Lenzo et al., 2019a) on the effect of the front-to-rear wheel torque distribution on the handling behaviour, with $M_z = 0$. In other words, it was possible to compare a FWD, RWD and AWD layouts with the very same equipment. Interestingly, the study showed that at low speed and high lateral acceleration, the RWD vehicle is more understeering than the FWD and the AWD ones (Fig. 20). This intuitively unexpected result was discussed and justified by a thorough analysis of the yaw moment contributions caused by the longitudinal forces of the front tyres (N_f in 50).

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State and Parameter Estimation for Vehicle Dynamics



Frank Naets

Abstract An accurate knowledge of the vehicle state and parameters is essential for current and future active vehicle applications. However, many of these quantities cannot be measured directly in commercial vehicles through the use of cost effective sensors. Estimation methods allow one to combine a (simple) model for the vehicle with available measurement data in order to assimilate a wide range of quantities of interests, ranging from slip states to tire parameters. In this chapter we will review various basic estimation algorithms, and discuss how they can lead to valuable estimators for vehicle dynamics applications by considering gradually increasing model complexity. This should provide the reader with the basic insights required to set up their own vehicle dynamics estimators.

1 Introduction

In this chapter we will cover the fundamentals and some extension for state-estimation in vehicle dynamic applications. State estimation approaches enable an assessment of a range of vehicle states and parameters during operation, with the need to measure them directly.

Vehicles are highly complex systems for which high performance requirements are ever more important to satisfy customer demands. In order to ensure these requirements are met, accurate data and information on the dynamic performance of the vehicle is required during:

- *Development/design*: In order to assess different performance metrics during the development and design of new vehicles, a range of measurements needs to be

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obtained. These measurement results can then be compared to a range of desired performance metrics. This data does not need to be available in real-time or online as it mainly serves for analysis by engineers and designers.

- *Operation*: During operation a range of measurements are performed on the vehicle for diagnostic (e.g. tyre punctures) and control (e.g. electronic stability program) purposes. These measurements need to be sufficiently robust and available at high rates (online or real-time) to ensure reliable operation of the vehicle.

From a vehicle dynamics perspective, several specific quantities of interest exist. On the one hand a range of kinematic information is relevant to assess and quantify the motion of the vehicle:

- global position of the vehicle;
- relative position with respect to other vehicles;
- longitudinal velocity and acceleration;
- lateral velocity and acceleration;
- vehicle slip angle;
- ...

These global kinematic quantities can be extended with detailed component motion information like suspension and wheel center motion, tire velocities, etc.

Besides these kinematic quantities, there is often an important focus on gaining insight in the dynamic quantities of the vehicle. Especially the forces acting in the tire-road contact patch play a crucial role in vehicle behavior and are key quantities for advanced vehicle control.

Besides the vehicle states, we are often also interested in gaining more insight in the actual vehicle parameters during operational conditions. Quantities like mass distribution, suspension stiffness, and equivalent tire stiffnesses provide key information about the vehicle. It is therefore highly valuable to have methodologies which can provide a quantified view on the evolution of these parameters, at a limited computational load.

The required accuracy and sampling rate for certain data is highly dependent on the purpose of the data: e.g. velocity information for anti-lock braking system (ABS) purposes versus navigation purposes. For the ABS system having the data available at sufficiently low noise levels, to avoid false triggering of the ABS system, and guaranteed high sampling rates is key to ensure safe operation of the vehicle. For the navigation system on the other hand, velocity information serves an informative purpose and small delays are perfectly acceptable from a driver perspective.

For all these quantities there is often an important lack of sensors which allow direct measurements. Or, the direct instrumentation might be prohibitively expensive for broader purposes besides some limited design validation testing. In these cases estimation and observer approaches offer a framework which allows for the indirect quantification of these variables based on the available sensor measurements and a model description of the system.

1.1 Sensors in Vehicles

Modern vehicles are typically equipped with a range of sensors which provide information on the operation of the vehicle. Several of the common sensors available in modern vehicles are:

- global positioning system (GPS) sensor
- Wheel speed transducer(s)
- Accelerometer(s): microelectromechanical system (MEMS) sensor which allow for low-frequency measurement of the vehicle longitudinal and lateral acceleration.
- Gyroscope(s): MEMS sensor which allow for at least yaw-rate measurement.

For automotive applications, several of these sensors are often combined in an inertial measurement unit (IMU), which also integrates some basic processing of the sensor data. It is important to highlight that the sensor modules for automotive applications are specifically designed for robustness in a range of operating conditions which vehicles typically encounter. This is in stark contrast to their laboratory grade counterparts which require a well controlled environment and intricate calibration. Also, from a cost perspective there is often strong push to limit the amount of sensors to the absolute minimum. Even though the absolute cost of many of these sensors seems limited, this does amount to a significant cost for the manufacturer due to the large production numbers in automotive.

An important issue is that none of the commonly available sensors provide direct information on many of the above mentioned quantities of interest. This even holds for seemingly trivial quantities like longitudinal velocity v^x :

- wheel speed can be inconsistent with vehicle velocity due to wheel slip;
- accelerometer data needs to be integrated to obtain velocities, which can lead to drift;
- GPS data needs to be differentiated with respect to time, which typically leads to high noise levels.

For high end test campaigns, dedicated sensorization can be set up. In these settings optical scanning measurement systems can be employed for accurate velocity measurements. Strain gauges or force wheels, as produced by Kistler, can be employed for accurate wheel force assessment. In practice these are very expensive systems which often require extensive setup and calibration (again leading to high usage costs), which make them inapplicable for regular operational conditions.

In recent years there has also been a trend toward sensors for collecting external information for autonomous driving purposes, like vision camera systems, radio detection and ranging (RADAR) systems, and laser imaging detection and ranging (LIDAR) systems. Currently there is still an open question how this sensor data can be exploited for dynamic vehicle purposes. Several researchers are investigating these possibilities (see Llorca et al., 2016; Kampelmühler et al., 2018), but these types of data are not further explored in this chapter.

1.2 Engineering Rules to Extract Quantities of Interest

Classically, simple and complex *engineering rules* (or *soft sensors*) have been employed to convert the measured quantities into (approximations of) the quantities of interest. A typical example is the evaluation of longitudinal vehicle velocity. As modern vehicles feature a rotational encoder on every wheel, we can decide on different rules to extract the actual vehicle velocity. A straightforward engineering rule for example extracts the longitudinal vehicle velocity v^x as the average of the corresponding no-slip velocities of the different wheels:

$$v^x = \frac{1}{4} (\omega^{fl} r^w + \omega^{fr} r^w + \omega^{rl} r^w + \omega^{rr} r^w), \quad (1)$$

where ω^{ij} represent the angular velocities of the front and rear ($i = f, r$), and left and right ($j = l, r$) wheels with their corresponding rolling radii r^{ij} . Very often these type of rules are still the most common approaches, but several open issues clearly exist. On the one hand these methodologies can struggle to account for varying operating conditions (large wheel slip, varying wheel diameters, ...). On the other hand it is not clear how to consistently set up rules to merge data coming from more distinct measurements. How could the rule from Eq. (1) be extended to also include accelerometer data? And how could this for example be leveraged to robustify the approach. Conceptually it is possible to set up engineering rules for all these cases, but this will involve large engineering and calibration effort. Estimators and observers provide a consistent framework to answer these questions.

1.3 Sensor Fusion

Many observer and estimator schemes practically accomplish sensor fusion (see Gustafsson, 2010) as they allow to combine information from multiple sensors. Sensor fusion generally enables more accurate and/or robust evaluation of the different quantities of interest. This is obtained by leveraging the redundant information present in the different sensors. The key in estimator and observers is the use of a *system model* in the fusion process to combine data coming from these different sensors. These models allow to couple the different sensor outputs to a common set of system states. Sensor fusion schemes are particularly interesting for automotive application as a range of sensor information is anyhow available on the CAN bus. Effective approaches should therefore utilize all data available for assessing a particular QOI.

1.4 Example: Longitudinal Vehicle Velocity

As an illustration of a typical state-estimation problem for vehicles, we consider the determination of longitudinal vehicle velocity. At first glance this seems like a trivial problem, as vehicle velocity is determined by rotational wheel speed sensors in common vehicle applications through the basic engineering rule:

$$\tilde{v}^x = r^w \omega^w \approx v^x. \quad (2)$$

Here the real longitudinal velocity v^x is approximated by the estimated longitudinal velocity $\tilde{v}^x$ using a perfect rolling assumption for the tire. However, several questions arise when using this strategy. First and foremost, the insight that under acceleration and braking significant wheel slip is present, poses an important limitation on the accuracy of this engineering rule. Secondly, also the equivalent wheel radius r^w can vary significantly and might have large uncertainty bounds. The resulting inaccuracy is high enough that for certain applications dedicated (expensive) sensors are deployed to get a more accurate assessment of the longitudinal velocity, like the pitot probes used on Formula 1 cars.

Moreover, when this basic rule is used, it is not immediately apparent how additional sensor information which might be available in the vehicle can be leveraged to improve the accuracy of the longitudinal velocity estimates. The introduction of ABS and electronic stability program (ESP) systems has led to the presence of wheel speed sensors on all four wheels and this information should be optimally leveraged. But besides this information, modern vehicles also have accelerometers which can provide additional information on the evolution of the vehicle velocity. Moreover, also (GPS) sensors provide information on the vehicle velocity, but this information is typically captured at a sampling frequency which is too low for exploitation in dynamic vehicle control.

This seemingly simple problem statement of longitudinal velocity estimation demonstrates the many questions which arise in assessing vehicle dynamics quantities. In the upcoming section we will explore how various observer and estimation methods can mitigate the issues raised above.

1.5 Summary

The estimator schemes presented in this chapter will allow us to:

- obtain difficult to measure quantities of interest (QOIs) like vehicle sideslip angle and tire forces;
- from a range of available sensors in the vehicle;
- without the need to set up an extensive number of engineering rules;
- by fusing their data through a system model.

This chapter first revises basic observer and estimation methods in Sect. 2. The application of Kalman filter based estimators for vehicle dynamics applications is discussed in Sect. 3 and this framework is further extended to the crucial topic tire force and parameter estimation in Sect. 4.

2 General Observer and Estimation Methods

In this section we provide a brief overview of general state observer and estimation methodologies. As the aim is mainly to attain a practical understanding for vehicle dynamics engineers, we will not focus on derivation of the presented equations or proofs of convergence. For the classical methods these are abundantly available in literature and relevant references will be provided for the interested reader. This section is therefore aimed at providing the practical building blocks to start constructing observers and estimators for vehicle dynamics applications. In this presentation we mainly focus on *physics driven* methods which rely on the user setting up and exploiting a model based on physical insight in the problem considered. Pure *data-driven* methods are out of scope for this presentation.

2.1 Physics Driven Observer and Estimation Schemes

The starting point for an observer/estimator is a real physical system $\mathcal{S}$ with n real states $\mathbf{x}^r \in \mathbb{R}^n$ and real inputs $\mathbf{u}^r \in \mathbb{R}^{n''}$. In our particular case this real system will be the vehicle and depending on which level of detail is expected, the states can vary from high level longitudinal velocity to detailed suspension vibrations. The real system is equipped with a range of sensors which provide real measurements $\mathbf{y}^r \in \mathbb{R}^{n'}$. In practice we only have these measurements, which typically do not correspond directly to the states, to infer the states of the system.

We also assume the availability of a model of the system. This model is a mathematical approximation of the behavior of the real system.¹ In this model, the system is characterized by the model state $\mathbf{x} \in \mathbb{R}^n$, which should be a good approximation of the real state of the system:

$$\mathbf{x} \approx \mathbf{x}^r. \quad (3)$$

In this chapter we distinguish three types of models which can be used to obtain QOIs for the vehicle from a set of available measurements:

- static models;
- linear kinematic and dynamic models;
- nonlinear kinematic and dynamic models.

¹ It is important to stress that in practice all models exhibit inaccuracies and are at best useful approximations of the real system behavior.

The first class of models assumes a direct relation between the measurements and the full state of the vehicle. The static models take the form of a simple measurement equation:

$$\mathbf{y} = \mathbf{h}(\mathbf{x}). \quad (4)$$

Under the assumption that there are at least as many measurements as there are relevant states ($n^y \geq n$), which is often not the case, and a full rank measurement, this model can be used to infer the states:

$$\mathbf{x} = \mathbf{h}^{-1}(\mathbf{y}), \quad (5)$$

where $\mathbf{h}^{-1}$ represents an inverse representation of the measurements. In practice a convenient way to set up this inverse relation is by directly setting up the inverse model through calibration. However a major downside of these direct approaches is the inability to exploit the information available in the evolution of the previous measurements. This generally leads to the requirement of more measurements to obtain the model states, as well as higher noise levels on the inferred quantities in contrast to methods which do account for this history.

In the frame of observers and estimators, the use of linear dynamic models are very popular. In discrete time from timestep k to $k + 1$, these models take the following form:

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u}_k, \quad (6)$$

$$\mathbf{y}_k = \mathbf{H}\mathbf{x}_k + \mathbf{D}\mathbf{u}_k, \quad (7)$$

where $\mathbf{A} \in \mathbb{R}^{n \times n}$ and $\mathbf{B} \in \mathbb{R}^{n \times n^u}$ represent the linear system dynamics. $\mathbf{H} \in \mathbb{R}^{n^y \times n}$ and $\mathbf{D} \in \mathbb{R}^{n^y \times n^u}$ represent the linear measurement model. When defining this discrete time model often the starting point will be a continuous time model:

$$\dot{\mathbf{x}} = \mathbf{A}^c \mathbf{x} + \mathbf{B}^c \mathbf{u}, \quad (8)$$

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{D}\mathbf{u}, \quad (9)$$

where numerous schemes can be used to perform the time-discretization of $\mathbf{A}^c$ and $\mathbf{B}^c$. In the following chapters we will typically first construct a continuous time model and perform the time discretization through a forward Euler scheme with timestep Δt^2 :

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \Delta t \dot{\mathbf{x}}_k, \quad (10)$$

such that

² Note that in general the forward Euler scheme requires small timesteps in order to obtain a stable solution. Due to the limited frequency content in vehicle dynamics model this is often not an issue, but care should be taken when considering high-dynamic phenomena through this scheme.

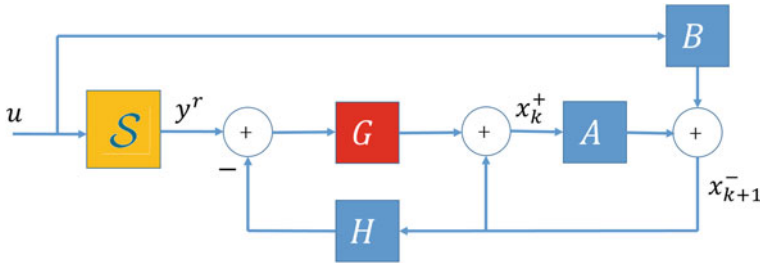


Fig. 1 General state observer/estimator scheme

$$\mathbf{A} = \mathbf{I}^{n \times n} + \Delta t \mathbf{A}^c, \quad (11)$$

$$\mathbf{B} = \Delta t \mathbf{B}^c. \quad (12)$$

Even though linear models are very simple, it is interesting to see that several key estimation problems for vehicle dynamics can be effectively captured through these models. This will be further demonstrated in Sects. 3 and 4.

However there are also many applications where the system dynamics cannot be accurately captured through a linear model structure. In this case a nonlinear model needs to be considered. These nonlinear discrete time models generally take the form:

$$\mathbf{x}_{k+1} = \mathbf{g}(\mathbf{x}_k, \mathbf{u}_k), \quad (13)$$

$$\mathbf{y}_k = \mathbf{h}(\mathbf{x}_k, \mathbf{u}_k), \quad (14)$$

where $\mathbf{g}$ and $\mathbf{h}$ represent nonlinear functions describing the dynamics and measurements. Again starting from the continuous time model:

$$\dot{\mathbf{x}} = \mathbf{g}^c(\mathbf{x}, \mathbf{u}), \quad (15)$$

$$\mathbf{y} = \mathbf{h}(\mathbf{x}, \mathbf{u}), \quad (16)$$

the time discretized system can be set up through a forward Euler scheme:

$$\mathbf{g} = \mathbf{x} + \Delta t \mathbf{g}^c(\mathbf{x}, \mathbf{u}). \quad (17)$$

A general observer or estimator scheme allows to approximately determine the full state of system from a (limited) set of measurements on the real system. These frameworks generally follow a feedback structure as shown in Fig. 1. The fundamental difference between an observer and an estimator is that the former assumes deterministic behavior, whereas the latter inherently aims to account for the uncertainty in the model and measurements, and often allows to extract additional momenta of the uncertainty distribution of the estimated states.

These observer/estimator frameworks deployed on the real system $\mathcal{S}$ consist of three main ingredients:

- **Model $\mathbf{A}$, $\mathbf{B}$, $\mathbf{H}$:** a mathematical description of the system state dynamics as well as of the measurement output as a function of the states.
- **Measurements $\mathbf{y}^r$:** the real measurement signals as recorded by the sensors on the physical system.
- **Estimator/observer gain $\mathbf{G}$:** The gain should enable an asymptotic approximation of the real state of the system from a series of measurements. The gain is determined in different ways depending on the observer/estimator scheme selected.

The resulting equations for this scheme fit a predictor-corrector. Here we make the distinction between the predicted quantities $(\cdot)^-$ and corrected quantities $(\cdot)^+$. The numerical evaluation for a linear model structure can then be summarized as:

- **Prediction:**

$$\mathbf{x}_{k+1}^- = \mathbf{A}\mathbf{x}_k^+ + \mathbf{B}\mathbf{u}_k \quad (18)$$

- **Correction:**

$$\mathbf{x}_{k+1}^+ = \mathbf{x}_{k+1}^- + \mathbf{G}(\mathbf{y}_{k+1}^r - \mathbf{H}\mathbf{x}_{k+1}^-) \quad (19)$$

The key difference between an observer or an estimator, lies in the the origin of the gain matrix $\mathbf{G}$. For observers, like the basic Luenberger observer, we assume a deterministic behavior and determine the gain in order to obtain good dynamic properties for the observer. In estimators, like the Kalman filter, the gain is determined in order to minimize the uncertainty on the corrected state estimates. From a practical perspective it is interesting to note that often estimator schemes are exploited in order to actually obtain a scheme that is more akin to an observer as many end users are not particularly interested in the obtained uncertainty distributions, but only consider the estimate for the mean as a pseudo-deterministic quantity. In these cases also the tuning of the estimator scheme can be tailored to obtain a desirable dynamic behavior (Cumbo et al., 2020).

In the following subsections we will consider several reference observer and estimator schemes which can be readily leveraged for vehicle dynamics applications.

2.1.1 The Luenberger Observer

The most basic observer scheme is the Luenberger observer approach (Luenberger, 1966). This approach provides an exact duality to feedback control problems and the design of the observer gain can therefore be based on common feedback control design methodologies (see e.g. Ellis, 2002). In a Luenberger observer, the observer gain $\mathbf{G}$ is chosen to obtain desirable dynamic properties for the estimator:

- Desired dynamic range: typically the gain is chosen such that the observer exhibits some low-pass filtering behavior to reduce the impact of noise on the measurements.
- Stability: the obtained observer scheme should be asymptotically stable to avoid spurious values in the estimated states. This can be guaranteed if the eigenvalues of the feedback system $(\mathbf{A} - \mathbf{GH})$ are within the unit-circle.

One particularly suitable approach to design these estimator seem to be in the pole-placement methods which are also often employed for controller design (as presented in Guan and Saif, 1991). However, in practice it can be challenging to achieve desirable trade-offs between the treatment of different measurements (sensor fusion) through this scheme, as for example the different noise levels for different sensors require a different treatment of these sensors. The Kalman filter, as described in the next sections, offers an effective answer to this problem. As the Kalman filtering scheme is the dominant approach in current state-estimation research for vehicle dynamics, we will not focus further on the observer framework and limit ourselves to a more detailed description of the estimation schemes.

2.1.2 The Kalman Filter

Contrary to the Luenberger observer, the Kalman filter is an estimation scheme, as it is founded on a stochastic basis. The Kalman filter was originally developed for the tracking of systems with random cumulative disturbances $\mathbf{q}$ (see Kalman, 1960a):

$$\mathbf{x}_{k+1} = \mathbf{Ax}_k + \mathbf{Bu} + \mathbf{q}. \quad (20)$$

Note that in the theory for the Kalman filter the disturbance $\mathbf{q}$ can follow any zero-mean uncertainty distribution, and is not limited to Gaussian distributions as is often wrongly stated.

In the Kalman filter it is assumed that the model matrices $\mathbf{A}$ and $\mathbf{B}$ are known, and for the unbiased disturbances $\mathbf{q}$ the covariance matrix $\mathbf{Q} \in \mathbb{R}_+^{n \times n}$ is known, but obviously the actual values of $\mathbf{q}$ are unknown. Similarly the Kalman filter assumes knowledge of the measurement equations $\mathbf{H}$ and the covariance matrix $\mathbf{R} \in \mathbb{R}_+^{n^y \times n^y}$ of the uncertainty distribution of the unbiased measurement noise $\mathbf{r}$:

$$\mathbf{y}_{k+1} = \mathbf{Hx}_k + \mathbf{r}. \quad (21)$$

Besides the states and the measurements, the Kalman filter also accounts for the (time-varying) state covariance matrix $\mathbf{P}_k \in \mathbb{R}_+^{n \times n}$. Even though in practice the theoretical conditions for the model and sensor noise are rarely met, the Kalman filter was shown to be powerful framework in numerous applications.

The evaluation of the Kalman filter follows the same predictor-corrector scheme as outlined before:

- Predict: First the expected state and state-covariance is predicted from the estimated previous states and known inputs:

$$\mathbf{x}_{k+1}^- = \mathbf{A}\mathbf{x}_k^+ + \mathbf{B}\mathbf{u} \quad (22)$$

$$\mathbf{P}_{k+1}^- = \mathbf{A}\mathbf{P}_k^+ \mathbf{A}^T + \mathbf{Q}. \quad (23)$$

Note that in contrast to the Luenberger observer, the state-covariance is predicted as well.

- Kalman gain: In contrast to the Luenberger observer where typically a fixed gain is exploited, the Kalman filter determines an optimal gain to minimize the trace of the estimated state-covariance. A full derivation of the Kalman gain is out of scope for this text, but can be found in literature (see Kalman, 1960b; Simon, 2006). The defining feature of the Kalman filter is the fact that this optimization does not need to be performed during the filtering, but has a closed form solution to enable a fast setup of the gain:

$$\mathbf{G} = \mathbf{P}_{k+1}^- \mathbf{H}^T (\mathbf{H}\mathbf{P}_{k+1}^- \mathbf{H}^T + \mathbf{R})^{-1}. \quad (24)$$

- Correct: The gain matrix for the Kalman filter $\mathbf{G}$ is employed to correct both the states and the state-covariance matrix:

$$\mathbf{x}_{k+1}^+ = \mathbf{x}_{k+1}^- + \mathbf{G}(\mathbf{y}^r - \mathbf{H}\mathbf{x}_{k+1}^-) \quad (25)$$

$$\mathbf{P}_{k+1}^+ = (\mathbf{I} - \mathbf{G}\mathbf{H}) \mathbf{P}_{k+1}^-. \quad (26)$$

Note that different schemes to compute the corrected covariance exist (Simon, 2006), several of which improve the conditioning of this operation, as it is imperative that the state-covariance matrix maintains a symmetric semi-definite structure. However, some of these implementation might come at an increased computational load, which might pose issues in embedded automotive implementations.

Even though the linear model assumption might seem highly restrictive in the frame of vehicle dynamics, we will see in Sects. 3 and 4 that several highly valuable vehicle dynamics estimators can be constructed from linear models. However, for several more advanced estimation problems nonlinear models become indispensable. For these cases, the extended Kalman filter is discussed in the following section.

2.1.3 The Extended Kalman Filter

In order to perform state-estimation on nonlinear systems, the regular linear Kalman filter can be extended into the *extended Kalman filter* (EKF) (see Jazwinski, 2007). This scheme starts from the (time-discretized) nonlinear model equations:

$$\mathbf{x}_{k+1} = \mathbf{g}(\mathbf{x}_k, \mathbf{u}_k) + \mathbf{q} \quad (27)$$

$$\mathbf{y}_k = \mathbf{h}(\mathbf{x}_k) + \mathbf{r}, \quad (28)$$

where $\mathbf{g} \in \mathbb{R}^n$ is a set of nonlinear equations for the state propagation and $\mathbf{h} \in \mathbb{R}^{n_y}$ is a set of nonlinear equations to represent the measurements. In this nonlinear case, local linearizations can be employed to construct the EKF:

$$\mathbf{A} = \frac{\partial \mathbf{g}}{\partial \mathbf{x}} \quad (29)$$

$$\mathbf{H} = \frac{\partial \mathbf{h}}{\partial \mathbf{x}}. \quad (30)$$

With these approximations, the extended Kalman filter again follows the predictor-corrector scheme:

- Predict: For the state-prediction the nonlinear state equations are employed, but the covariance prediction relies on the approximate linearized equations to avoid more expensive evaluations:

$$\mathbf{x}_{k+1}^- = \mathbf{g}(\mathbf{x}_k^+, \mathbf{u}), \quad (31)$$

$$\mathbf{A} = \left(\frac{\partial \mathbf{g}}{\partial \mathbf{x}} \right) |_{\mathbf{x}_k^+}, \quad (32)$$

$$\mathbf{P}_{k+1}^- = \mathbf{A} \mathbf{P}_k^+ \mathbf{A}^T + \mathbf{Q}. \quad (33)$$

In this approximation it is important to note that there is no approximation for the state-prediction, and only the covariances exhibit approximate dynamics.

- Kalman gain: Starting from the linearizations from the prediction step, the evaluation of the Kalman gain follows the same procedure as for the linear Kalman filter:

$$\mathbf{H} = \left(\frac{\partial \mathbf{h}}{\partial \mathbf{x}} \right) |_{\mathbf{x}_{k+1}^-}, \quad (34)$$

$$\mathbf{G} = \mathbf{P}_{k+1}^- \mathbf{H}^T (\mathbf{H} \mathbf{P}_{k+1}^- \mathbf{H}^T + \mathbf{R})^{-1}. \quad (35)$$

- Correct: In the corrector step, the nonlinear measurement equations are employed to update the states:

$$\mathbf{x}_{k+1}^+ = \mathbf{x}_{k+1}^- + \mathbf{G} (\mathbf{y}^r - \mathbf{h}(\mathbf{x}_{k+1}^-)) \quad (36)$$

$$\mathbf{P}_{k+1}^+ = (\mathbf{I} - \mathbf{G} \mathbf{H}) \mathbf{P}_{k+1}^- \quad (37)$$

With these slight modifications from the linear Kalman filter, a framework is created to tackle a wide range of nonlinear estimation problems. The trade-off is that this nonlinear framework lacks the mathematically rigorous foundation of the linear Kalman filter, as many of the underlying stochastic concepts cannot be directly trans-

lated from the linear to the nonlinear domain. In practice where often the Kalman filter is used as an easier-to-tune observer, this poses little issues. However, if more rigorous stochastic results are required, this is not be best scheme.

2.1.4 Other Nonlinear Kalman Filters

As a result of the lack of theoretical rigor of the extended Kalman filter, a range of alternative nonlinear estimators based on the Kalman filtering concepts have been developed over years. Generally they aim to address two key shortcomings of the extended Kalman filter:

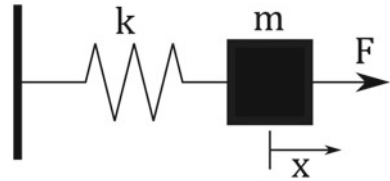
- It is often not straightforward to obtain the exact (analytical) derivatives of the state-propagation $\mathbf{A}$. On the one hand this can be the result of a complex or non-accessible (e.g. commercial software) state equation. On the other hand, this could be the result of a more complex time-integration rule, e.g. an implicit integration (see Cuadrado et al., 2012), which makes it difficult to extract explicit derivatives.
- The lack of mathematical rigour of the extended Kalman filter will lead to some error in the predicted value, which is the mean of the expected values. Even though these errors are small in practice compared to the many other approximation errors which are present in the model, this can lead to issues in particular applications. This approximation error becomes even worse for the estimated covariance in strongly nonlinear problems.

In order to alleviate these issues sampling-based Kalman filters (or *sigma-point Kalman filters*) can be employed to mitigate these issues (see Lefebvre et al., 2004). These approaches rely on a direct sampling of the uncertainty distribution rather than a linearization of the local dynamics. The two most notable schemes are:

- **Unscented Kalman filter:** this approach uses a minimal number of samples to approximate the uncertainty distribution of the states up to the second moment (see Julier & Uhlmann, 2004). Over the past decades this scheme has become a popular and flexible methodology for a range of nonlinear problems. A potential drawback is the more involved tuning of this approach. Depending on the exact implementation of the analytical derivatives this approach can be either more or less expensive computationally, but on average the computational load is similar.
- **Particle filter:** this approach uses a large number of samples to get higher moment approximations of the uncertainty distribution of the estimated states (see Carpenter et al., 1999). The primary aim for this scheme is to increase the accuracy of the estimated uncertainty distribution, but for practical vehicle dynamics applications, the computational load is generally too high.

In this chapter we will focus on the application of the extended Kalman filter for vehicle dynamics applications. However, there is little limitation to use alternative sampling-based approaches in conjunction with the approaches presented here.

Fig. 2 Single mass-spring-damper system



2.2 Kinematic Versus Dynamic Models for Estimation

For the above description we did not yet specifically consider the nature of the models employed for the estimator scheme. In practice we can distinguish between two types of models commonly employed for vehicle dynamics applications, which we denote as *kinematic* and *dynamic* models (see Viehweger et al., 2020).

In the kinematic model approach, a description is set up which is independent of an actual force balance and only relies on kinematic relations and integration rules. A key benefit of this approach is that many, often uncertain, parameters in vehicles (mass, tire parameters, etc.) are not necessary. On the other hand this does pose limitations on the prior information which can be accounted for in the model, and limits to some extent the types of sensors which can be fused through the estimator. As we will discuss in Sect. 2.3, these models also tend to be more prone to observability issues than the dynamic model approaches. Moreover, the number and types of states which can be extracted from this approach are typically relatively limited as there needs to be an explicit kinematic relation between the different states. Let us for example consider a simple mass-spring-damper model as shown in Fig. 2.

The kinematic model can be described in continuous time as:

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{u} \\ \ddot{u} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ \dot{u} \end{bmatrix} + \begin{bmatrix} 0 \\ a_u \end{bmatrix} = \mathbf{A}^c \mathbf{x} + \mathbf{B}^c \mathbf{u}. \quad (38)$$

This model represents that the derivative of the position is the velocity, and the acceleration a_u is considered as an input. It is clear that this model requires no or very limited parameters, so it is very general purpose. For example the same model could be used for vehicle position and velocity. In the case where the acceleration is not measured directly, this can simply be considered as an additional model uncertainty, described by an increased model covariance. However, as this approach relies so strongly on an accurate acceleration input, high errors can result if these do not meet the desired accuracy. Moreover, as will be demonstrated in Sect. 2.3, this model type does require position level measurements to ensure long term estimation stability, which can be difficult to realize in practice.

In the dynamic model approach, the model for the estimator relies on the translational and rotational momentum balance equations of the system. This typically implies that a range of additional parameters are required in order to set up the model, such as mass, rotational inertia of the vehicle (and wheels), tire parameters, etc. In practice it can be challenging to reliably determine these parameters for a

given operational condition. In order to alleviate uncertainty on these parameters, state/parameter estimation (as discussed in Sect. 4) can be exploited. An important benefit of these approaches is that they enable the inclusion of more a-priori information. They also allow for fusing a wider variety of sensors (e.g. force sensors). Finally, these approaches typically also allow the estimation of a wider range of (internal) states. This can be very valuable as in this way the estimator effectively amplifies the available data with respect to the direct measurements.

The continuous time dynamic model for the mass-spring-damper system can be summarized as:

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{u} \\ \ddot{u} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix} \begin{bmatrix} u \\ \dot{u} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{f}{m} \end{bmatrix} = \mathbf{A}^c \mathbf{x} + \mathbf{B}^c \mathbf{u} \quad (39)$$

In this model, the first row represents the basic integration of the velocities, but the second row represents the dynamic force balance for the mass for an applied force f . Even for this simple system, it is apparent that additional parameters like the mass m , stiffness k and damping c are required in order set up the estimator. Any error on these parameters leads to a model error which is actually multiplicative and does not match the basic Kalman filtering assumptions. The presented model is also much more specific to the particular application and the resulting estimator cannot directly be reused in other applications. As an explicit force measurement is difficult in most applications, we will often need to consider the force as an additional noise term. Alternatively we can estimate it as an *augmented state*, as is often of interest as we will discuss in Sect. 4. However, an important benefit of this scheme is that the additional prior information which goes into the model allows to relax on the required measurements to obtain an observable problem, as will be discussed in Sect. 2.3.

For automotive applications, we will see that both approaches can be exploited. Which approach is favorable will depend on the specific application of the estimator.

2.3 Observability for Reliable State Observations and Estimates

Whenever a state needs to be observed or estimated, its reliability is essential. This reliability will be dependent both on the physical system, the modeling assumptions, and the available sensors. A minimal requirement for the estimated states is that we are assured that the error on those states remains bounded. In order to enable a theoretical analysis of this property we introduce the concept of *observability*. If a system is observable, this implies that we can estimate all the states of the system with a bounded error, whereas if the system is unobservable, some or all of the system states have an infinite error. Fortunately most systems exhibit some damped behavior, such that the response remains bounded by definition, and hence the error cannot go to infinity. In this case we say that the system is *detectable*, which is a less strict requirement than observability. However, one can immediately notice that the

resulting estimates might not be particularly informative if they are only detectable because it implies that the behavior of the observed states is not necessarily correlated to the real states, but merely remains bounded.

Even though the observability and detectability are conceptual properties of the system and the sensors, in practice we evaluate it from the model we have available for the system. The assumption we have employed for the model will therefore crucially impact the conclusion on observability. This then also implies that we can render an unobservable problem observable by introducing additional prior knowledge in the model. As we will see later on, this is often a crucial aspect in the engineering design of a vehicle dynamics estimation scheme.

Besides the conceptual limitations of an unobservable problem, the observability also has an impact on the stability of the Kalman filter. This issue will be discussed in the next section. Finally we will briefly review several methods for numerically analyzing the observability of a system and how they can be leveraged in practical estimator design.

2.3.1 Observability Interpretation

For automotive applications, long term stability of the estimator is essential. In the case of dedicated experimental test this requirement can be relaxed, but for production purposes it needs to be ensured that an estimation scheme does not run into numerical issues resulting from near infinite values.

It can be shown that the observability of a system is a sufficient requirement for a Kalman filter to remain stable over time, as the covariance matrix remains bounded. Towards practical automotive dynamics applications it is interesting to notice that observability is not a necessary condition, and when choosing the model uncertainty correctly a stable unobservable estimator can be obtained (see Naets et al., 2017).

The observability for continuous and discrete time systems can be defined as (quoted from Simon, 2006):

A continuous-time system is observable if for any initial state $\mathbf{x}(0)$ and any final time $t > 0$ the initial state can be uniquely determined by knowledge of the input $\mathbf{u}(\tau)$ and measurements $\mathbf{y}(\tau)$ for all $\tau \in [0, t]$.

A discrete-time system is observable if for any initial state $\mathbf{x}_0$ and any final time k the initial state can be uniquely determined by knowledge of the input $\mathbf{u}_i$ and measurements $\mathbf{y}_i$ for all $i \in [0, k]$.

It can be shown that these conditions are sufficient to obtain a Kalman filter with finite estimator covariance $\mathbf{P}$.

Of course this is a rather mathematical view on a seemingly practical problem. To gain some more practical insight, we now consider two variations of the mass-spring-damper we previously considered, and we turn this system into two different mass-spring configurations, as shown in Fig. 3.

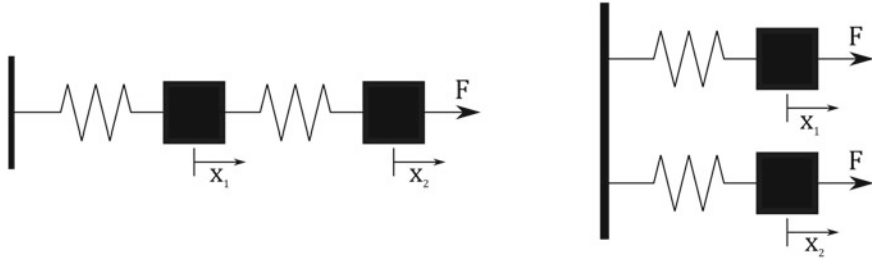


Fig. 3 Two double mass-spring configurations

For the sake of simplicity, we assume there is a zero-mean random excitation acting on both masses of this undamped system, a single acceleration and velocity measurement is available on mass 2, and both spring stiffnesses and masses are the same.

Intuitively it can be seen immediately that system 2 will be unobservable, even without doing any numerical analysis. In this case it would be impossible to infer any information on the current state of mass 1 from the available model information or data. For system 1 on the other hand, it is not equally intuitive to assess the observability and we will need to conduct a more rigorous analysis. First of all we can summarize the different models we can use for this system. As we are interested in the position and velocities of both masses the state vector $\mathbf{x}$ becomes:

$$\mathbf{x} = \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix}, \quad (40)$$

and the available measurements are the velocity $\dot{x}_2$ and acceleration $\ddot{x}_2$ on the second mass. The resulting continuous and discrete time models for the kinematic and dynamic approach are:

- **Kinematic model:** In this model there is no knowledge of the forces resulting from the springs and the continuous time model becomes:

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \ddot{x}_2 \end{bmatrix}, \quad (41)$$

$$y = [0 \ 0 \ 0 \ 1] \mathbf{x} \quad (42)$$

Note that in this case, the acceleration measurement serves as the input measurement, and only the velocity measurement can be employed as a regular measure-

ment equation. For the time discretization we assume the forward Euler scheme as described in Eq. (10).

- **Dynamic model:** For the dynamic model equations in continuous time we obtain:

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -2k/m & 0 & k/m & 0 \\ 0 & 0 & 0 & 1 \\ k/m & 0 & -k/m & 0 \end{bmatrix} \mathbf{x}, \quad (43)$$

$$\mathbf{y} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ k/m & 0 & -k/m & 0 \end{bmatrix} \mathbf{x}, \quad (44)$$

where k are the spring stiffnesses and m is the mass. For this case we do need to have information on the stiffness and mass, which we assume to be available. As we assume there is no direct input force measurement (which is difficult to obtain in practice) we assume all inputs to the dynamic equations as noise. In contrast to the kinematic model, we now get two explicit measurement equations. For the time discretization we again assume the forward Euler scheme as described in Eq. (10).

Even for this case it is not trivial anymore to assess the observability. A first observation that needs to be made is that no explicit measurement needs to be performed on each degree-of-freedom in order to obtain an observable system. This is an important and fundamental insight, as it implies that with an effective sensor selection and model setup, we can obtain accurate estimates for all dynamically coupled states in a system from only a limited set of measurements. In order to actually assess the observability of different model-sensor combinations we need to employ formal metrics. The two most common metrics, which we will discuss hereafter, are the Kalman observability criterion and the Popov-Belevitch-Hautus (PBH) criterion.

2.3.2 Kalman Observability Criterion

The most common used criterion for assessing the observability of a model-sensor combination is the *Kalman observability criterion* (see Kalman, 1970). In this approach the observability is assessed through the Kalman observability matrix $\mathbf{O}_{Kal}$ (assuming a discrete time system):

$$\mathbf{O}_{Kal} = \begin{bmatrix} \mathbf{H} \\ \mathbf{HA} \\ \vdots \\ \mathbf{HA}^{n-1} \end{bmatrix}. \quad (45)$$

The system is observable if $\mathbf{O}_{Kal}$ is of full column rank n .

The main benefit of this approach is that it is convenient to numerically evaluate this criterion numerically on a computer. For example in Matlab this matrix can be easily generated and the *rank*-command can be used to evaluate the observability. However, an important downside from an engineering perspective is the interpretability of this matrix. If it turns out that the system is unobservable, it is difficult to infer simply by looking at the structure of this matrix. Let us for example consider the kinematic model from Eqs. (41) and (42) in discrete time form:

$$\mathbf{A} = \begin{bmatrix} 1 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta t \end{bmatrix}. \quad (46)$$

For this case, the Kalman observability matrix becomes:

$$\mathbf{O}_{Kal} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (47)$$

which obviously implies that the current model-sensor combination is not observable. Unfortunately we cannot immediately use this result to assess which additional sensor should be added to make the system observable. Now, look back at Eq. (41), it is actually completely logical that we cannot observe the motion of the first mass, as this becomes completely decoupled from the second mass using the kinematic approach. For the unobservability of the displacement of the second mass, it is important to assess if an initial error on the initial displacement can ever be corrected by measuring the velocity. Obviously this is not the case. In order to know the sensors we should minimally add, this framework provides little guidance and for this the PBH criterion will be more suitable. For comparison we can now also fill in the observability matrix for the dynamic model from Eq. (43) in discrete time form:

$$\mathbf{A} = \begin{bmatrix} 1 & \Delta t & 0 & 0 \\ -2k\Delta t/m & 1 & k\Delta t/m & 0 \\ 0 & 0 & 1 & \Delta t \\ k\Delta t/m & 0 & -k\Delta t/m & 1 \end{bmatrix} \quad (48)$$

and for comparative purposes, we account only for the velocity measurement. The Kalman observability matrix then becomes:

$$\mathbf{O}_{Kal} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ \frac{k\Delta t}{m} & 0 & -\frac{k\Delta t}{m} & 1 \\ \frac{2k\Delta t}{m} & \frac{k\Delta t^2}{m} & -\frac{2k\Delta t}{m} & 1 - \frac{k\Delta t^2}{m} \\ \frac{3k\Delta t}{m} - \frac{3k^2\Delta t^3}{m^2} & \frac{3\Delta t^2 k}{m} - \frac{3k\Delta t}{m} + \frac{2k^2\Delta t^3}{m^2} & 1 - \frac{3\Delta t^2 k}{m} \end{bmatrix}. \quad (49)$$

Even though it is more difficult to see analytically, this is a matrix of full rank and hence this approach is observable. This is an important insight as the cost of needing to determine additional parameters for a dynamic model, can in many cases be offset by needing to instrument a system more extensively with additional sensors. This is particularly true for vehicle applications where sensor cost should be kept to a minimum and preferably only available sensors would be reused for estimation purposes. However, again it is clear that this resulting Kalman observability matrix is relatively complex to interpret from an engineering perspective.

2.3.3 Popov-Belevitch-Hautus Criterion

An alternative approach for assessing observability (or actually detectability) is through the Popov-Belevitch-Hautus (PBH) observability matrix $\mathbf{O}_{PBH}$ (see Belevitch, 1968). This observability matrix can be constructed and evaluated both for continuous time and discrete time systems:

$$\mathbf{O}_{PBH} = \begin{bmatrix} s\mathbf{I} - \mathbf{A} \\ \mathbf{H} \end{bmatrix}, \quad (50)$$

and the system is observable if $\mathbf{O}_{PBH}$ is of full column rank for all s in the complex domain. As $s\mathbf{I} - \mathbf{A}$ is by definition of full rank except for when s takes the value of an eigenfrequency, the observability matrix only needs to be evaluated at these eigenfrequencies. Moreover, it is known that for damped eigenmodes, the resulting $s\mathbf{I} - \mathbf{A}$ will also be non-singular. It is therefore necessary to detect which undamped modes are present in the model and assess if the available sensors allow to observe those modes. From this description it is apparent that the PBH criterion will be more difficult to evaluate reliable in an automated procedure. However, it does provide much more insight by assessing the resulting matrices analytically and often enables one to select a set of suitable additional measurements to render a problem observable. For example, let us revisit the kinematic model for the double mass-spring system and perform the observability analysis for the continuous time model:

$$\mathbf{O}_{PBH} = \begin{bmatrix} s-1 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s-1 \\ 0 & 0 & 0 & s \\ \hline 0 & 0 & 0 & 1 \end{bmatrix} \quad (51)$$

This system has potential issues when $s = 0$, in which case the first and third column become zero columns and thus singular. In order to avoid this, a measurement for these states can be added. Therefore it is necessary to add the position measurements for both masses in order to obtain an observable system.

2.3.4 Observability of Nonlinear Systems

For nonlinear problems two different types of observability can be evaluated: local or global observability (see Byrnes & Martin, 1988). Unfortunately the relation between these measures and the stability of the estimates are not so clear as in the case of linear systems. In general it can be seen that local observability over the entire configuration space is a more difficult condition to meet than global observability. This is because the linearization on which the local observability is based, omits some of the potential feed-through of information which can occur through the nonlinear behavior in a global observability analysis. Evaluating global observability requires the use of Lie-derivatives and an analytical assessment of the rank of the Lie-derivative matrix (Hermann & Krener, 1977). In practice this is very difficult to evaluate. We therefore resort to the more pragmatic local observability (Krener, 2003). In order to assess the local observability, we evaluate an observability criterion for different linearizations of the nonlinear model around different points along a reference trajectory of the system. If the system is linearly observable for these linearizations, this can typically be considered sufficient for a stable estimation.

It is important to highlight that in many applications, a nonlinear system might be locally observable in certain configurations, and not in others. Strategies to resolve this will be further discussed in Sect. 4. This is for example often an issue when parameters need to be estimated alongside the states and where the dynamics governed by these parameters are not always excited.

2.3.5 What About Unobservable Systems

In practice many problems are unobservable. This is especially true for nonlinear problems where local unobservabilities can occur. A typical example is when lateral tyre stiffness needs to be estimated. It is clear that during straight driving, no information can be inferred on the stiffnesses as neither lateral slip nor forces are being generated. However, these straight driving cases are also not of particular interest and hence can be mitigated pragmatically in order to obtain a stable estimation scheme.

In the case of a non-observable problem, several strategies can be pursued. First of all the chosen model formulation can be revisited. If there is further prior information on the system behavior, this can be taken into account in order to resolve the unobservability. A typical example is the above described shift from a kinematic model to a dynamic model structure. The main benefit of this approach is that this does not need to incur any additional cost on the deployed implementation of the estimator. Secondly, the sensor selection can be revisited (see Cumbo et al., 2020). From this perspective two choices need to be made. On the one hand the type of sensor can be selected such that they provide the additional information required to obtain an observable system. As discussed above, the PBH observability criterion provides a more practical framework to identify which additional measurements would be required in order to obtain an observable system. Obviously the main downside of this approach is that there is typically an additional cost related to adding more sen-

sors for the estimator, which is often difficult to justify in the highly cost-competitive automotive market. On the other hand, also the sensor placement on the vehicle can be reconsidered. As an intuitive example it can be seen that an accelerometer located at the center-of-gravity will not measure any contribution from the yaw motion of vehicle, whereas an accelerometer with an offset with respect to the center-of-gravity will also pick up the yaw acceleration. Sensor placement therefore has an important impact on the final (observability) performance of the estimator. The above approaches allow to obtain an observable system, but other approaches exist which can pragmatically resolve the instabilities resulting from the unobservability of the system by somehow omitting the impact of the unobservable state (combinations):

- A first method is the reduced order Kalman filter (see Simon, 2007). In this approach the unobservable modes of the system are eliminated from the equations of motion by performing a projection on the observable space. This is an interesting approach, which mainly has merit for linear problems, where the observable space remains constant. For automotive applications where issues typically arise for nonlinear estimation problems, the potential of this scheme is not clear and has not yet been applied.
- A second potential method is to use a (scheduled) model covariance $\mathbf{Q}$ tuning (see Naets et al., 2017). By putting the model covariance for unobservable states to zero, these are effectively treated as deterministic variables and the estimator remains stable. This is an effective approach to counter instabilities resulting from locally unobservable systems, where the model covariance matrix can be made a function of the states or inputs of the system to ensure that it goes to zero as unobservable configurations are approached.
- A final approach to turn an unobservable problem into an observable one, is the addition of *dummy measurements* (see Naets et al., 2015). In this framework, additional artificial measurement equations are added to prevent certain states to drift off:

$$\begin{bmatrix} \mathbf{H} \\ \mathbf{H}^{dummy} \end{bmatrix} \mathbf{x} = \begin{bmatrix} \mathbf{y}^r \\ \mathbf{y}^{dummy} \end{bmatrix}, \quad (52)$$

where $\mathbf{H}^{dummy}$ links unobservable states to constant values in $\mathbf{y}^{dummy}$. This explicit introduction of additional a-priori information does require crucial application-dependent insight from the engineer in selecting representative $\mathbf{y}^{dummy}$ values. The impact of these dummy measurements on the final estimated states is tailored by adjusting the respective measurement covariances $\mathbf{R}^{dummy}$. The more these dummy covariances are reduced, the more the estimated dynamics of the system will be inhibited. In order to get reliable results from this approach, significant tuning effort is typically required.

2.4 Conclusion

In this section we summarized the Kalman filter estimation framework in the context of general model based observers and estimators. In practice one of the main benefits of the Kalman filter are the relatively intuitive tuning of the estimator and the straightforward extension of the methodologies to nonlinear problems. Also the relatively low computational cost plays an important role for practical applications, especially in automotive applications where algorithms often need to run on embedded hardware. Several key features related to the types of the model used have been discussed, and it has been shown that a trade-off exists between general applicability (and lack of parameters) in the model and more detailed dynamic descriptions, which also have repercussion on the observability of the problem. The observability is a key property that needs to be ensured for any estimator, and several strategies to obtain stable estimators starting from these properties have been presented. In the following sections we will now explore how these concepts can be deployed for a range of vehicle dynamics applications. Through these cases we will demonstrate that this relatively simple estimation framework enables a wide range of impactful vehicle dynamics state and parameter estimators.

3 Kalman Filter Based State Estimators for Vehicle Dynamics

In this section, we investigate the Kalman filter based state estimation for common vehicle states. In this respect we first consider two decoupled cases, the longitudinal and lateral motion. Next we consider the combined longitudinal and lateral motion.

For vehicle dynamics applications, it is important to highlight that in general the global position is not of interest (this might not be the case for example for autonomous applications). The key quantities of interest are the local vehicle speeds, vehicle and tire slip rates and angles.

3.1 Reference Data Description

Throughout this section and Sect. 4, we will use experimental data obtained by Flanders Make from a Range Rover Evoque (see Naets et al., 2017; Viehweger et al., 2020).³ The vehicle is instrumented with a high-end optical velocity measurement system which can provide reliable reference data to compare the estimators. Several sensors, like accelerations, wheels speeds, yaw-rate, etc. are logged from common sensor systems for use in the presented estimation schemes. The instrumented vehicle

³ The authors gratefully acknowledges the Flanders Make DecisionS research group for supplying the presented data set.

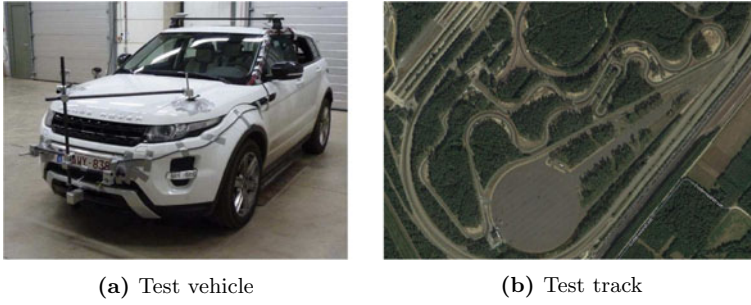


Fig. 4 Experimental Range Rover Evoque vehicle and test track (from Naets et al., 2017)

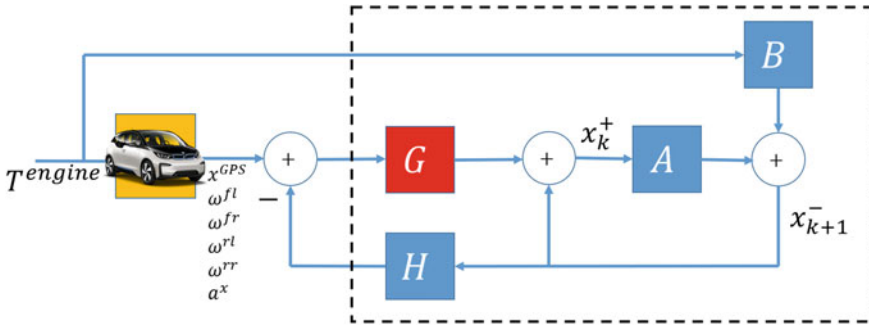


Fig. 5 Longitudinal vehicle state Kalman filter

and the handling test track on which the presented data was obtained, is shown in Fig. 4. This test case represents a generally dynamic driving scenario in which different representative operational conditions are encountered, with both linear and nonlinear vehicle behavior.

3.2 Decoupled Vehicle State Estimation: Longitudinal Vehicle States

For the longitudinal state estimation for vehicles, we exploit the Kalman filter architecture as summarized in Fig. 5, where the engine torque T^{engine} is assumed as a known input.

The main aim of this estimator is to obtain a reliable longitudinal velocity v^x . As has been noted in the introduction, this is difficult due to different potential errors which exist in the indirect measurements of the actual velocity of the center-of-gravity. Three approaches will be discussed to demonstrate the key difference between different choices which can be made in the model definition and the impact it has on the final estimator design. First we present a purely kinematic approach in

Sect. 3.2.1. Next we discuss a dynamic approach without explicit wheel rotational states in Sect. 3.2.2, which leads to a fully linear model. Finally we consider an approach with explicit wheel rotational states in Sect. 3.2.3, which inherently leads to a nonlinear estimation problem due to the typical longitudinal slip definition.

3.2.1 Longitudinal Vehicle State Estimation: Kinematic Approach

As discussed before, the most basic model approach which can be employed for the longitudinal velocity estimation, is a purely kinematic model. For the longitudinal velocity v^x as state, the model becomes:

$$\dot{\mathbf{x}} = \dot{v}^x = [0] v^x + [1] a^x, \quad (53)$$

where the output of a longitudinal accelerometer a^x is considered as input to the model. Note that this is exactly the same model as we used for the example of the mass-spring system and again we do not require any vehicle parameters to perform the evaluation. The time discretization through the forward Euler scheme from Eq. (10) becomes:

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \Delta t a^x. \quad (54)$$

The measurements which can be exploited for this case are the longitudinal velocity measurements consist of the four wheel speed sensors:

$$y^1 = \omega^{fl} = v^x / r^w, \quad (55)$$

$$y^2 = \omega^{fr} = v^x / r^w, \quad (56)$$

$$y^3 = \omega^{rl} = v^x / r^w, \quad (57)$$

$$y^4 = \omega^{rr} = v^x / r^w, \quad (58)$$

with r^w the assumed wheel radius. As the accelerometer is already employed for the input measurement, it should not be reused as an additional measurement for the Kalman filter correction. It can be verified that for this configuration, an observable problem is obtained.

To set up the longitudinal velocity estimator, the linear Kalman filter is employed as outlined in Sect. 2.1.2:

$$\mathbf{x}_{k+1}^- = \mathbf{A} \mathbf{x}_k^+ + \mathbf{B} \mathbf{u} \quad (59)$$

$$\mathbf{P}_{k+1}^- = \mathbf{A} \mathbf{P}_k^+ \mathbf{A}^T + \mathbf{Q} \quad (60)$$

$$\mathbf{G} = \mathbf{P}_{k+1}^- \mathbf{H}^T (\mathbf{H} \mathbf{P}_{k+1}^- \mathbf{H}^T + \mathbf{R})^{-1} \quad (61)$$

$$\mathbf{x}_{k+1}^+ = \mathbf{x}_{k+1}^- + \mathbf{G} (y^r - \mathbf{h}(\mathbf{x}_{k+1}^-)) \quad (62)$$

$$\mathbf{P}_{k+1}^+ = (\mathbf{I} - \mathbf{G} \mathbf{H}) \mathbf{P}_{k+1}^-, \quad (63)$$

where for the given scheme:

$$\mathbf{x} = v^x, \quad (64)$$

$$\mathbf{A} = [1], \quad (65)$$

$$\mathbf{u} = [a^x], \quad (66)$$

$$\mathbf{B} = [\Delta t], \quad (67)$$

$$\mathbf{H} = \begin{bmatrix} 1/r^w \\ 1/r^w \\ 1/r^w \\ 1/r^w \end{bmatrix} \quad (68)$$

Note that in this case all equations reduce to simple scalar equations. The tuning on this filter is very straightforward as $\mathbf{Q}$ reduces to a single scalar value which needs to be tuned. The measurement covariance matrix $\mathbf{R}$ is obtained from the sensor data sheets, or from a straightforward calibration.

3.2.2 Longitudinal Vehicle State Estimation: No Wheel DOFs

For the longitudinal velocity estimator, we can also consider a dynamic model approach. The inputs which are required for this model are the torques T^i acting on the different wheels for $i = fl, fr, rl, rr$ resulting from the drivetrain and brake actuation. Approximations of these quantities can be made available on the vehicle CAN system from engine maps and ABS systems.

As no information is assumed available on the force generation in the tire contact patch, the wheel torques are assumed to be fully transferred through the contact patch. The continuous-time equation of motion for a vehicle with mass m and tire radius r becomes:

$$[\dot{v}^x] = [0][v^x] + [1/m] \left(\sum_{i=1}^4 T^i / r^i \right). \quad (69)$$

and with the forward Euler scheme from Eq. (10) we get the discrete time system we need for the Kalman filter:

$$v_{k+1}^x = v_k^x + [1/m] \left(\sum_{i=1}^4 T_k^i / r^i \right) \Delta t. \quad (70)$$

Note that this is a very simple model to express the motion, and many disturbances will cause (severe) errors on this model. Here we even omit relatively simple effects like aerodynamic drag, as the quadratic velocity dependance would already lead to a nonlinear model.

Available sensors and corresponding measurement equations, available for common vehicles, are:

- Rotational wheel speed measurements:

$$y^1 = \omega^{fl} = v^x / r^w \quad (71)$$

$$y^2 = \omega^{fr} = v^x / r^w \quad (72)$$

$$y^3 = \omega^{rl} = v^x / r^w \quad (73)$$

$$y^4 = \omega^{rr} = v^x / r^w \quad (74)$$

- Accelerometer measurement: using the dynamic equations we can evaluate the resulting torque as:

$$a^x = [1/m] \left(\sum_{i=1}^4 T^i / r^i \right), \quad (75)$$

but this provides little additional information on the vehicle states. Alternatively we can use the time integrator relations as a measurement equation:

$$y^5 = a^x \approx (v_{k+1}^x - v_k^x) / \Delta t. \quad (76)$$

Even though the presented approach exploits a dynamic model, very little prior information is included and performance can be expected to be on par with a kinematic model approach.

Following the linear Kalman from Sect. 2.1.2, we develop a basic longitudinal velocity estimator where for the given scheme:

$$\mathbf{x} = v^x, \quad (77)$$

$$\mathbf{A} = [1], \quad (78)$$

$$\mathbf{u} = \begin{bmatrix} T^{fl} \\ T^{fr} \\ T^{rl} \\ T^{rr} \end{bmatrix}, \quad (79)$$

$$\mathbf{B} = [1/(mr^w) \ 1/(mr^w) \ 1/(mr^w) \ 1/(mr^w)], \quad (80)$$

$$\mathbf{H} = \begin{bmatrix} 1/r^w \\ 1/r^w \\ 1/r^w \\ 1/r^w \\ 1/\Delta t \end{bmatrix} \quad (81)$$

Note that in this case again all equations reduce to simple scalar equations and the tuning of the model covariance $\mathbf{Q}$ reduces to a single scalar value which needs to be tuned.

Finally it is trivial to determine that this problem is fully observable as under the current assumptions the single state is directly measurable in each assumed sensor. The proposed scheme will therefore effectively serve as a sensor fusion scheme for five sensors providing conceptually similar information. The uncertainty on the different measurements can then be effectively traded off through the Kalman filter.

However, it is important to analyze from an engineering perspective when the different assumptions employed in this scheme are expected to fail. The key assumptions which can cause a range or errors are:

- As only the wheel torques are accounted for in the dynamic model, effects like aerodynamic drag, rolling resistance, etc. will lead to a non-zero-mean error on the model. This violates one of the basic assumptions of the Kalman filter and will cause bias errors in the velocity estimates.
- The wheel velocity measurements assume perfect rolling. This is obviously a significant approximation of reality in higher slip cases, and poor results can be expected whenever the friction limit is approached.

This final assumption can be (partially) mitigated by including a tire model in the estimator. This is explored in the next section.

3.2.3 Longitudinal Vehicle State Estimation: Wheel DOFs

In order to improve the transient behavior, the previous model can be extended by adding the wheel rotational velocity DOFs ω , such that the full state vector becomes:

$$\mathbf{x} = \begin{bmatrix} v^x \\ \omega^{fl} l \\ \omega^{fr} r \\ \omega^{rl} l \\ \omega^{rr} r \end{bmatrix}. \quad (82)$$

For this extended state vector, the continuous time equations of motion $\mathbf{g}^c$ become:

$$\begin{bmatrix} \dot{v}^x \\ \dot{\omega}^{fl} l \\ \dot{\omega}^{fr} r \\ \dot{\omega}^{rl} l \\ \dot{\omega}^{rr} r \end{bmatrix} = \begin{bmatrix} (f^{x,fl} + f^{x,fr} + f^{x,rl} + f^{x,rr} - c^d (v^x)^2 / 2) / m \\ -f^{x,fl} / I^w \\ -f^{x,fr} / I^w \\ -f^{x,rl} / I^w \\ -f^{x,rr} / I^w \end{bmatrix} + \begin{bmatrix} 0 \\ T^{fl} / I^w \\ T^{fr} / I^w \\ T^{rl} / I^w \\ T^{rr} / I^w \end{bmatrix}, \quad (83)$$

where I^w represents the rotational inertia for the wheels and f^x represents the longitudinal tire forces as a function of the vehicle states. As the tire forces are generally a nonlinear function of the states, we obtain a nonlinear system of equations. As a result, it does not impose any additional complexity to also include the aerodynamic drag through the drag coefficient c^d .

As has been discussed before, different tire models exist, with different levels of complexity and accuracy. As it is generally difficult to obtain reliable parameters for the vehicle model, we employ the linear tire model approach:

$$f^x = c^x s, \quad s = \frac{\omega r^w - v^x}{v^x}, \quad (84)$$

where c^x represents the longitudinal tire stiffness. Notice that even for this simple model, a nonlinear relationship results with respect to the relevant longitudinal states.

This change in the equations of motion, also leads to a change in the acceleration measurement equation:

$$\begin{aligned} y^5 = a^x = & (f^x(\omega^{fl}, v^x) + f^x(\omega^{fr}, v^x) + f^x(\omega^{rl}, v^x)) \\ & (+ f^x(\omega^{rr}, v^x) - c^d(v^x)^2/2) / m \end{aligned} \quad (85)$$

This is again a nonlinear measurement equation due to the relationships for the longitudinal tire forces f^x . Notice that alternatively the numerical differentiation scheme from Eq. (76) could still be employed as measurement equation.

In order to convert to the time-discretized equations, the forward Euler scheme from Eq. (10) is again employed:

$$\mathbf{x}_{k+1} = \mathbf{g}(\mathbf{x}_k, \mathbf{u}_k) = \mathbf{x}_k + \Delta t \mathbf{g}^c(\mathbf{x}_k, \mathbf{u}_k). \quad (86)$$

An important point of attention here is the smaller time step which will be required compared to the model without tire forces. The relatively low inertia of the wheels and high longitudinal stiffness leads to high frequency content in the dynamics which requires as small time step to ensure stable integration.

As a result of the nonlinear components in this model, we now employ the extended Kalman filter from Sect. 2.1.3:

$$\mathbf{x}_{k+1}^- = \mathbf{g}(\mathbf{x}_k^+, \mathbf{u}), \quad (87)$$

$$\mathbf{P}_{k+1}^- = \mathbf{A} \mathbf{P}_k^+ \mathbf{A}^T + \mathbf{Q}, \quad (88)$$

$$\mathbf{G} = \mathbf{P}_{k+1}^- \mathbf{H}^T (\mathbf{H} \mathbf{P}_{k+1}^- \mathbf{H}^T + \mathbf{R})^{-1}, \quad (89)$$

$$\mathbf{x}_{k+1}^+ = \mathbf{x}_{k+1}^- + \mathbf{G} (\mathbf{y}^r - \mathbf{h}(\mathbf{x}_{k+1}^-)), \quad (90)$$

$$\mathbf{P}_{k+1}^+ = (\mathbf{I} - \mathbf{G} \mathbf{H}) \mathbf{P}_{k+1}^-, \quad (91)$$

with Eq. (86) for $\mathbf{g}$, and assuming the linear tire model, the tangent model matrix becomes:

$$\mathbf{A} = \begin{bmatrix} 1 - \Delta t A_{11} & \frac{\Delta t c^x r^w}{v^x m} & \frac{\Delta t c^x r^w}{v^x m} & \frac{\Delta t c^x r^w}{v^x m} & \frac{\Delta t c^x r^w}{v^x m} \\ \frac{\Delta t c^x \omega^{fl} r^w}{2(v^x)^2 I^w} & 1 - \frac{\Delta t c^x r^w}{v^x I^w} & 0 & 0 & 0 \\ \frac{\Delta t c^x \omega^{fr} r^w}{2(v^x)^2 I^w} & 0 & 1 - \frac{\Delta t c^x r^w}{v^x I^w} & 0 & 0 \\ \frac{\Delta t c^x \omega^{rl} r^w}{2(v^x)^2 I^w} & 0 & 0 & 1 - \frac{\Delta t c^x r^w}{v^x I^w} & 0 \\ \frac{\Delta t c^x \omega^{rr} r^w}{2(v^x)^2 I^w} & 0 & 0 & 0 & 1 - \frac{\Delta t c^x r^w}{v^x I^w} \end{bmatrix}, \quad (92)$$

$$A_{11} = \left(\frac{c^x \omega^{fl} r^w}{2(v^x)^2 m} + \frac{c^x \omega^{fr} r^w}{2(v^x)^2 m} + \frac{c^x \omega^{rl} r^w}{2(v^x)^2 m} + \frac{c^x \omega^{rr} r^w}{2(v^x)^2 m} + \frac{2c^d v^x}{m} \right), \quad (93)$$

and similarly the linearized measurement matrix becomes:

$$\mathbf{H} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -A_{11} & \frac{c^x r^w}{v^x m} & \frac{c^x r^w}{v^x m} & \frac{c^x r^w}{v^x m} & \frac{c^x r^w}{v^x m} \end{bmatrix}. \quad (94)$$

The tuning of the model covariance $\mathbf{Q}$ for this filter now becomes more involved than the previous case, as a full set of matrix equations is now obtained. Some practical considerations for the tuning of the model covariance are:

- $\mathbf{Q}^{v_x v_x}$ is mostly dependent on the expected model mismatch, and can be relatively high in practical cases.
- $\mathbf{Q}^{\omega \omega}$ will dependent both on the model mismatch as well at the torque uncertainty
- cross-variances are typically set to zero to make tuning easier.

This estimation framework can again be evaluated in terms of observability to ensure that sufficient sensors are available. To perform this assessment we rely on the PBH criterion for the linearized system in the continuous time form:

$$\mathbf{O}_{PBH} = \begin{bmatrix} s + A_{11} & -\frac{c^x r^w}{v^x m} & -\frac{c^x r^w}{v^x m} & -\frac{c^x r^w}{v^x m} & -\frac{c^x r^w}{v^x m} \\ -\frac{c^x \omega^{fl} r^w}{2(v^x)^2 I^w} & s + \frac{c^x r^w}{v^x I^w} & 0 & 0 & 0 \\ -\frac{c^x \omega^{fr} r^w}{2(v^x)^2 I^w} & 0 & s + \frac{c^x r^w}{v^x I^w} & 0 & 0 \\ -\frac{c^x \omega^{rl} r^w}{2(v^x)^2 I^w} & 0 & 0 & s + \frac{c^x r^w}{v^x I^w} & 0 \\ -\frac{c^x \omega^{rr} r^w}{2(v^x)^2 I^w} & 0 & 0 & 0 & s + \frac{c^x r^w}{v^x I^w} \\ \hline 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -A_{11} & \frac{c^x r^w}{v^x m} & \frac{c^x r^w}{v^x m} & \frac{c^x r^w}{v^x m} & \frac{c^x r^w}{v^x m} \end{bmatrix}. \quad (95)$$

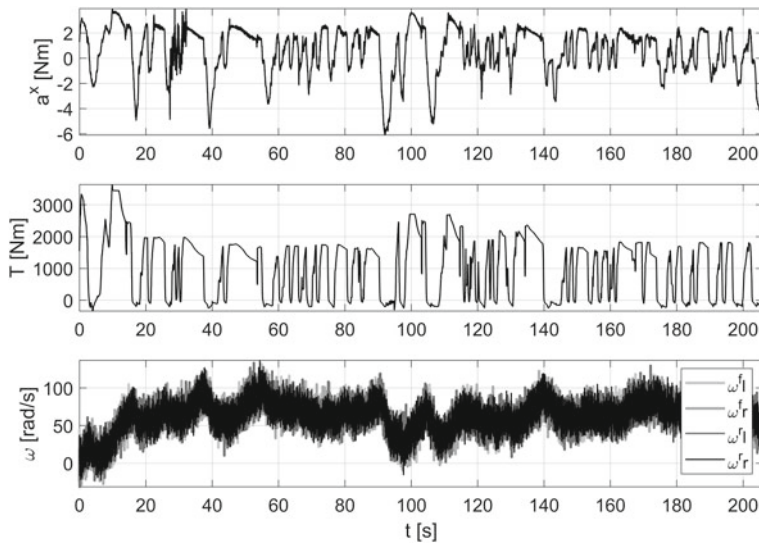


Fig. 6 Measurement channels for longitudinal velocity estimation

From this observability matrix, it is clear that given these measurements, no issues are expected for any particular states of the longitudinal vehicle motion, so the approach is fully locally observable. The only case which could cause practical issues is $v^x = 0$, as the proposed tire model is not valid for a stationary vehicle and an alternative tire model should be employed for these cases.

3.2.4 Comparison of Results for Longitudinal Velocity Estimators

We will now compare some key differences in the response of these different estimators and key tuning aspects. We assume the availability of (an approximate) motor torque, longitudinal acceleration measurements and wheel speed measurements. These measurements are shown in Fig. 6. Note that for the wheel speed measurements we added additional artificial noise to highlight some of the estimation effects more clearly.

Let us first consider the kinematic estimator. It has been highlighted before that this approach could be susceptible to low frequency drift if the impact of the measurement equations is not sufficient. On the other hand we could expect that noise on the wheel speed measurements leads to noisy velocity estimates if too much (relative) confidence is put on the wheel speed measurements. In Fig. 7 we compare the longitudinal velocity estimates from the kinematic estimator for different values of the model covariance Q . In this figure, it is apparent that drift occurs when too much confidence (low Q) is placed on the model, as the drift in the accelerometer measurement gets amplified over time. On the other hand, if too little confidence is

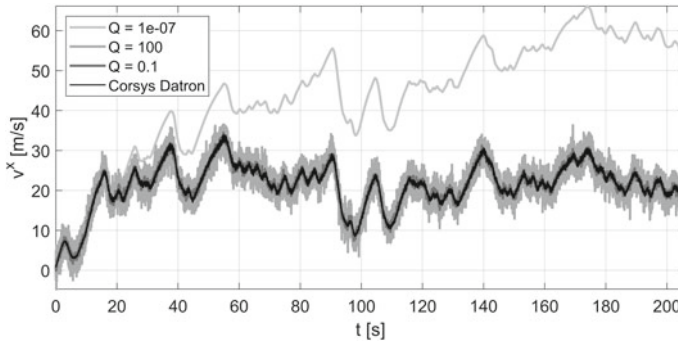


Fig. 7 Longitudinal velocity estimates for kinematic estimator with different values for the model covariance Q

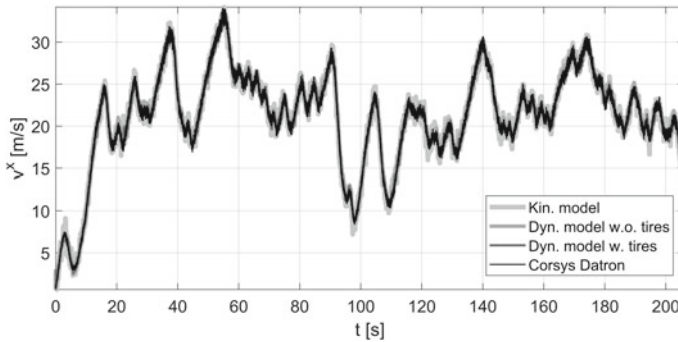


Fig. 8 Longitudinal velocity estimate comparison for different estimators

place on the model (high Q), the filtering properties go lost and a noisy result occurs. By finding a good balance for Q a very accurate noise-free estimate can be obtained.

For the three different estimation schemes discussed above, the longitudinal velocity estimates are shown in Fig. 8. Here it is clear that the different approaches present very similar results, and the noise on the different measurements can be effectively eliminated. It is important to note that the sampling rate of the measurement data is $\Delta t = 10$ ms. This is fine as a timestep for the kinematic estimator, and the one without tire model. However, in the case of the dynamic model with a tire model, this timestep is too large and would lead to (numerical) instability in the integration. For the dynamic model with tire, the measurement data is resampled at $\Delta t = 0.1$ ms, which again leads to stable results.

It should also be noted that besides the vehicle longitudinal velocity state, the last approach also has four additional states for the wheels. Due to the faster dynamics, we set the model uncertainty on the wheel velocities four order of magnitude larger than for the longitudinal velocity. The resulting wheel speed estimates are shown in Fig. 9, and compared to the real measurements. It is clear that this estimator

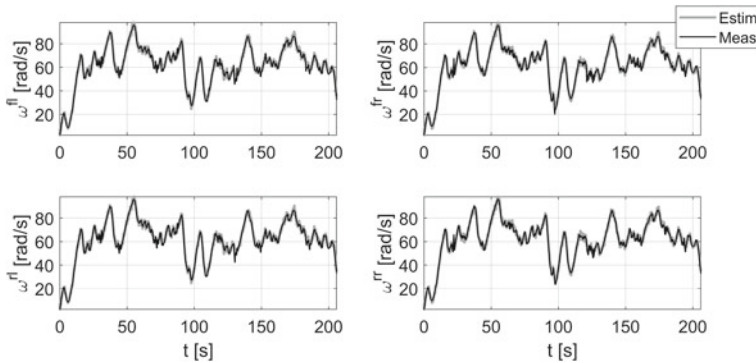


Fig. 9 Wheel speed estimates for dynamic longitudinal vehicle model with tires

effectively reduces the (artificial) noise on the wheel speed measurements which was used for the estimation from Fig. 6. This is obviously very useful for different applications where detailed wheel speed velocity could be required, but comes at the cost of a more complex estimator.

3.3 Lateral Vehicle State Estimation

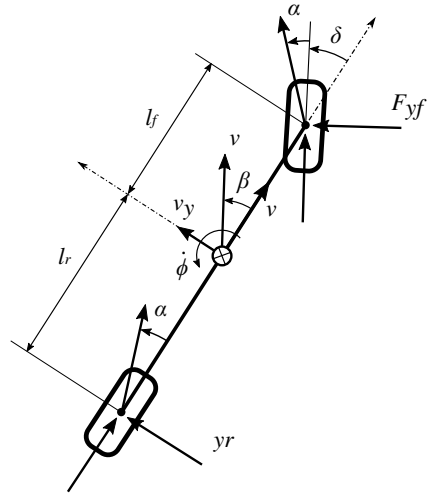
A key quantity for vehicle dynamics assessment is the side-slip angle β :

$$\beta = \arctan\left(\frac{v^y}{v^x}\right) \approx \frac{v^y}{v^x}, \quad (96)$$

as this provides essential information on the current stability of the vehicle motion. However, direct sensors for this quantity (like optical systems) are very expensive such that indirect estimator strategies are very appealing. The most basic model to cover this quantity has two degree-of-freedom: lateral vehicle velocity v^y and yaw rate $\dot{\psi}$. The longitudinal velocity v^x is assumed to be reliably known, for example from one of the previously discussed estimators, and it serves as an input to the lateral model.

In order to describe the lateral dynamics, the bicycle model, as developed by Segel (1956) is employed. Figure 10 shows the different quantities that make up this model. The different variables in this bicycle model are:

- $\dot{\psi}$: the yaw rate,
- v^x and v^y : the longitudinal and lateral center-of-gravity (COG) vehicle velocities projected onto a frame that rotates with the vehicle, and v is the resulting full velocity vector,
- β : the vehicle sideslip angle,

Fig. 10 The bicycle model

- α_f and α_r : the front and rear wheel sideslip angles,
- $f^{x,f}$, $f^{y,f}$: the longitudinal and lateral front tire force and $f^{x,r}$, $f^{y,r}$ are the longitudinal and lateral rear tire force (all described in a tire-attached frame),
- δ : the equivalent front steering angle,
- l^f and l^r : the respective distances from the COG to the front and rear axle.

The assumption is made that the steering angle is small, which allows to decouple the longitudinal and lateral forces generated at the front wheels. This allows to only consider the lateral vehicle dynamics in the estimator. These longitudinal tire forces are eliminated from the lateral model and wheel torque measurements are not necessary as a result of this assumption. In its most general form, the lateral vehicle dynamics can now be formulated as:

$$\dot{v}^y = \frac{1}{m} (f^{y,f} + f^{y,r}) - v^x \dot{\psi} \quad (97)$$

$$\ddot{\psi} = \frac{1}{I^{zz}} (l^f f^{y,f} - l^r f^{y,r}) \quad (98)$$

where m is the vehicle mass and I^{zz} is the yaw moment of inertia (both of which are assumed known). The longitudinal velocity v^x appears in Eqs. (97) and (98) as an input. However, in order to be rigorous the estimator of the longitudinal motion (in case this is used) should be combined by extending the state, such that any of the schemes presented in Sect. 3.2 can be added to these equations of motion. The main difficulty which arises in this case, is that the resulting scheme could get strongly nonlinear. In the following framework we assume this can be analyzed in a decoupled fashion such that a fully linear estimator is obtained.

The dynamic equations require a tire force model to assess f^y for the front and rear axle. We again assume a simple linear tire model where the resulting axle load

due to the lateral tire force f^y is given by:

$$f^y = -c^y \alpha, \quad (99)$$

with the equivalent axle cornering stiffness c^y and wheel sideslip angle α . Under the small steering and slip angle approximation, the front (α^f) and rear (α^r) sideslip angles are given by:

$$\alpha^f = \frac{v^y + l^f \dot{\psi}}{v^x} - \delta, \quad \alpha^r = \frac{v^y - l^r \dot{\psi}}{v^x}. \quad (100)$$

Note that for v^y the effect of the yaw rate is considered, whereas it is neglected for v^x , as the contribution in the latter case due to the yaw rate is assumed to be relatively small. Under the assumption of the same cornering stiffness at the front and the rear, the lateral tire forces can be evaluated as:

$$f^{y,f} = -\frac{c^y}{v^x} (v^y + l^f \dot{\psi}) + c^y \delta, \quad (101)$$

$$f^{y,r} = -\frac{c^y}{v^x} (v^y - l^r \dot{\psi}), \quad (102)$$

and the resulting lateral equations of motion are:

$$\begin{bmatrix} \ddot{v}^y \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} -2\frac{c^y}{v^x}/m & (l^r - l^f)\frac{c^y}{v^x}/m - v^x \\ -(l^f - l^r)\frac{c^y}{v^x}/I^v & -((l^r)^2 + (l^f)^2)\frac{c^y}{v^x}/I^v \end{bmatrix} \begin{bmatrix} v^y \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} c^y/m \\ l^f c^y/I^z \end{bmatrix} \delta. \quad (103)$$

As the longitudinal velocity is a *time-varying parameter* in this model, the resulting structure is that of a linear model time-varying model. In this scheme we require a steering angle measurement. This is typically obtained from a steering mounted encoder combined with a kinematic map of the steering system in order to obtain the left and right wheel angles (respectively δ_l and δ_r). The wheel angle for the dynamic model is obtained as the average angle of both front wheels:

$$\delta = \frac{\delta_l + \delta_r}{2}. \quad (104)$$

This continuous time model can now be converted to a discrete time model using the forward Euler scheme from Eq. (10):

$$\mathbf{x}_{k+1} = \begin{bmatrix} 1 + \Delta t 2\frac{c^y}{v^x}/m & \Delta t (l^r - l^f)\frac{c^y}{v^x}/m - v^x \\ \Delta t (l^f - l^r)\frac{c^y}{v^x}/I^v & 1 + \Delta t ((l^r)^2 + (l^f)^2)\frac{c^y}{v^x}/I^v \end{bmatrix} \mathbf{x}_k + \begin{bmatrix} \Delta t c^y/m \\ \Delta t l^f c^y/I^z \end{bmatrix} \delta_k. \quad (105)$$

Care should again be taken in selecting a sufficiently small time step Δt to ensure a stable integration.

For the measurements for the lateral state estimator, the following sensors are assumed:

- Wheel speed measurements:

$$y^1 = \omega^{fl} = (v^x - t/2\dot{\psi})/r^w \quad (106)$$

$$y^2 = \omega^{fr} = (v^x + t/2\dot{\psi})/r^w \quad (107)$$

$$y^3 = \omega^{rl} = (v^x - t/2\dot{\psi})/r^w \quad (108)$$

$$y^4 = \omega^{rr} = (v^x + t/2\dot{\psi})/r^w \quad (109)$$

- Accelerometer measurement (which measures the absolute acceleration):

$$y^5 = a^y = (2\frac{c^y}{v^x}/m)v^y + ((l^r - l^f)\frac{c^y}{v^x}/m)\dot{\psi} + c^y/m\delta \quad (110)$$

- Yaw rate sensor (this is typically a MEMS gyroscope):

$$y^6 = \omega^z = \dot{\psi}. \quad (111)$$

- GPS positioning could potentially also be used, but again offers some challenges in terms of sampling rate. This sensor is therefore omitted in this example.

This model structure allows us to apply the regular linear Kalman filter from Sect. 2.1.2 with:

$$\mathbf{x} = \begin{bmatrix} v^y \\ \dot{\psi} \end{bmatrix}, \quad (112)$$

$$\mathbf{A} = \begin{bmatrix} 1 + \Delta t 2\frac{c^y}{v^x}/m & \Delta t(l^r - l^f)\frac{c^y}{v^x}/m - v^x \\ \Delta t(l^f - l^r)\frac{c^y}{v^x}/I^v & 1 + \Delta t((l^r)^2 + (l^f)^2)\frac{c^y}{v^x}/I^v \end{bmatrix}, \quad (113)$$

$$\mathbf{u} = [\delta], \quad (114)$$

$$\mathbf{B} = \begin{bmatrix} \Delta t c^y/m \\ \Delta t l^f c^y/I^z \end{bmatrix}, \quad (115)$$

$$\mathbf{H} = \begin{bmatrix} 0 & -\frac{t}{2r^w} \\ 0 & \frac{t}{2r^w} \\ 0 & -\frac{t}{2r^w} \\ 0 & \frac{t}{2r^w} \\ 2\frac{c^y}{v^x}/m & (l^r - l^f)\frac{c^y}{v^x}/m \\ 0 & 1 \end{bmatrix} \quad (116)$$

The tuning for this estimator now involves two covariances (again assuming cross terms zero) $\mathbf{Q}^{v^y v^y}$ and $\mathbf{Q}^{\dot{\psi} \dot{\psi}}$. Both of these covariances need to account for inaccuracies in the vehicle model assumptions, parameters (also longitudinal velocity), and steering inputs.

From this estimator, the vehicle sideslip angle β can be computed as a post-processed result:

$$\beta = \tan^{-1} \left(\frac{v^y}{v^x} \right). \quad (117)$$

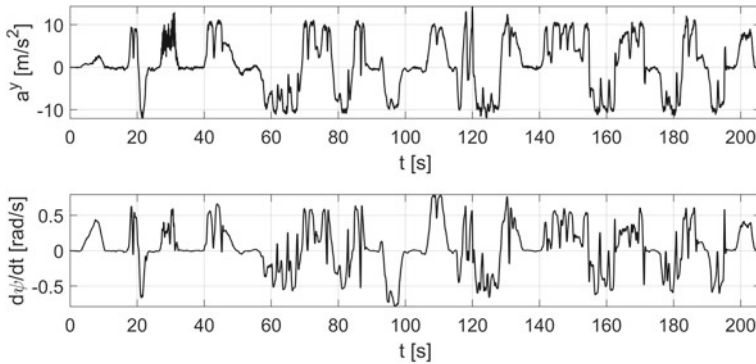


Fig. 11 Lateral acceleration and yaw rate measurements

Overall this scheme leads to a relatively simple linear estimator scheme to provide estimates on a rather complex dynamic phenomenon. The main issue with this scheme is the requirement to have reliable information on the parameters, especially the lateral tire stiffness. This is further complicated by the fact that this stiffness varies significantly as the friction limit is approached. Different strategies to alleviate this issue are discussed in Sect. 4.

3.3.1 Results for Lateral State Estimation

Starting again from the reference data from Sect. 3.1, we can perform the state estimation for the lateral vehicle motion. We now add the yaw rate measurement and lateral acceleration measurement, as shown in Fig. 11, for the estimation. From these data, we can run the lateral state estimator, which leads to the state estimates shown in Fig. 12. These results show that there is very good tracking for the yaw rate $\dot{\psi}$, which is to be expected as this measurement is used directly. For the lateral velocity v^y on the other hand it is clear that this is often not tracked very accurately. However, it has to be kept in mind that this is a very simple model which neglects any nonlinear tire effects. It is moreover necessary to obtain the equivalent tire stiffness, which is typically not straightforward. In this approach for example the same stiffness with a value of $c^y = 60 \text{ kN/rad}$ is chosen for the front and the rear. However, as we will see in the next section, this assumption of equal stiffness at the front and the rear is not very accurate, and neither is the assumption of linear tire behavior for this test.

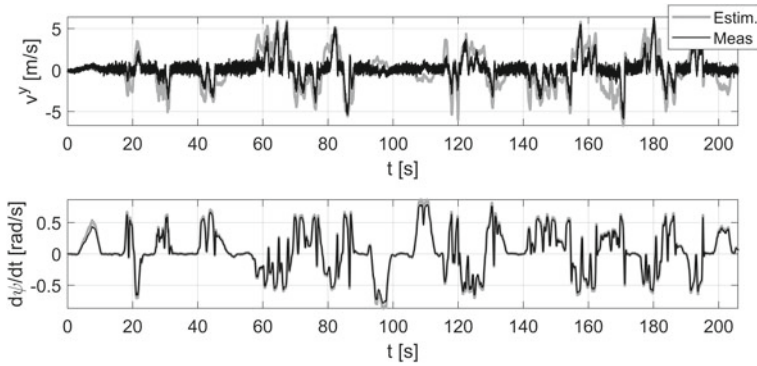


Fig. 12 Lateral state estimates

4 Kalman Filter Based Estimators for Vehicle Dynamics with Unknown Tire Models

In the previous section we described how several basic state estimators for vehicles can be constructed. However, an important limitation in practice is the lack of knowledge of the parameters of the vehicle. In particular the equivalent tire stiffness form a major bottleneck. Even if extensive testing for calibration is performed, then still these parameters vary strongly depending on the tire, road, and environmental conditions. In this section we therefore explore methods which allow to not only estimate the states, but also the tire forces or parameters. Through these methods the above issues can be circumvented and more robust approaches can be obtained. The tradeoff is that in general more measurements are required in order to ensure an observable system.

4.1 Coupled State/Input and State/Parameter Estimation

In order to enable continuous correction of (biased) unknown inputs $\mathbf{u}^u$ or unknown parameters $\mathbf{p}^u$, we *augment* the states with these augmentation states $\mathbf{x}^a$. This leads to the augmented state vector $\mathbf{x}^*$

$$\mathbf{x}^* = \begin{bmatrix} \mathbf{x} \\ \mathbf{u}^u \end{bmatrix} \quad \text{or} \quad \mathbf{x}^* = \begin{bmatrix} \mathbf{x} \\ \mathbf{p}^u \end{bmatrix}. \quad (118)$$

With this approach, the unknown inputs or parameters are estimated along with the other states.

In order to allow for these estimates, state equations also need to be added for these augmentation states $\mathbf{x}^a$. The continuous time model which we assume in this

work, and which is most often employed, is the random walk model:

$$\dot{\mathbf{x}}^a = 0 + \mathbf{q}^{a,c}, \quad (119)$$

which implies that the unknown state remains constant except for some unknown random velocity disturbance $\mathbf{q}^{a,c}$. The equivalent time-discrete model is obtained after performing the forward Euler scheme:

$$\mathbf{x}_{k+1}^a = \mathbf{x}_k^a + \mathbf{q}^{a,d}, \quad (120)$$

where $\mathbf{q}^{a,d}$ now represents the randomized variation in the augmentation states over a single time step.

For a linear system, the continuous time full augmented system equations with unknown inputs $\mathbf{u}^u$ become:

$$\dot{\mathbf{x}}^* = \begin{bmatrix} \mathbf{A}^c & \mathbf{B}^c \\ 0 & 0 \end{bmatrix} \mathbf{x}^*, \quad (121)$$

$$\dot{\mathbf{x}}^* = \mathbf{A}^{c,*} \mathbf{x}^*. \quad (122)$$

This approach maintains the linearity of the non-augmented model, and the discrete time system is trivially extracted.

Starting from a parameterized linear system $\mathbf{A}^c(\mathbf{p}^u)$, $\mathbf{B}^c(\mathbf{p}^u)$, the continuous time augmented system equations for unknown parameters become:

$$\dot{\mathbf{x}}^* = \begin{bmatrix} \mathbf{A}^c & \frac{\partial \mathbf{A}^c}{\partial \mathbf{p}^u} \mathbf{x} + \frac{\partial \mathbf{B}^c}{\partial \mathbf{p}^u} \mathbf{u} \\ 0 & 0 \end{bmatrix} \mathbf{x}^* + \begin{bmatrix} \mathbf{B}^c \\ 0 \end{bmatrix} \mathbf{u}, \quad (123)$$

$$\dot{\mathbf{x}}^* = \mathbf{A}^{c,*} \mathbf{x}^* + \mathbf{B}^{c,*} \mathbf{u}. \quad (124)$$

Notice that even in the case where the regular system behaves linear, the augmented system becomes a nonlinear model due to dependence of $\mathbf{A}^{c,*}$ on the parameters. These types of problems therefore inherently require a nonlinear estimator approach, and here we will assume an extended Kalman filter as discussed in Sect. 2.1.3.

When deploying these augmented estimation schemes, there is a fundamental difference in the tuning for unknown inputs and parameters:

- For the unknown inputs $\mathbf{x}^a = \mathbf{u}^u$, it can be expected that these exhibit strong variations in many applications. The tire forces for example can vary very rapidly. This implies that a large model covariance needs to be assigned for the corresponding state equations in order to ensure proper tracking of the states and obtain good accuracy.
- For the unknown parameters $\mathbf{x}^a = \mathbf{p}^u$, on the other hand, it can be expected that these exhibit relatively slow variations. For example in the case of tire stiffness, the equivalent stiffness can be expected to not vary much until the tire starts saturating. This then in turn implies that the corresponding model covariance should be rather low.

For all these augmented estimation schemes, an important point of attention is the observability of the system. As we will see in the following cases, these approaches typically require a more careful sensor selection or specific adaptations to ensure observability.

We will now deploy these two different approaches for the lateral vehicle estimation to account for the uncertainty on the tire behavior: lateral state/force estimation, and lateral state/tire parameter estimation.

4.2 Lateral State/Force Estimation

For the lateral estimator with unknown tire forces, we again start from the bicycle model as outlined in Sect. 3.3. The choice for the bicycle model is essential when it comes to tire force estimation. This choice implies that only average tire forces for the front and rear axle can be obtained, rather than separate forces for the left and right side as well. This is however a conscious choice as trying to split as well between left and right forces would inevitably lead to an unobservable system. This can simply be seen from the assessment that the two degrees-of-freedom which are available to describe the lateral motion are insufficient to differentiate between four forces, and hence any linear combination of these forces would be feasible. The only approach to ensure that more forces could be determined is by extending the degrees-of-freedom of the system by (for example) introducing additional compliance in the vehicle suspension model (see Risaliti et al., 2019).

The augmented state vector for the combined lateral state and force estimation then becomes:

$$\mathbf{x}^* = \begin{bmatrix} v^y \\ \dot{\psi} \\ f^{y,f} \\ f^{y,r} \end{bmatrix}. \quad (125)$$

In this approach a direct estimation of the lateral tire forces $f^{y,f}$ and $f^{y,r}$ is obtained by modeling their evolution as a random walk process. This approach will be referred to as the force estimation (FE) approach in the remainder of this text. Starting from the previously described bicycle model, this approach leads to the following continuous-time model for the estimator:

$$\dot{\mathbf{x}}^* = \begin{bmatrix} \dot{v}^y \\ \ddot{\psi} \\ \dot{f}^{y,f} \\ \dot{f}^{y,r} \end{bmatrix} = \mathbf{g}^c(\mathbf{x}^*, \mathbf{u}) = \begin{bmatrix} \frac{1}{m} (f^{y,f} + f^{y,r}) - v^x \dot{\psi} \\ \frac{1}{I^z} (l^f f^{y,f} - l^r f^{y,r}) \\ 0 \\ 0 \end{bmatrix}, \quad (126)$$

where now only the vehicle mass m and yaw inertia I^z need to be known as parameters, such that:

$$\mathbf{A}^{c,*} = \begin{bmatrix} 0 & -v^x & 1/m & 1/m \\ 0 & 0 & l^f/I^z & -l^r/I^z \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad (127)$$

and a simple linear system is obtained. Note that here we consider the longitudinal velocity as an (exact) input. Alternatively the longitudinal velocity estimator could be integrated with the lateral one, which leads to a (mildly) nonlinear system (see Naets et al., 2017).

For this estimation approach, four different types of measurements are considered:

- Yaw rate measurement from a (MEMS) gyroscope:

$$y^1 = \dot{\psi}. \quad (128)$$

- Lateral acceleration measurement from a (MEMS) accelerometer. For the model with unknown forces, the measurement equation becomes:

$$y^2 = \frac{1}{m} (F_{yf} + F_{yr}), \quad (129)$$

- Four wheel speed measurements from wheel mounted encoders:

$$y^3 = v^{x,rl} = v^x - \frac{t^r}{2} \dot{\psi}, \quad (130)$$

$$y^4 = v^{x,rr} = v^x + \frac{t^r}{2} \dot{\psi}, \quad (131)$$

$$y^5 = v^{x,fl} = \left(v^x - \frac{t^f}{2} \dot{\psi} \right) \cos(\delta^l) + (v^y + l^f \dot{\psi}) \sin(\delta^l), \quad (132)$$

$$y^6 = v^{x,fr} = \left(v^x + \frac{t^f}{2} \dot{\psi} \right) \cos(\delta^r) + (v^y + l^f \dot{\psi}) \sin(\delta^r), \quad (133)$$

with t^r and t^f respectively the rear and front track width. In contrast to the regular lateral state estimator, we now also account for the orientation of the front wheels in the analysis of the velocity, such that they also contain a contribution from the lateral velocity. This is for the sake of observability as we will discuss hereafter. Notice that in general that these equations only hold for limited longitudinal tire slip.

Even though we have set up a linear model equation, the measurement equations are nonlinear.

The locally linearized PBH observability matrix for the continuous time system $\mathbf{O}_{PBH}$ is:

$$\mathbf{O}_{PBH} = \begin{bmatrix} s & v^x & -1/m & -1/m \\ 0 & s & -l^f/I^z & l^r/I^z \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & s \\ \hline 0 & 1 & 0 & 0 \\ 0 & 0 & 1/m & 1/m \\ 0 & -t^r/2 & 0 & 0 \\ 0 & t^r/2 & 0 & 0 \\ \sin(\delta^l) & -t^f \cos(\delta^l)/2 + l^f \sin(\delta^l) & 0 & 0 \\ \sin(\delta^r) & t^f \cos(\delta^r)/2 + l^f \sin(\delta^r) & 0 & 0 \end{bmatrix}. \quad (134)$$

The main observability issues for augmented state approaches typically arise at $s = 0$. This implies that low frequency drift will happen in the estimator. In this case the issue occurs when no steering input is applied i.e. $\delta^l = \delta^r = 0$. For this particular case, the lateral velocity v^y becomes non observable (i.e. the first column becomes a zero-column). Moreover, it is interesting to note that if we had used the simpler equations for the front wheel velocities, the lateral velocity would have been unobservable under any condition as the first column would always be the zero row at $s = 0$. This again demonstrates how important it is to properly assess the model and measurement assumptions and resulting equations. A possible approach to mitigate this issue is the addition of GPS velocity data. However, for common GPS sensors, the accuracy is rather poor such that the final estimates would still be very unreliable. If the car is cornering, at least the yaw rate and acceleration measurements are required in order to obtain an observable system. However, the fusion with the wheel speed measurements and/or GPS velocity measurement provides valuable additional information which leads to better estimation accuracy by lowering the estimation covariance and because they prevent drift due to accelerometer bias. The above remarks indicate that stability issues can occur during (prolonged) straight driving for this state/force estimation approach. As has been discussed in Naets et al. (2017) there is no straightforward approach to alleviate these stability issues.

4.3 Lateral State/Tire Parameter Estimation

In an alternative scheme (see also Naets et al., 2017) the lateral tire forces are obtained through a simple linear tire model where the resulting axle load due to the lateral tire force f^y is given by:

$$f^y = -2c^y\alpha, \quad (135)$$

with tire cornering stiffness c^y and wheel sideslip angle α . Under a small angle approximation, the front (α^f) and rear (α^r) sideslip angles are given by:

$$\alpha^f = \frac{v^y + l^f \dot{\psi}}{v^x} - \delta, \quad \alpha^r = \frac{v^y - l^r \dot{\psi}}{v^x}. \quad (136)$$

With these equations, the lateral forces can be evaluated in the bicycle model. In the current scheme we will consider the stiffness parameters as augmentation states, which leads to the augmented state vector:

$$\mathbf{x}^* = \begin{bmatrix} v^y \\ \dot{\psi} \\ c^{y,f} \\ c^{y,r} \end{bmatrix}. \quad (137)$$

This approach will be referred to as the stiffness estimation (SE) approach in the remainder of this work.

The evolution of the unknown cornering stiffness is again modeled as a random walk to account for variable tire behavior. This general framework should allow corrections for tire nonlinearity (e.g. saturation), road friction changes, tire wear, etc. Starting from the tire model in Eq. (135), the stiffness can be interpreted as a secant stiffness rather than a tangent stiffness. This is important to note, as in general the secant stiffness will exhibit smaller variations than the tangent stiffness (which will e.g. go to zero for a saturated tire). Substituting the tire model Eqs. (135) and (136) in model Eq. (126) results in the following continuous-time model for the estimator:

$$\dot{\mathbf{x}}^* = \begin{bmatrix} \dot{v}^y \\ \ddot{\psi} \\ \dot{c}^{y,f} \\ \dot{c}^{y,r} \end{bmatrix} = \mathbf{g}^{C,*}(\mathbf{x}^*, \mathbf{u}) = \begin{bmatrix} \frac{-2(c^f + c^r)}{mv^x} v^y - \left(\frac{2(c^f l^f - c^r l^r)}{mv^x} + v^x \right) \dot{\psi} + \frac{2c^f}{m} \delta \\ \frac{-2(c^f l^f - c^r l^r)}{I^z v^x} v^y - \frac{2(c^f l^f{}^2 + c^r l^r{}^2)}{I^z v^x} \dot{\psi} + \frac{2c^f l^f}{I^z} \delta \\ 0 \\ 0 \end{bmatrix}, \quad (138)$$

with input vector $\mathbf{u}$:

$$\mathbf{u} = \begin{bmatrix} \delta \\ v^x \end{bmatrix}. \quad (139)$$

In contrast to the FE approach, we now obtain an inherently nonlinear problem, such that a nonlinear Kalman filter is required. Again here the longitudinal vehicle estimator could be integrated with the lateral state/parameter estimation.

For this estimator the same sensors are used as in the FE previous approach. For the given states, measurements y^1, y^3, y^4, y^5, y^6 remain the same as for the FE approach. The SE scheme only has an influence on the resulting equation for the lateral acceleration measurement:

$$y^2 = a^y = \frac{-2(C_f + C_r)}{mv_x} v_y - \frac{2(l_f C_f - l_r C_r)}{mv_x} \dot{\psi} + \frac{2C_f}{m} \delta. \quad (140)$$

We can again analyze the observability of this estimator scheme for the locally linearized PBH criterion:

$$\mathbf{O}_{PBH} = \begin{bmatrix} s + \frac{2(c^f + c^r)}{mv^s} & \frac{2(c^f l^f - c^r l^r)}{mv^s} + v^x & \frac{2v^y + 2l^f \dot{\psi}}{mv^s} - \frac{2\delta}{m} & \frac{2v^y - 2l^r \dot{\psi}}{mv^s} \\ -\frac{2(c^f l^f - c^r l^r)}{l^z v^s} & s + \frac{2(c^f l^{f2} + c^r l^{r2})}{l^z v^s} & \frac{2l^f v^y + 2l^{f2} \dot{\psi}}{l^z v^s} - 2\frac{l^f}{l^z} \delta & \frac{-2l^r v^y + 2l^{r2} \dot{\psi}}{l^z v^s} \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & s \\ \hline 0 & 1 & 0 & 0 \\ -\frac{2(c^f + c^r)}{mv^s} & -\frac{2(c^f l^f - c^r l^r)}{mv^s} & -\frac{2c^f v^y + 2c^f l^f \dot{\psi}}{mv^s} - \frac{2\delta}{m} & \frac{2c^r v^y - 2c^r l^r \dot{\psi}}{mv^s} \\ 0 & -t^r/2 & 0 & 0 \\ 0 & t^r/2 & 0 & 0 \\ \sin(\delta^f) & -t^f \cos(\delta^f)/2 + l^f \sin(\delta^r) & 0 & 0 \\ \sin(\delta^r) & t^f \cos(\delta^f)/2 + l^f \sin(\delta^r) & 0 & 0 \end{bmatrix}. \quad (141)$$

Again here, difficulties can be expected for steady state drift at $s = 0$. We now see that this system becomes unobservable in pure straight driving when $\delta = \dot{\psi} = v_y = 0$, and in this case the tire stiffnesses become unobservable as the final two columns become all zeros. Notice that this is already a much more involved condition than the one for the FE approach, where it was sufficient if merely the steering wheel angle was zero to make the system unobservable. This behavior is in line with the physical insight that there is no information about the lateral dynamics in this case, allowing the cornering stiffnesses to have any value while producing the same vehicle response. However, whenever the states deviate from a pure straight driving condition (which is typically the case in real life scenario's) the system becomes observable again.

The above remarks indicate that stability issues can occur during (prolonged) straight driving for both the FE and SE approach, even though the exact unobservability is more unlikely for the SE approach. In order to ensure stability for the estimator in the case of a temporarily unobservable configuration, a pragmatic scheduling scheme can be developed by modifying the model covariance to zero for the unobservable states, as was proposed by Naets et al. (2017). In the case of the SE approach this is not expected to have much influence on the final results for the estimator as anyhow the tire stiffness is expected to be constant (i.e. linear tire behavior) for small lateral motion. However, this approach does not translate well to the FE estimator, as there is no reason to expect the lateral velocity to behave particularly constant for small deviations around the straight driving configuration, and this approach would for example inherently miss the impact of sidewind on the vehicle. From this perspective of practical stability, and as we will also demonstrate in the following section, the SE approach behaves superior to the FE approach.

4.3.1 Results for Lateral State and Tire Force/Parameter Estimation

Using this augmented approach, we can now repeat the lateral state estimation from Sect. 3.3.1. The lateral velocity and yaw rate are shown in Fig. 13. These results show that the SE approach manages to very accurately approximate the measured

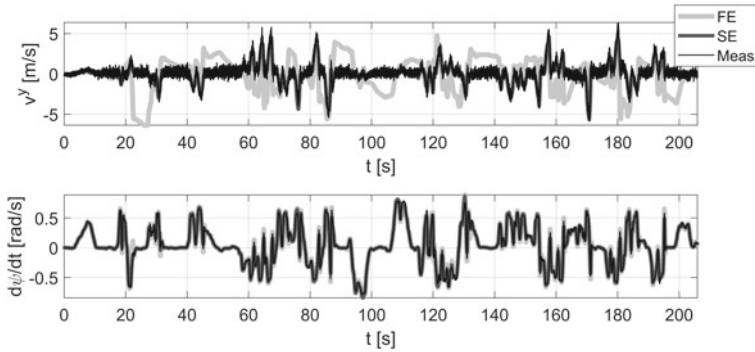


Fig. 13 Lateral state estimates from augmented state estimators

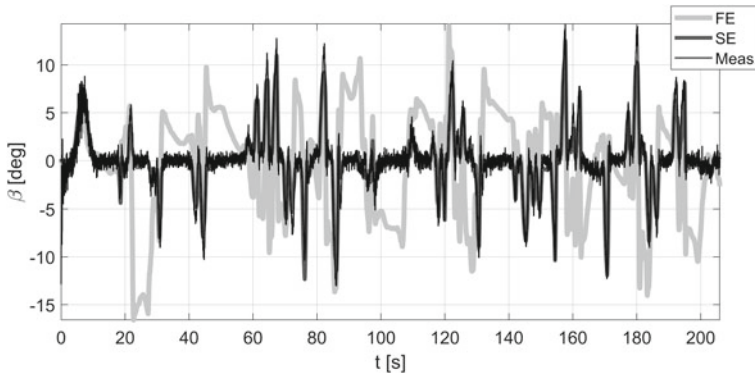


Fig. 14 Vehicle sideslip angle from augmented state estimators

lateral velocity, significantly better than the regular lateral state estimator. The FE approach, on the other hand, shows much larger deviations. This also translates in the vehicle body slip angles which can be post-processed from this data, as shown in Fig. 14. It is now also interesting to compare the equivalent lateral axle forces are the front and the rear for both the FE and SE estimation scheme. These forces are shown in Fig. 15. Even though both estimators show very different estimates for the lateral velocity and yaw rate, the resulting lateral force estimates are very close. In the next section we will investigate how we can employ this information to identify the vehicle tire models. As a result of the poor approximation of the states however, the data obtained from the FE approach is not suitable for this identification. As we will see, the data resulting from the SE approach can yield meaningful tire models.

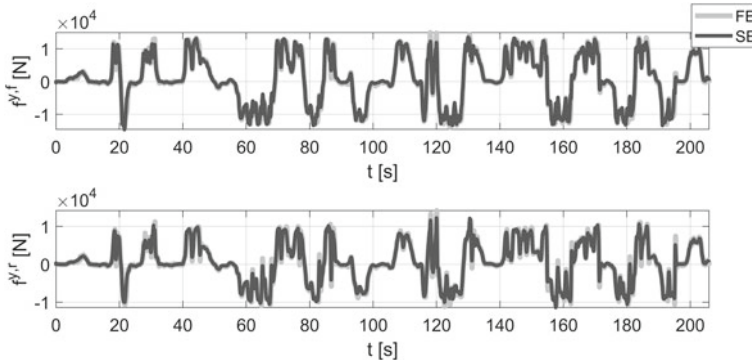


Fig. 15 Estimated lateral forces for augmented state estimators

4.4 Post-processing for Tire Model Extraction

From both of the joint estimators shown here, characteristics can be extracted for the estimated lateral tire forces $\bar{f}^y$ as a function of the estimated slip angles $\bar{\alpha}$ at the tires (see also Naets et al., 2017). The operational assessment of these characteristics for a given vehicle is very valuable as this is the type of data for which classically highly dedicated tests would need to be performed for a specific tire. In a post-processing of the estimation results, these estimated characteristics can be employed to perform the fitting of a more complex tire model which also accounts for effects like saturation.

We propose to apply a general nonlinear least-squares fitting method for the tire parameters. This leaves us with a very flexible framework to estimate different tire models. This framework aims to find the parameters for the tire model $\mathbf{p}^t$ which best fit the estimated characteristic. Potentially a weighted least squares problem exploiting the variance from the estimator could be performed. However here we chose not to do so, as the covariance values are known to not be reliable in general (see also Lefebvre et al., 2004). The minimization problem to solve is:

$$\min_{\mathbf{p}^t \in \mathbb{R}^m} \frac{1}{2} \left\| \hat{\mathbf{f}}^y(\bar{\alpha}, \bar{\mathbf{f}}^z, \mathbf{p}^t) - \bar{\mathbf{f}}^y \right\|_2^2 \quad (142)$$

where the parameter vector $\mathbf{p}^t$ are the parameters for the tire model $\hat{f}^y$ which needs to be identified. Note that depending on the tire model, also an estimate of the normal tire load $\bar{f}^z$ might be required. The different time estimates are combined in different vectors. For the tire model this becomes:

$$\hat{\mathbf{f}}^y = \begin{bmatrix} \hat{f}^y(\bar{\alpha}_1, \bar{f}_1^z, \mathbf{p}^t) \\ \vdots \\ \hat{f}^y(\bar{\alpha}_n, \bar{f}_n^z, \mathbf{p}^t) \end{bmatrix}, \quad (143)$$

for n time samples. The slip angles α can be evaluated for the front and rear axle according to Eq. (136). In order to estimate the normal load, a simple approximation suffices as the used bicycle model does not distinguish between the effect of the right and left tire during cornering, and hence the lateral load transfer is not taken into account. In this case the vertical load transfer is only based on the fore-aft transfer with the measured accelerations:

$$\bar{f}^{z,f} = mg \frac{l^r}{l^r + l^f} - a^x m h^{COG}, \quad (144)$$

$$\bar{f}^{z,r} = mg \frac{l^f}{l^r + l^f} + a^x m h^{COG}. \quad (145)$$

In these equations g is the gravitational acceleration and h^{COG} is the constantly assumed height of the COG with respect to the road. For the data obtained from the FE approach, the $\bar{f}^y$ vector is directly retrieved from the estimator:

$$\bar{\mathbf{f}}^y = \begin{bmatrix} f_1^y \\ \vdots \\ f_n^y \end{bmatrix}. \quad (146)$$

In the case of the SE approach, the $\bar{f}^y$ vector is obtained as:

$$\bar{\mathbf{f}}^y = \begin{bmatrix} c_1^y \bar{\alpha}_1 \\ \vdots \\ c_n^y \bar{\alpha}_n \end{bmatrix}. \quad (147)$$

An important point is the selection of the number of time samples n to perform this identification. A long sampling window in time will probably capture more of the tire behavior, thus ensuring a reliable fitting of the saturation parameters. However, as the tire behavior is varying in time due to varying road conditions, tire wear, etc. the window should not be too long. Otherwise it can be expected that most common tire models will fit poorly as they are not designed for these varying conditions. In order to make the identification more efficient, sample points related to (close to) straight driving phenomena can be omitted as these will typically be rather noisy and carry little information on the actual tire behavior.

Many methods exist to solve nonlinear least-squares problems of the above presented form, see for example Gill and Murray (1978), and here we employ a trust-region Gauss-Newton approach. This algorithm is based on the Jacobian of the identified tire forces:

$$\mathbf{J}_p = \frac{\partial \hat{\mathbf{f}}^y}{\partial \mathbf{p}^t}. \quad (148)$$

The algorithm is summarized for this application in Algorithm 1.

As the number of parameters for most tire-models are relatively limited (less than ten), the inversion of the squared Jacobian does not lead to large computational loads. The optimization is terminated when negligible improvement is obtained for the fit. Many alternative schemes can of course be applied, and many numerical tools exist to perform this type of optimization. A key aspect in these optimization is to have good initial guess for the parameters $\mathbf{p}_0$. Below we will discuss several tire models which could be used in this scheme and some good starting values for the parameters.

Some typical tire model which can be employed to perform this fitting are:

- *Bilinear tire model*: the bilinear tire model is a basic model which accounts for saturation of the tire. This simple physics based model assumes linear tire behavior until the friction limit is reached. The lateral tire force (f^y) equations for this model are given by:

$$f^y(\alpha, f^z, p_t) = \begin{cases} c^y \alpha, & \alpha \leq (f^z \mu / c^y) \\ \mu f^z, & \alpha > (f^z \mu / c^y) \end{cases}. \quad (149)$$

Algorithm 1 Trust-region Gauss-Newton for tire model fitting.

Input: Estimated tire sideslip angle α , estimated normal load $\bar{f}^z$, estimated tire-forces $\bar{\mathbf{f}}^y$

```

1: Initialize  $\mathbf{p}_0$ 
2:  $\hat{\mathbf{f}}_0^y = \hat{\mathbf{f}}^y(\bar{\alpha}, \bar{f}^z, \mathbf{p}_0)$ 
3:  $k = 0$ 
4: while  $\|\hat{\mathbf{f}}_{k-1}^y - \bar{\mathbf{f}}^y\| - \|\hat{\mathbf{f}}_k^y - \bar{\mathbf{f}}^y\| \geq tol$  do
5:    $\mathbf{J}_p = \mathbf{J}_p(\bar{\alpha}, \bar{f}^z, \mathbf{p}_k)$ 
6:    $\Delta = -(\mathbf{J}_p^T \mathbf{J}_p)^{-1} \mathbf{J}_p^T (\hat{\mathbf{f}}_k^y - \bar{\mathbf{f}}^y)$ 
7:    $a = 0.5, k = k + 1,$ 
8:    $\mathbf{p}_k = \mathbf{p}_{k-1} + \Delta$ 
9:    $\hat{\mathbf{f}}_k^y = \hat{\mathbf{f}}^y(\bar{\alpha}, \bar{f}^z, \mathbf{p}_k)$ 
10:  while  $\|\hat{\mathbf{f}}_{k-1}^y - \bar{\mathbf{f}}^y\| \geq \|\hat{\mathbf{f}}_k^y - \bar{\mathbf{f}}^y\|$  do
11:     $\Delta = a \Delta$ 
12:     $\mathbf{p}_k = \mathbf{p}_{k-1} + \Delta$ 
13:     $\hat{\mathbf{f}}_k^y = \hat{\mathbf{f}}^y(\bar{\alpha}, \bar{f}^z, \mathbf{p}_k)$ 
14:  end while
15: end while
```

Two parameters need to be determined, being the linear tire stiffness c^y and the friction coefficient μ , such that:

$$\mathbf{p}^t = \begin{bmatrix} c^y \\ \mu \end{bmatrix}. \quad (150)$$

The Jacobian $\mathbf{J}_p$ of this tire force with respect to the parameters can be obtained as:

$$\mathbf{J}_p(\alpha, \mathbf{f}^z, \mathbf{p}^t) = \begin{cases} \begin{bmatrix} \alpha, 0 \end{bmatrix}, & \alpha \leq (f^z \mu / c^y) \\ \begin{bmatrix} 0, f^z \end{bmatrix}, & \alpha > (f^z \mu / c^y) \end{cases}. \quad (151)$$

This Jacobian, which is very low cost to evaluate, enables a particularly efficient fitting. A good initial guess for the initial parameters $\mathbf{p}_0^t$ for the bilinear model is:

$$\mathbf{p}_0^t = \begin{bmatrix} \max(\bar{f}^y / \bar{f}^z) \\ \max(\bar{f}^y / \bar{\alpha}) \end{bmatrix}. \quad (152)$$

- *Dugoff tire model*: this model is a more advanced physics inspired tire model with only two parameters, but which provides a gradual transition from the linear tire operation to the saturated behavior, as was discussed by Dugoff et al. (1969) and Guntur and Sankar (1980). The governing equations for the lateral tire model (assuming zero longitudinal slip) are:

$$\lambda = \begin{cases} \frac{f^z \mu}{2c^y \tan(\alpha)}, & \alpha \neq 0 \\ 1, & \alpha = 0 \end{cases}, \quad (153)$$

and

$$f^y(\alpha, f^z, \mathbf{p}^t) = \begin{cases} c^y \tan(\alpha)(2\lambda - \lambda^2), & \lambda < 1 \\ c^y \tan(\alpha), & \lambda \geq 1 \end{cases}. \quad (154)$$

The physically interpretable parameters for this model are the linear lateral tire stiffness c^y and the tire-road friction μ . In this formula, λ determines if saturation effects come into play, but the course of the saturation behavior cannot be adjusted through the two parameters. The parameter vector $\mathbf{p}^t$ for this model is:

$$\mathbf{p}^t = \begin{bmatrix} c^y \\ \mu \end{bmatrix}. \quad (155)$$

It is interesting to notice that even though these parameters are physically the same as the bilinear model, the fitting process is much more nonlinear. The Jacobian J_p can be evaluated as:

$$J_p(\alpha, f^z, \mathbf{p}^t) = \begin{cases} \begin{bmatrix} \tan(\alpha)((2\lambda - \lambda^2) + c^y(2 - 2\lambda)\frac{\partial \lambda}{\partial c^y}), \\ c^y \tan(\alpha)(2\frac{\partial \lambda}{\partial \mu} - 2\lambda\frac{\partial \lambda}{\partial \mu}) \end{bmatrix}, & \lambda < 1 \\ \begin{bmatrix} \tan(\alpha), 0 \end{bmatrix}, & \lambda \leq 1 \end{cases}, \quad (156)$$

with

$$\frac{\partial \lambda}{\partial c^y} = \begin{cases} \frac{-f^z \mu \tan(\alpha)}{8(c^y \tan(\alpha))^2}, & \alpha \neq 0 \\ 0, & \alpha = 0 \end{cases}, \quad (157)$$

$$\frac{\partial \lambda}{\partial \mu} = \begin{cases} \frac{f^z}{2c^y \tan(\alpha)}, & \alpha \neq 0 \\ 0, & \alpha = 0 \end{cases}. \quad (158)$$

For the parameter initialization, the same selection can be made as for the bilinear model for $\mathbf{p}_0^t$:

$$\mathbf{p}_0^t = \begin{bmatrix} \max(\bar{f}^y / \bar{f}^z) \\ \max(\bar{f}^y / \bar{\alpha}) \end{bmatrix}. \quad (159)$$

- *Magic formula*: the magic tire formulas proposed by Pacejka and Bakker (1992) are popular data based tire models. These models offer a modular definition where more and more nonlinear tire effects can be accounted for, with associated increase in the number of parameters. A possible downside of this approach is the lack of direct physical interpretation of the model parameters compared to the other two models. An important benefit of these models is the possibility to model the transition from static to sliding friction, which is of paramount importance e.g. in ABS control. In this work, the most basic magic formula employing four parameters is used:

$$f^y(\alpha, f^z, \mathbf{p}^t) = (f^z D) \sin(C \arctan(B(1 - E)\alpha + \arctan(B\alpha))), \quad (160)$$

with four unknown parameters $\mathbf{p}^t$:

$$\mathbf{p}^t = \begin{bmatrix} B \\ C \\ D \\ E \end{bmatrix}. \quad (161)$$

Also here the Jacobian $\mathbf{J}_p$ can be readily obtained:

$$\mathbf{J}_p(\alpha, f^z, \mathbf{p}^t) = \begin{bmatrix} (f^z D) \cos(C \arctan(B(1 - E)\alpha + E \arctan(B\alpha))) \frac{C}{1 + (B(1 - E)\alpha + E \arctan(B\alpha))^2} \left((1 - E)\alpha + \frac{E}{1 + (B\alpha)^2} \alpha \right) \\ (f^z D) \cos(C \arctan(B(1 - E)\alpha + E \arctan(B\alpha))) \arctan(B(1 - E)\alpha + E \arctan(B\alpha)) \\ f^z \sin(C \arctan(B(1 - E)\alpha + E \arctan(B\alpha))) \\ (f^z D) \cos(C \arctan(B(1 - E)\alpha + E \arctan(B\alpha))) \frac{C}{1 + (B(1 - E)\alpha + E \arctan(B\alpha))^2} (-B\alpha + \arctan(B\alpha)) \end{bmatrix}^T. \quad (162)$$

The advantage over the previous models is the fact that this function is differentiable as it is fully continuous, which can be beneficial for the speed of convergence. For the magic formula, a generic set of initial parameters, corresponding to an average tire, can be used:

$$\mathbf{p}_0^t = \begin{bmatrix} 10 \\ 1.9 \\ 1 \\ 0.97 \end{bmatrix}. \quad (163)$$

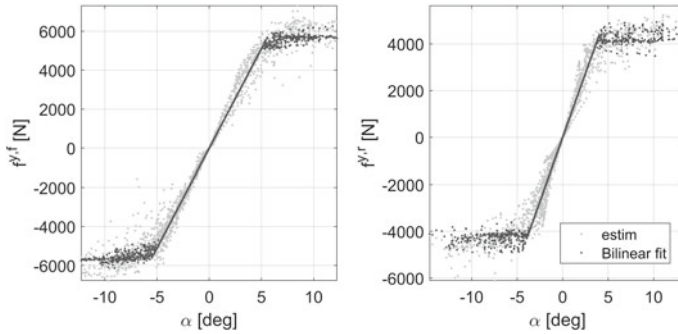


Fig. 16 Bilinear tire model identification

Many other tire models can be used in this context, but these three basic models provide a good trade-off with respect to accuracy and number of parameters.

4.4.1 Results for Identified Tire Models

Starting from the estimated quantities in Sect. 4.3.1, we can now perform the tire model identification as discussed here. Given the unreliable nature of the lateral states obtained for the FE approach, we here employ the data resulting from the SE estimation.

The identification results for the bilinear model for the front and rear tires are shown in Fig. 16. The resulting tire-slip/force samples clearly show a very typical behavior as can be expected for a tire. Moreover, it is also clear that saturation of the tire occurs, which explains why the lateral estimator with a simple linear model did not provide consistent estimates. The bilinear model can easily be fitted for the front and rear tires. The obtained equivalent front stiffness is $c^{y,f} = 58.5 \text{ kN/rad}$ and the equivalent rear stiffness $c^{y,r} = 62.5 \text{ kN/rad}$. This is slightly different from the values used in the regular lateral state estimator, and indicates that the vehicle will be slightly more understeered than the model used for the lateral estimator.

Similarly, the Dugoff tire model, as shown in Fig. 17, and Pacejka tire model, as shown in Fig. 18, can be fitted to this data. The Pacejka model clearly shows the best fit with the experimental data, which is to be expected as it also has the most parameters for fitting the nonlinearity of the tire.

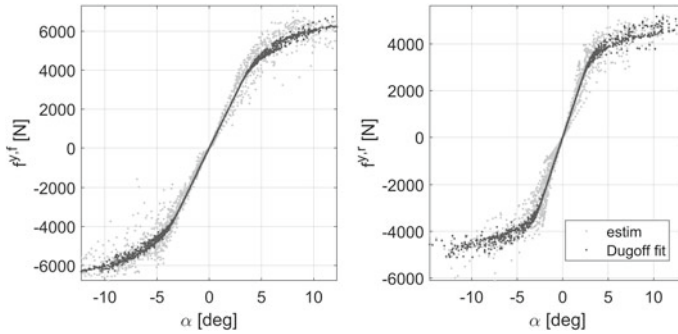


Fig. 17 Dugoff tire model identification

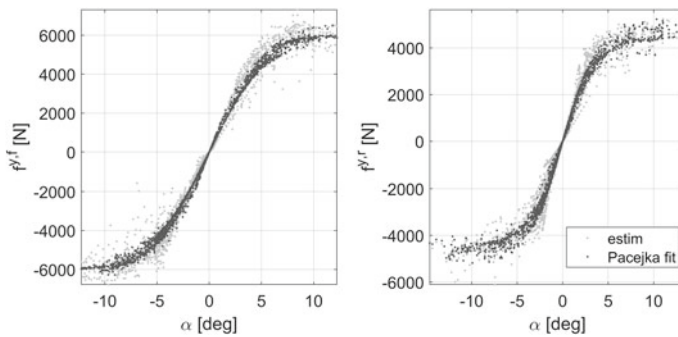


Fig. 18 Pacejka tire model identification

5 Conclusion

In this chapter we have investigated how state-estimation methods, and more in particular the Kalman filter, can be exploited for vehicle dynamics. Starting from the basic background on estimation methods, several noteworthy applications have been presented. First we deployed the basic theory to give a comparison for different ways to estimate longitudinal vehicle velocity, ranging from a simple linear kinematic model to a nonlinear model with tire forces. The methodology is then applied to lateral vehicle state estimation, to show that this relatively complex phenomenon can be approximated by a linear model and hence a linear estimator. However, the main pitfall for this approach are the rather uncertain vehicle parameters, and in particular the uncertain tire parameters. In order to alleviate this issue, we presented the augmented Kalman filter scheme which can be exploited to estimate inputs (like tire forces) or parameters (like tire stiffnesses). The joint estimators for lateral state/tire force and lateral state/tire parameters are presented. Several arguments, starting from basic observability theory, are given to highlight the benefits of the lateral state/tire parameter estimation approach, and these are supported by experimental results. We

have finally shown a framework which can be exploited to extract a full nonlinear tire model, starting from these operational estimation which require only minimal instrumentation.

The Kalman filter based estimators presented in this chapter only pose the basics of what can be achieved through state-estimation approaches in vehicles. However, they already demonstrate the large amount of information which can be gained from a limited sensor set through the judicious selection of a vehicle model and a proper design of an estimator.

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Automated Driving Vehicles



Kyongsu Yi and Bongsob Song

Abstract This chapter introduces a full scope of autonomous vehicle technology ranging from perception to control. While a literature review for commercial environment sensors and sensor fusion to both model-based and data-driven approaches is summarized in Sect. 2, detailed decision and motion planning strategies for urban driving situations as well as highway are described. Moreover, some experimental results are demonstrated in Sect. 3. In Sects. 4 and 5, lateral and longitudinal control methods are proposed respectively in the hierarchical structure. The several upper controllers are designed to track the desired path and their performances are compared experimentally. The torque controller for a steering actuator as a lower controller is designed based on nonlinear control techniques. Similarly, the upper and lower controller for longitudinal control with consideration of human driver characteristics is proposed for better naturalistic driving performance. Finally, a scenario-based approach for validation of the autonomous vehicle technology including all proposed algorithms is investigated in the end.

1 Introduction

1.1 *The Role of the Driver*

Although automotive vehicle driver was considered to be a job with special skills in 1970s in Korea, nowadays driving a car on complex urban roads is not a challenging task that most people can safely conduct without significant difficulties. In driving a car, safety is perhaps the biggest concern. In order to drive a car safely, all the dynamic driving tasks should be conducted successfully by the driver. The dynamic

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driving tasks mean all of the real-time functions required to operate a vehicle in on-road traffic and consist of:

- Object and event detection, recognition, and classification
- Object and event response
- Real-time mission planning
- Steering, turning, lane keeping, and lane changing
- Acceleration and deceleration
- Enhancing conspicuity: lighting, signaling and gesturing, etc.

The role of an automotive vehicle driver consists of navigation, guidance and control. The navigation implies choice of route and route planning from the start point to the goal destination. Guidance means the choice of course on roads to achieve vehicle safety, driving comfort, and driving effectiveness. Control implies applying steering wheel, throttle, brake pedal inputs for the generation of vehicle motions in path tracking, turning, lane changing with safe clearance to adjacent moving vehicles and static obstacles. Although human driving may be far from optimal [Macadam], an attentive driver can successfully conduct all the dynamic driving tasks for safe and comfort driving on normal roads. It has long been recognized that more than ninety percent of the vehicle accidents are due to the human failure, i.e., drivers' attention switched away from the driving task results in most of the vehicle accidents. In the case when a driver experiences abnormal road condition such as black ice or split tire-road friction, the driver loses control of the vehicle and encounters an accident due to unpredictable and uncontrollable dynamic behavior of the vehicle (Fig. 1).

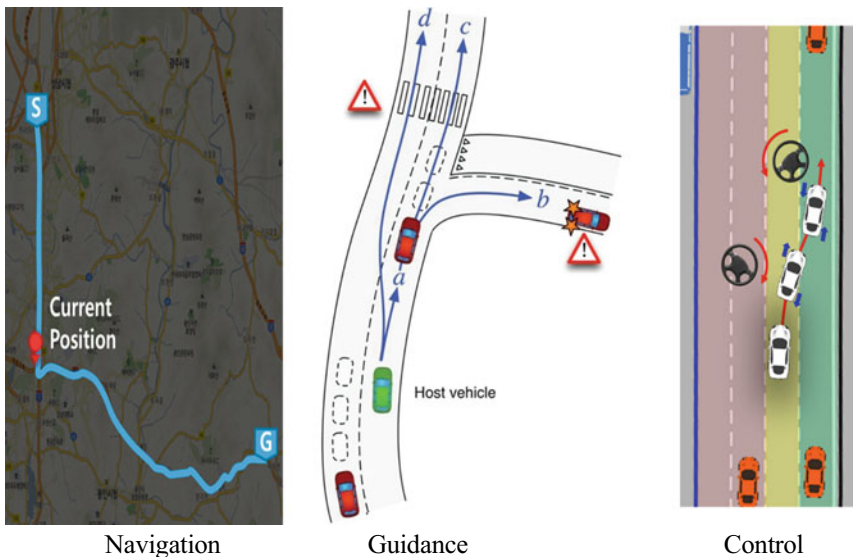


Fig. 1 The role of the driver: navigation, guidance, and control [Gordon and Lidberg, Vehicle System Dynamics (2015)]

1.2 Advanced Driver Assistance Systems and Automated Driving Systems

In order to assist a human driver and prevent vehicle collision or reduce severity of the collision, many advanced driver assistance systems have already been commercialized.

The commercialization is largely due to a significant progress in sensors, actuators and real time software technologies for automotive vehicles. In addition, recently, level 2 automated driving systems have been introduced by several auto manufacturers and level 3 and 4 automated driving systems are expected to appear in between 2020 and 2024. The higher levels of automation will not be viable with an autonomous approach but will require a cooperative approach with V2V and V2I cooperation. Therefore, the potentially large changes in transportation and urban development are going to be taken even further in the future.

Automated driving vehicles developed by Future Mobility Technology Center of Seoul National University, Waymo and Daimler Benz are shown in Fig. 2. A typical configuration of environment sensors, controller, and actuators of an automated driving vehicle is shown in Fig. 3. An automated driving system implies a combination of the hardware and software that conduct all aspects of the dynamic driving task for a vehicle. As illustrated in Fig. 3 the hardware consists of the environment sensors such as radars, vision cameras, LiDAR, and around view monitoring camera, GPS, controller, and vehicle interface. Radar and vision camera have been already commercialized for ADAS applications and LiDAR is expected to be commercialized soon for automotive applications.

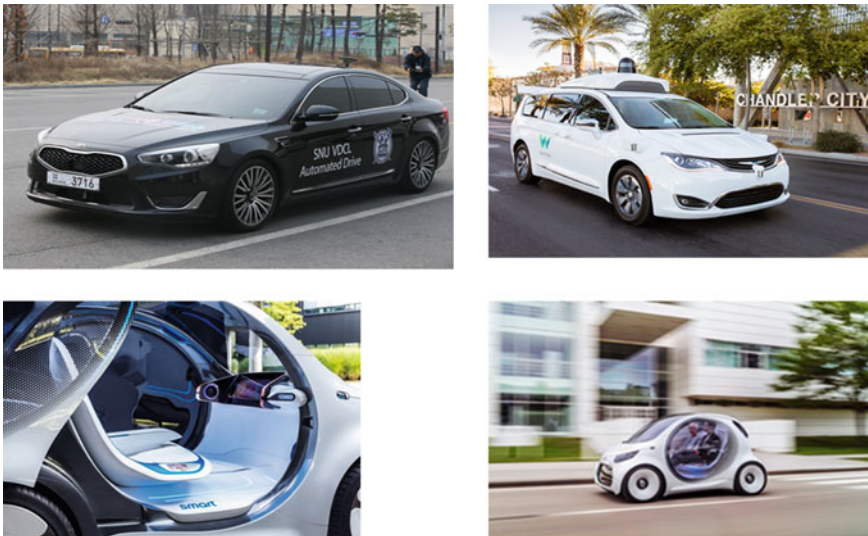


Fig. 2 Automated driving vehicles (*Photo FMTC SNU, Waymo, Daimler Benz*)

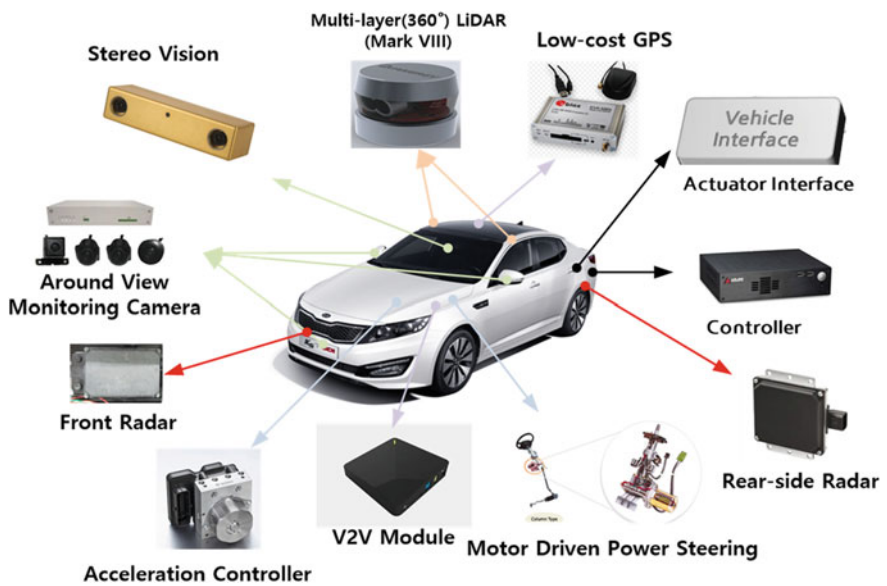


Fig. 3 Automated driving vehicle sensors, controller, and actuators

The Configuration of Functional Modules of an Automated Driving Vehicle System is illustrated in Fig. 4. Key components of the automated driving system software are localization, perception, decision and planning, and control modules. An automated driving vehicle needs to know where it is in the world, using some local coordinate system. It also need to know the geometry of stationary features such as lane marks, stop line, road boundaries, and intersections. The position and orientation or heading angle of an automated driving vehicle is needed in real time. Although the obvious way to obtain the position and heading information of an automated vehicle is via satellite navigation systems, it is very expensive and insufficiently available to obtain them with accuracy sufficient for automated driving in urban environments.

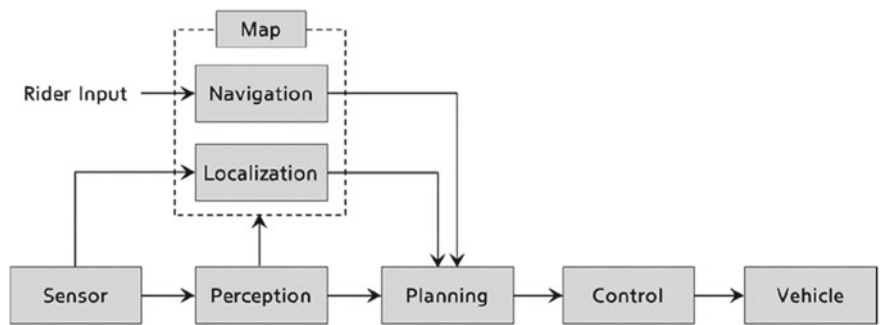


Fig. 4 Configuration of functional modules of an automated driving vehicle

A reasonable and sufficient pose information can be obtained by combining all the information from a low-cost GPS, vision sensors, and onboard vehicle sensors such as wheel speed sensors, accelerometer and a yaw rate sensor.

The perception module detects moving and static obstacles, lane marks, and stop lines, tracks the moving targets and generates static obstacle map for driving mode decision and motion planning of the automated driving vehicle. This information is obtained by fusing all the information from the environment sensors.

The Decision and planning module determines driving mode such as lane keeping, lane change, clearance control, intersection path generation and tracking, etc., and generates desired motion of the automated vehicle.

The control module determines the vehicle control inputs to be applied to the vehicle to generate desired vehicle motion so that the vehicle tracks the desired path with acceptable deviation error and maintains safe clearance to the target vehicle. The vehicle control inputs are the throttle, brake and steering wheel angle or vehicle acceleration command and steering angle, depending on the vehicle interface module.

1.3 Concluding Remarks

It has been well recognized that great benefit can be obtained by the use of autonomous driving vehicles. Autonomous self-driving vehicle-based smart mobility systems can be a solution to many problems we are encountering in an urban mega city nowadays. The benefits from the urban autonomous mobility ranges various aspects: improved road safety, decrease in pollution, freed up space, increased traffic efficiency, reduced cost of mobility, less waiting time, equitable access to mobility, and productivity boost, etc. [Berdichevskiy].

Still there exist technical challenges to be overcome in order to achieve ubiquitous autonomous driving functionality. Definitely significant technical progresses such as environment perception systems, intention inference with artificial intelligence, and validation of automated driving safety functionality will be conducted in the near future.

Automated vehicles can be a good solution to mitigate the impact of the pandemic in the area of mobility. A self-driving and, maybe, self-sanitizing vehicle, e.g. could provide a kind of protective mobility shell for the vulnerable society members that would be superior to any kind of public transport in terms of infection safety [Meyer]. There is a lot of potential in connected automated vehicles in the post-corona era (Fig. 5).

2 Sensor Fusion

The sensor fusion is one of the key technologies for autonomous vehicles (AVs) and evolves rapidly due to introduction of innovative technologies, e.g., deep learning

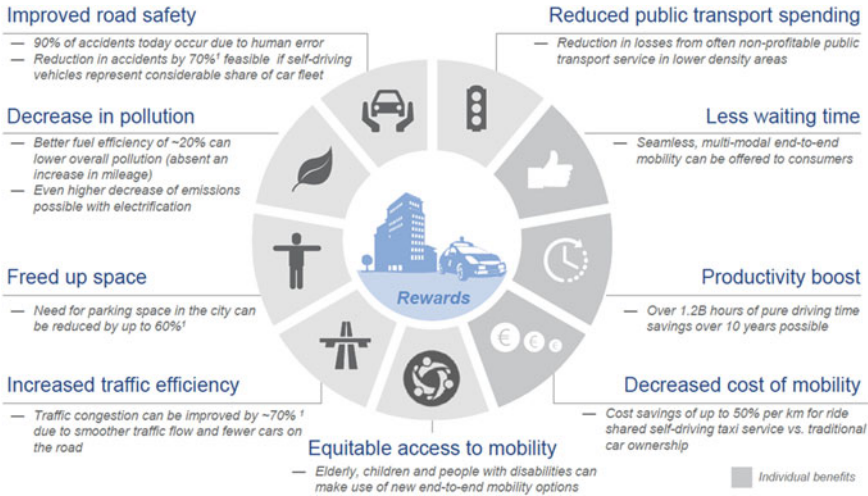


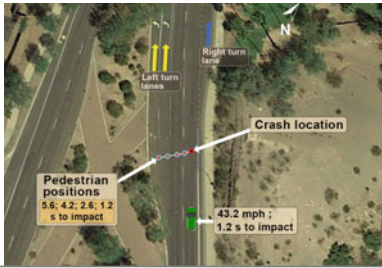
Fig. 5 Why autonomous smart mobility: benefits from urban autonomous mobility (from Berdichevskiy, 2017)

(DL), high resolution sensors, cloud, and high definition (HD) map. At the same time, the higher reliability of perception is required to guarantee the safety of the autonomous vehicle. For instance, it was reported by National Transportation Safety Board (NTSB) that the autonomous vehicle tested and operated by Uber struck a pedestrian in Tempe, Arizona on March 18, 2018 (Becic et al., 2019). As shown in a road scene of Table 1, the female pedestrian walked her bicycle east across Mill Avenue and was identified as an *unknown* and stationary object at 1.5 s before collision. Although the unknown object was partially on the path of the sport utility vehicle (SUV), the appropriate collision avoidance was not activated, i.e., the suppression of its motion plan (action suppression) was initiated and continued during a given time period (e.g., one second), and the automatic emergency brake (AEB) system was thus not engaged. At 1.2 s a prior, despite the correction of object classification to bicycle, the AEB was not activated because a fallback strategy to the unexpected event was not appropriate (i.e., the action suppression was not designed safely with respect to unknown scenarios). The accident motivated to initiate collaborative interaction of Volvo and Uber to enhance the safety. It may be expected to include collaborative sensor fusion, which combine sensor fusion of lidar and camera by Uber (see environmental sensors on the roof of autonomous vehicle in Table 1) with one of radar and camera by Volvo (see also the in-vehicle sensors in the table).

As inferred in this accident, three aspects for sensor fusion are discussed in this section. The first hypothesis is that there is no sensor with 100% detection performance. This motivates the following considerations for sensor fusion:

- Sensor characteristics: state-of-the-art sensors (e.g., a lidar with 360° field of view, a high-resolution radar, a HD camera), detection characteristics of commercial

Table 1 Selected parameters recorded by the ADS (Becic et al., 2019)

Time (s) relative to impact	Classification and path prediction	Sensors/accident scenario
−2.5	Classification: bicycle by lidar Path prediction: the left through lane (adjacent to the SUV), not on the path of the SUV	Self Driving Vehicle (SDV) Overview The mounted radar units provide a 360° 3-dimensional scan of the environment Side and rear facing cameras assist in a continuous view of the lane, vehicle's surroundings Roof mounted ultrasonic sensors detect objects in close proximity and measure their dimensions Forward facing cameras assist driver both visually and for lane keeping, for vehicle control, including monitoring lane, traffic lights, and signals Custom designed hardware and storage allow for real time processing of data while a fully integrated sensing system monitors components running software 
−1.5	Classification: <i>unknown</i> by lidar Path prediction: <i>static</i> , <i>partially</i> on the path of the SUV	
−1.2	Classification: bicycle by lidar Path prediction: the travel lane of the SUV, <i>fully</i> on the path of the SUV	

- sensors (e.g., ghost targets coming from a radar, performance degradation of a camera due to weather and light conditions)
- Sensor configuration: combination of heterogeneous sensors (i.e., radar, lidar, and vision), the number of sensors and the corresponding installation position
 - Use of virtual sensors: in-vehicle sensors such as speed, steering wheel, yaw-rate sensors, HD map, V2X communication, traffic information on cloud, cloud-connected strategy.
- The second hypothesis is that adaption of data-driven approaches or DL is inevitable. It is related to the following questions:
- Sensor fusion architecture: end-to-end sensor fusion, a hybrid structure of model-based and data-driven approaches or an integrated approach
 - Methodologies: recent techniques of machine learning and deep learning, generation of database, update and/or upgrade strategy.

The last hypothesis is that there is still false perception (e.g., false classification, recognition, and scene understanding) resulting from sensor fusion. Since the performance of sensor fusion is directly related to decision of safety assessment and fallback (i.e., minimal risk condition management), it is required to safeguard the sensor fusion in the presence of performance degradation or unexpected hazards. With consideration of the three hypotheses, the sensor fusion thus evolves into multi-source and heterogeneous information fusion based on different physical and virtual sensors, self-learning, and real-time performance monitoring. It aims at a more comprehensive and reliable perception and recognition of the environment or target in order to avoid the limitations and uncertainties of a single sensor.

2.1 Sensor Fusion Configuration

Before stating a sensor fusion problem, it is a prerequisite to specify sensor configuration as well as sensor characteristics. For advanced driver assistance systems (ADAS) such as adaptive cruise control (ACC), AEB, and parking assistance systems (PAS), a millimeter-wave radar and a camera have been commercialized for ACC and AEB, and either multiple ultrasonic sensors or their combination of ultrasonic and camera sensors are generally used for PAS. For the AVs, it is easily expected that a larger set of sensors with high resolution and performance may be necessary. In the literature, seven kinds of sensors are primarily considered, i.e., cameras (mono/stereo/surround/infrared), radar (long/medium/short range), a global positioning system (GPS/DGPS/RTK-GPS), inertial measurement unit (IMU), lidar (single/multiple channels), ultrasonic (short/medium range), and communication (V2I/V2V/V2P) module (Wang et al., 2019). It is furthermore shown that their detection coverage and the corresponding applications are introduced in Fig. 6. It is inferred that the radar, lidar, and camera are at least needed for 360 field of view (FOV) of AVs. It is also noted that the installation positions may be limited due to their size and design aspect (e.g., no sensor on the roof) and auto calibration is necessary due to re-installation (e.g., resulting from A/S, mis-orientation due to physical collision) if AVs are deployed at full scale. These issues are also related to the problem statement of sensor fusion.

Various sensors have their advantages and disadvantages, so there are often different task divisions in the automated driving system (Girardin & Mounier, 2015).

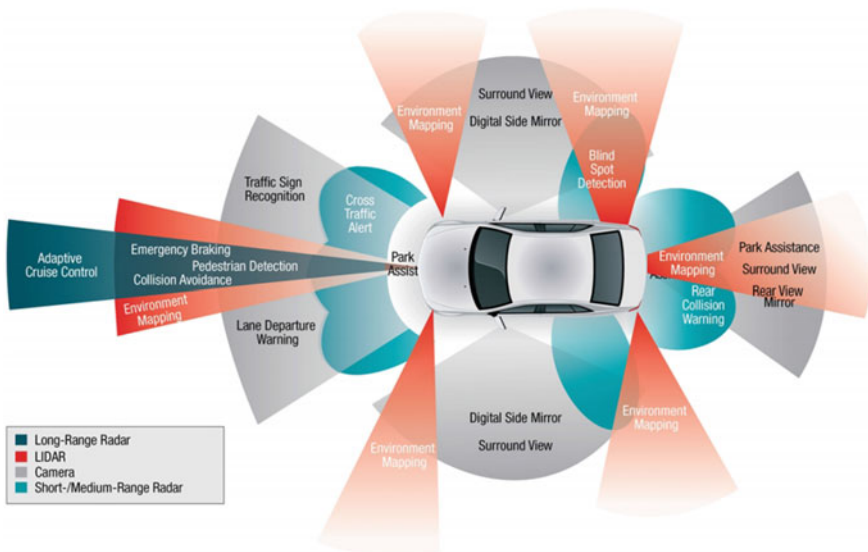


Fig. 6 Sensor configuration (source SAE International)

Their pros and cons are summarized in Table 2 (Wang et al., 2019). The detection capability and reliability of different sensors are limited in different environments, and sensor fusion can thus improve the accuracy of object detection and recognition. Possible sensor configuration for AV is exemplified in Table 2. It is noted that many different combinations of sensors with respect to specific functions of AVs will be introduced in the market.

Currently a primary sensor combination of commercial radar and camera are the most used for ADAS (i.e., AV level 1 or 2) because this combination can obtain excellent resolution as well as the distance information of the surrounding objects. That is, the camera can obtain the contour, texture, and color distribution of the object and the radar has longer wave lengths, which can penetrate rain, snow, and fog. However, there are still the drawbacks as shown in Fig. 7. The radar has no classification of detected objects, relatively low accuracy of lateral position measurement (thus resulting in discrepancy between measurements of radar and camera as shown in Fig. 7b, d) and is more susceptible to clutter interference (see also in Fig. 7a–d). The camera has light interference and performance degradation to bad light condition and weather (see also in Fig. 7e, f). The application of stereo and depth cameras allows image data to have depth information and one of infrared camera is used to obtain further thermal imaging information and to sense potential pedestrians and animals, but there is still a long way to go for a high accuracy (Ma et al., 2017). It is

Table 2 Sensor configuration of AV and sensor characteristics

AV	Sensor configuration (source: Yole development)	Sensor characteristics (Wang et al., 2019)
Level 2	– Ultrasonic sensor: 4 – Long Range Radar: 1 – Camera: 1	• Ultrasonic sensor – Inexpensive – Low resolution – Inapplicable for high speed • Radar – Long working distance – Available for radial velocity – Applicable for all-weather – Generating false alarm easily • Camera – Excellent discernibility – Available lateral velocity – Heavy calculation burden – Light interference – Weather susceptible – Unavailable for radial velocity easily • Lidar – Wide field of view (FOV) – High range and angle resolution – Insufferable for bad weather
Level 3	– Ultrasonic sensor: 8 – Radar: 5 (Long: 1, Short: 4) – Lidar (180° FOV): 2 – Camera: 4 – GPS	
Level 4	– Ultrasonic sensor: 10 – Radar: 8 (Long: 2, Short: 6) – Camera: 8 (Long: 2, Stereo: 1, Surround: 5) – Lidar (360° FOV): 1 – GPS, IMU, V2X	
Level 5	– Ultrasonic sensor: 10 – Radar: 8 (Long: 2, Short: 6) – Camera: 11 (Long: 4, Stereo: 2, Surround: 5) – Lidar (360° FOV): 1 – GPS, IMU, V2X	

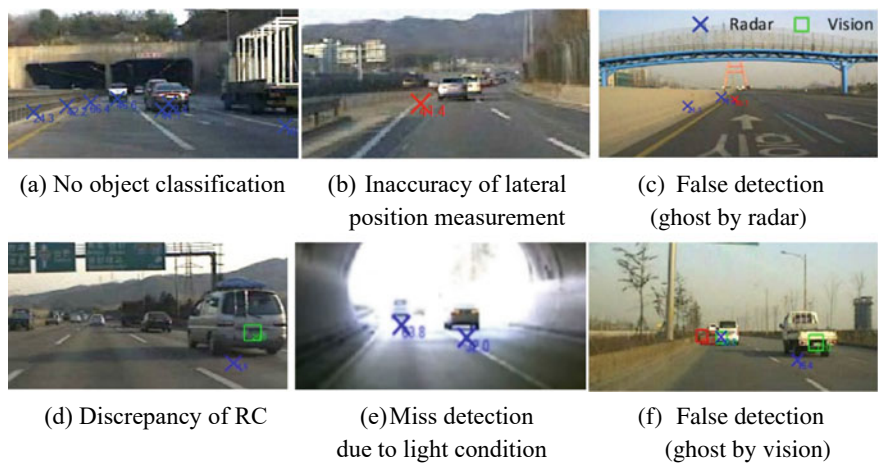


Fig. 7 Sensor characteristics of radar and vision (*source* Delphi and Mobileye)

thus said that the combination of radar and camera may be not enough for AV level 3 and above (Divakarla et al., 2018).

As sensor technology evolves rapidly, the high-resolution radar is commercialized for Advanced ACC or rear collision warning and mitigation (see also in Fig. 8a). Furthermore, the performance of lidar is growing gradually and it can work continuously in day and night, which provides high-resolution and long-distance 3-D data

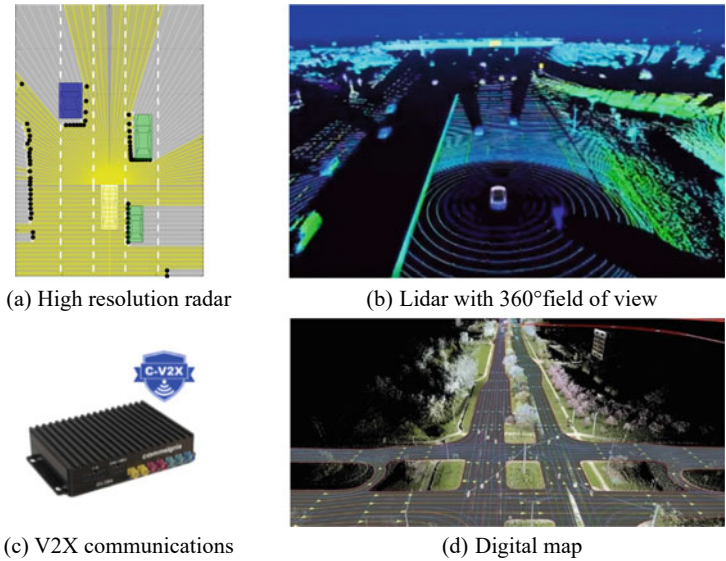


Fig. 8 Additional and virtual sensors (*source* Delphi, Velodyne, Qualcomm, HERE)

except for severe weather conditions as shown in Fig. 8b. However, the price of the high-performance product is still relatively high. There are additional heterogeneous sensors for AVs. A lidar and HD or infrared cameras may be adopted for the level 3 and above, thus resulting in the new combinations, e.g., radar-lidar (RL), camera-lidar (CL), radar-camera-lidar (RCL), have been developed. For instance, image information with depth can get from the combination of CL (Chen et al., 2017), and some studies uses the RCL configuration to improve safety redundancy (Chavez-Garcia & Aycard, 2015). More applications will be reviewed in Sects. 2.2 and 2.3.

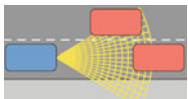
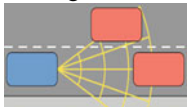
High definition (HD) or digital map and on-board V2X communications allow vehicles to share location information in real time. For example, the digital map is used to reconstruct road curvature in real time with the application to a curve over-speed warning system (Li et al., 2008). Most applications focus on localization (Fayyad et al., 2020). As shown in Fig. 8b, d, the combination of lidar and digital map has been used for simultaneous localization and mapping (SLAM) to improve position information of AVs. Finally, the cooperative active safety systems based on vehicle-to-vehicle (V2V) or vehicle-to-infrastructure (V2I) communications are demonstrated by National Highway Traffic Safety Administration (NHTSA) such as do-not-pass warning, emergency electronic brake lights, and intersection movement assist (Ahmed-Zaid et al., 2011). With the application of 5G or other higher frequency communication technologies, the data transmission rate of the connected AV will increase considerably. Although many studies are devoted to the fusion of multi-sensor data like radar, camera, and lidar, it is necessary to combine the sensing technology and vehicle communication and map technology to obtain sufficient reliability redundancy (Wang et al., 2019).

2.2 *Model-Based Approach*

The sensor fusion is classified into three-fold: raw-data fusion, feature fusion, and track-based fusion (Lindl & Walchshäusl, 2006). Roughly speaking, the raw-data fusion combines several raw data of radar, camera or lidar and the feature level fusion merges various features such as edges, corners, rectangles, or segments after the raw data are processed (see also multiple extended objects in Table 3). Although there are numerous studies of the low-level (i.e., raw-data and feature) fusion methods in the literature (Takizawa et al., 2004), the method based on deep learning has more advantages in accuracy of detection and classification without an artificial feature extraction process. That is, the fusion process based on features is embedded into the neural network architecture. Therefore, the very few researches on the low-level fusion based on the model-based approach was introduced in the recent literature due to the emergence of deep learning (Wang et al., 2019). The recent sensor fusion methods based on the deep learning will be introduced in Sect. 2.3.

The track-based (or high-level) fusion fuses object (or track) level data which are processed independently from each sensor. As summarized in Table 3, the sensor fusion transforms sensory data into semantic information via detection, tracking,

Table 3 Sensor fusion level and process

	Detection	Classification	Recognition
Low-level 	Multiple extended object – 2D/3D detection – Multiple object tracking	– Vehicle – Guardrail – Pedestrian – Cyclist – Lane mark – Drivable area – Traffic sign	– Function-specific (e.g., primary vehicle for ACC) – Maneuver (e.g., cut-in/out, lane-change) – Interaction aware (e.g., yield, traffic rule)
High-level 	Multiple-point object – Data association – Data fusion		

classification, and recognition of road agents. It is thus assumed in the high-level sensor fusion that the detection is already applied on each sensor data and it is tracked over time to provide best estimates on position and velocity of a point (a so-called track). The examples of the multiple-point tracks are shown in Fig. 7. Once the multiple-point objects are generated and estimated via data association and fusion of the multiple-point tracks, the object classification is aimed to identify the road agent (e.g., vehicles, pedestrians, cyclists, road infrastructure, etc.). Finally, the recognition is designed to deal with specific functions of AVs. For example a primary vehicle, which is the closest preceding vehicle in-lane, is recognized for adaptive cruise control (ACC). Cut-in and cut-out maneuvers may be recognized for traffic jam assistance (TJA) and the lane-change is identified for lane change assist (LCA) systems. For AVs, it is also necessary to recognize interaction between ego and surrounding vehicles such as yield and cut-in intention.

In the multiple-point object process in Table 3, Kalman filter-based methods are the most popular among model-based approaches (Jagannathan et al., 2018). The methods are classified by what type of model is used and what estimation technique is applied. For instance, to model object motion in terms of position, velocity and/or angular velocity, the various dynamic models such as constant velocity (CV), constant acceleration (CA), coordinated turn, Ornstein–Uhlenbeck, and Singer models are commonly used (Bar-Shalom et al., 2001; Blackman & Popoli, 1999; Li & Jilkov, 2003). Furthermore, significant progress in filtering, e.g., Kalman filter, extended Kalman filter, unscented Kalman filter, Gaussian sum filter, and particle filter, has been made during the last several decades (Vo et al., 2015). If one of the filtering algorithms mentioned previously is applied to a single dynamic model, it is called as a single-model filter. Despite great success of the single-model filter, it is limited because the motion of a maneuvering object involves multiple dynamic models. The multiple model approach is an effective filtering algorithm for maneuvering targets in which the continuous kinematic state and discrete mode or model are estimated. The interacting multiple model (IMM) and variable-structure IMM (VS-IMM) estimators are two well-known filtering algorithms for maneuvering targets (Bar-Shalom et al., 2001, Bar-Shalom et al., 2011, Li et al., 2000, Li & Jilkov, 2005, Mazor et al., 1998).

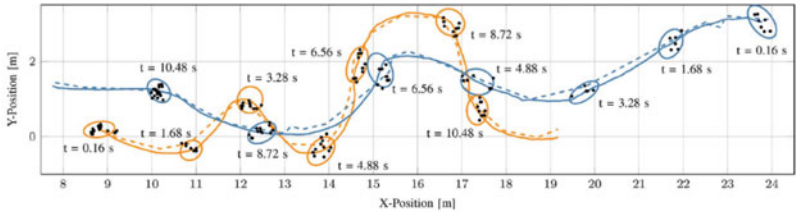
Based on various models and estimation techniques, numerous tracking algorithms have been proposed in the literature for data association and fusion. For instance, the global nearest neighbor (GNN) was intuitively appealing and simple to implement. It is however susceptible to track loss and consequently exhibits poor performance when targets are not well separated (Blackman & Popoli, 1999). Based on the probabilistic estimation technique such as Kalman filter, the probabilistic data association filter (PDAF) and joint probabilistic data association filter (JPDAF) have been used for single and multiple target tracking for more than four decades (Bar-Shalom & Tse, 1975; Bar-Shalom et al., 2011). Furthermore, integrating the interacting multiple model (IMM) with PDAF and JPDAF, IMM-PDAF and IMM-JPDAF has been applied for multiple target tracking for sensor fusion of radar and vision (Liu et al., 2008; Schreier et al., 2015).

Another approach for multiple extended object tracking is the multiple hypothesis tracking (MHT) which is a deferred-decision approach to association-based target-tracking data and is used for sensor fusion of radar and lidar (Lee et al., 2019). While the computational cost of the MHT is much higher than that of the GNN or JPDAF, it is shown in the literature that the MHT works significantly better than the GNN and JPDAF for tracking scenarios with low signal to noise ratio (SNR) and closely spaced targets (Blackman, 2004; Blackman & Popoli, 1999). Furthermore, the random finite set (RFS) based algorithms such as probability hypothesis density (PHD) filter, Gaussian Mixture PHD (GM-PHD) filter, have received considerable interest during the last two decades (Koch, 2008; Mahler, 2003; Vo & Ma, 2006). They are applied to AV applications based on single lidar or simulation data (Granstrom & Orguner, 2012; Granstrom et al., 2012; Orguner et al., 2011). It is demonstrated that a sub-optimal RFS-based filter called the cardinalized probability hypothesis density (CPHD) filter has comparable performance to MHT with much lower computational cost (Svensson et al., 2009). Moreover, a Gaussian inverse Wishart (GIW) method is applied using multi-target filters based on the random finite set (RFS) framework, thus resulting in GIW-PHD filter (Granstrom & Orguner, 2012), GIW-CPHD (Lundquist et al., 2013), and gamma Gaussian inverse Wishart CPHD (GGIW-CPHD) filter.

Recently proposed algorithms for multiple extended object tracking are the generalized labelled multi-Bernoulli (GLMB) filter and labelled multi-Bernoulli (LMB) filter (Reuter et al., 2014). It is shown that the GLMB outperforms both the PHD and CPHD filters with higher computational cost (Vo et al., 2014). In Kunz et al., (2015), LMB is proposed with three Ibeo Lux laser sensors (or lidar), mono camera, and three radars. Integrating GLMB with the GGIW model, the GGIW-GLMB can estimate the kinematics and extents of multiple extended targets in clutter, with the advantage of producing full target tracks (Beard et al., 2015). While more sensor fusion methods are based on heterogeneous sensors, the ones above are based on high-level fusion of homogeneous sensors, e.g., three Ibeo Lux laser sensors (or lidar) are preferred to low-level fusion or multiple extended object tracking (Beard et al., 2015). As an example, it is shown in Fig. 9 that the GGIW-LMB filter is applied to pedestrian tracking under the assumption that the measurements are distributed normally around the target centroid. It is also validated that two close objects are distinguished and tracked over time.



(a) Pedestrian tracking scenario: The two pedestrians are getting close at $t=6.5s$



(b) Ground truth (dashed), estimated trajectories (solid), and pedestrians at selected time instants (ellipsoid).

Fig. 9 Tracking result of GGIW-LMB (Beard et al., 2015)

Finally, the classification and recognition in Table 3 are discussed. The classification is mainly based on camera and the data association follows to fuse other sensors such as radar or lidar. The vision-based classification (or object detection and scene understanding) is one of the most active research areas in deep learning society. Therefore, it is recommended that the readers refer to many survey papers and references in the literature. One of the major challenges in sensor fusion is to detect dangerous situations and it requires predicting the likely evolution of the current traffic situation and assessing how dangerous that future situation might be. This role is called recognition in this section. It is classified into three model types: function-specific, maneuver-based and interaction-aware as shown in Table 3. The function-specific recognition depends on what function is intended. As described in Sect. 2.1, a primary vehicle is the most interested in the adaptive cruise control (ACC) system. Furthermore, both left and right vehicles in rear will be critical for lane change assistance (LCA) systems. Therefore, the vehicle recognition for surrounding vehicles may be defined as shown in Fig. 10. That is, three vehicles in front are defined as Front Vehicle In-lane/Left lane/Right lane (FVI/FVL/FVR) and the FVI is a primary vehicle for ACC. Similarly, the recognition of Rear Vehicle In-lane/Left lane/Right lane (RVI/RVL/RVR) via sensor fusion allows the LCA and rear collision warning system to work properly.

The maneuver-based and interaction-aware classification comes from an increasing degree of abstraction for motion modeling as shown in Fig. 11 (Lefèvre et al., 2014). While the physics-based motion models are mainly used in the detection and tracking process of Table 3, the maneuver-based models are more advanced

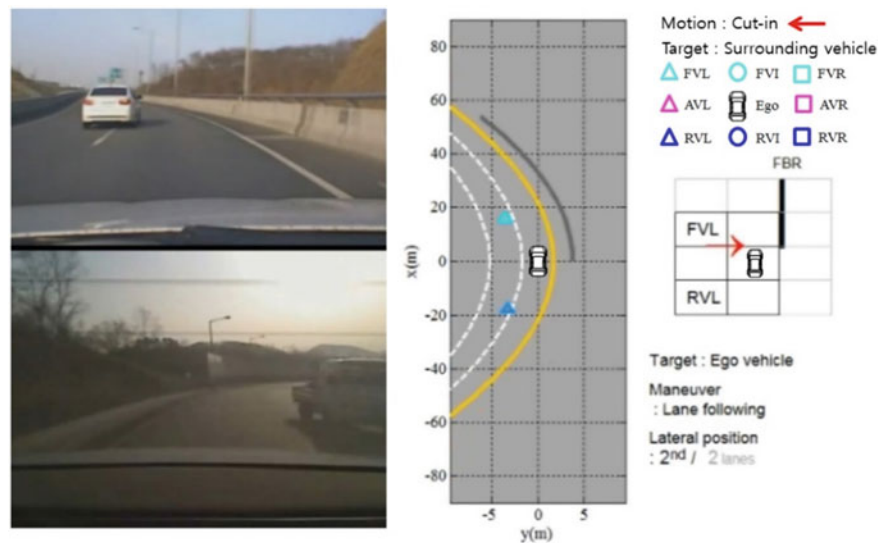


Fig. 10 Representation of vehicle recognition and maneuver for cut-in scenario-triangles (vehicle), grey solid line in the middle and Front Barrier in Right lane (FBR) in the last (guardrail), white dashed line and yellow solid line (lane marks), red arrow (maneuver)

	Target	Variables	Challenges	Tools
Symbolic	Interaction-aware models	- Social conventions. - Joint activities. - Communications.	- Detecting interactions. - Identifying interactions. - Combinatorial explosion.	- Coupled HMMs. - Dynamically-linked HMMs. - Rule-based systems.
	Maneuver-based models	- Intentions. - Perception. - Surrounding objects and places.	- Unobservability. - Complexity of intentional behavior.	- Clustering. - Planning as prediction. - Hidden Markov Models. - Goal oriented models - Reinforcement Learning.
Metric	Physics-based models	- Kinematic and dynamic properties.	- State estimation from noisy sensors. - Sensitivity to initial conditions.	- Kalman Filters. - Monte Carlo sampling.

Fig. 11 Abstraction level of motion modeling (Lefèvre et al., 2014)

and consider that the future motion of a vehicle also depends on the maneuver that the driver intends to perform, e.g., cut-in or cut-out intention. The interaction-aware model considers the inter-dependencies between vehicles’ maneuvers and traffic rules. For example, the preceding vehicle may wait temporarily at the intersection on green light if it cannot cross the intersection in the given time in a traffic jam situation.

Recent methods based on stochastic and data-driven approaches have been introduced in the literature for the recognition for maneuver-based and interaction-aware models, e.g., Hidden Markov Models (HMM) (Aoude et al., 2012, Berndt et al., 2008, Streubel et al., 2014), machine learning (Kumar et al., 2013), and Coupled Hidden Markov Models (CHMMs) (Agamennoni et al., 2012; Oliver & Pentland, 2000). While the more technical details can be referred to Lefèvre et al. (2014) and references therein, an example to describe what outputs from sensor fusion are represented is shown in Fig. 10. With the sensor configuration of a front and two rear radars, and a front monocular camera, a high-level sensor fusion is applied for symbolic representation. Two vehicles in front and rear are tracked over time and classified as a vehicle via tracking and classification process in Table 3. Furthermore, both lane mark and guardrail are classified. Then, two vehicles are recognized as FVL and RVL respectively and the FVL is identified in the cut-in maneuver on the curved road.

2.3 Data-Driven Approach

Due to applications of the data-driven approaches, i.e., deep learning or deep neural networks (DNN), the performance of 2D object detection for autonomous driving has greatly improved. For instance, an average precision (AP) of more than 90% on the well-established “KITTI” object detection benchmark has been achieved (Geiger et al., 2012). Furthermore, the 3D object detection methods introduce a third dimension to the localization and size regression, revealing depth information in world coordinates (Arnold et al., 2019). The AP of about 80% on the KITTI benchmark has been reported recently. Numerous networks for the object detection have been introduced and open to public.

Among many sensor configurations introduced in Sect. 2.1, the camera-lidar (CL) fusion is the most popular in the 3D object detection. While point clouds of the lidar do not provide texture information, which is valuable for object classification, monocular images cannot capture depth values. Moreover, the density of point clouds tends to reduce quickly as the distance from the sensor increases, while images can still detect far vehicles and objects. In order to compensate for disadvantages of two sensors, three types of fusion architectures are introduced for data-driven approaches: early fusion, late fusion, and deep fusion (Chen et al., 2017).

As shown in Fig. 12a, the early fusion combines features from multiple sensors in the input stage. For instance, an Aggregate View Object Detection (AVOD) which merge the bird-eye view (BEV) from the lidar and RGB channels from the monocular camera with a region proposal network (RPN) is shown in Fig. 13 (Ku et al., 2018). Although the detailed description can be referred in the reference, it is noted that the transfer learning is used in the sense that the encoder is modeled after VGG-16 (Simonyan & Zisserman, 2014) with a reduction of the number of channels is used as an encoder in AVOD. Finally, other methods such as multi-view 3D object detection network (MV3D) (Chen et al., 2017) and Deep3DBox (Mousavian et al., 2017)

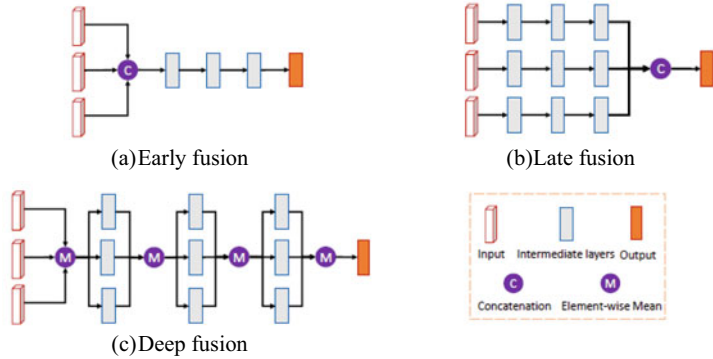


Fig. 12 Architectures of sensor fusion for data-driven approaches (Chen et al., 2017)

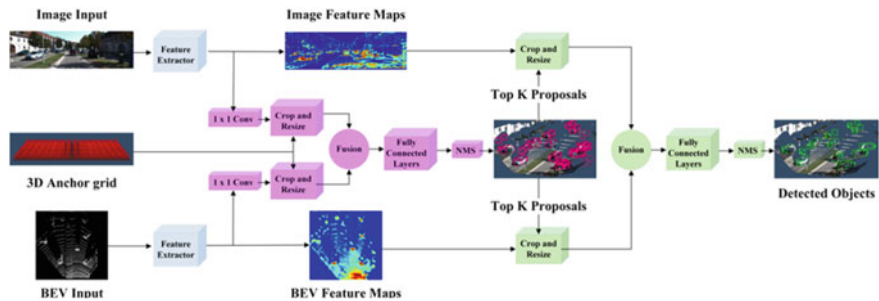


Fig. 13 An example of early fusion architecture (Ku et al., 2018)

are shown in Fig. 14. The MV3D fuses lidar and camera image with deep fusion architecture, which will be discussed shortly, and the Deep3DBox uses only mono camera and its performance is relatively lower than other sensor fusion approaches.

The late fusion obtains features using separate subnetworks independently and combines decision-level features in the prediction stage as shown in Fig. 12b. It is remarked that the concept is similar with high-level sensor fusion in the model-based approach. The late fusion at different stages of a 3D pedestrian detection pipeline was introduced in Schlosser et al. (2016). It utilizes horizontal disparity, height, and angle (HHA) features from lidar as shown in Fig. 15. Then RGB and HHA features are fused. It is shown that the late fusion yields the best performance with significant computational cost when the KITTI dataset and benchmarks are utilized.

As shown in Fig. 12c, the deep fusion extracts feature in intermediate layers from deep neural network and fuses features with element-wise mean operation at intermediate levels. Its concept is similar with feature fusion or hybrid fusion in the model-based approach. As shown in Fig. 16, the MV3D was proposed to fuse bird-eye and front view projections of lidar and the RGB channels of a forward monocular camera (Chen et al., 2017). As mentioned in the AVOD above, the base network of

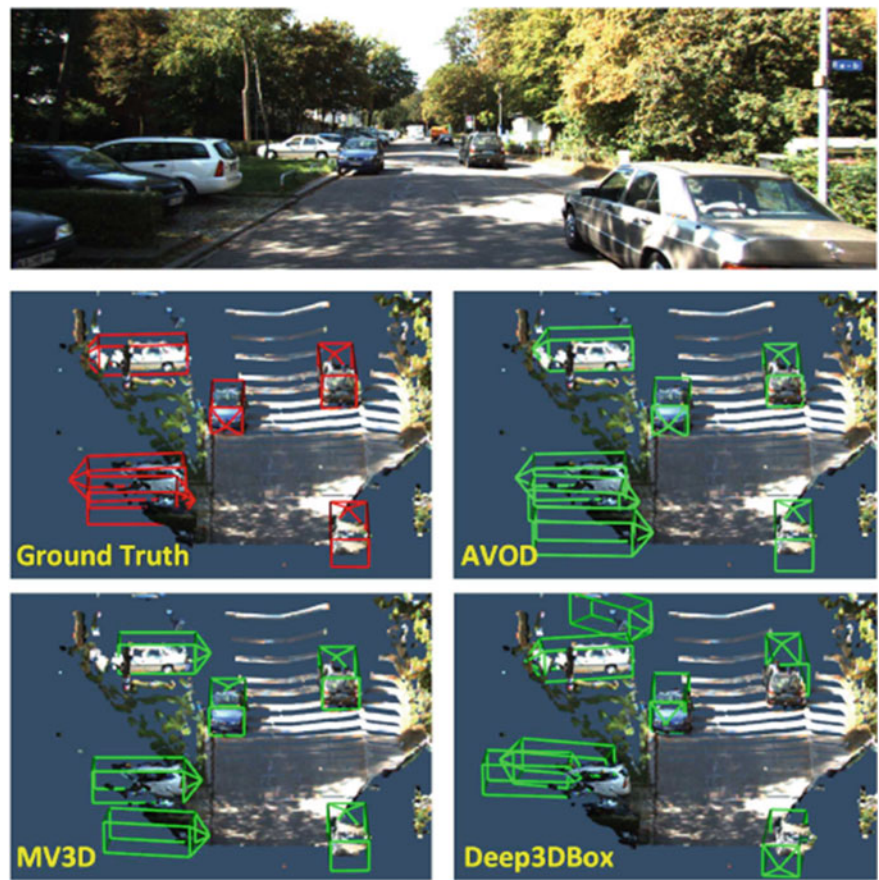


Fig. 14 A comparison between MV3D, Deep3DBox, AVOD (Ku et al., 2018)

MV3D is also built on the VGG-16 with several modifications. Based on the KITTI benchmark, the performance of deep fusion (i.e., MV3D) is compared with the ones of 3DOP (Chen et al., 2015) and VeloFCN (Li et al., 2016) in Chen et al. (2017) to show the advantage of CL sensor fusion. It is noted that the 3DOP is based on only stereo camera and the VeloFCN is on only lidar. Although only a few sensor fusion examples based on deep learning are introduced in this section, it is concluded that many sensor fusion methods have been developed recently with respect to fusion architecture (based on image detection networks) and additional information (e.g., BEV, front view, 3D anchor) from lidar.

Although most of sensor fusion methods based on data-driven approach or deep learning focus on either raw-data or feature fusion, the data association of multiple-point objects listed in Table 3, is formulated as the multidimensional assignment problem (MDAP). The goal is to find the optimal assignment of measurements to tracks so that the total assignment cost is minimized (Emami et al., 2018). Hung and

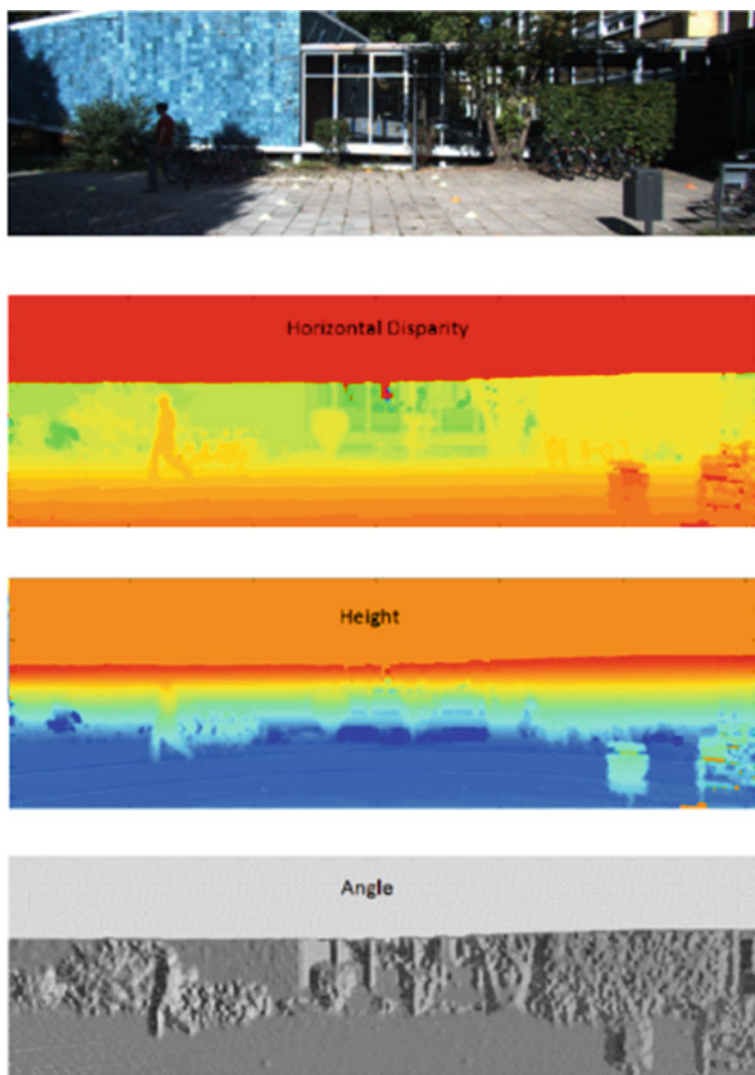


Fig. 15 Horizontal disparity, height, and angle for an image (Schlosser et al., 2016)

Wang (2003) considered recurrent neural network (RNN) to solve the assignment problem, however, the number of iterations to achieve an optimal solution increased with the problem scale. One of the RNN limits is that it can only be used in a clear and reliable environment without clutter or missed detection (Dequaire et al., 2018; Milan et al., 2016; Ondruska & Posner, 2016). Liu et al. (2019) extend an LSTM network of data association problems with clutter and missed detections using simulated experimental scenario. Suppose there are 5 targets placed in a 2D space, and their trajectories would be crossing each other at certain time (see in Fig. 17a). As the

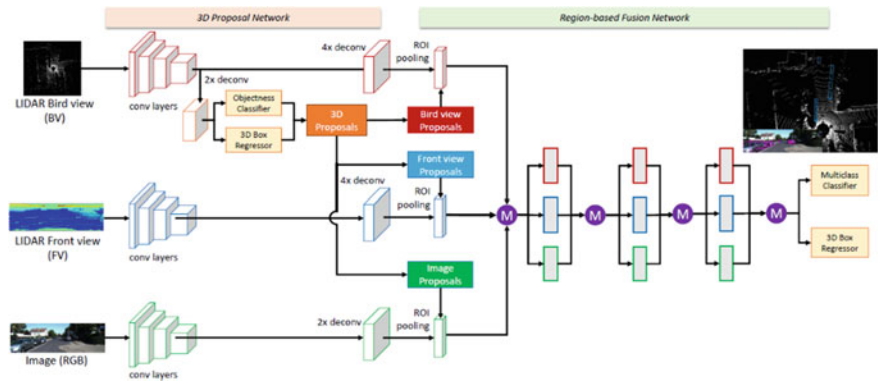


Fig. 16 Example of deep fusion architectures (Chen et al., 2017)

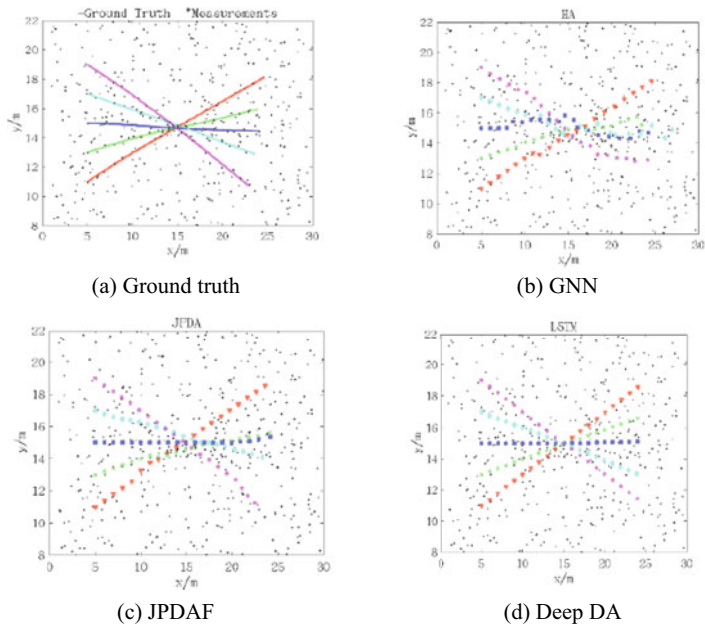


Fig. 17 Performance comparison of data association for multiple-point objects

proposed deep data association framework is shown in Fig. 18, RNN (Milan et al., 2016) is used for the tracking filter and a pair-wise distance matrix generated from all measurements and state prediction of each target is preprocessed as the input of LSTM network. Then, measure-to-track association probability is calculated by the LSTM networks, and each target is updated by the associated measurement to maintain a track. It is shown in Fig. 17 that the performance of DeepDA is compared to

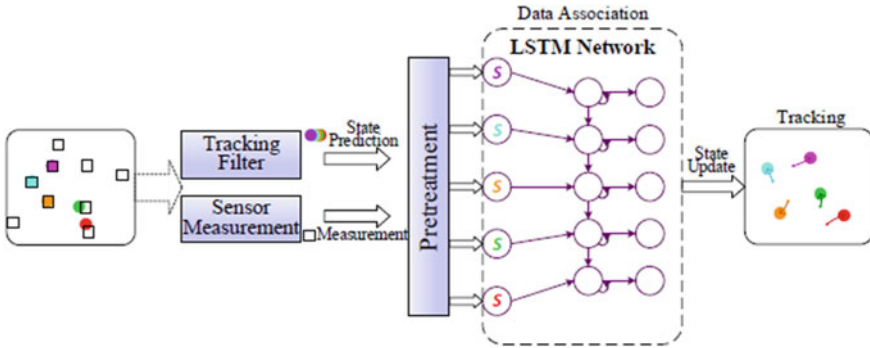


Fig. 18 Overview of DeepDA data association framework

one of GNN and JPDAF. The DeepDA has a more robust association on intersecting targets, while the GNN is more sensitive to clutter than other methods.

2.4 Safeguarding Sensor Fusion

Although most of researchers may agree with the last hypothesis that there is still false perception resulting from sensor fusion, there are few studies to solve the so-called safeguarding problem of sensor fusion. The problem is also closely related to verification and validation in Sect. 6. Without hardware redundancy of sensors, the analytical redundancy technique is one of the most challenging problems for AVs. Since the data-driven approaches may not work well for unknown or unexpected scenarios (i.e., untrained scenarios), it is obvious that as many dangerous situations as possible should be considered in the training stage of sensor fusion. However, it is rare to face the real dangerous situations on the road, which implies lack of reliability of the data-driven sensor fusion based on field of test (FOT) data. Therefore, a large and comprehensive data set of dangerous situations is prerequisite to enhance reliability of data-driven sensor fusion. One of the on-going approaches is to use virtual clone to consider more various situations. Among many visualization tools in the industry, one example is shown in Fig. 19 to compare their similarity between images and simulations (or virtual clones). It is also remarked that the virtual clone allows us to consider bad weather (rain/snow) and various light conditions including day/night as well as critical scenarios (e.g., based on Generative Adversarial Network, GAN).

Another approach is to consider a parallel architecture for safeguarding sensor fusion. In the research area of fault detection and diagnosis (FDD), some hybrid approaches are proposed to integrate model-based and data-driven approaches (Gao et al., 2015). Although most of them are applied to FDD of a single hardware and only machine learning techniques such as principal component analysis (PCA) and support vector machine (SVM) are used, it is expected that the parallel architecture



Fig. 19 Frames from 5 real KITTI videos (first column) and the corresponding virtual clones (second column) (*source* Gaidon et al., 2016)

or hybrid approach will be adopted for performance monitoring of the deep-learning based sensor fusion. For instance, output of a commercial camera used for ACC and AEB includes the confidence, which indicates performance or assurance of detection. Although the detailed technology is not published, it is known that the deep learning (DL) is applied to object and lane detection. Furthermore, the confidence for lane mark detection is shown in the right plot of Fig. 20. In our on-going study, it is shown that one of bank observers may predict the confidence (see also the dotted line in the right plot) but does not cover all performance degradation of DL-based lane mark detection. Therefore, a set of observers based on different dynamic models (e.g., constant velocity (CV) and acceleration (CA) models) and observer techniques (e.g., Kalman filter, unknown input observer, disturbance observer) have been tested. Moreover, their analytical residuals are used to identify performance degradation (or confidence) in the framework of SVM. That is, in the parallel architecture, a combination of model-based and machine learning techniques is running for performance monitoring of the DL-based detection. We expect that the related research will be promoted rapidly for early adoption and commercialization of DL in AVs.

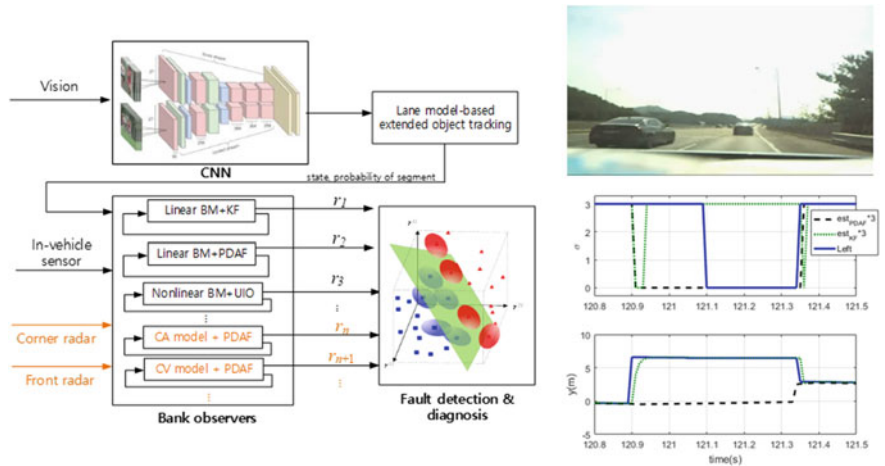


Fig. 20 Architecture and example of lane mark monitoring- sensor output (solid line) and model-based decision (dashed and dotted lines)

3 Motion Planning for Autonomous Driving

This section describes decision-making and motion planning for autonomous driving. The overall architecture of an autonomous vehicle is illustrated in Fig. 21. An automated driving algorithm consists of three consecutive parts. The first part is ‘Environment Representation’ that identifies current global position, detects moving objects, and generates static obstacle map using sensor data. The second part is ‘Motion Planning’ that computes a safety envelope, desired driving path and desired speed profile. The third part is ‘Vehicle Control’ that determines throttle, brake and steering controls to follow the desired vehicle motions, i.e. the path and speed profile.

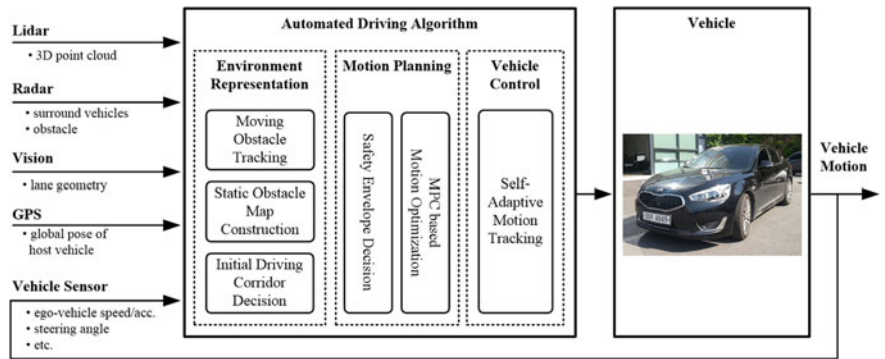


Fig. 21 The overall block diagram of the autonomous vehicle from Kim (2016)

3.1 Decision and Motion Planning for Autonomous Vehicles

Motorway driving patterns consist of “lane keeping” and “lane change”, as illustrated in Fig. 22. The lane keeping includes path tracking and clearance control. In the case of the lane change, two types of lane changes exist: discretionary and mandatory.

In general, the lane changes on the highway are discretionary. A discretionary lane change is conducted based on traffic flows of subject and adjacent lanes. Sometimes, an autonomous vehicle must change the lane to pass through an interchange: off-ramp (split) or on-ramp (merge) cases. In such cases, the autonomous vehicle must plan the motions to change the lane in a range of designated region. We call such a lane change scenario as a mandatory lane change.

At a signalized or an unsignalized intersection, additional decision and motion planning modes are needed because lane keeping or lane change maneuvers cannot fully cover the motion at such roads. For example, when the driver turns left at the unsignalized intersection while there exist other approaching vehicles, the autonomous vehicle needs to monitor and infer others’ future motions. Therefore, the motion planning algorithm at intersections includes additional driving modes in addition to the lane keeping and lane change modes.

(1) Lane keeping and Lane change

This subsection presents the motion planning for keeping the vehicle within the current lane and safely changing to the adjacent lane. This subsection is structured as follows. First, lane keeping and lane change scenarios are defined. Second, the mode decision between lane keeping and lane change modes is described. Third, the key concepts of the motion planning for lane keeping and lane change are described.

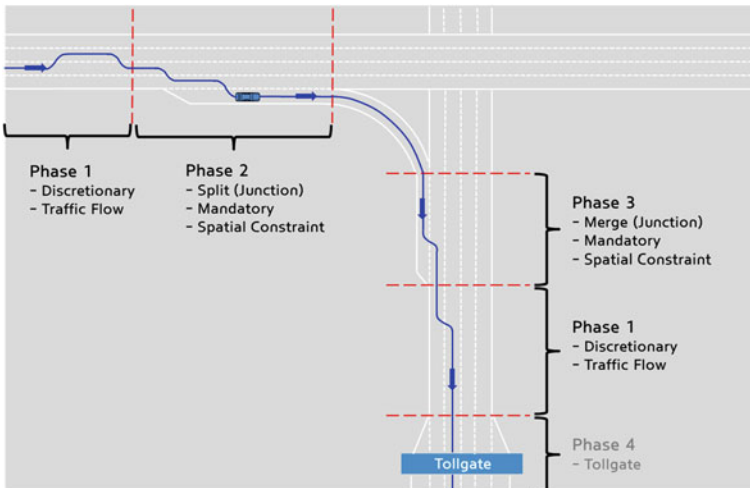


Fig. 22 Motorway autonomous driving patterns

First, we define lane keeping and lane change scenarios. The lane keeping scenarios are divided into three cases: (1) Drives within the current lane (Normal lane keeping); (2) Clearance control while keeping the lane; and (3) Path modification to maintain a safe lateral clearance. The lane change scenarios are divided into two cases: (1) Discretionary lane change and (2) Mandatory lane change. For example, when drivers have to pass the on/off-ramp, the drivers must change the lane to enter interchange. We designate such situation as a mandatory lane change. In contrast, when the traffic flow speed of the adjacent lane is faster than the speed of the preceding vehicle, the drivers do not have to change the lane but often change to the adjacent lane. We designate such situation as a discretionary lane change.

Based on the aforementioned definitions of lane keeping and lane change scenarios, it can be claimed that the driving on highways and main roads consists of lane keeping and lane change scenarios. The drivers can reach from one interchange to the other interchange by only keeping the lane or keeping the lane with the discretionary lane change. The autonomous vehicles change the lane to enter the interchange by utilizing the mandatory lane change algorithm.

Second, the mode decision between lane keeping and lane change is described. The mode decision algorithm is essential to design the motion planning algorithm for lane keeping and lane change. Figure 23 presents the conceptual logic flow of the mode decision algorithm. The default mode of the mode decision algorithm is the lane keeping mode. During this mode, the algorithm constantly finds the safe region to change the lane and determines possible space to cut-in. If the lane change maneuver is required (lane change demand check), then the algorithm checks whether the vehicle can change the lane immediately. If possible, the driving mode is changed to

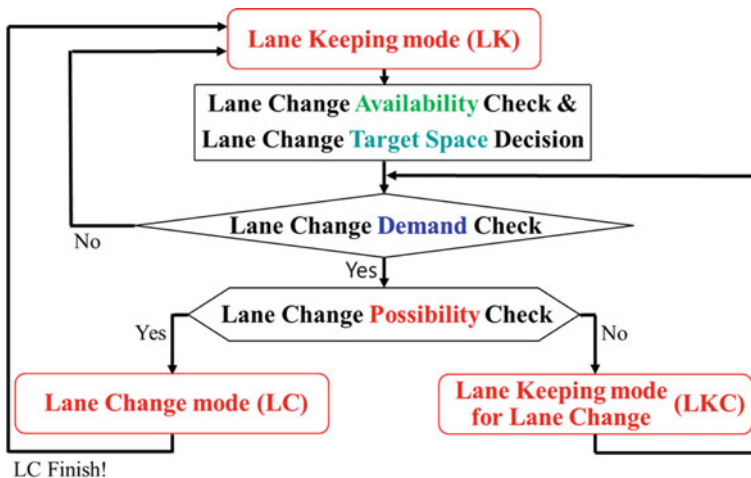


Fig. 23 Driving mode decision: Lane keeping mode and Lane change mode from Chae and Yi (2020)

the lane change mode. If not, the algorithm plans the longitudinal motion to facilitate the lane change (Lane keeping mode for lane change).

Third, the key concepts of the lane keeping and lane change algorithms are described. The lane keeping algorithm is described first, and subsequently the lane change algorithm is described.

The lane keeping scenarios are divided into three cases: (1) Drives within the current lane (Normal lane keeping); (2) Clearance control while keeping the lane; and (3) Path modification for safe lateral clearance.

The normal lane keeping case is a situation where the autonomous vehicle makes a decision to keep the lane with set speed because the safety of the ego-vehicle will not be violated. Figure 24 presents a good example for the normal lane keeping. In this scenario, the oncoming vehicle is predicted to maintain its lane and the parked vehicles do not intrude on the current lane. Therefore, it is safe to keep the current lane with set speed as illustrated in Fig. 24.

The clearance control during lane keeping is a situation where there exists a preceding vehicle on the current lane. In this case, the ego-vehicle should maintain the clearance, not cruise with set speed. For example, in urban driving scenario, it is prevalent that the preceding vehicle stops at a red light so the ego-vehicle must be stopped as shown in Fig. 25. The ego-vehicle decelerates to maintain the clearance between the preceding and the subject vehicles.

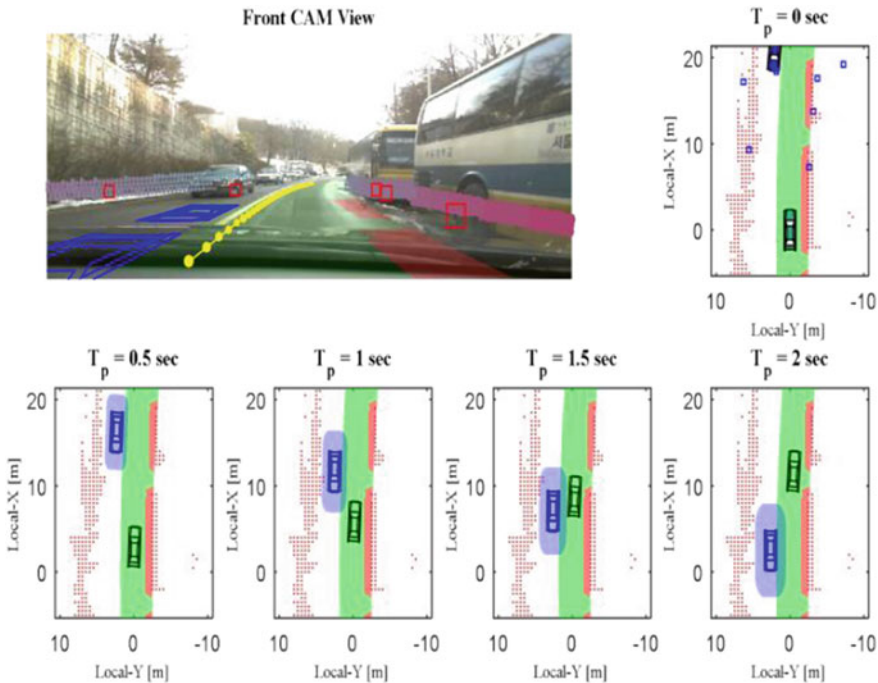


Fig. 24 Normal lane keeping case Kim (2016)

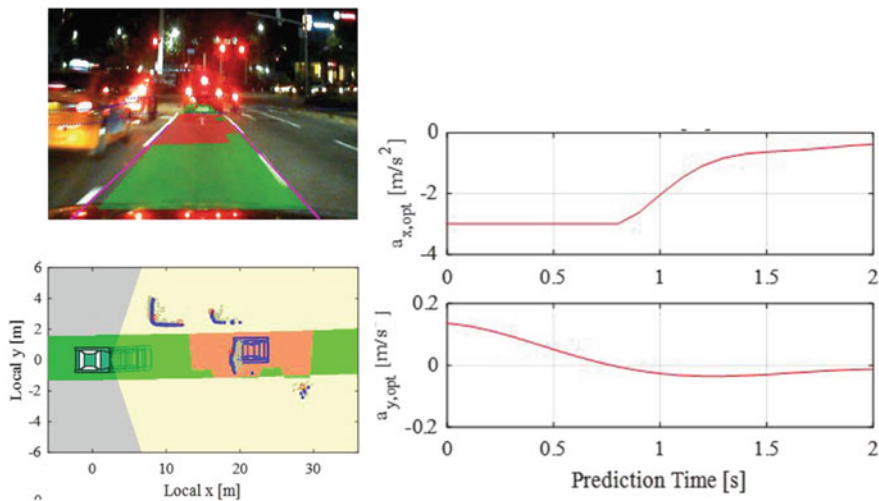


Fig. 25 Clearance control during lane keeping (2016)

The path modification during lane keeping is a situation where there exists a static obstacle (such as cones, parked vehicles, unexpected obstacles, etc.) on the current lane. In this case, the path modification is needed to safely avoid the static obstacle and continue the autonomous driving. A good example scenario is given in Fig. 26. In this case, a taxi parks on the side of the current lane. To avoid the taxi safely, the autonomous vehicle decides to slightly change the planned path.

Second, decision making and motion planning to change the lane is presented. Lane changes constitute key parts in autonomous driving at the highways as illustrated in Fig. 22.

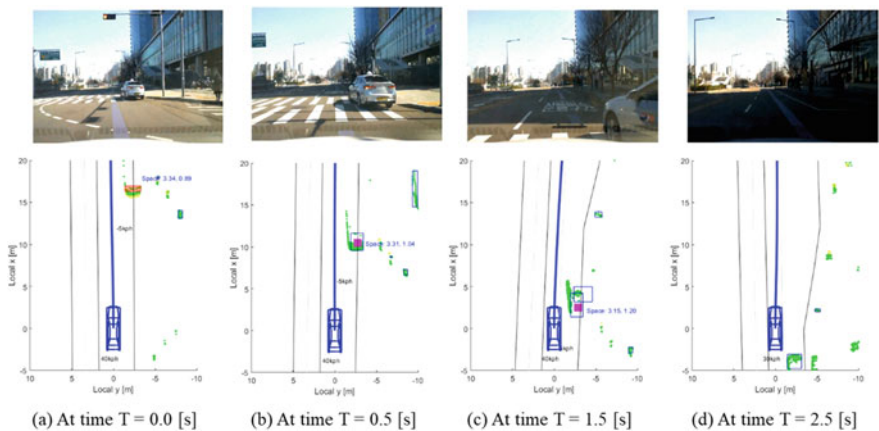


Fig. 26 Path modification during lane keeping

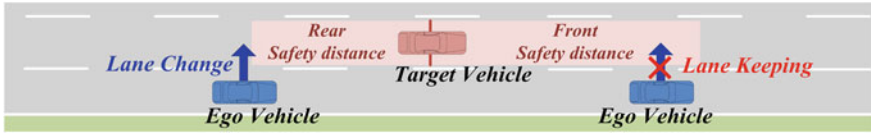


Fig. 27 Safety distance from Chae and Yi (2020)

To change the lane safely, there must exist a certain amount of space to change the lane. To quantify the size of such space, we adopt the notion of the safety distance. The safety distance is the distance for quantifying the unsafe region to change the lane as illustrated in Fig. 27.

Front/Rear safety distance is located in front of/behind the vehicle on the adjacent lane (target vehicle). If the ego-vehicle is located right next to the front/rear safety distance, then the ego-vehicle cannot change the lane safely. If the ego-vehicle's side is not occupied by the safety distance, then the ego-vehicle can change the lane.

Determining the size of the safety distance is important for passengers' comfort. If the size of the safety distance is too short or too long, the passengers think that the autonomous vehicle drives too aggressive or too defensive.

To design the lane change algorithm that shows anthropomorphic autonomous behaviors, we conduct 1,500 km driving and analyze the data. Figure 28 presents the data. In Fig. 28, each line represents each lane change data, and the 'o' and 'x' indicates the start and end of the lane change. Figure 28 clearly shows that there exists common danger area that drivers do not attempt to change the lane. When the relative speed is negative (i.e., clearance increases as time goes by, e.g., slower target vehicle when the ego-vehicle is ahead of the target vehicle), the human drivers feel safe to change the lane if the clearance between the target and ego vehicle is larger than 10 m. When the relative speed is positive (i.e., clearance decreases), the safe clearance to change the lane increases as the relative speed increases.

Based on the results of the driver data analysis, the safety distance is defined as:

$$SD_{LC} = \begin{cases} \max[(v_{OV} - v_{SV}), 0] \cdot \tau_{LC, 1} + c_{LC} & \text{if } p_{SV} > p_{OV} \\ \max[(v_{SV} - v_{OV}), 0] \cdot \tau_{LC, 1} + c_{LC} & \text{otherwise} \end{cases} \quad (1)$$

The parameters $\tau_{LC, 1}$, c_{LC} are set to 0.94 s and 10 m based on the data in the Fig. 28.

By using the safety distance, we can draw unsafe region to change the lane around the target vehicle. By excluding the unsafe region in the adjacent lane, the safe region can be obtained as illustrated in Fig. 29. The red and blue regions indicate the unsafe and safe region to change the lane, respectively.

As illustrated in Fig. 29, the autonomous vehicle should choose one among multiple safe regions. We design the lane change algorithm to choose one safe region that is wide and close to the ego-vehicle.

In the case of the discretionary lane change, the algorithm decides to change the lane when the speed of the preceding vehicle is lower than that of the target vehicle.

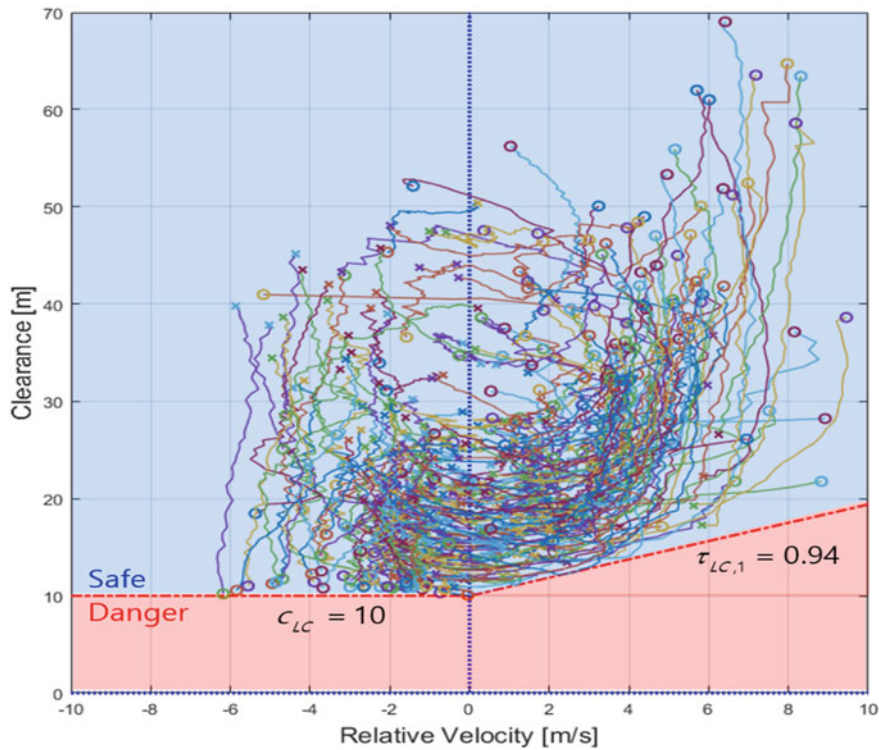


Fig. 28 Lane change the driving data from Chae et al. (2020)

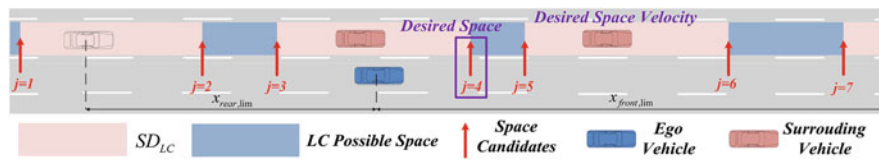


Fig. 29 Lane change the driving data from Chae and Yi (2020)

Since the law enforces not to continuously remain on the first lane, the vehicle must return to the original lane. Figure 30 provides the schematic flow of the discretionary lane change.

In the case of the mandatory lane change (driving on the off-ramp and on-ramp), the algorithm also considers the remaining distance to lane drops as illustrated in Fig. 31. The algorithm needs to prepare a contingency stop motion plan for avoiding an accident at the lane drops. If the remaining distance is not enough to change the lane into the safe region, then the mandatory lane change is conducted in stop-first lane change when possible.

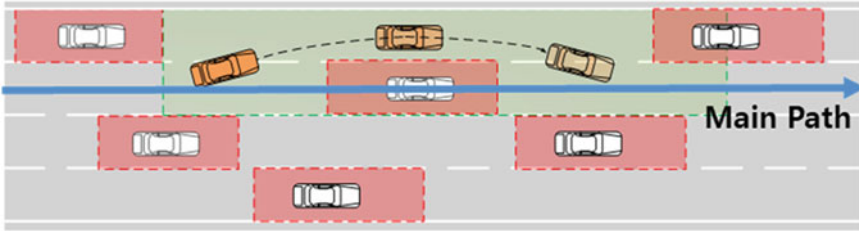


Fig. 30 Discretionary lane change



Fig. 31 Mandatory lane change at the on-ramp

(2) Signalized intersection

This subsection presents the motion planning at signalized intersections. Specifically, the scenario that will be discussed is motion planning for multi-lane left turn as presented in Fig. 32.

Because of the traffic light signals, the signalized intersections are more organized than the unsignalized intersections. Most cases are very clear and safe for planning the subject vehicle's motion to obey the traffic light signals. Moreover, when the subject vehicle obeys the traffic light signals, the subject vehicle's motion at the signalized intersections is nothing but lane keeping (at straight or curved roads) or lane change.

In contrast to such cases, multi-lane left turn is a relatively dangerous scenario. This is because lanes are not clearly drawn at the intersections, so the surrounding vehicles often unintentionally intrude to the subject vehicle's lane as illustrated in Fig. 33. To ensure the safety in this scenario, we need to predict the target vehicles' motion at a multi-lane left turn scenario, and plan the safe motion when the target vehicle intrudes the subject vehicle's lane. Based on this reasoning, the motion planning algorithm for multi-lane left turn consists of two parts: (1) prediction of surrounding vehicles, (2) motion planning to avoid a collision with the surrounding vehicles.

(3) Urban unsignalled intersection

In urban driving environments, uncontrolled intersections are risky areas because there is no traffic signals or road signs to control the traffic. Each driver needs to infer the others' inference and plan the vehicle's motion to avoid a collision. In this sense, the motion planning at unsignalized intersections includes two issues: (1) Intention inference, (2) Planning motion not to collide with other road participants.

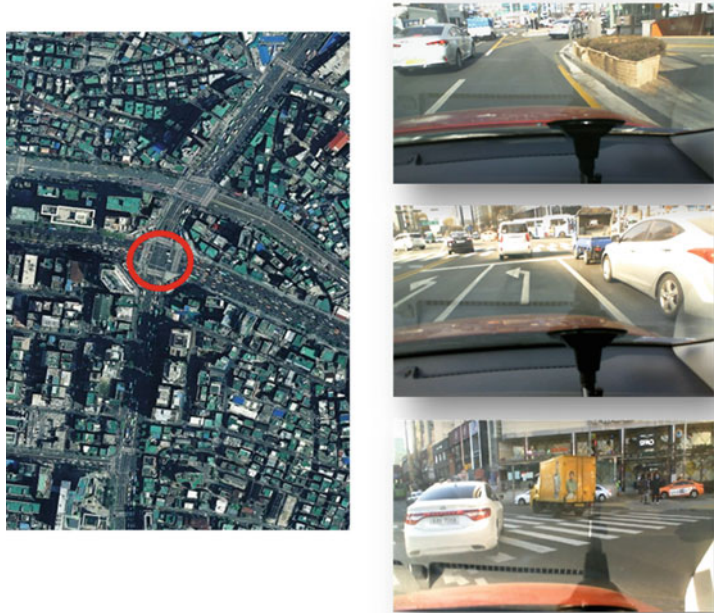
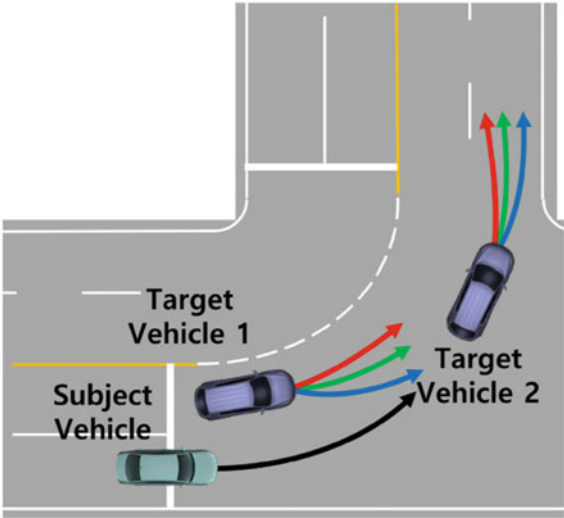


Fig. 32 Multi-lane left turn intersection from Jeong et al. (2020)

Fig. 33 Surrounding vehicle unintentionally intrudes to the ego-vehicle's lane from Jeong et al. (2020)



In urban area, there are two kinds of the unsignalized intersection: (1) the multi-lane unsignalized intersections and (2) the narrow intersection. In the multi-lane intersection, the driver or the automated driving system can be fully aware of other approaching vehicles in the intersection. In contrast, in the narrow intersection as shown in Fig. 34, the driver or the automated driving system cannot perceive the other road participants until the vehicle enters the intersection due to the side walls or the static obstacles. Therefore, the motion planning methods at the multi-lane and narrow intersection must be different and will be described separately.

In the multi-lane unsignalized intersections, the surrounding vehicles' inference estimation is important. The issue of intention inference includes the predicted behaviors of other vehicles (cross, decelerate, stop) and the predicted motions of other vehicles (positions, speed, etc.). To design the motion planning algorithm at unsignalized intersections that show anthropomorphic autonomous behaviors, we collect and analyze driving data at unsignalized multi-lane intersections.

To quantitatively analyze the data, the parameters for intersection automated driving are described. The parameters for intersection automated driving are presented in Fig. 35, and the description of each parameter is given in Table 4. The clearance between the approaching vehicle and the host vehicle is defined with respect to the conflict point in Fig. 35. As described in the Table 4, the clearance is a sum of the approaching and the host vehicles' distances to the conflict point.

Based on the defined parameters in Table 4, the human driving data are analyzed in Fig. 36. The family of blue/red lines presents the data when the vehicle cross the intersection before/after the other vehicle cross the intersection, respectively. Note that the x axis is d_{DTC} . d_{DTC} is positive (negative) before (after) the vehicle passes the conflict point. For the 'Cross First' case, the vehicle drives at 30–40 km/h with -0.5 – 1.0 m/s² longitudinal acceleration. In contrast, for the 'Cross Later' case, the vehicle



Fig. 34 Example of the narrow intersection from Akagi and Raksincharoensak (2015)

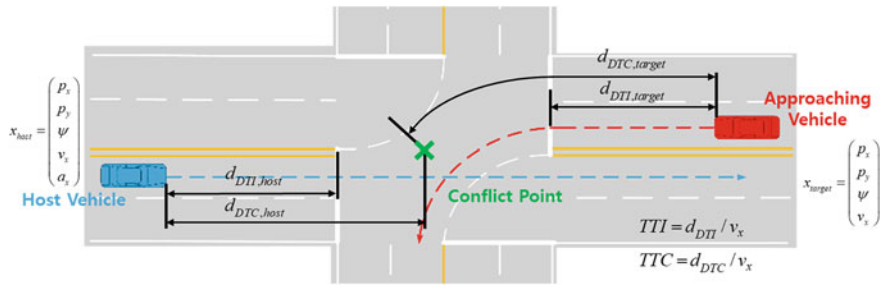


Fig. 35 Parameters for Multi-lane Intersection Automated Driving from Jeong and Yi (2019)

Table 4 Lane keeping: sensor range and maximum relative speed

Variables	Definition
d_{DTI}	Distance to intersection, remaining distance to intersection entrance
d_{DTC}	Distance to conflict point, remaining distance to conflict point
$c = d_{DTC,host} + d_{DTC,target}$	Clearance, actual clearance between vehicles
$TTI = d_{DTI}/v_x$	Time to intersection, remaining time to intersection entrance based on current motion
$TTC = d_{DTC}/v_x$	Time to conflict point, remaining time to conflict point based on current motion
$TTIC$	Time to intersection cross, remaining time to intersection crossing based on the predicted states of vehicles

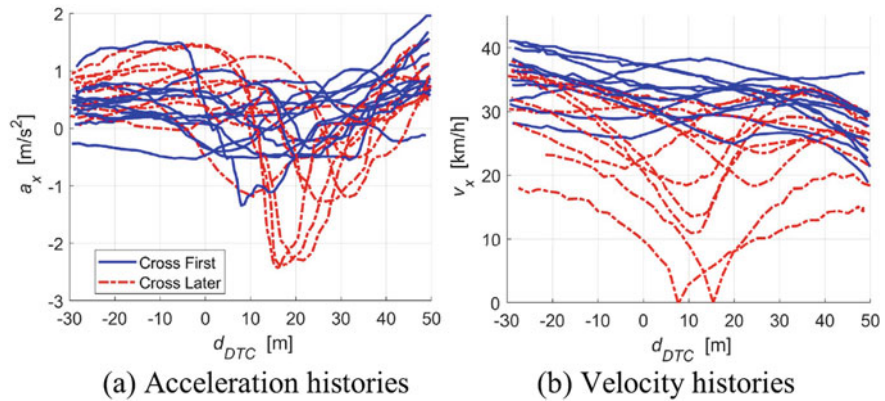


Fig. 36 Parameters for Intersection automated driving from Jeong and Yi (2019)

decelerates up to -2.5 m/s^2 before entering an intersection. Also, ‘Cross Later’ case can be categorized into two situations. First, the vehicle totally stops before the conflict point. Second, the vehicle decelerates first and subsequently accelerates after the other vehicle passes the conflict point. In conclusion, there exists three different inferences at unsignalized intersections: (1) Pass (2) Decelerate (3) Stop.

To estimate other approaching vehicles’ interferences based on the driving data, K-means clustering method is utilized. As illustrated in Fig. 37, three different models can be obtained: Pass model, Decelerate model, and Stop model. By comparing the current motions of the others with these models, the current motions can be classified into the corresponding models. Moreover, by using such model, the predicted motions can be calculated.

The subject vehicle’s motions at unsignalized intersections can be divided into three different modes as illustrated in Fig. 38. First, in the approach phase, the

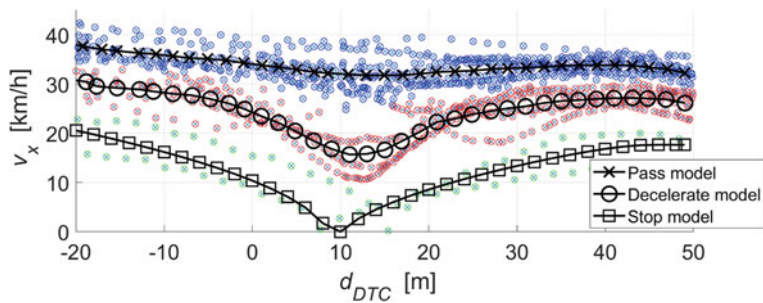


Fig. 37 Driver behavior model from Jeong and Yi (2019)

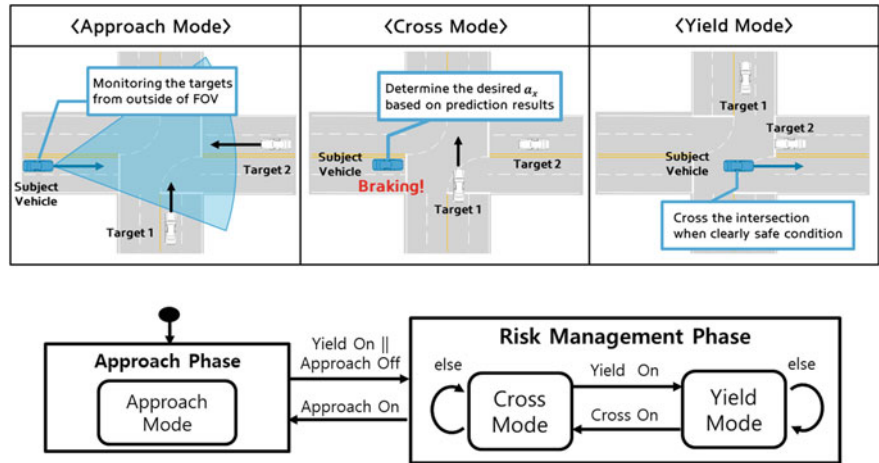
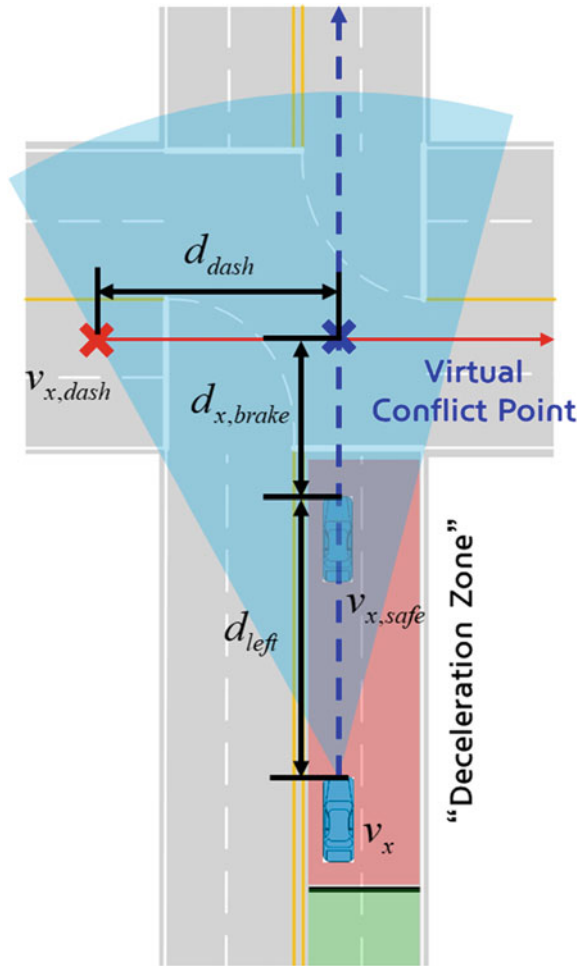


Fig. 38 Driver behavior model from Jeong and Yi (2019)

vehicle approaches to the intersection with mild longitudinal deceleration as illustrated in Fig. 39. To ensure the safety at unsignalized intersections, decelerating before entering the intersection is imperative. This is because the sensors’ field of view (the blue sector) is limited, and the autonomous vehicle cannot perceive vehicles that approach to the intersection. In this phase, it is assumed that a virtual vehicle (red x in Fig. 39) approaches the intersection with constant speed at the boundary of sensors’ field of view. Therefore, the target deceleration is calculated to avoid a collision with such objects at a conflict point (blue x in Fig. 39).

Second, when the approaching vehicles are within the sensors’ field of view and detected, the algorithm falls into risk management phase as illustrated in Fig. 38. In this phase, the algorithm monitors the other vehicles’ motions and decides to

Fig. 39 Motion planning in the approach phase from Jeong and Yi (2019)



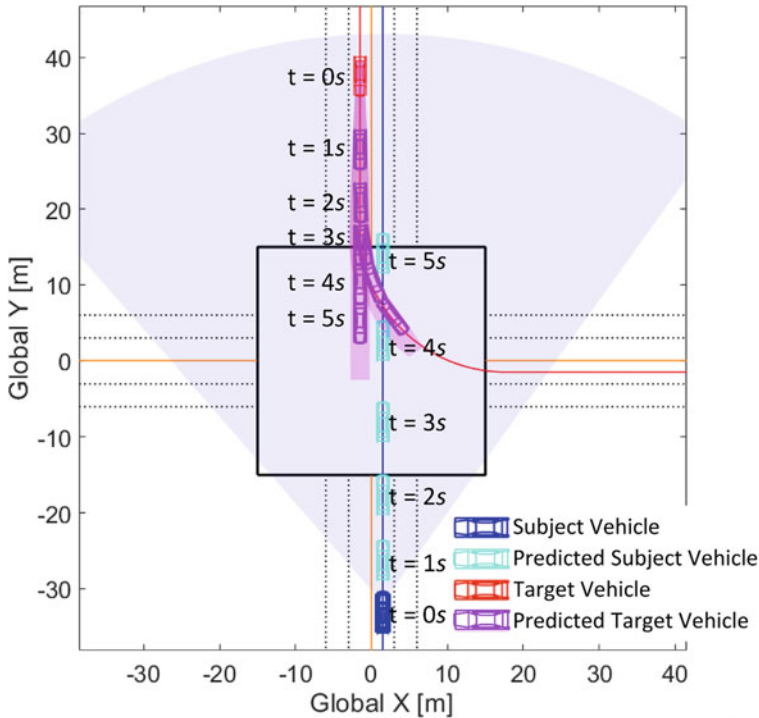


Fig. 40 Example scenario for the risk management phase from Jeong and Yi (2019)

cross the intersection first (Cross mode) or yield to other approaching vehicle (Yield mode).

Figure 40 presents an example scenario to elucidate the risk management phase. In this case, the subject vehicle tries to cross the intersection, and the target vehicle’s behavior is expected either to go straight or turn left.

To ensure the safety, the subject vehicle must have a plan to cope with the situation if the target vehicle decides to turn left. In such situation, two collision-free options are available for the subject vehicle. First, the subject vehicle can accelerate enough to cross the intersection before the target vehicle turns left. Second, the subject vehicle can decelerate appropriately to yield the intersection to the left turning target vehicle.

Figure 41 illustrates the target vehicle’s prediction and the subject vehicle’s future motion. The target vehicle’s motion is predicted by means of extended Kalman filter, and the subject vehicle’s motion is predicted when the driving mode is ‘Cross mode’. The red plane in Fig. 41 presents the subject vehicle’s driving area. Since the subject vehicle is located on $x = 2$ m, the red plane is the vertical plane $x = 2$ m. The space cut through by this red plane is the region of interest because we want to plan the subject vehicle’s motion to avoid a collision with the target vehicle.

The cross section of the target vehicle’s predicted motions is presented in Fig. 42. The red polygon represents the conflict region where the collision can occur. To avoid

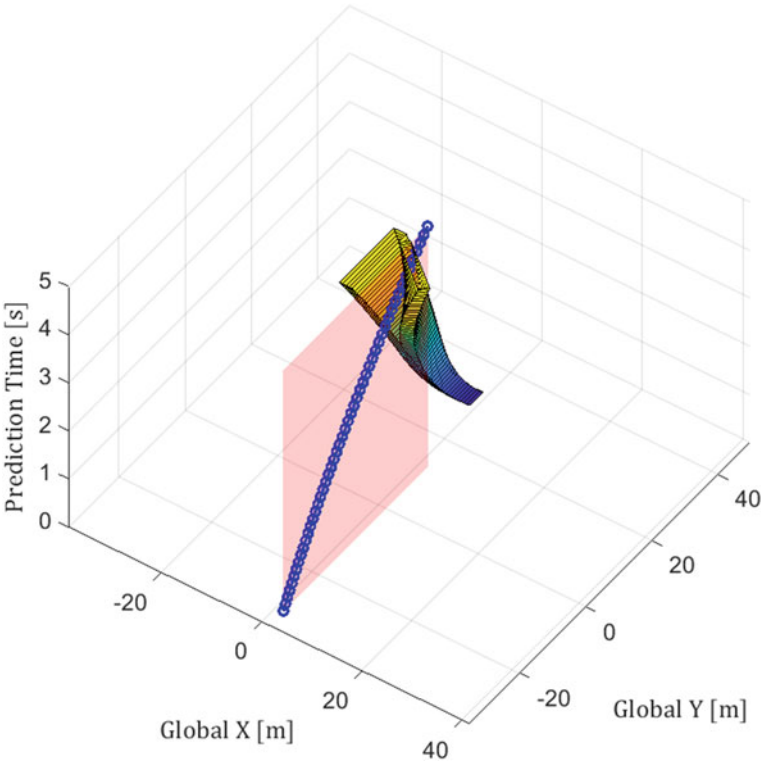


Fig. 41 Target vehicle’s predicted motion and Subject vehicle’s motion from Jeong and Yi (2019)

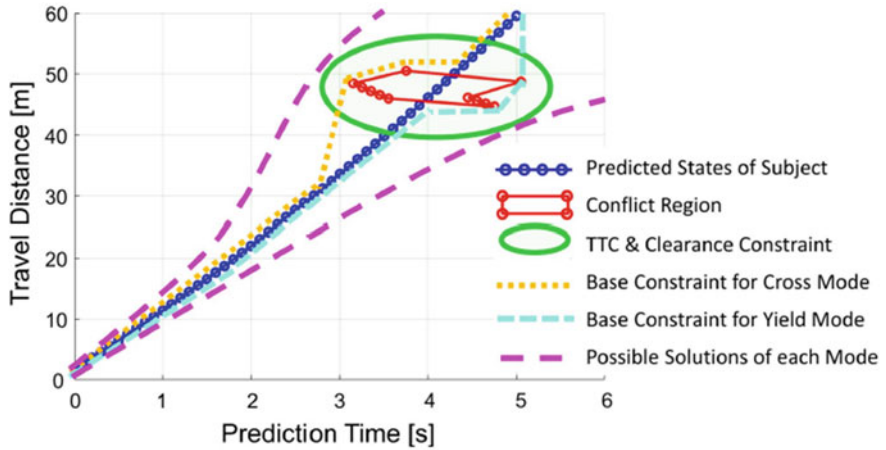


Fig. 42 Cross section about the subject vehicle’s trajectory from Jeong and Yi (2019)

a collision, two constraints that enclose the red polygon can be drawn. The upper constraint is that the subject vehicle must obey when the subject vehicle crosses the intersection prior to the target vehicle. The lower constraint is that the subject vehicle must obey when the subject vehicle yields the intersection to the target vehicle. Based on these constraints, two possible solutions are illustrated in Fig. 42. The upper/lower magenta lines are safe motions to cross the intersection safely before/after the target vehicle, respectively.

The motion planning approach at the narrow unsignalized intersection is different with that at the multi-lane intersection. This is because the autonomous vehicle cannot perceive other approaching vehicles until enters the narrow intersection (blind corners), and there is not enough space to avoid a collision in lateral direction. Due to the nature, the motion planning at the narrow intersection needs to be defensive and is limited to the longitudinal motion planning. Specifically, the longitudinal motion is designed to appropriately decelerate before the vehicle enters the intersection even if the other road participants are observed. By doing so, the vehicle can avoid or mitigate a collision even if the other road participants suddenly enter the intersection. Note that the motion planning approach at the narrow intersection below is an original work of Akagi and Raksincharoensak (2015).

To design the motion planning algorithm that shows socially compliant maneuvers, the data are collected and analyzed. Figure 43 presents the speed profiles of 6 different driving instructors. $X = 0$ m indicates the center of the intersection. Figure 43 clearly shows that all drivers decelerate before entering the intersection. Sometimes, the drivers decelerate independently of the position of the intersection. This is because there exists pedestrians or oncoming vehicles.

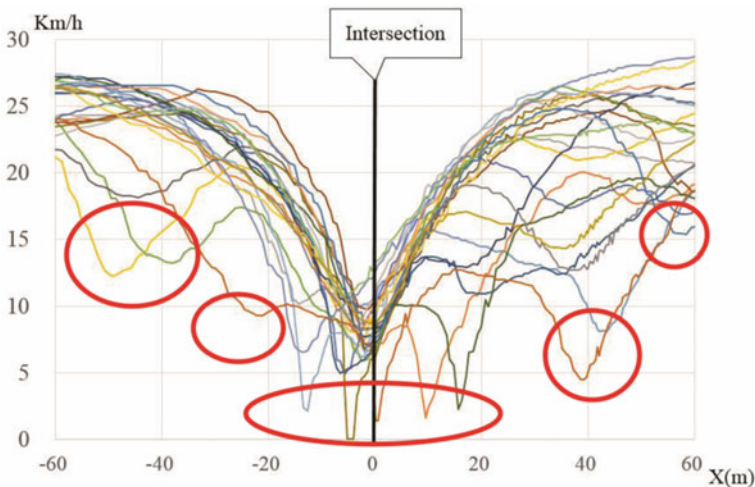
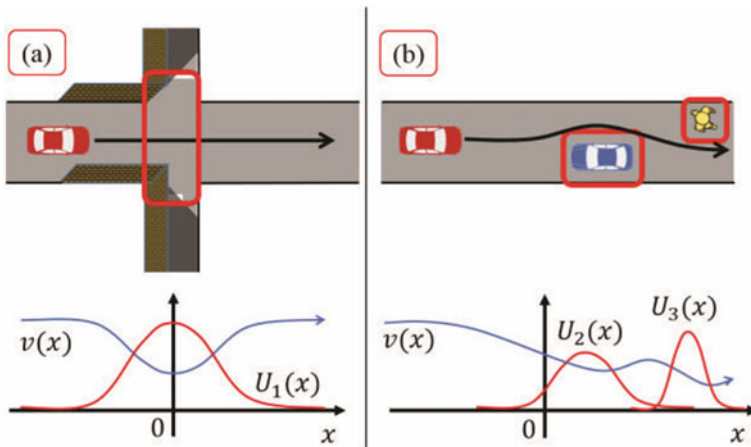


Fig. 43 Speed profiles of the expert drivers passing the urban narrow unsignalized intersection. Red circles: deceleration points to pass the oncoming vehicles and pedestrians from Akagi and Raksincharoensak (2015)



Blue lines: vehicle's speed. Red lines: risk potential values

(a) Passing the unsignalized intersection.

(b) Passing the oncoming vehicles and pedestrians

Fig. 44 Intersection-position-dependent potential functions from Akagi and Raksincharoensak (2015)

To model such intersection position-dependent motion, risk potential functions are adopted as illustrated in Fig. 44. The shaping parameters of the potential function are obtained from the drivers' data. Based on the potential field theory, the decelerating force $F(x)$ can be calculated as:

$$F(x) = -\frac{\partial U(x)}{\partial x} \quad (2)$$

The longitudinal motion planning based on the potential function is investigated via vehicle tests. The results are presented in Fig. 45. As illustrated in Fig. 45, the potential field function successfully mimics the human drivers' motion at the narrow intersection.

3.2 Safe Driving Envelope Decision and Motion Optimization

The Safe Driving Envelope Decision algorithm determines the vehicle's drivable area based on the prediction of the moving objects, the positions of the static objects, and the lane and then finds the maximum polytope within the drivable area by means of quadratic programming. The maximum polytope is the safe driving envelope. The output of the Safe Driving Envelope Decision algorithm is illustrated in Fig. 46.

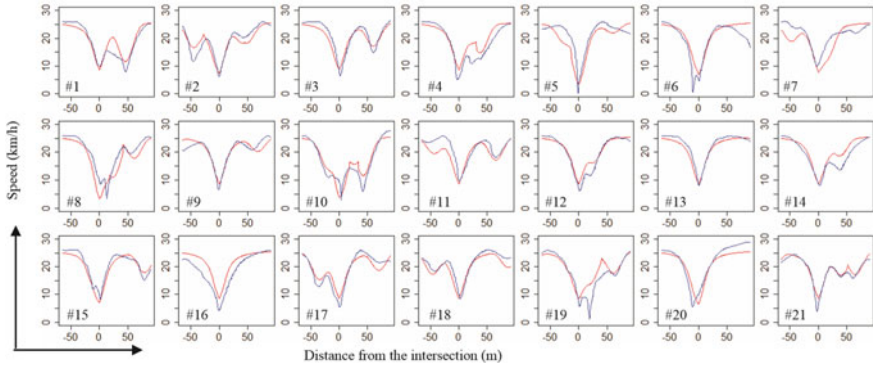


Fig. 45 Comparison of the potential-field-based motion planning method and the human drivers' motions from Akagi and Raksincharensak (2015)

The vehicle's motion is planned at each time step by means of Stochastic Model Predictive Control (SMPC). The SMPC problem is formulated as follows:

$$\begin{aligned}
 & \min \sum_{k=0}^{N_p-1} \mathbb{E} \left(\|x_{t+k+1,t} - x_{ref,k+1}\|_{\mathbf{Q}}^2 + \|u_{t+k,t}\|_{\mathbf{R}}^2 \right) \\
 & s.t. \quad x_{t+k+1,t} = \mathbf{A}_{l,k} x_{t+k,t} + \mathbf{B}_{l,k} u_{t+k,t} + \mathbf{D}_k w_k \\
 & \quad \Pr(g_{k+1}^T x_{k+1} \leq h_{k+1}) \geq p \\
 & \quad u_{k,t} \leq u_{\max} \\
 & \quad u_{k,t} \geq u_{\min} \\
 & \quad x_{t,t} = x(t) \\
 & \quad (k = 0, \dots, N_p-1) \\
 & \text{where, } x = [v_x \ v_y \ \gamma \ e_\psi \ e_y \ s]^T \text{ and } u = [\delta_f \ a_x]^T
 \end{aligned} \tag{3}$$

Figure 47 graphically shows the definitions of the variables in (3). The cost function of the SMPC is the expectation of the state errors and the control inputs. The constraints of the SMPC are the vehicle dynamics with noise, a chance constraint for the vehicle to drive within the safe driving envelope, two control input constraints and an initial condition. This control problem is a stochastic problem because the predicted motions of the moving objects, the positions of the static obstacles, and the vehicle states are stochastic variables.

The problem of SMPC is that solving the optimization problem of (3) in real-time is impossible. To circumvent the above difficulty, the method in Kouvaritakis, Cannon, Rakovic, and Cheng (2010) is adopted. The proposed method is able to convert the chance constraint into the simple linear constraint with a slack variable if the stochastic properties of the chance constraint are given. The interested reader of this converting method should read through Kouvaritakis, Cannon, Rakovic and

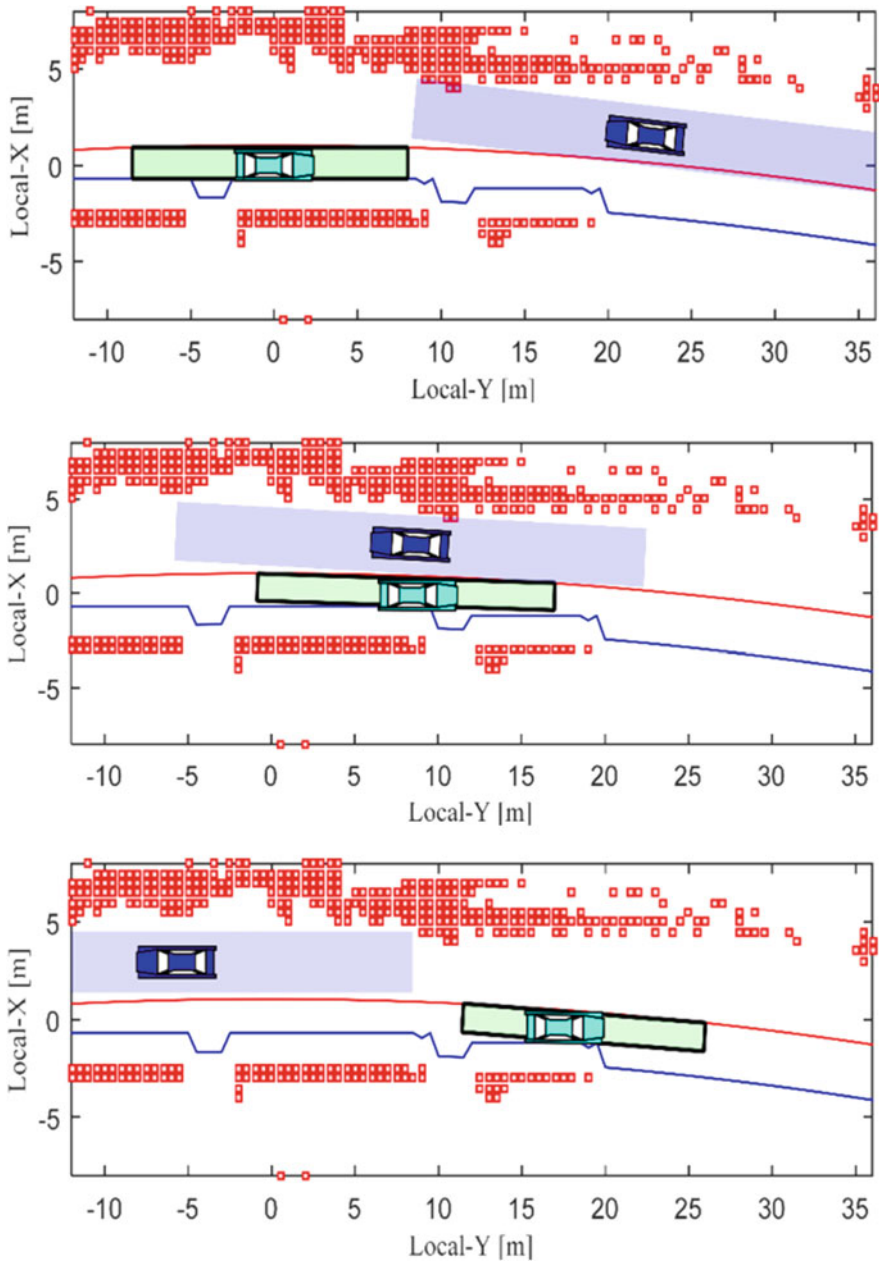
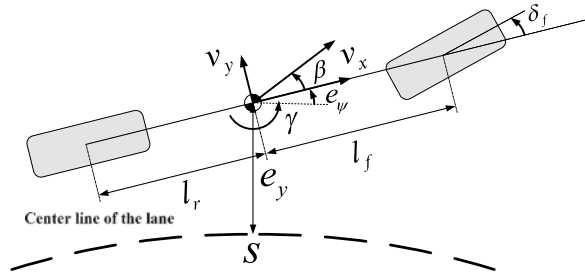


Fig. 46 The safe driving envelope (green box) from Suh (2016)

Fig. 47 Vehicle control model from Suh (2016)



Cheng (2010). Based on such method, the SMPC of (3), a computationally intractable problem, can be simplified into a linear model predictive control problem as follows:

$$\begin{aligned}
 \min \quad & \sum_{k=0}^{N_p-1} \left(\|z_{t+k+1,t} - z_{ref,k+1}\|_{\mathbf{Q}}^2 + \|K_{t+k,t} z_{t+k,t} + b_{t+k,t}\|_{\mathbf{R}}^2 + \lambda \|\rho\|^2 \right) \\
 \text{s.t.} \quad & z_{t+k+1,t} = \mathbf{A}_{l,k} z_{t+k,t} + \mathbf{B}_{l,k} b_{t+k,t} \\
 & g_k^T z_k \leq h_k - \gamma_{\max,k} + \rho \\
 & u_{k,t} \leq u_{\max} \\
 & u_{k,t} \geq u_{\min} \\
 & x_{t,t} = x(t) \\
 & (k = 0, \dots, N_p-1)
 \end{aligned} \tag{4}$$

where $\gamma_{\max,k}$ is the parameter related to the probability p and the known stochastic property. By setting p in the given normal distribution, the $\gamma_{\max,k}$ can be determined as illustrated in Fig. 48.

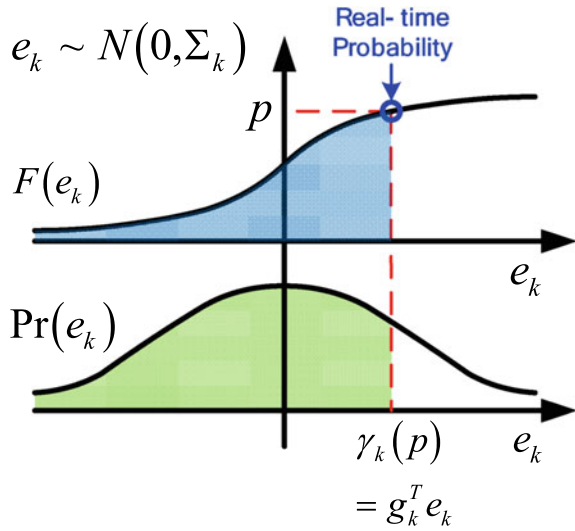
By using a solver in Domahidi (2012), the optimization problem in (4) is solved in real-time. For more details of the SMPC utilized in this paper, the interested readers should read Suh (2016).

3.3 Conclusion and Future Works

Decision and motion planning for fully automated driving have been described. In addition to Lane keeping/lane change in motorway environments, intention-inference-based decision and motion planning for urban intersection autonomous driving have been described.

Risk detection and decision/motion planning in snowy or rainy road environments are the on-going topics of research. Dynamic stability monitoring and control for road-off/lane-off safety, smooth motion control for ride comfort, and naturalistic motion planning for motion sickness are also the topics of future research.

Fig. 48 The performance of the multi traffic driving status estimation algorithm from Suh (2016)



4 Automatic Steering Control for Autonomous Vehicle Path Tracking

This section provides design, stability, and experimental validation of the automatic steering control system. The steering control algorithm in this chapter is defined as ‘the algorithm that provides a steering torque to track pre-designed path (desired path).’ The subsequent sections study a steering control framework that is realizable, practical, and theoretically stable. Moreover, the final section will provide the steering control framework to achieve autonomous drifting.

The block diagram of the steering control algorithm is given in Fig. 49. The steering control algorithm consists of two parts: (1) path tracking algorithm and (2) torque controller.

The path tracking algorithm uses the desired path and vehicle motion states to calculate a steering wheel angle that reduces error between vehicle position states

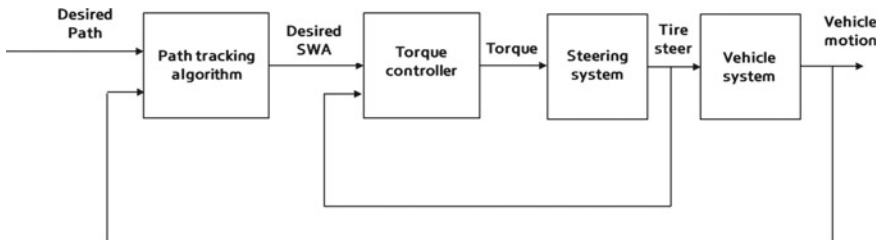


Fig. 49 The block diagram of the steering control system

and the desired path. The torque controller determines the steering torque to achieve the calculated steering wheel angle of the path tracking algorithm.

4.1 Path Tracking Algorithm

The path tracking algorithm determines a steering wheel angle input to track the desired path. A number of researchers conducted research in path tracking, and the traits of each method are well-known in the autonomous vehicle research field. Specifically, Snider (2009) empirically evaluated five methods (Pure pursuit, Stanley, Kinematic steering, Linear Quadratic Regulator with Feedforward input, and preview steering controller) regarding three performance characteristics (robustness to disturbances, steady state error, and overshoot response of the vehicle). Table 5 is the evaluation results in Snider (2009).

In this chapter, a pure pursuit algorithm is adopted to design the path tracking control algorithm.

Pure pursuit algorithm

The design of the path tracking algorithm includes modeling of vehicle lateral motions. As far as mild maneuvering region of autonomous vehicles is concerned, Ackerman geometry is an appropriate model for controlling vehicle lateral motion. This is because the Ackerman geometry fits reasonably well at low speeds and

Table 5 Empirical comparison of tracking results in Snider (2009)

Tracking method	Robustness to disturbances	Steady state error	Overshooting
Pure pursuit	Good	Significant as speed increases	Moderate as speed increases
Stanley	Fair	Significant as speed increases	Moderate as speed increases
Kinematic	Poor	Significant as speed increases	Moderate as speed increases Significant during rapidly changing curvature
LQR with FF	Poor	Minimal until much higher speeds	Significant during rapidly changing curvature
Preview	Fair	Minimal until much higher speeds	Moderate in during rapidly changing curvature

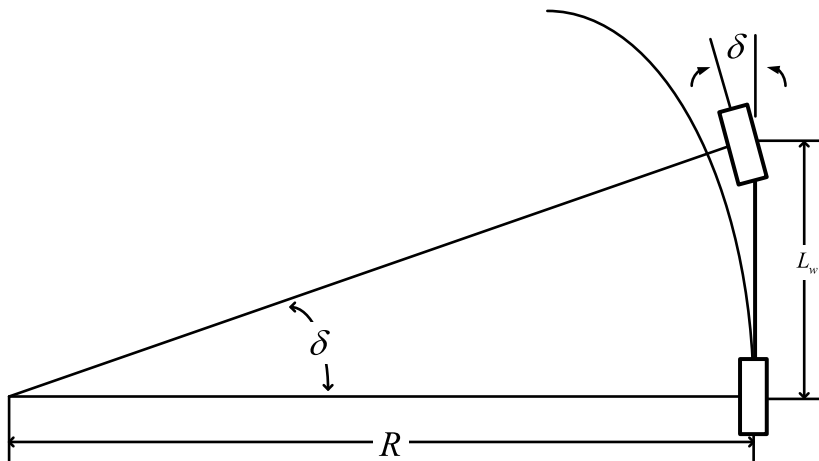


Fig. 50 Ackerman geometry

moderate steering angles. The Ackerman geometry is a kinematic model that represents the relationship between the steering angle and the vehicle lateral motions. Under the assumption described below, the Ackerman geometry can be derived.

Consider a bicycle model in Fig. 50. In the bicycle model, left and right wheels are lumped together and treated as one wheel. Moreover, assume that the velocity at each wheel is in the direction of the wheel. This assumption fits well when the vehicle is not at higher speed and higher lateral acceleration. Under these assumption, the steering angle of the front wheel can be derived as:

$$\delta = \frac{L_w}{R} \quad (5)$$

where δ is a steering angle of the front wheel, L_w is a wheelbase of the vehicle, and R is cornering radius of the vehicle.

Based on the Ackerman geometry, the Pure pursuit algorithm is designed. The Pure pursuit is a path tracking algorithm that works by calculating the curvature-to-go that will move a vehicle from its current position to the goal position on the desired path (see Coulter, 1992). To calculate the curvature-to-go, the algorithm must set a goal point on the desired path beforehand. After choosing the goal point, the algorithm can draw the circle that links the current position to the goal point and thereby calculates the curvature-to-go.

In the last part of this section, the key equations of the Pure pursuit algorithm will be derived, and the experimental results will be presented.

Consider the vehicle in Fig. 51. Because the road desired path is a set of X, Y coordinates, the road desired path in Fig. 51 is presented as equally spaced points. As shown in Fig. 51, the goal point (P_k in the figure) is the closest point to the one located at a preview distance L_p ahead of the vehicle current position. After finding

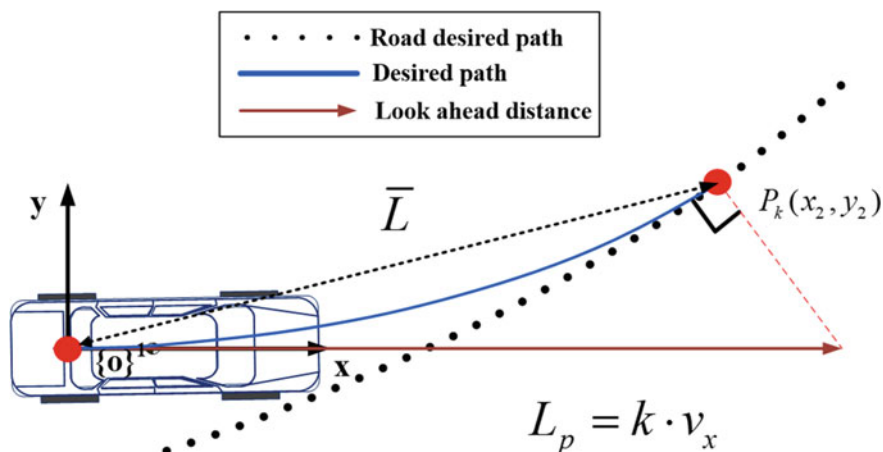


Fig. 51 Choose the goal point among X, Y coordinate of the desired path

the goal point, one can draw the circle that connects the vehicle current position and the goal point. The blue line in Fig. 51 denotes this circle. $\bar{L}$ is the length of the line that links the vehicle's current position to the goal point.

Consider the cornering kinematics and the bicycle model in Fig. 52. From the isosceles triangle in Fig. 52, the following equation can be derived:

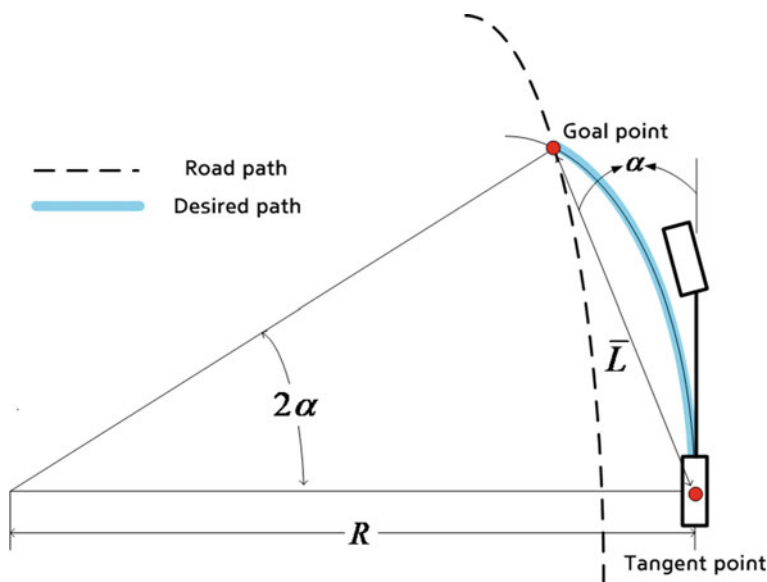


Fig. 52 Calculate the curvature-to-go

$$R \sin(\alpha) = \frac{\bar{L}}{2} \quad (6)$$

From (5, 6), the relationship between the curvature-to-go and the steering angle can be formulated:

$$\delta = \frac{L_w}{R} = \frac{2L_w \sin(\alpha)}{\bar{L}} \quad (7)$$

As shown in the Fig. 52, α in (6, 7) is the angle between the x-axis of the local coordinate system and the line that connects the vehicle's position to the goal point. To calculate the angle α , we need to rotate the road desired path in the global coordinate system to the local coordinate system as shown in Fig. 51. In the local coordinate system, the angle α can be easily derived:

$$\sin(\alpha) = \frac{y_2}{\bar{L}} \quad (8)$$

Based on (7, 8), the steering command of the Pure Pursuit algorithm can be formulated as follows.

$$\delta = \frac{2L_w y_2}{\bar{L}^2} \quad (9)$$

Because of the steering gear ratio N , the desired steering wheel angle can be derived as:

$$\theta_{des} = N \cdot \delta \quad (10)$$

Vehicle test results: Pure pursuit algorithm

With the Pure Pursuit algorithm as the path tracking algorithm, we test to verify the path tracking performance. Our testbed is shown in Fig. 53. Our testbed is equipped with environment sensors for localization and perception purposes, global positioning system (RT 3002), on-board computers (Autobox) and actuators to control the vehicle.

The test scenario is the path tracking at low speed (8–12 km/h) on high curvature road (minimum radius 10 m). The road desired path is the centerline of the lane.

The test results are shown in Figs. 54 and 55. Figure 54 shows the trajectory of the vehicle. From the large curvature ($R = 10$ m) to the zero curvature (straight line), the Pure pursuit algorithm shows reasonably good performance in path tracking. Figure 55 shows the vehicle speed, steering wheel angle, and the lateral error of the vehicle. As illustrated in Fig. 55, the longitudinal speed is almost constant in this test, and the maximum lateral error is below 0.3 m. Consider that the lane width is 3.6 m, the error 0.3 m is below 10%. The Pure Pursuit algorithm calculates appropriate steering wheel angles to follow the pre-defined desired path.



Fig. 53 Vehicle testbed: Hyundai K7

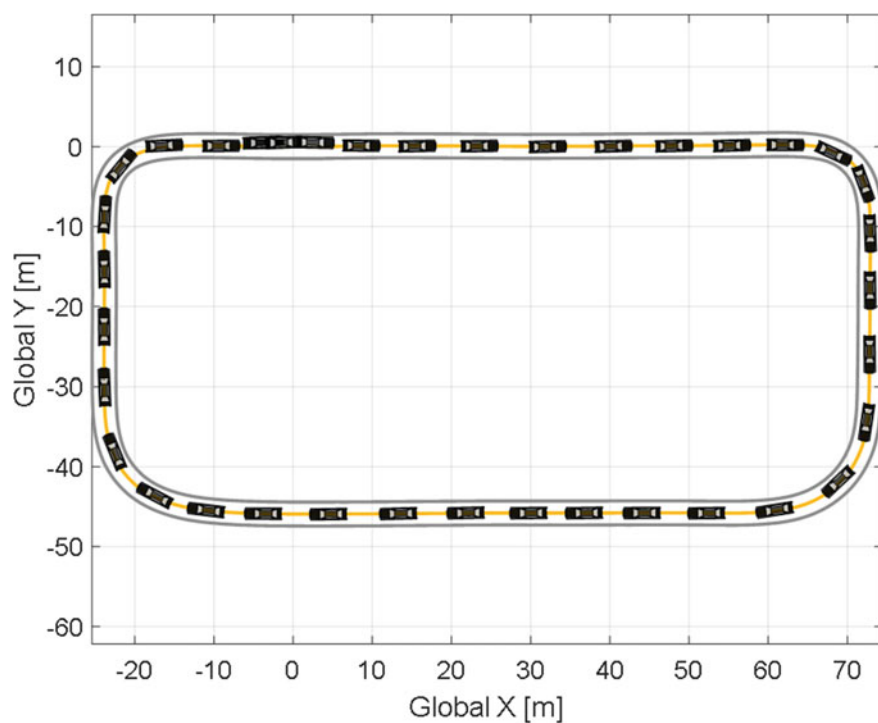
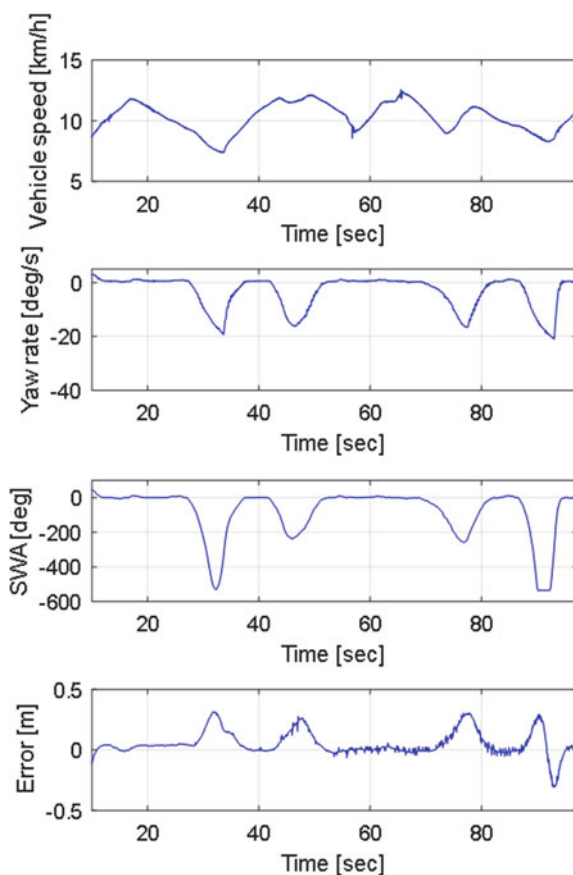


Fig. 54 Trajectory of the vehicle

Fig. 55 Vehicle speed, steering wheel angle, and lateral error



In this section, the Pure Pursuit algorithm is presented. The key equations of the Pure Pursuit algorithm are derived, and the vehicle tests were conducted to verify the Pure Pursuit algorithm. The test results show that the Pure Pursuit algorithm offers reasonable path tracking performance with simple equations.

4.2 Torque Controller

The torque controller determines the steering motor torque to achieve the desired steering wheel angle, the output of the Pure pursuit algorithm.

Simplified steering system model

The steering motor exerts the torque input, and the torque input is transmitted through the steering column and rack system. Finally, the torque input steers the front wheels.

To design a model-based torque controller, we need to model the steering mechanism. One of the simplest and most reasonable models is the simplified steering system model in Gillespie (1992). This model approximates the steering system as the spring-damper system as illustrated in Fig. 56.

Based on this model, the steering dynamics can be formulated as follows:

$$J_{eq}\ddot{\theta}_{sw} + B_{eq}\dot{\theta}_{sw} + K_{eff}\theta_{sw} = T_{mot} - F_d \quad (11)$$

where θ_{sw} is the steering wheel angle; J_{eq} (equivalent inertia), B_{eq} (equivalent damping coefficient), K_{eff} (effective compliance) are parameters of the steering system; T_{mot} is the steering motor torque input; and F_d is the total external force which is the sum of rack force, friction forces of steering system, and road disturbance.

Torque Controller: Estimation and Adaptive sliding mode controller

The block diagram of the torque controller is presented in Fig. 57. As illustrated

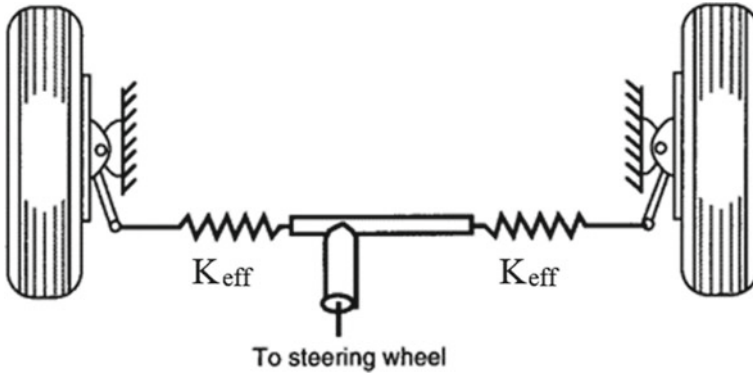


Fig. 56 Simplified steering system model from Gillespie (1992)

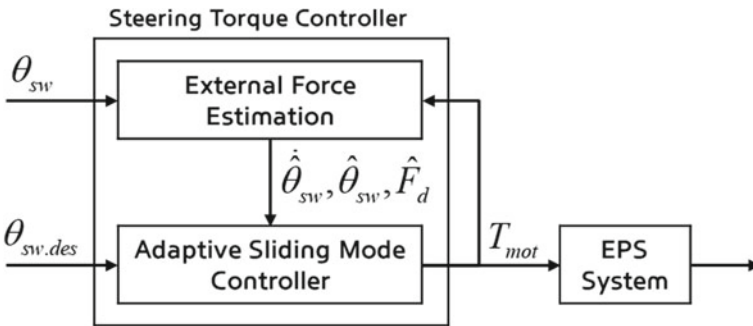


Fig. 57 The block diagram of the torque controller

in the block diagram, the torque controller consists of two parts: (1) External force estimation and (6) Adaptive sliding mode controller.

The external force estimation estimates the total external force F_d and the first time derivative of the steering wheel angle θ_{sw} to calculate the desired torque input. These two signals are estimated by means of a Kalman filter. Because the dynamics of the external force does not exist, the random walk model is adopted to represent the time derivative of the total external force. Based on the steering system model in Eq. (11) and the random walk model of the external force, the following process model can be derived.

$$\dot{x} = \begin{bmatrix} \ddot{\theta}_{sw} \\ \dot{\theta}_{sw} \\ \dot{F}_d \end{bmatrix} = Ax + Bu = \begin{bmatrix} -\frac{B_{eq}}{J_{eq}} & -\frac{K_{eff}}{J_{eq}} & -\frac{1}{J_{eq}} \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_{sw} \\ \theta_{sw} \\ \dot{F}_d \end{bmatrix} + \begin{bmatrix} \frac{T_{mot}}{J_{eq}} \\ 0 \\ \omega \end{bmatrix} \quad (12)$$

where ω is gaussian noise of the random walk model. In this system, the steering wheel angle is only measurement. Thus, the measurement model can be formulated as follows.

$$z = Hx = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_{sw} \\ \theta_{sw} \\ F_d \end{bmatrix} \quad (13)$$

Three parameters of the steering model in (11) should be identified to calculate the reasonable torque input. Among the three parameters, two parameters $\hat{J}_{eq}$ and B_{eq} are nearly constant according to Gillespie (1992). Therefore, the effective compliance K_{eff} is the only parameter to be identified in real-time. In our torque control framework, the adaptive sliding mode control is utilized to identify the parameter K_{eff} and calculate the steering torque input (that can achieve the desired steering wheel angle) at the same time.

By employing the adaptive sliding mode control, the following control law and the adaptation law can be derived.

Control law

$$e(t) = \theta_{sw.des}(t) - \theta_{sw}(t)$$

$$s(t) = \lambda e(t) + \dot{e}(t)$$

$$T_{mot} = \hat{B}_{eq}\dot{\theta}_{sw} + \hat{K}_{eff}\theta_{sat} + \hat{F}_d + \hat{J}_{eq}(\ddot{\theta}_{sw.des} + \dot{e}) + k \cdot \text{sat}(s/\Phi)$$

Adaptive law

$$\hat{K}_{eff} = \frac{1}{\rho_K} \cdot s \cdot \theta_{sw} \quad (14)$$

where $e(t)$ is the steering wheel angle error; $s(t)$ is the sliding variable of the adaptive sliding mode controller; and ρ_K is the adaptive gain.

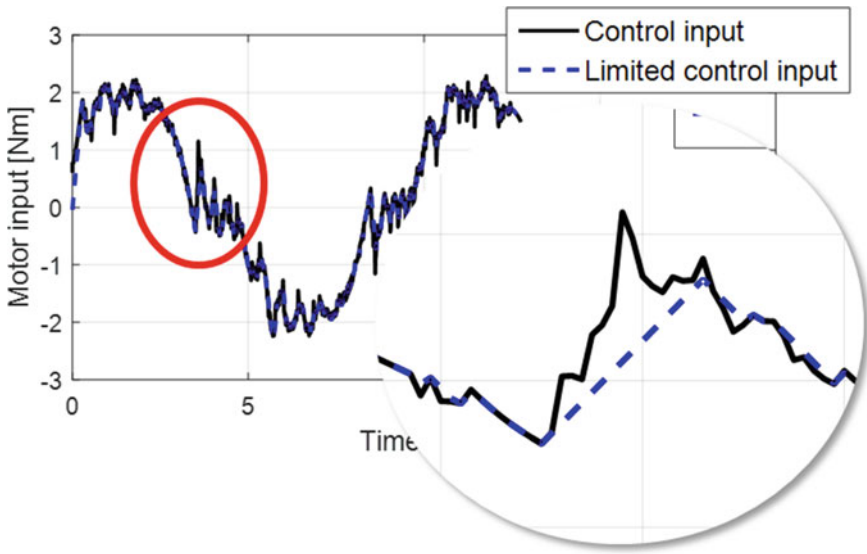


Fig. 58 Difference between torque control command and actual torque output

Vehicle test results: Torque controller

Vehicle tests were conducted to verify the tracking performance of the torque controller. The testbed is the same vehicle in Fig. 53. The steering motor in this vehicle has the limitation in the torque and torque rate. The torque and torque rate limit of the vehicle is ± 8 Nm and ± 8 Nm/s, respectively. Due to the limitation, there is difference between the actual torque input and the torque command as illustrated in Fig. 58.

Sine steering tests were conducted to test the torque controller, and the torque controller was compared with the Proportional-Integral (PI) controller. The test results of three different tests are presented in Figs. 59, 60 and 61, respectively. In Fig. 59, the torque controller was tested at vehicle speed 10 km/h, frequency 0.1 Hz, and peak steering wheel angle 45° . The lower-right figure of Fig. 59 shows the effective compliance from the adaptive sliding mode controller. The upper-left figure shows the force from effective compliance and the adaptation force. As illustrated in these figures, the adaptive sliding model controller can estimate the effective compliance to minimize the error between adaptation force and effective compliance. Because of this adaptation and the estimated external force, the proposed torque controller shows better tracking performance than PI controller.

The proposed controller shows robust performance against vehicle speed and sine frequency. Figure 60 shows the vehicle test results when the vehicle speed is 20 km/h, frequency is 0.1 Hz, and the peak steering wheel angle is 45° . The test results show that the proposed controller successfully tracks the desired steering wheel angle. Figure 61 shows the vehicle test results when the vehicle speed is 30 km/h, frequency

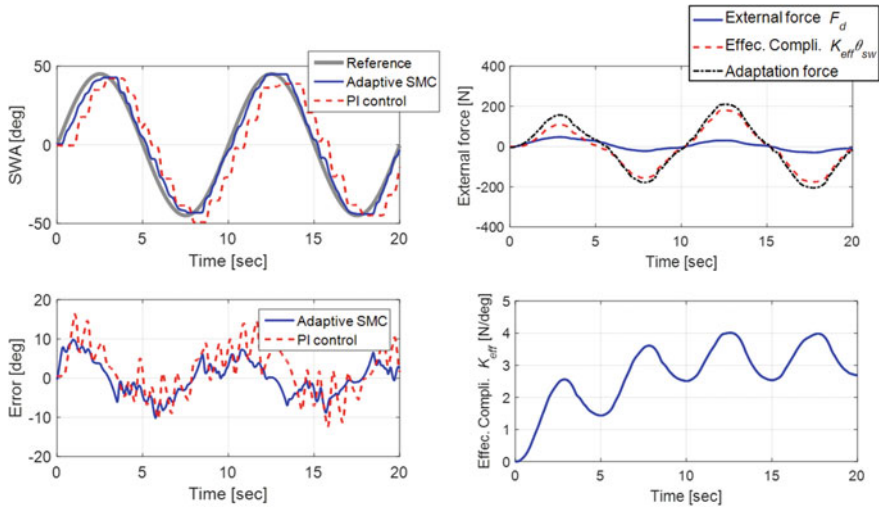


Fig. 59 Vehicle test results: 10 km/h, 0.1 Hz, 45°

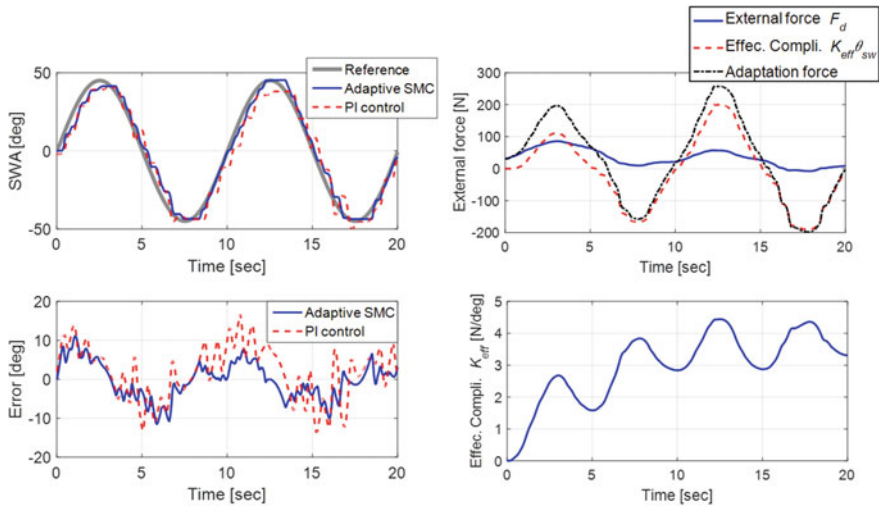


Fig. 60 Vehicle test results: 20 km/h, 0.1 Hz, 45°

is 0.1 Hz, and the peak steering wheel angle is 45°. The test results show that the proposed controller successfully tracks the desired steering wheel angle.

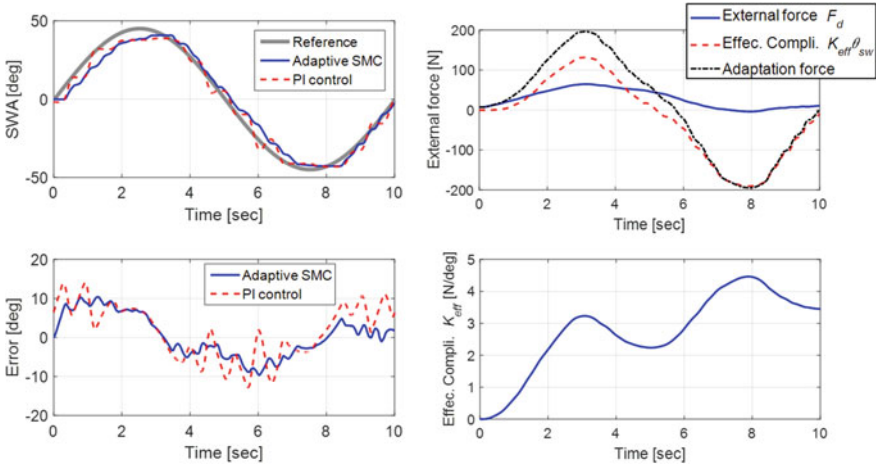


Fig. 61 Vehicle test results: 20 km/h, 0.1 Hz, 45°

4.3 Vehicle Test Results: Automatic Steering Control

This section presents the vehicle test results to test the overall performance of the path tracking algorithm and the torque controller. The test vehicle in this section is a light commercial van, Hyundai Solati equipped with industrial computer that provides the road desired path, dSpace MicroAutobox in which the pure pursuit algorithm and the proposed torque controller are downloaded, and differential global positioning system.

The vehicle test was conducted in K-city, the test site of automated vehicles. The test course is a combination of the urban and main way roads. The consecutive turns are in the urban road, and the cornering radius of such roads is ~15 m. In the main way road, lane changes were conducted with the vehicle speed over 50 km/h (Fig. 62).

The vehicle test results in urban roads are presented in Figs. 63 and 64. In the urban road of the test course, the average vehicle speed is around 30 km/h. The steering wheel angle of the Pure Pursuit algorithm and the steering torque of the proposed torque controller are illustrated in this figure. In this scenario, the maximum steering wheel angle is 300°. This figure shows that the automatic steering control algorithm (the integration of the Pure Pursuit algorithm and the torque controller) tracks the road desired path well.

Figure 64 shows the performance of the torque controller in the urban road. The results show that the torque controller can track the desired steering wheel angle of the Pure Pursuit algorithm under low vehicle speed and high curvature conditions.

The vehicle test results in mainway roads are presented in Figs. 65 and 66. In the mainway road of the test course, the average vehicle speed is over 50 km/h. The results show that the control algorithm controls well the vehicle to track the road desired path under high speed and low curvature conditions. In Fig. 66, the performance of the

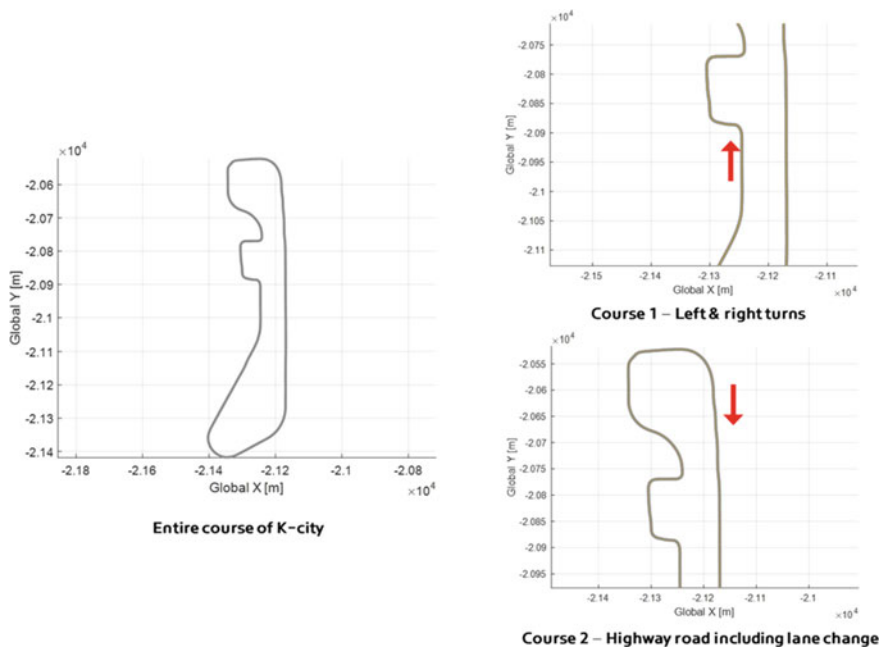


Fig. 62 Test course in K-city

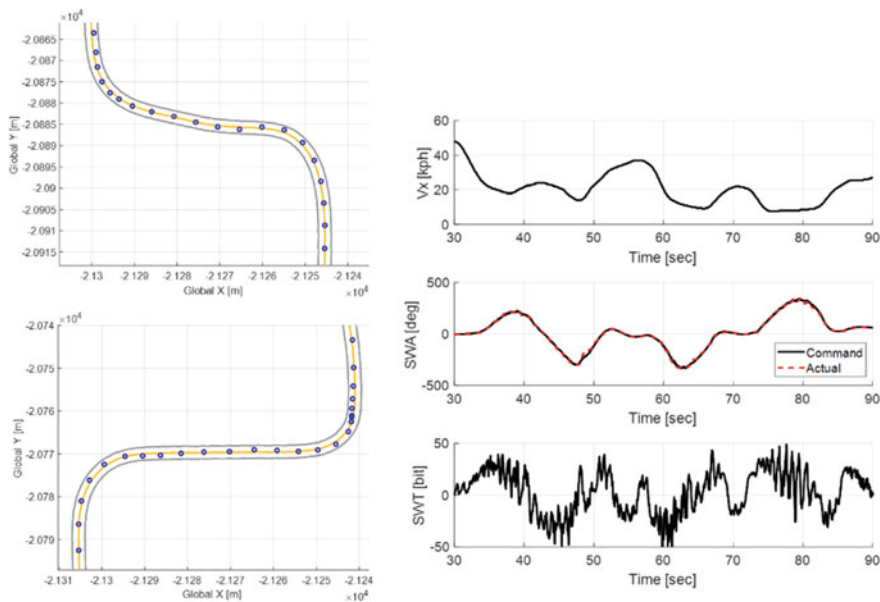


Fig. 63 Vehicle test results in the urban road

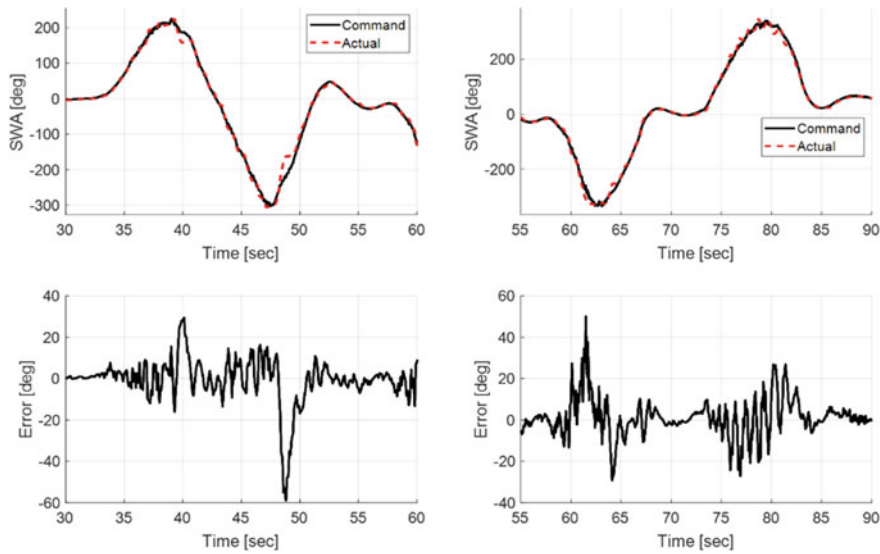


Fig. 64 The performance of the torque controller in the urban road

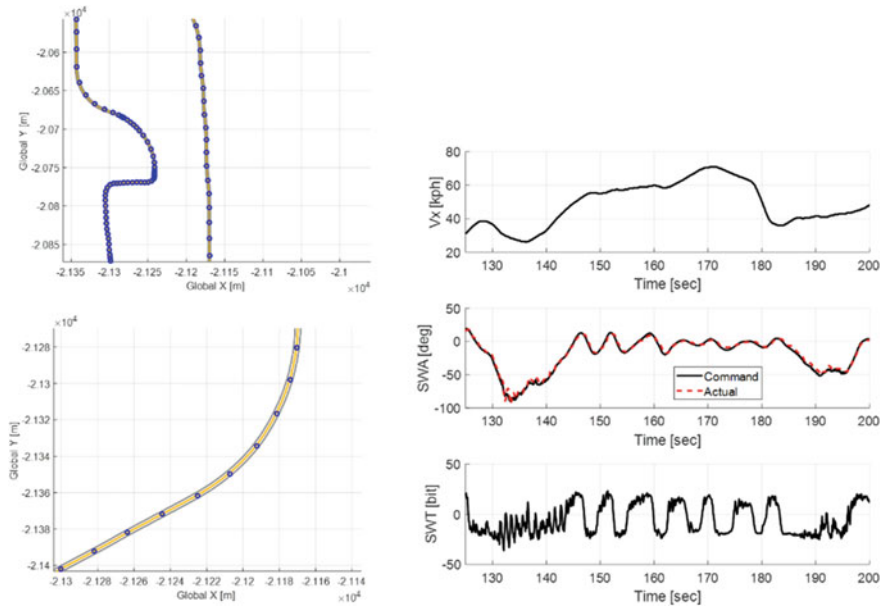


Fig. 65 Vehicle test results in the main road

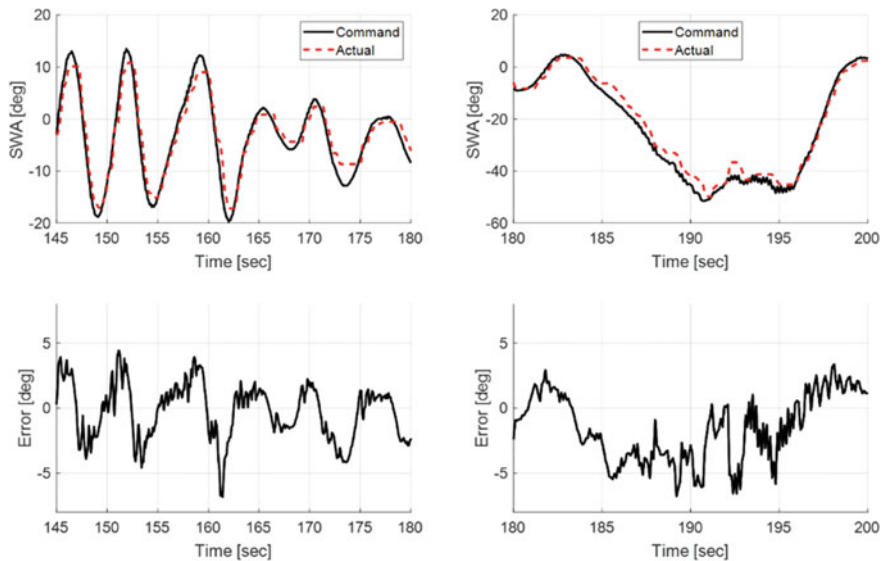


Fig. 66 The performance of the torque controller in the mainway road

proposed torque controller are given. Compared to the performance in Fig. 64 (Urban road), the performance in Fig. 66 shows better steering angle tracking performance as illustrated in the steering wheel angle error. This is because the friction force of the steering system, which is difficult to estimate, increases as a vehicle speed decreases, and large friction forces deteriorate the tracking performance.

4.4 Stability Analysis

This section presents the stability analysis of the path tracking control algorithm. The results of the stability analysis provide the stability proof and the range of the preview distance that can regulate the path tracking error despite the presence of actuator delays.

Steering system identification

The overall block diagram of the automatic steering control can be illustrated as Fig. 67. To prove the stability of the path tracking algorithm in our system, the mathematical model of steering system is identified first.

The steering system model's parameters are identified through step response tests. To be specific, we exert step steering input and acquire the data to model the steering system based on the method in Garnier (2003). The mathematical model we utilized is a transfer function in (15). In summary, by conducting step steering input tests as Fig. 68, we obtain the model parameters of the steering system.

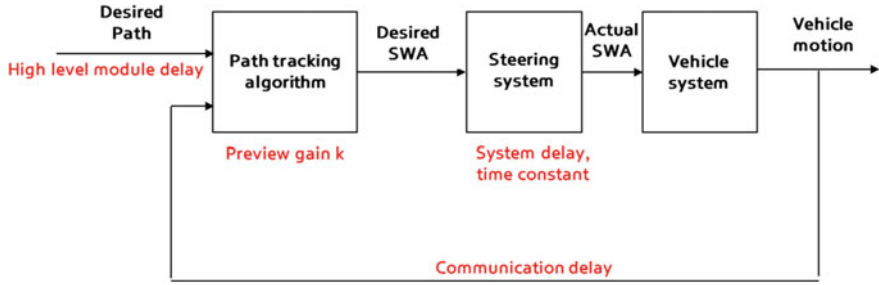


Fig. 67 Block diagram of the automatic steering control system

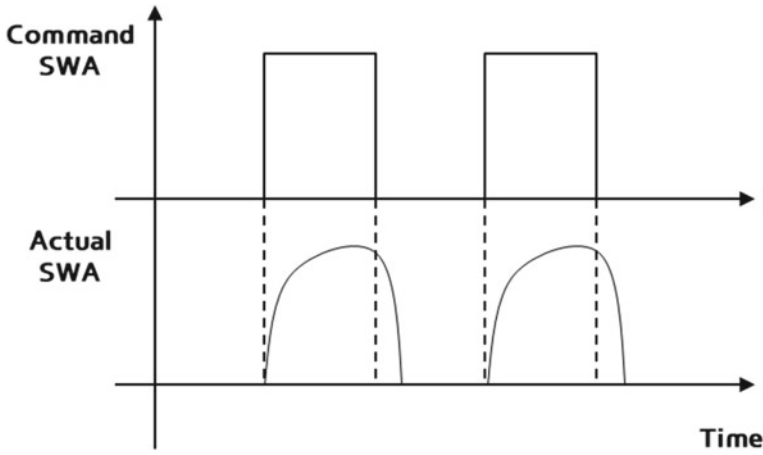


Fig. 68 Step steering test for the system identification

$$\frac{\theta_{swa}(s)}{\theta_{cmd}(s)} = \frac{z_1 s^{n-1} + z_2 s^{n-2} + \dots + p_n}{p_1 s^n + p_2 s^{n-1} + \dots + p_{n+1}} e^{-T_d s} \quad (15)$$

The identification results are shown in Fig. 69. The desired steering wheel angle is presented as red solid line in the figure. Due to the actuator system delay, communication delay, and the actuator dynamics, the actual steering wheel angle (magenta dashed dotted line) is delayed. With the method in Garnier (2003), we are able to model the response of the steering system. As illustrated in Fig. 69, the identified model shows similar response with the actual steering wheel angle. The identified model can be summarized in transfer function form as:

$$\frac{\theta_{swa}(s)}{\theta_{cmd}(s)} = \frac{35.22s + 118.08}{s^2 + 38.68s + 118.40} e^{-0.05s} \quad (16)$$

Stability analysis from Heredia and Ollero (2007)

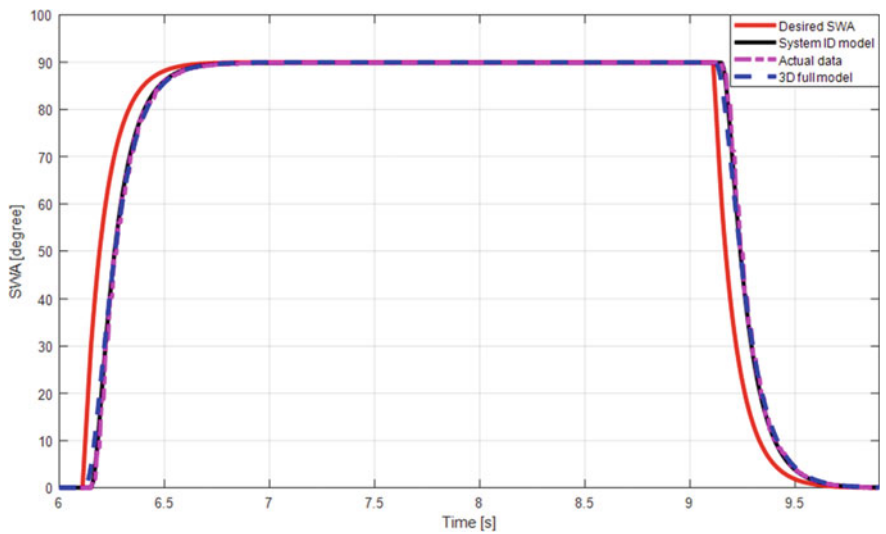
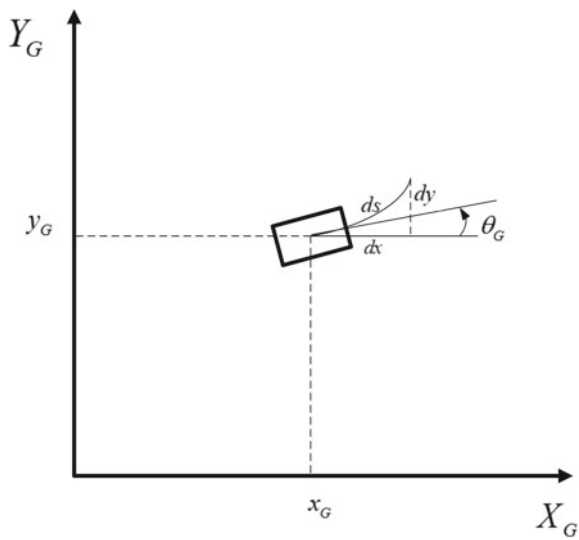


Fig. 69 Step steering test for the system identification

This section provides the stability of the closed-loop steering control system. The point mass model is utilized to represent the vehicle motions, and the Pure Pursuit path tracking algorithm is a steering control algorithm.

The differential equation of the point mass model can be derived from the kinematics. As illustrated in Fig. 70, the infinitesimal element of the global coordinates of the point mass model can be derived as follows:

Fig. 70 Cornering kinematics of the point mass model



$$\begin{aligned}
dx_G &= \cos \theta_G \cdot ds \\
dy_G &= \sin \theta_G \cdot ds \\
d\theta_G &= \kappa_{act} \cdot ds
\end{aligned} \tag{17}$$

From (17), the differential equations of the vehicle cornering kinematics can be formulated as:

$$\begin{aligned}
\dot{x}_G(t) &= V \cos \theta_G(t) \\
\dot{y}_G(t) &= V \sin \theta_G(t) \\
\dot{\theta}_G(t) &= V \kappa_{act}(t)
\end{aligned} \tag{18}$$

To derive the closed-loop dynamics, we incorporate the Pure Pursuit algorithm into the point mass model. Basically, the output (or the control input) of the Pure Pursuit algorithm is a steering wheel angle. However, to simplify the stability proof, we assume that the steering wheel angle from the Pure Pursuit algorithm can be replaced with the desired road curvature in (6). Considering the steering system delay, we assume that the relationship between vehicle's curvature in (18) and the desired curvature from the Pure Pursuit algorithm is first-order dynamics as follows:

$$\dot{\kappa}_{act}(t) = -\frac{1}{T} [\kappa_{act}(t) - \kappa_{act,input}(t - \tau)] \tag{19}$$

where T is a steering system time delay constant, τ is an overall system time delay, κ_{act} is the cornering curvature of the vehicle and $\kappa_{act,input}$ is the desired road curvature of the Pure Pursuit algorithm in (6).

In the case of the straight driving, the lateral error and the heading error are defined in Fig. 71. To derive the road desired curvature of the Pure Pursuit algorithm, we need to calculate y_2 as we did before at **Pure Pursuit algorithm** subsection in the Sect. 2 (see Fig. 3 for the definition of y_2). From the geometry of the path tracking kinematics in Fig. 23, the following equation is derived.

$$y_2 = e_y \cos e_\psi - \sqrt{L^2 - e_y^2} \sin e_\psi \tag{20}$$

From (6), (8), and (20), the desired road curvature is derived as follows:

$$\kappa_{act,input}(t) = \frac{2}{VT \cdot L^2} \left(y \cos \theta - \sqrt{L^2 - y^2} \sin \theta \right) \tag{21}$$

From (18) and (19), a three-dimensional dynamic equation can be derived. We nondimensionalize the states and the parameters of the dynamic equation as follows:

$$y = \frac{e_y}{VT}, \theta = e_\psi, \kappa = VT \kappa_{act}, \kappa_{input} = VT \kappa_{act,input},$$

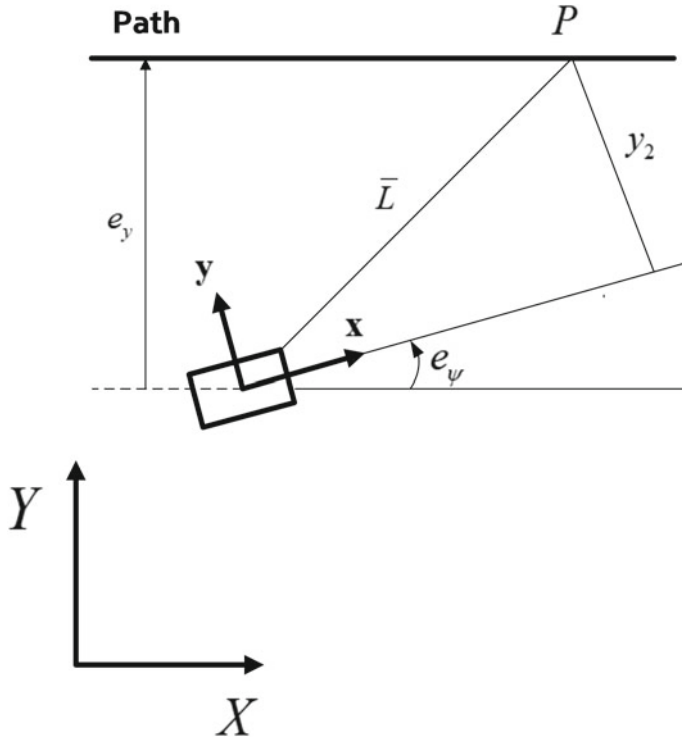


Fig. 71 Tracking the straight road

$$\tau = \frac{\tau_{act}}{T}, t = \frac{t_{act}}{T}, L = \frac{\bar{L}}{VT} \quad (22)$$

We set a lateral error, a heading error, and a curvature as state variables of the dynamic equation. Based on the nondimensionalized states (22), the path tracking dynamics can be derived as follows:

$$\begin{aligned} \dot{y}(Tt) &= -\sin \theta(Tt) \\ \dot{\theta}(Tt) &= \kappa(Tt) \\ \dot{\kappa}(Tt) &= -\kappa(Tt) + \kappa_{input}(Tt - T\tau) \end{aligned} \quad (23)$$

If we assume that there does not exist delay in the system delay, the equilibrium point of the closed-loop dynamics is analyzed as follows:

$$\begin{aligned} 0 &= -\sin \theta(t) \\ 0 &= \kappa(t) \\ 0 &= -\kappa(t) + \frac{2}{L^2} \left(y \cos \theta - \sqrt{L^2 - y^2} \sin \theta \right) \end{aligned} \quad (24)$$

There exist two equilibrium points in the closed-loop dynamics (24), $(y \ \theta \ \kappa)_{eq} = (0 \ 0 \ 0)$ or $(0 \ \pi \ 0)$. To analyze the stability of the closed-loop system's equilibrium point, the nonlinear system (24) is linearized around the first equilibrium point.

$$\begin{bmatrix} \dot{y} \\ \dot{\theta} \\ \dot{\kappa} \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ \varphi_y & \varphi_\theta & \varphi_\kappa - 1 \end{bmatrix} \begin{bmatrix} y \\ \theta \\ \kappa \end{bmatrix}$$

$$\varphi_y = \left. \frac{\partial \kappa_{input}}{\partial y} \right|_0 = \frac{2}{L^2}, \varphi_\theta = \left. \frac{\partial \kappa_{input}}{\partial \theta} \right|_0 = -\frac{2}{L}, \varphi_\kappa = \left. \frac{\partial \kappa_{input}}{\partial \kappa} \right|_0 = 0 \quad (25)$$

The characteristic equation of the linearized dynamics (25) is formulated as follows:

$$P(s) = s^3 + s^2(1 - \varphi_\kappa) - s\varphi_\theta + \varphi_y \quad (26)$$

The Routh-Hurwitz stability criterion is conducted to analyze the stability. All elements in the first column of the Routh array must be positive to make the system stable.

$$\begin{aligned} 1 - \varphi_\kappa &> 0 \\ \varphi_y &> 0 \\ -\varphi_\theta(1 - \varphi_\kappa) - \varphi_y &> 0 \end{aligned} \quad (27)$$

Based on the inequalities (27), the stable condition can be derived as follows:

$$L = \frac{\bar{L}}{VT} = \frac{kV}{VT} > 1, \quad k > T \quad (28)$$

Note that this inequality (28) is under the zero system delay assumption. If the system has a delay, then the inequality (28) is re-formulated as:

$$L = \frac{\bar{L}}{VT} > fcn(\tau = \frac{\tau_{act}}{T}), \quad \bar{L} > VT \times fcn(\frac{\tau_{act}}{T}) \quad (29)$$

The graphical results of the Eq. (29) can be plotted in Fig. 72. As illustrated in Fig. 72, the preview distance must increase as the system time delay increases to stabilize the path tracking dynamics.

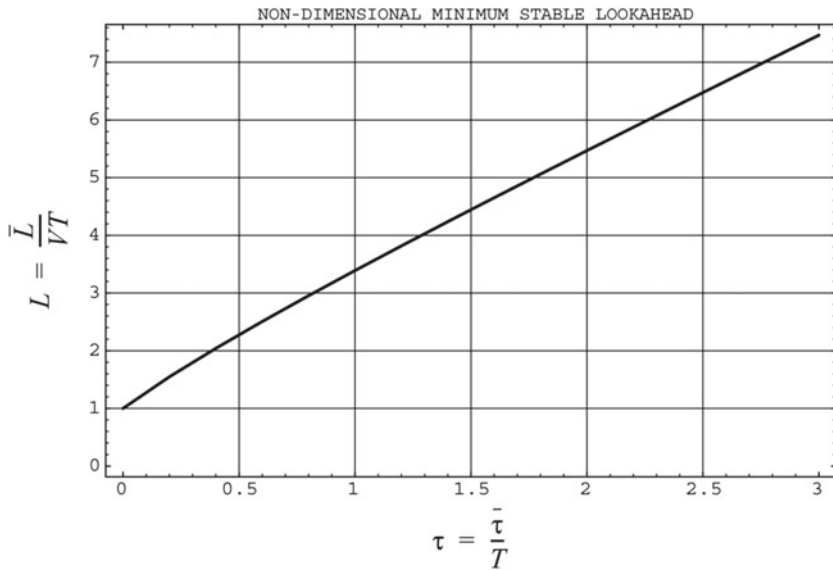


Fig. 72 Preview distance and system delay from Heredia and Ollero (2007)

4.5 Conclusion

The overall automatic steering control algorithm is presented in this chapter. Among the five methods (Pure pursuit, Stanley, Kinematic steering, Linear Quadratic Regulator with Feedforward input, and preview steering controller), the Pure Pursuit algorithm is described in detail and implemented on a test vehicle. For the steering system control, external force estimation and adaptive sliding mode control are proposed. The overall automate steering control algorithm is experimentally validated with a light commercial 4-door van. The stability of the closed-loop system is analyzed, and the results show that the proposed control framework can follow the track with bounded path tracking error in various driving speed/road environments including U-turn.

5 Speed and Clearance Control Algorithm for Autonomous Vehicle Longitudinal Control

The longitudinal control algorithm for autonomous vehicles will be presented in this section. The longitudinal controller of an autonomous vehicle calculates the gas and brake pedal inputs to control the longitudinal dynamics of the vehicle.

One of the renowned longitudinal control systems is Smart Cruise Control (SCC) system. The SCC system has two control modes ((1) Following control and (2) Free

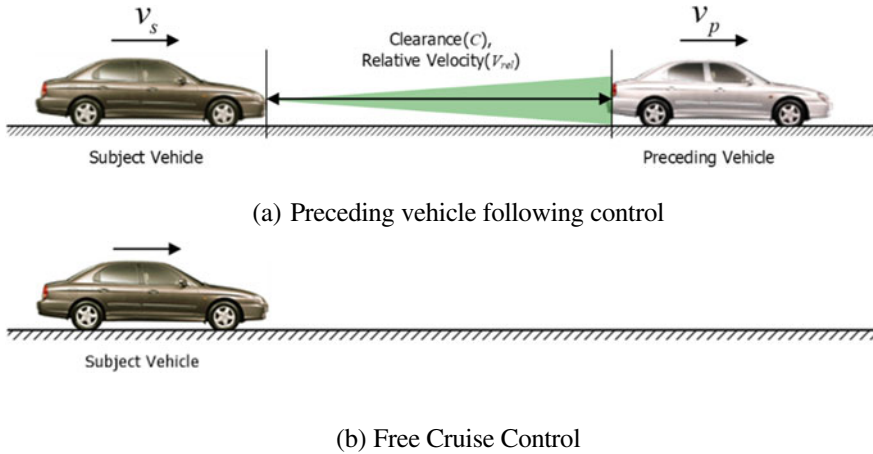


Fig. 73 Smart cruise control

Cruise Control) depending on the presence of the preceding vehicles as illustrated in Fig. 73. When the SCC performs Following Control mode (or when there exists a preceding vehicle), the system utilizes a relative velocity and the clearance to follow the preceding vehicle by maintaining a pre-defined distance. When the SCC is in Free Cruise Control mode, the system controls the vehicle to maintain pre-defined speed.

Our longitudinal control algorithm for automated driving is similar to the way in which the SCC system controls the vehicle in longitudinal direction. The longitudinal control algorithm consists of High and Low level control algorithms. The high level control algorithm determines the longitudinal acceleration depending on control modes: Following control mode and Desired speed tracking mode, while the low level control algorithm calculates the actuator control inputs to achieve the desired longitudinal acceleration. Throughout this section, we will describe a design and experimental validation of such longitudinal control algorithm.

5.1 Vehicle Model for Longitudinal Control

In this section, the vehicle model to control the longitudinal dynamics will be presented. The vehicle model consists of an actuator model and a longitudinal vehicle dynamic model.

Actuator model

The actuator model consists of an accelerator/brake pedal motor model (from motor input to pedal position) and a vehicle system model (from pedal position to acceleration) as illustrated in Fig. 74. “FC Algorithm” implies preceding vehicle following control algorithm that computes the desired acceleration.

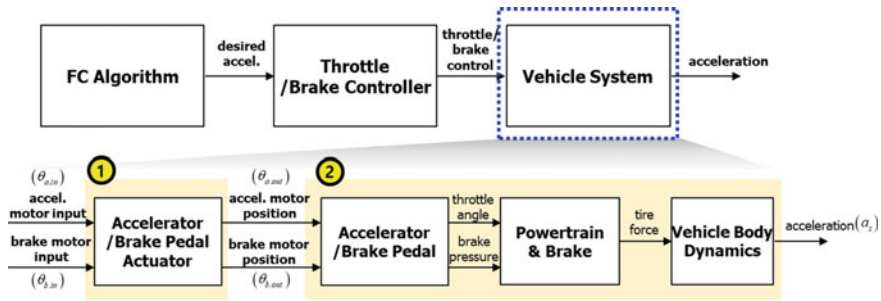


Fig. 74 Vehicle system for longitudinal control

In many cases, in order to control pedal inputs, electric motors are embedded as illustrated in Fig. 75. The accelerator/brake pedal motor model is to represent the motor's dynamics. The pedal actuator can be modelled as a first-order delay with a gain as follows:

$$\frac{\theta_{i.out}}{\theta_{i.in}} = \frac{K}{1 + \tau_{i.p}s} \quad (30)$$

where $\theta_{i.in/out}$ is an input/output of the motors ($i = \{\text{accelerator, brake}\}$), $\tau_{i.p}$ is a time constant, and K is a gain.

To identify the model parameters in (30), multiple experiments are conducted. There are two test scenarios: Step input test and Half-sine input test. Through these tests, we recognize that the accelerator pedal actuator has very small delay, while the brake pedal actuator has large delay. This is because the brake pedal has a large reaction force while the accelerator pedal has very small reaction force.

The test results for brake pedal inputs are presented in Fig. 76. By using a parameter identification tool, the parameters of brake pedal motor model can be derived. The

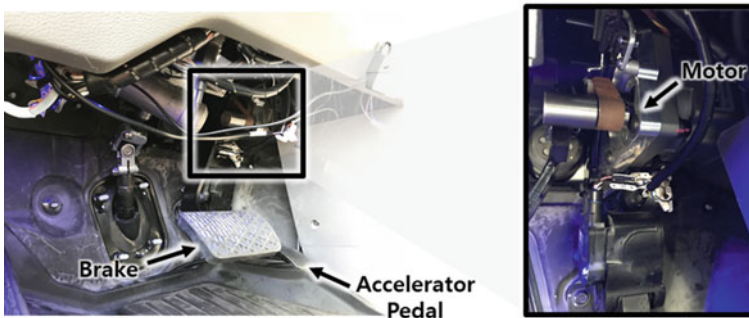


Fig. 75 System hardware interface: Throttle and Brake pedal inputs

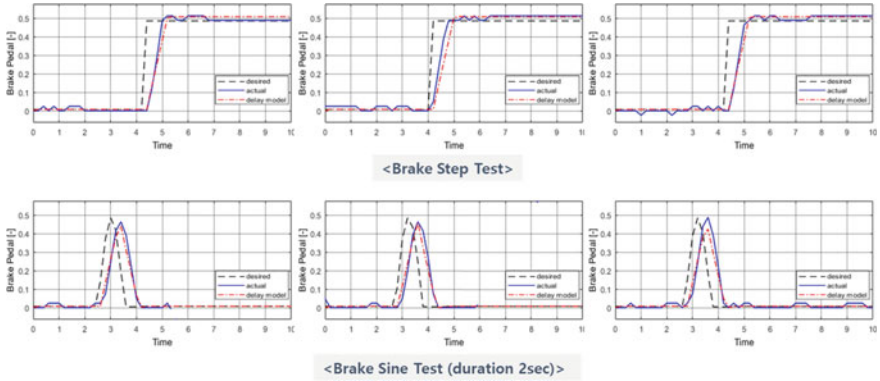


Fig. 76 System identification: Brake pedal actuator

obtained parameters are $K = 1.1$ and $\tau_{brake.p} = 0.05$. Moreover, as shown in Fig. 76, the response of the first-order brake model well fits the actual response.

The remaining part of the actuator model is a vehicle system model. This model represents the dynamics between the pedal inputs and the acceleration output. By conducting vehicle tests as shown in Fig. 77, we model the vehicle system model as a first-order time delay with a dead time as follows:

$$\frac{a_x}{\theta_{i.out}} = \frac{Ke^{-\tau_{i.dead}s}}{1 + \tau_{i.a}s} \quad (31)$$

where $\theta_{i.out}$ is an output of the motors ($i = \{\text{acceleration, deceleration}\}$), $\tau_{i.dead}$ is a dead time constant, $\tau_{i.a}$ is an acceleration constant, and K is a gain.

To identify the model parameters in (31), vehicle test results are analyzed. By using a parameter identification tool, the parameters can be identified. The identified parameters are dead time constants: $\tau_{acceleration/brake.dead} = 0.5 \text{ s}/0.1 \text{ s}$, and time constants $\tau_{acceleration/brake.a} = 0.5 \text{ s}/0.01 \text{ s}$.

In conclusion, the actuator model consists of the pedal motor model and the vehicle system model. The pedal motor model is first-order delay dynamics, and the vehicle system model is first-order delay dynamics with dead time. The model parameters are identified via vehicle tests.

Longitudinal vehicle dynamic model

The longitudinal vehicle dynamic model is presented in this sub-section. Considering mild driving region of autonomous vehicles, we assume that the longitudinal slip is negligible. Under this assumption, the relationship between engine/brake torque and longitudinal force, F_x , can be formulated as follows:

$$F_x = \begin{cases} \frac{1}{r_g R_{eff}} T_e, & \text{for Engine} \\ -\frac{1}{R_{eff}} T_b, & \text{for Brake} \end{cases} \quad (32)$$

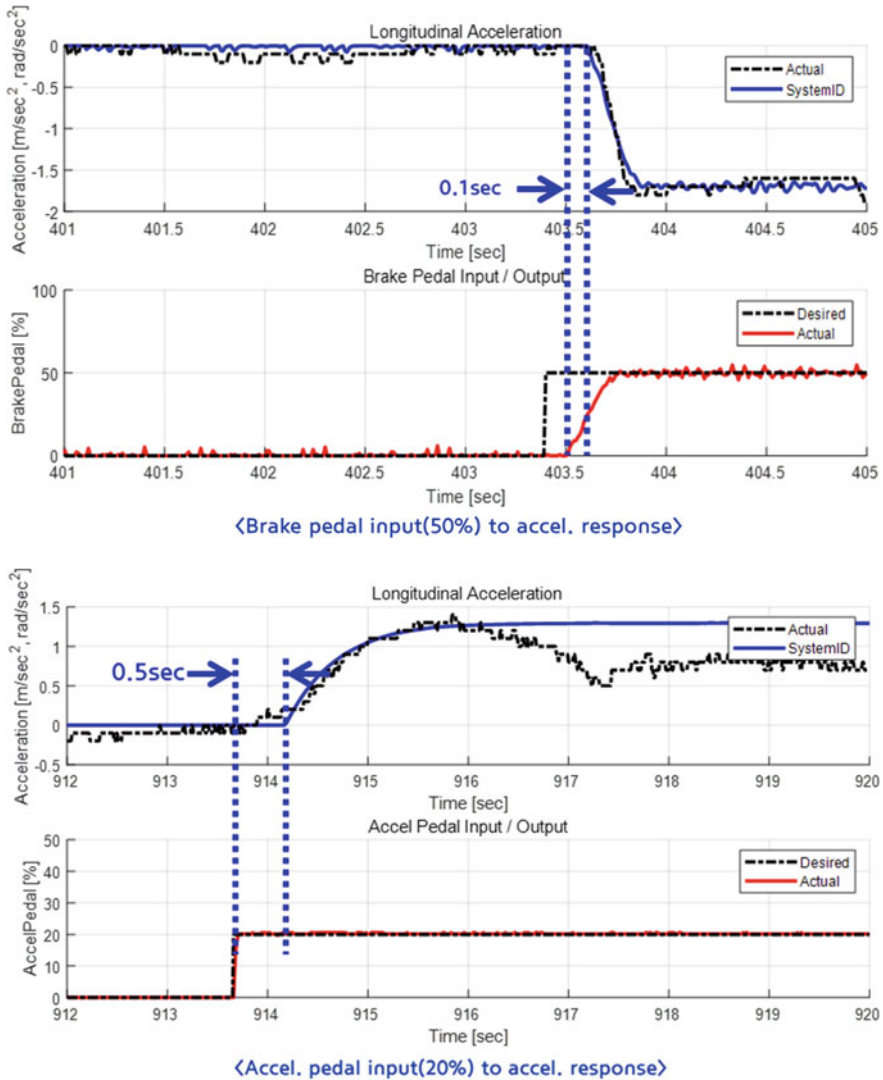


Fig. 77 System identification: Pedal inputs to Acceleration modeling

where $T_{e/b}$ is engine/brake torque, R_{eff} is a tire effective radius, and r_g is a total gear ratio from engine shaft to driving wheels. As analyzed in the previous sub-section, the relationship between engine/brake torques and the throttle/brake inputs is a first-order dynamics with dead time. In this section, for further stability analysis, we assume that the dead time can be neglected, and the relationship can be simplified as a first-order dynamics. Considering time delay in longitudinal dynamics and control algorithm is our future research topic. Therefore, (32) can be re-formulated as follows:

$$F_x = \begin{cases} \frac{1}{r_g R_{eff}} \cdot \frac{k_{th}}{1 + \tau_{th}s} u_{th}, & \text{for Engine} \\ -\frac{1}{R_{eff}} \cdot \frac{k_b}{1 + \tau_b s} u_b, & \text{for Brake} \end{cases} \quad (33)$$

where $u_{th/b}$ is throttle/brake inputs, $\tau_{th/b}$ is a time constant of the first-order dynamics, and $k_{th/b}$ is a gain.

The free body diagram of longitudinal vehicle dynamics is provided in Fig. 78. Based on Fig. 78, the longitudinal vehicle dynamic equation is derived as follows:

$$\begin{aligned} ma_x &= F_x - F_R \\ &= (F_{xf} + F_{xr}) - (F_a + R_{xf} + R_{xr} + mg \sin \theta) \end{aligned} \quad (34)$$

where a_x is a longitudinal acceleration, m is a mass of the vehicle, F_a is the aerodynamic drag force, $R_{xf,r}$ are the rolling resistances of front and rear tires, g is the gravitational acceleration, and θ is the road grade.

From (33) and (34), the vehicle model for longitudinal control can be reformulated as follows:

$$\begin{aligned} ma_x &= F_x - F_R \\ &= \begin{cases} \frac{1}{1 + \tau_{th}s} \left(\frac{k_{th}}{r_g R_{eff}} u_{th} \right) - F_R, & \text{if throttle} \\ \frac{1}{1 + \tau_b s} \left(-\frac{k_b}{R_{eff}} u_b \right) - F_R, & \text{if brake} \end{cases} \end{aligned} \quad (35)$$

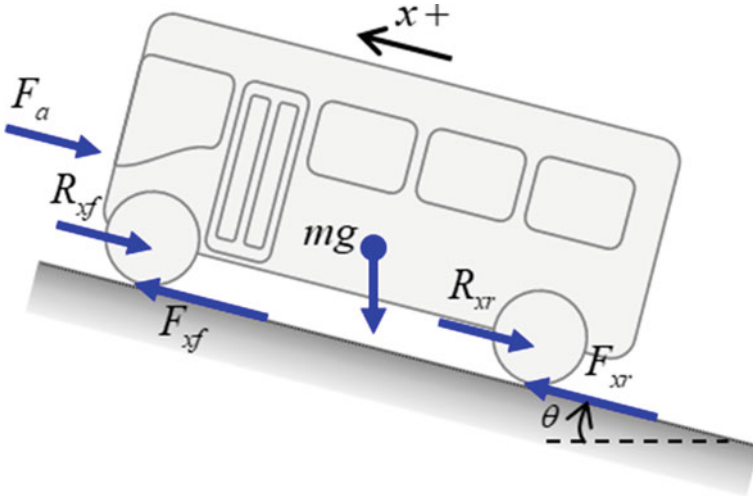


Fig. 78 Vehicle longitudinal dynamics

5.2 High Level Control: Desired Acceleration Design

The high level longitudinal control algorithm determines the desired acceleration depending on control modes: Following control and Free Cruise control. In the preceding-vehicle following control mode as illustrated in Fig. 79, the algorithm utilizes the longitudinal speeds, v_s, v_p , relative distance i.e. clearance, c and the relative speed, v_r to regulate the clearance error (difference between actual clearance c and desired clearance c_{des}) and minimize the relative speed (error between ego-vehicle's speed v_s and preceding vehicle's speed v_p). In the free cruise control, the algorithm maintains pre-set speed.

The free cruise control mode is a special case of the following control mode. By replacing the clearance error with zero and setting the preceding vehicle's speed as a pre-set speed, the following control algorithm can be changed into the free cruise control algorithm. Therefore, the following control algorithm is only described in this sub-section.

Driving behavior analysis

Next sections describe the analysis of human data to set the parameters in the control algorithm. It is important to mimic how human drivers drive in developing the autonomous control algorithm and tuning the parameters of the algorithm. This is because the behavior and the motion of autonomous vehicles should be predictable and acceptable to passengers. In our longitudinal control algorithm, in order to incorporate the human driving characteristics into the control algorithm, the parameters in the desired clearance and the control input are tuned.

Desired clearance

To identify the clearance that human drivers prefer, the human driving data of 125 people for 523 min is analyzed. The collected manual driving data is presented in Fig. 80. Based on the driving behavior of human drivers, the desired clearance can be modelled as follows:

$$c_{des} = c_0 + \tau \cdot v_p \quad (36)$$

where c_0 is a minimum clearance and τ is a time gap. The relationship between the preceding vehicle's speed and the clearance is proportional. The time gap that

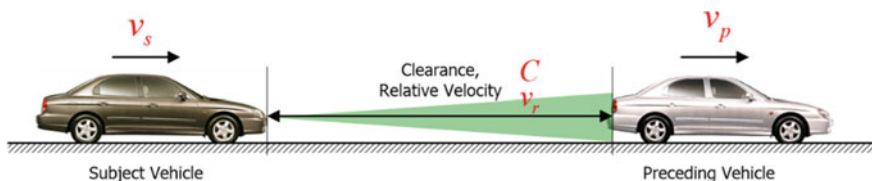
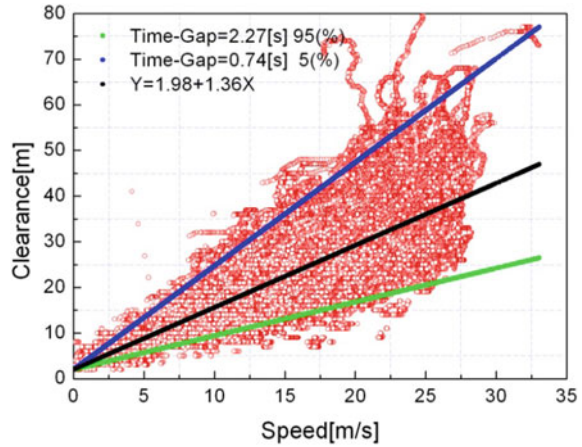


Fig. 79 Control logic: following control

Fig. 80 Time gap of collected manual driving data



human drivers prefer varies from 0.74 to 2.27 s. The mean value of the time gap is 1.36 s. By tuning the time gap within the range, the desired clearance can be designed to show similar performance with human drivers. The minimum clearance also can be obtained through the collected data in Fig. 80. Based on the data, the minimum clearance (y intercept) is 1.98 m.

Clearance error dynamics, Control law, and Control gain from data

Based on the desired clearance, the error dynamics can be formulated as follows:

$$\begin{aligned}
 \dot{x} &= \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} \tau \\ -1 \end{bmatrix} u + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w, \\
 x &= [c_{des} - c, v_p - v_s]^T, \\
 u &= a_s, \\
 w &= a_p
 \end{aligned} \tag{37}$$

where a_s is the acceleration of the subject vehicle and a_p is that of the preceding vehicle.

As presented in (37), the control input, u , for high level control is the longitudinal acceleration because the goal of the high level control algorithm is to determine the desired longitudinal acceleration of the subject vehicle. The acceleration of the preceding vehicle is treated as a disturbance because this signal cannot be measured with on-board sensors. Recently, utilizing vehicle-to-vehicle (V2V) communication is actively discussed and it is possible to receive the longitudinal acceleration of the preceding vehicle with V2V communication.

The control objective of the high level algorithm is to regulate the clearance error in (37). To achieve this goal, the control law is designed in form of full state feedback control as follows:

$$u = a_s = -k_1 \cdot (c_{des} - c) - k_2 \cdot (v_p - v_s) \quad (38)$$

where k_1 and k_2 are feedback gains.

It is noteworthy that the control gains, k_1 and k_2 , in (38) are functions of the ego-subject vehicle's speed, v_s . This is because human drivers show different driving behaviors depending on the ego-vehicle's speed as illustrated in Fig. 81. As illustrated in Fig. 81, human drivers tend to smaller acceleration as the vehicle speed increases. To incorporate such characteristics into the control law in (38), the control gains are differently designed with respect to the ego-vehicle speed as conceptually presented in Fig. 82.

The control law is a full state feedback as described in (38), and when the errors are large, the desired acceleration can be too large to feel comfortable. To avoid such large acceleration issue, the maximum acceleration is analyzed using collected data. The results are presented in Fig. 83. The data show that the maximum acceleration

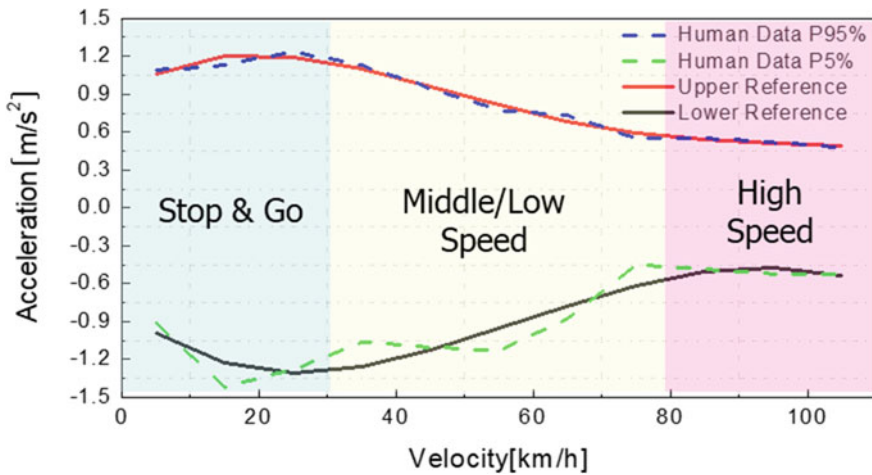
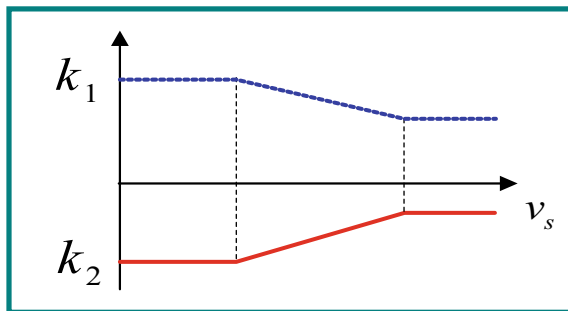


Fig. 81 Gain tuning to mimic human drivers

Fig. 82 Control gain with respect to ego-vehicle's speed



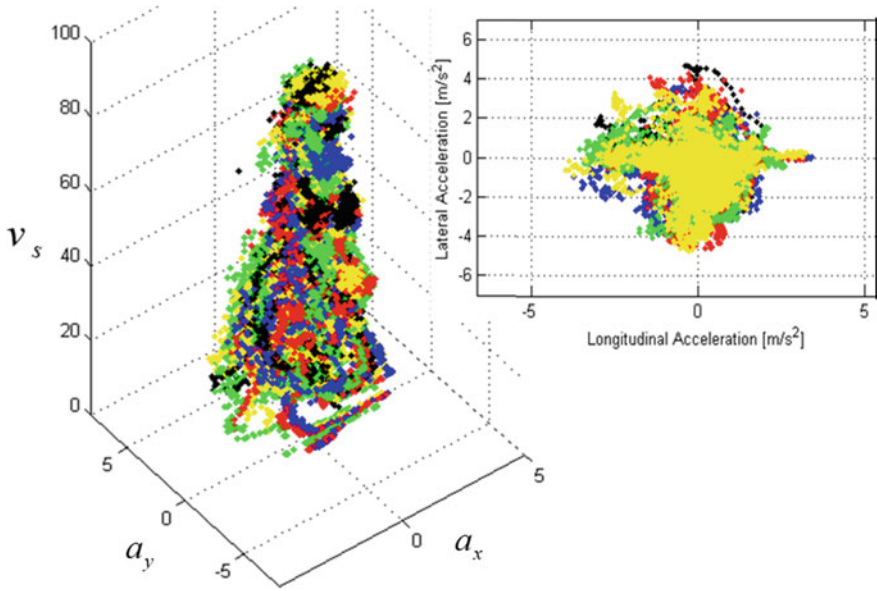


Fig. 83 Limit acceleration to mimic human drivers

drivers use becomes smaller as the vehicle speed increases. To mimic such driving characteristics, the constraints in acceleration are adopted as follows:

$$a_{des} = \begin{cases} a_{\max}(v_s) & \text{if } a_s > a_{\max}(v_s) \\ a_s & \text{if } a_{\min}(v_s) \leq a_s \leq a_{\max}(v_s) \\ a_{\min}(v_s) & \text{if } a_s < a_{\min}(v_s) \end{cases} \quad (39)$$

Sometimes, human drivers do not specify the desired clearance as (36) in controlling longitudinal dynamics. Rather than specifying the desired clearance, they just maintain the clearance that is comfortable for them. To mimic such driving characteristics, the simple proportional control law in (38) is modified as follows:

$$a_{des} = -k_1 \cdot e_c - k_2 \cdot (v_p - v_s)$$

$$e_c = \begin{cases} c_{des,\max} - c & \text{if } c > c_{des,\max} \\ c_{des,\min} - c & \text{else if } c < c_{des,\min} \\ 0 & \text{else} \end{cases} \quad (40)$$

The control law in (40) is to maintain the vehicle within minimum and maximum clearance $c_{des,\min}$ & $c_{des,\max}$, or graphically maintain the vehicle within the green region in Fig. 84. The minimum and maximum clearance can be designed with the collected driving data to mimic human drivers' driving characteristics as illustrated in Fig. 84.

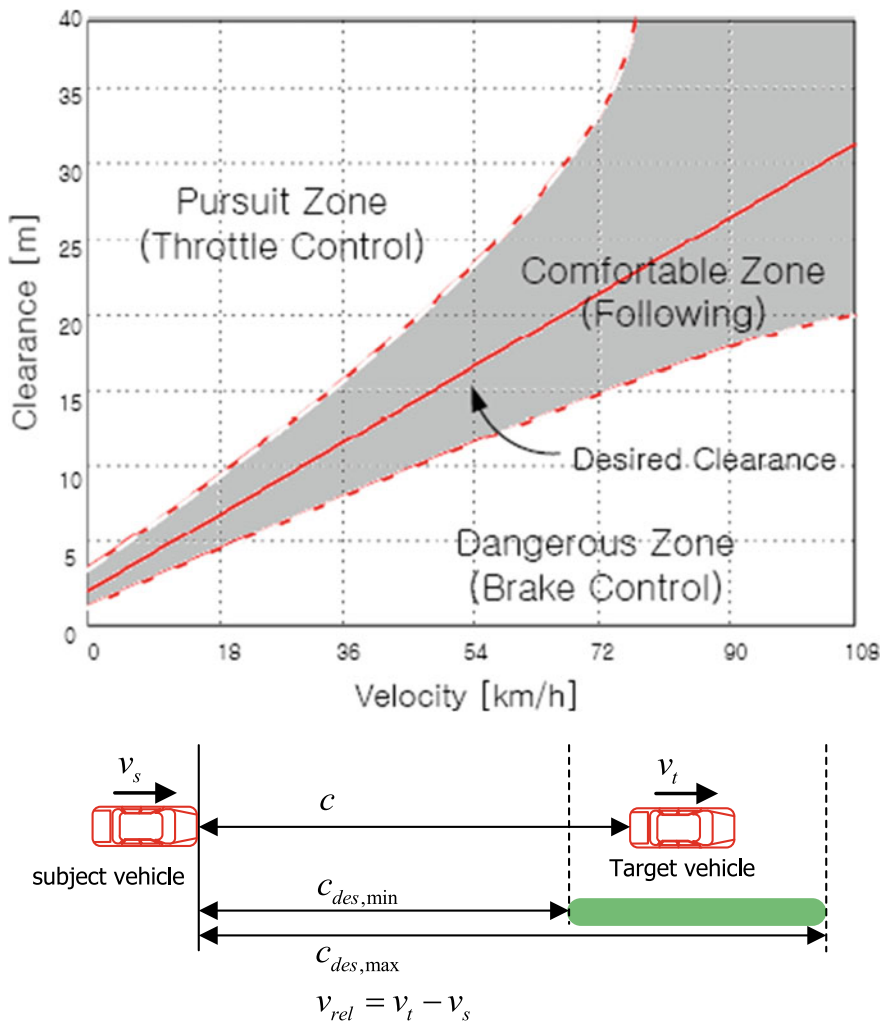


Fig. 84 Control mode and desired clearance from Yi and Moon (2004)

For more details of High level control, readers can refer to Moon, Moon, and Yi (2009).

5.3 Low Level Control: Determine Actuator Inputs

The low level control algorithm determines actuator inputs to achieve the high level control algorithm's desired acceleration. To calculate such actuator inputs, the parameters of the actuator models should be identified. However, identifying these parameters for different vehicles is almost impossible. Therefore, in this sub-section, we propose the adaptive low level control algorithm, which we call as 'Model-Free Control Algorithm'.

Model-Free Control Algorithm

The longitudinal vehicle dynamics in (35) is re-formulated as follows:

$$\dot{a}_x = \begin{cases} \frac{1}{\tau_{th}} \left(-a_x + \frac{\overbrace{k_{th}}^{\rho_{th}}}{m r_g R_{eff}} u_{th} - \frac{\overbrace{F_R}^g}{m} \right), & \text{if throttle} \\ \frac{1}{\tau_b} \left(-a_x - \frac{k_b}{\underbrace{m R_{eff}}_{\rho_b}} u_b - \frac{F_R}{m} \right), & \text{if brake} \end{cases} \quad (41)$$

In this model, ρ_{th} , ρ_b are vehicle-dependent parameters, and g is a vehicle state-dependent parameter. In the proposed algorithm, these parameters are adapted. The control law and the adaptation law are designed as follows:

$$u_i = \frac{1}{\hat{\rho}_i} \left(\tau_i \dot{a}_x + a_x + \hat{d}_g \right), \quad i \in \{th, b\} \quad (42)$$

$$\begin{aligned} \frac{d\hat{\rho}_{th}}{dt} &= -\frac{\gamma_1}{\tau_{th}} e_1 u_{th}, \quad \frac{d\hat{\rho}_b}{dt} = -\frac{\gamma_2}{\tau_b} e_1 u_b, \quad \frac{d\hat{g}}{dt} = \frac{\gamma_3}{\tau_i} e_1 \\ e_1 &= a_x - a_{x,des} \end{aligned} \quad (43)$$

To show that the overall system with the control law in (42) and the adaptation law in (43) is stable, we set the Lyapunov function that is a positive definite function of errors as follows:

$$V(e, \phi) = \frac{1}{2} e_1^2 + \frac{1}{2\gamma} \left((\hat{\rho}_i - \rho_{i,des})^2 + (\hat{g} - g_{des})^2 \right) \quad (44)$$

By taking first-time derivative of the Lyapunov function in (44) and incorporating the acceleration dynamics of (41), the designed control input of (42) and (43) into the first-time derivative, the following process is straightforward.

$$\begin{aligned}
\dot{V}(e, \phi) &= e_1 \dot{e}_1 + \frac{1}{\gamma} \left((\hat{\rho}_i - \rho_{i,des}) (\dot{\hat{\rho}}_i - \dot{\rho}_{i,des}) + (\hat{g} - g_{des}) (\dot{\hat{g}} - \dot{g}_{des}) \right) \\
&= \frac{1}{\tau_i} (-e_1^2 + e_1 e_2 u_i - e_1 e_3) + \frac{1}{\gamma} \left(\underbrace{(\hat{\rho}_i - \rho_{i,des})}_{e_2} (\dot{\hat{\rho}}_i - \dot{\rho}_{i,des}) + \underbrace{(\hat{g} - g_{des})}_{e_3} (\dot{\hat{g}} - \dot{g}_{des}) \right) \\
&= \frac{1}{\tau_i} (-e_1^2 + e_1 e_2 u_i - e_1 e_3) + \frac{1}{\gamma} \left(e_2 \left(-\frac{\gamma}{\tau_i} e_1 u_i - \dot{\rho}_{i,des} \right) + e_3 \left(\frac{\gamma}{\tau_i} e_1 - \dot{g}_{des} \right) \right) \\
&= -\frac{1}{\tau_i} e_1^2 - \frac{1}{\tau_i} e_2 \dot{\rho}_{i,des} - \frac{1}{\tau_i} e_3 \dot{g}_{des}
\end{aligned} \tag{45}$$

As described in (41), ρ_{th} , ρ_b are vehicle-dependent parameters, and g is vehicle state-dependent parameters. When these parameters do not change in time (ideal case), the results of (45) show that the first order time derivative of the Lyapunov function is negative semi-definite. Since the adaptation law in (43) is proportional to the acceleration error e_1 , it cannot be concluded that the parameter errors converge to zero. However, since the function is a negative definite function of e_1 , based on La Salle's invariance theorem, we can conclude that e_1 will converge to zero.

However, in many cases, the parameters (ρ_{th} , ρ_b and g) can be changed in time. In such cases, the proposed algorithm can maintain the error e_1 within some bounds. Let us bound the parameter errors as below, then the first time derivative of the Lyapunov function can be formulated as follows:

$$\begin{aligned}
&\text{Assumption : } |\dot{\rho}_{i,des}| < \varepsilon_1, |\dot{g}_{des}| < \varepsilon_2 \\
\dot{V}(e, \phi) &= -\frac{1}{\tau_i} e_1^2 - \frac{1}{\tau_i} e_2 \dot{\rho}_{i,des} - \frac{1}{\tau_i} e_3 \dot{g}_{des} \\
&< -\frac{1}{\tau_i} e_1^2 + \frac{1}{\tau_i} |e_2| \varepsilon_1 + \frac{1}{\tau_i} |e_3| \varepsilon_2
\end{aligned} \tag{46}$$

The results in (46) show that the error e_1 can be bounded if the parameter errors are bounded. Specifically, the error e_1 is bounded as follows:

$$|e_1| < \sqrt{|e_2 \varepsilon_1| + |e_3 \varepsilon_2|} \tag{47}$$

Vehicle test results: Low level control

The low level controller is validated via vehicle tests with Hyundai Solati as a testbed. The configuration of the testbed is illustrated in Fig. 85.

The test results are presented in Fig. 86. As illustrated in the longitudinal velocity and acceleration, the low level control algorithm well tracks the desired values. Moreover, the low level control algorithm successfully achieves the desired engine torque and deceleration by adapting the parameters as illustrated in Fig. 86.

For more details of High level control, readers can refer to Kim, Kim, Shu, and Yi (2015)



Fig. 85 Vehicle testbed: Hyundai Solati

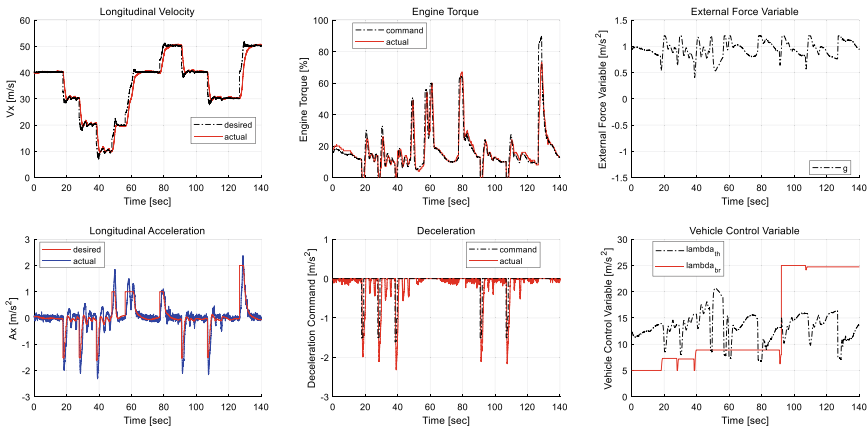


Fig. 86 Vehicle test results: low level longitudinal controller

5.4 Simulation Study

The simulation studies are conducted to investigate overall performance of the proposed algorithm. Three different cases are studied. In the first case, we assume that the control algorithm can exert an acceleration input directly on the vehicle, so the low level controller can be omitted. Therefore, the first case is for verifying the high level control algorithm only. In the second case, the integrated system (high and low level control algorithm) is investigated. In the third case, the autonomous emergency brake system is presented and tested.

Simulation study: Acceleration interface

First scenario is a simple vehicle following control scenario. The longitudinal speeds of the preceding and the subject vehicle are initially 50 km/h. Then, the preceding vehicle decreases its speed to 20 km/h with 0.3 g deceleration, and the subject vehicle subsequently reduces speed to prevent a collision. In this case, the control input of the vehicle is an acceleration so the control input of the high level controller can be

directly exerted. The simulation results are shown in Fig. 87. As illustrated in this figure, the controller can track the desired clearance and longitudinal speed of the preceding vehicle.

A more severe case is also conducted to investigate the performance of the proposed controller. The simulation results are shown in Fig. 88. As illustrated in this figure, the preceding vehicle now reduces the speed to 0 km/h; the controller can track the desired clearance and eventually the vehicle can stop without a collision.

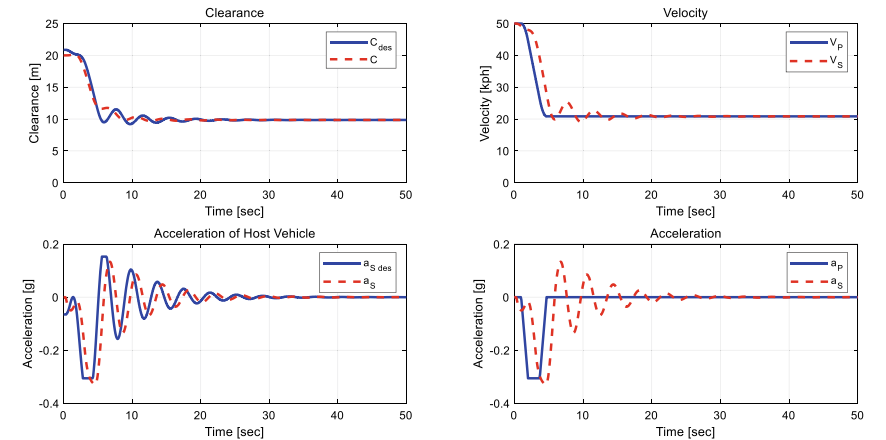


Fig. 87 Preceding vehicle deceleration (from 50–20 km/h)

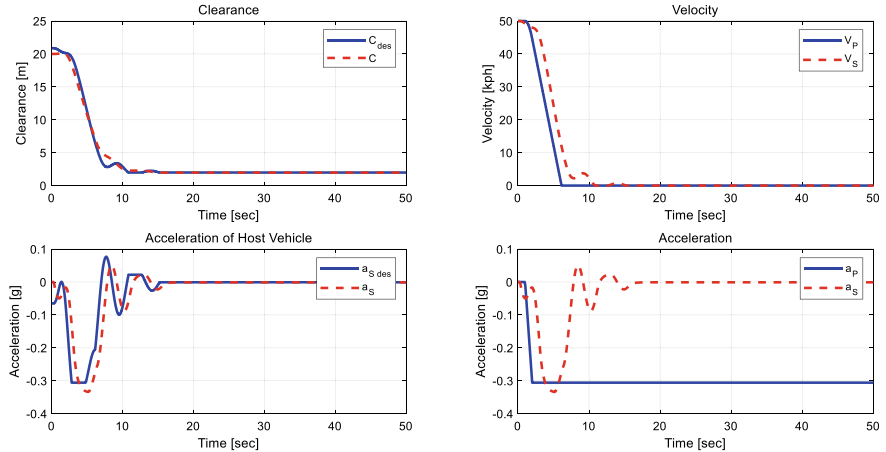


Fig. 88 Preceding vehicle deceleration (from 50–0 km/h)

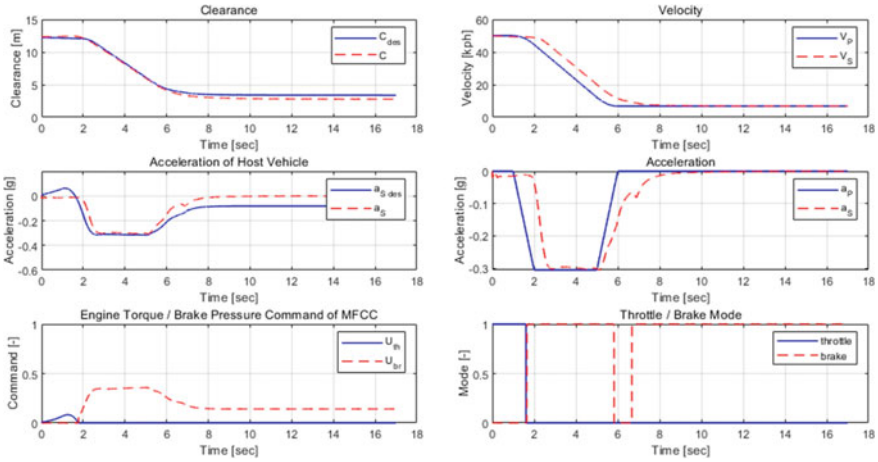


Fig. 89 Preceding vehicle deceleration (from 50–10 km/h)

Simulation study: High and Low level controller

While the first scenario is for investigating the High level controller, the second scenario is to investigate the integrated system (High and Low level controller). Now the control inputs of the vehicle are gas/brake pedal inputs. The longitudinal speeds of the preceding and the subject vehicle are initially 50 km/h. Then, the preceding vehicle decreases its speed to 10 km/h with deceleration 0.3 g and the subject vehicle subsequently reduces speed to prevent a collision. In this case, the control input of the vehicle is an acceleration so the control input of the High level controller can be directly exerted. The simulation results are shown in Fig. 89. As illustrated in this figure, the controller now shows more delayed response than the first case, but the results show that the controller can track the desired clearance and longitudinal speed of the preceding vehicle.

The proposed controller is also validated when there exists -15% road grade and the host vehicle's mass is doubled. The simulation results are presented in Fig. 90. As illustrated in this figure, the proposed control algorithm shows promising performance in this extreme case. This is because our low level controller is adaptive control algorithm that can change the parameters as the driving environment changes.

Simulation study: Autonomous Emergency Braking (AEB)

AEB is a system that can detect a potential forward collision and activate vehicle's braking system to decelerate the vehicle for avoiding or mitigating a collision. In this section, AEB control logic is tested to show the effectiveness and the necessity of AEB system. The simulation scenario is similar to the scenarios in previous subsections. The longitudinal speeds of the preceding and the subject vehicle are initially 50 km/h. Then, the preceding vehicle suddenly decreases its speed to 10 km/h with

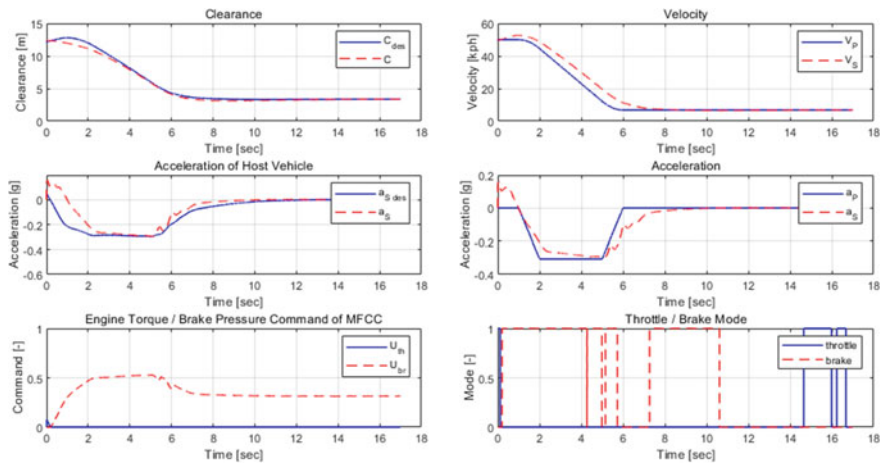


Fig. 90 Preceding vehicle deceleration on graded road (from 50–10 km/h)

large deceleration, 0.5 g, and the subject vehicle subsequently reduces speed to prevent a collision.

The simulation results are presented in Figs. 91 and 92. When the vehicle that is not equipped with AEB system, minimum clearance cannot be maintained in this large deceleration scenario as shown in Fig. 91. However, with AEB system, the vehicle can maintain the minimum clearance as illustrated in Fig. 92. Therefore, it can be concluded that the AEB system is necessary to prevent a forward collision and effective when the preceding vehicle suddenly decelerates with large deceleration.

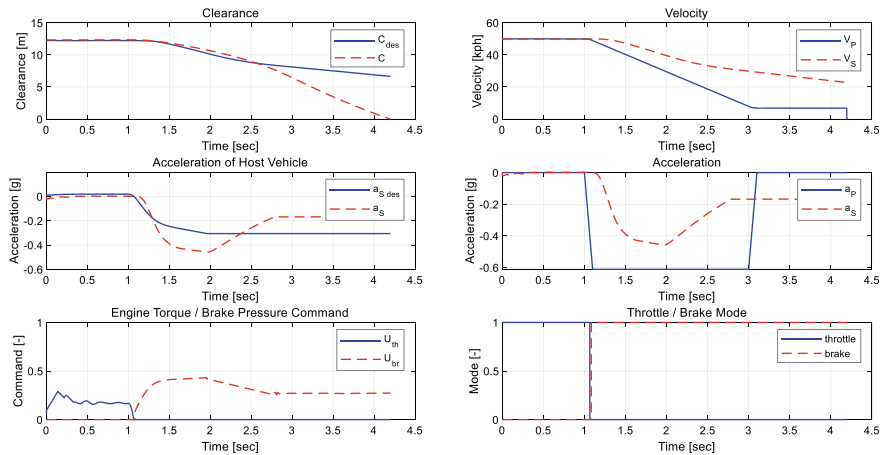


Fig. 91 Without AEB: Deceleration with high ax (from 50–0 km/h)

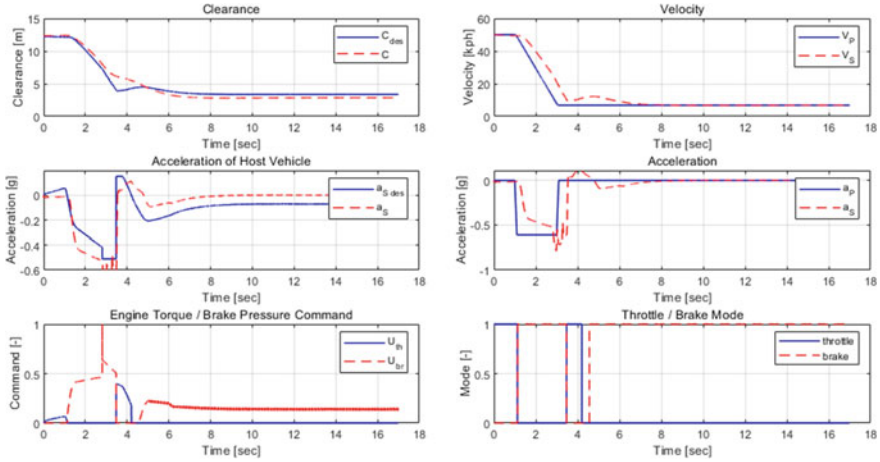


Fig. 92 With AEB: Deceleration with high a_x (from 50–0 km/h)

5.5 Conclusion

The longitudinal control algorithm for autonomous vehicles is presented in this section. The proposed control algorithm consists of high and low level control algorithms. The high level control algorithm determines the desired acceleration to track the desired clearance and speed that is designed based on the human drivers' data. The low level controller calculates the gas/brake pedal inputs by means of adaptive control algorithm. The vehicle test results show that the low level control algorithm can track the desired acceleration from the high level control algorithm. The proposed algorithm (both high and low level control algorithm) is investigated via computer simulations. The results show that the proposed algorithm can control the vehicle in longitudinal direction, and the effectiveness and necessity of the AEB system is also studied.

6 Verification and Validation

As numerous autonomous vehicles (AVs) in various stages of development have recently been demonstrated and tested on public roads (Dennis & Spulber, 2017), more attention has been paid to the safety issues. It is moreover said that there is a huge difference between developing a few autonomous vehicles to run thousands (or millions) km in reasonable benign conditions with professional drivers and producing a fleet of millions of vehicles to run in an unconstrained world (Koopman & Wagner, 2016). In other words, it is challenging to define a high-enough safety level for the autonomous vehicles to be deployed at full scale (Kalra & Paddock, 2016).

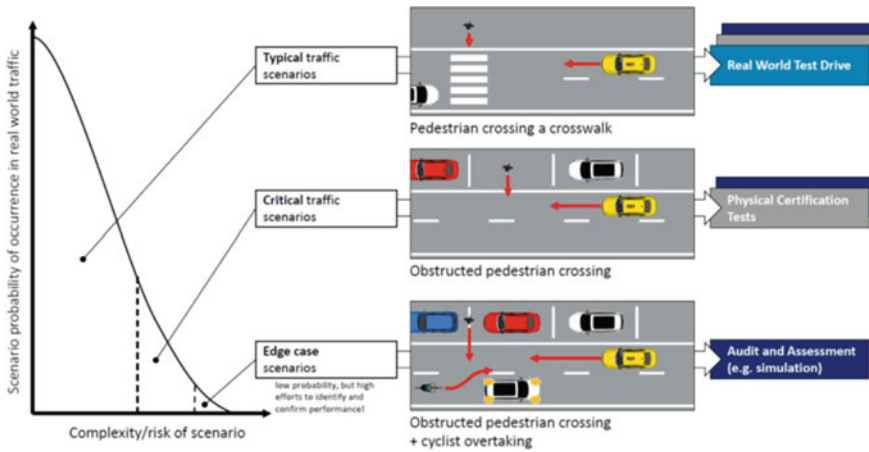


Fig. 93 Three pillars for validation and certification of AVs (*source* OICA, 2019)

New innovative technologies and methods are required to guarantee the level of safety and reliability which allows for safe market introduction of AVs. For instance, a future certification concept for AVs, so-called three pillars, is proposed: real-world test drive, physical certification tests and audit and assessment (OICA, 2019). As shown in Fig. 93, both real-world test drive and physical certification tests have been adapted for self-certification regime and new car assessment programs (NCAP) of advanced driver assistance systems (ADAS). In addition, the audit and assessment via virtual tests and simulations may be necessary to find edge case scenarios which are dependent on the various stage of development and objectives of AVs and to accelerate verification and validation (V&V) with respect to time and cost.

Combinations of ADAS systems such as lane-keep, lane-change, and adaptive cruise control seem close to highly or fully autonomous driving. However, the autonomous vehicle must have significant additional complexity to deal with unexpected emergency situations. Since there is no driver to control both wheel and brake when something goes wrong, it must be designed to a higher Automotive Safety Integrity Level, or ASIL (international organization for standardization (ISO) 26,262-3, 2011). The unexpected situations, i.e., the edge case scenarios in Fig. 93, are rare/have extremely low probability and so it is hard to find them in advance. For example, according to the fatal scenario of AV introduced in Table 1, the pedestrian walked her bicycle at night east across northbound Mill Avenue, which is separated from southbound Mill Avenue by a center median containing trees, shrubs, and brick landscaping, along with signage warning pedestrians to use the crosswalk (Becic et al., 2019). The edge case scenario may be related with road infrastructure (e.g., road geometry, center median, traffic signage), legislation (e.g., jaywalking), and light condition (e.g., day/night) as well as dynamic objects (i.e., pedestrian and bicycle). Therefore, it is easily expected how fast the number of feasible scenarios will increase exponentially if all possible elements of the scenario are considered.

In consequence, it was proved in the accident that the additional complexity to deal with the unexpected situations is not safe enough and the challenging question is how to validate it in terms of process and methodology before the market introduction of AVs.

It is well-known that the V development model has been used widely for V&V and has become the reference model to form the basis of ISO 26262 (ISO 26262-2, 2011). While a process for development of requirements for a system, the software architecture (integration), and software units on the left side of the V-model, test cases are executed to validate every requirement on the level of components, subsystems, systems, and vehicle tests on the right side (see in Fig. 94). It is well known that the V framework and functional safety reflect accepted practices for ensuing automotive safety.

A more methodical approach is however required to ensure safe AVs at scale, e.g., to deal with the unexpected situations or edge case scenarios. Only if all requirements are tested with test cases and the system is verified for a required quality, the system may be released to the market. To ensure that the AV is properly tested, a sufficient number of test cases have to be executed. Determining the necessary number of test cases for the sufficient AV tests is still an open research question. There are

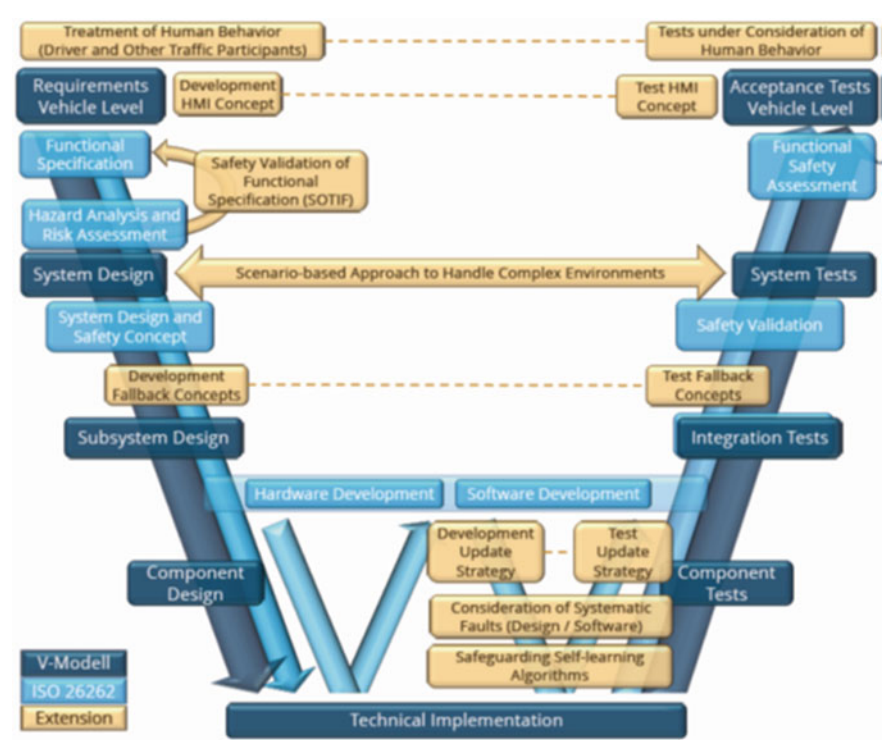


Fig. 94 Extension of functional safety (source PEGASUS Project)

several related projects shown in the literature. For instance, PEGASUS project in Germany proposed the extension of functional safety including safety of the intended functionality (SOTIF) (refer to ISO/PAS 21448, 2019 and see also in Fig. 94).

Since there are numerous technical challenges and open questions in V&V for AVs, it is necessary to focus on some specific topics in this section. First, although performance of perception of AVs is critical for safety and validation, the perception related methods for V&V are not discussed. It is in general known that mathematical sensor models for virtual environments and simulations are not accurate enough to describe uncertainty or randomness of commercial sensor characteristics, e.g., ghost target resulting from radar signal reflection, false or no measurement to stationary objects including guardrail, and false discrimination between two objects with close range-rate. In other words, it is limited to develop virtual environments for V&V of perception and sensor fusion. It is thus preferable at current stage to test them in the real-world test drive and physical certification tests (Fig. 93).

The V&V issues for decision and control of AVs are discussed here. That is, three key problems are reviewed in this section: scenario-based approach, adaptation of deep learning, and accelerated evaluation.

6.1 Scenario-Based Approach

The NCAP test procedures for many of ADAS systems were standard tests, i.e., the test conditions are predefined equally for all vehicles. The test scenarios were in general derived through statistical analysis of traffic accidents, which may include either critical or edge case scenarios. Moreover, the recent tests were customized to consider the fact that not all vehicles are created with the same performance specifications. For example, the customized rollover test is designed to consider differences on steering system gear ratio, tire friction limit, and roll dynamics (Peng & Leblanc, 2012). It is inferred that the test and validation for AVs need to be not only customized, but also based on the *worst-case* philosophy: verification and validation (V&V) in the worst-case philosophy must generate the worst-case concrete scenarios to explore the weakness of AVs in an iterative and/or probabilistic fashion within a specified operational design domain (ODD).

In the systematic validation method of Schuldt et al. (2018), the test scenario generation for ADAS has four steps: analysis of the system, systematic test scenario generation, test case execution, and test case evaluation. Especially for the systematic test scenario generation, definition of the scenario or a scenario model is prerequisite before generating the test scenarios. The definition of scenarios began with a series of research on pre-crash scenarios (IGLAD, 2019; Najm & Smith, 2007; Najm et al., 2007). Then, its extensions were adapted to cooperative ADAS (Najm et al., 2013a, 2013b) and AVs. As shown in Fig. 95, either 4-layer (Schuldt et al., 2018) or 5-layer models in the PEGASUS project have been proposed. Furthermore, the standard for a complete set of scenarios, the so-called OpenSCENARIO 2.0, has been proposed by the Association for Standardization of Automation and Measuring systems (ASAM)

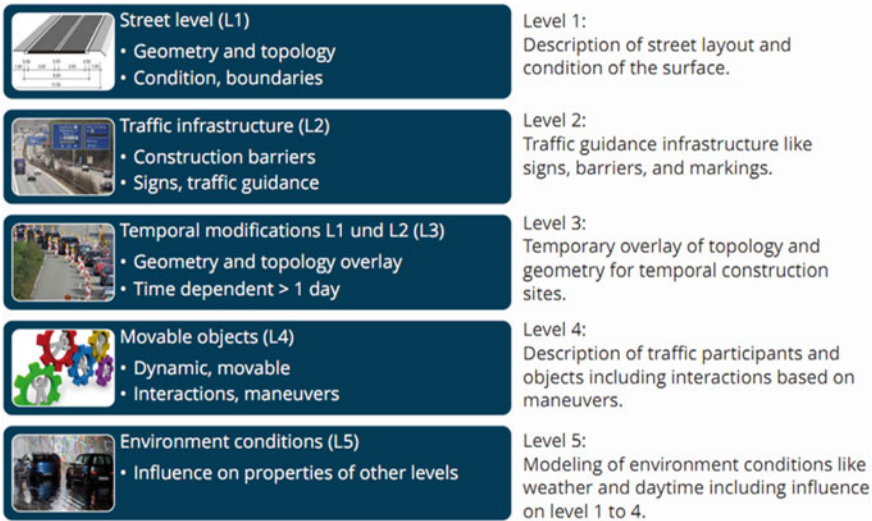


Fig. 95 5-layer model for scenarios (source PEGASUS Project)

(Amid et al., 2020). It is interesting to remark that the number of components in each model layer is directly related to the number of scenarios coming from their combinatorial generation. This problem will be discussed later in Sect. 6.2.

Once a scenario model is chosen, a set of parameters (or parameter space) need to be specified and they are also related to the number of test scenarios. Following terminologies of the *logical* and *concrete* scenario referred to the PEGASUS project (Winner et al., 2019), the logical scenario is based on the scenario model in Fig. 95 and the concrete scenario is derived from parameter space for the given logical scenario. The scenario categories are shown in Table 6. It is easily expected that the combinatorial explosion of test scenarios is getting worse if both scenario model layer and parameter space are considered simultaneously.

The next question is therefore how to select “appropriate” test scenarios among numerous ones since the consideration of all combination of possible scenarios is impractical. The appropriate scenarios should reflect the *worst-case* philosophy mentioned earlier in terms of completeness. The completeness is aimed to generate the worst-case scenarios to explore the weakness of AVs. It thus results in systematic methods to improve computation efficiency dramatically. In the level of functional and logical scenarios, the probabilistic concept shown in Fig. 93 is used widely rather than combinatorial scenarios based on the scenario model in Fig. 95.

Three test scenario cases based on the probability are listed in Table 6: typical, critical, and edge cases. The typical scenario case is most popular and in general based on naturalistic field operational test (N-FOT) or real-world test drive data. Virtual environments, which are similar to real road infrastructure, are generated and virtual mileage accumulation is conducted in this environment. More details will be discussed later in the accelerated evaluation. The choices of critical scenarios

Table 6 Scenario categories with respect to probability and level of description

Description	Probability		Critical	Edge case
	Typical			
Functional	• N-FOT – Car-following – Cut-in – Vulnerable road users (VRU)		• Statistical data for NCAP • Standards and guidelines • Accident data • iGLAD	• Accident cases of ADAS • Manual transition cases under automation • AV accidents cases
Logical				
Concrete ^a	Stationary condition - ego = 40 km/h - other = 20 km/h - Offset = 3.7m to left - Length = 15 m - Shape = sinusoidal Trigger - distance = 20m 			

^asource OpenSCENARIO 2.0 (Amid et al., 2020)

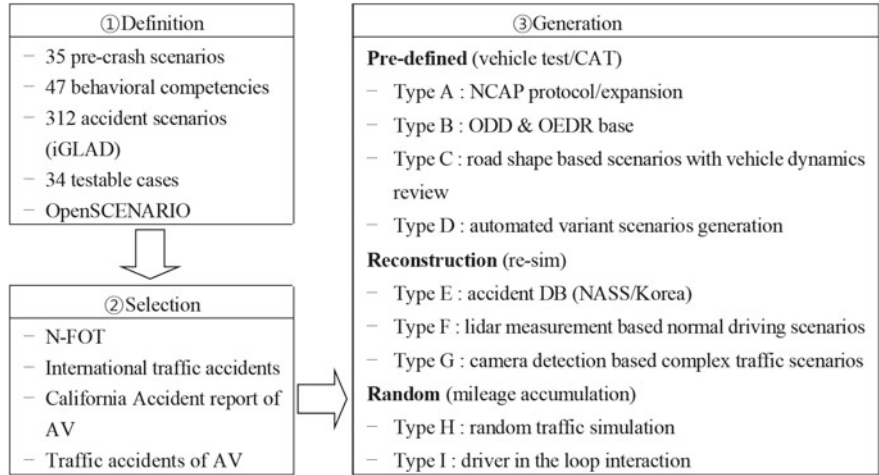


Fig. 96 A systematic process for scenario generation

are primarily based on the crash databases. Any driver is likely to see dangerous or crashed events on the road. A large fleet of vehicles will be likely to experience all such types of events and perhaps more. Combinations of adverse events and driving conditions are simply too numerous to be enumerated in a classical written requirements specification. The test matrix approach in the literature is based on statistical analysis of traffic accidents for critical scenario selection (Zhou & Re, 2017, Lee et al., 2020a, 2020b). For example, based on fatality or severity of the accident data, they are categorized into the predefined logical scenarios, e.g., pre-crash scenarios (see also in the definition in Fig. 96), iGLAD code (see also in the scenario 601 and 321 in Table 6), and their frequency of occurrence is analyzed to select critical scenarios (Lee et al., 2020a, 2020b). It is however noted that the accident or critical scenarios are based on accidents under manual driving, not autonomous driving.

It is hardest to find the edge cases (or events) and they are dependent on software as well as hardware of AVs. While the systematic methods are seldom introduced in the literature, the accident reports are highlighted to assess safety performance of AVs. As listed in Table 6, ADAS or L1/L2 automation systems have been introduced in the market, and some fatal accidents (e.g., ones of Tesla Autopilot) were informed on the news. Another well-known report is a California accident report of AV which is published annually and all AV-related accidents (e.g., Waymo’s accident with a bus) are reported. Other fatal accidents of the AV (e.g., Uber’s accident in Table 1) are open to public and gathered internationally. Therefore, it is a key challenge to find the edge case scenario in the complexity of a scenario model and the variance of software and hardware of AVs.

Next, in the level of concrete scenarios, the systematic test scenario generation is divided into three approaches (see also in Fig. 96): pre-defined, reconstruction, and

random methods (Yoo, 2019). Pre-defined scenario generation approach generates only limited scenarios (e.g., constant velocity or stopped vehicle) according to ADAS systems (EuroNCAP, 2020). The reconstruction approaches generate and illustrate the virtual driving scenarios based on real driving data with lidar and camera (Van der Made et al., 2015; Wagener & Katz, 2018). Finally, the random methods accumulate the massive scenarios to search the collision situations with Monte Carlo simulation or deep learning-based approaches (Zhao, 2016; Jenkins et al., 2018). Before introducing the approaches, the definition of concrete scenario is required and its example for cut-in maneuver based on OpenSCENARIO 2.0 is shown in Table 6. It is noted in the proposed standard that the parameter space has both stationary condition and trigger for a set of *concrete* scenarios in the given cut-in (*logical*) scenario. While the analysis of the system for concrete scenario generation aims at identifying parameters, which have a significant impact on the system under test, more detailed analysis is required for AVs. For example, definition of ODD, which determines how detailed is the ODD, is closely related to complexity level of scenario as well as the number and range of parameters in the parameter space.

Finally, it may be necessary to propose a key performance index (KPI) to identify the significant impact parameters and to measure the coverage and relevance of the generated concrete scenarios. For instance, if a set of concrete scenarios becomes the reduced set to improve computational efficiency, it is required to make sure that the reduced set is *relevant* or *equivalent* in the sense that the reduced set includes all possible critical scenarios in the original set with respect to certain KPI.

6.2 Accelerated Evaluation

The evaluation methods to accelerate the procedure are categorized into three-folds: test matrix approach, worst-case scenario evaluation, and Monte Carlo simulations (Zhao, 2016). The test matrix method is well-known to have a procedure that is repeatable, reliable and can be finished relatively quickly. As shown in Table 9, the scenarios are usually selected based on crash databases in which most of the crashes were caused by drivers. For instance, based on 10,364 records in second strategic highway research program (SHRP) 2 naturalistic driving study (NDS) database, 637 critical events on highway are selected and 99% of them (631 events) are covered with the 21 test scenarios, the so-called test case catalogue (Zhou & Re, 2017). Then, simulations for the concrete scenarios with reasonable range of parameters are performed to find safety performance border (or limit) as shown in Fig. 97.

Once the test (i.e., logical) scenarios are selected, it is necessary to generate the concrete scenarios for simulations systematically, while the parameter space may be designed by knowledge of experts. The three-step generation processes are proposed in Schuldt et al. (2018): equivalence class generation, boundary value analysis, and combinatorial test case generation. These processes are based on black-box test methods which do not use the internal structure or the source code of the test algorithm. In the equivalence class generation, one of main challenging problems is

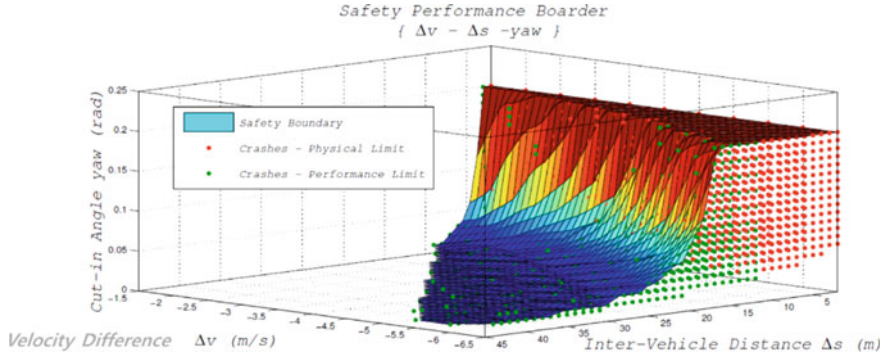


Fig. 97 Safety performance limit of the cut-in concrete scenarios (Zhou & Re, 2017)

that there is a chance that a value between two discretized parameters may generate a failure in the test object. However, finding an effective and efficient method for the discretization of parameters and the subsequent collection of discretized parameters in equivalence classes is still an open question and is also a domain specific question (Schuldt et al., 2018). For example, six parameters in stationary conditions and trigger are defined and assigned in the cut-in scenario in Table 6. The equivalence class generation is used to discretize the impact parameters into discrete values in and out of the range of the specification. A more specific example with the impact parameter space will be discussed in Sect. 6.3.

The boundary value analysis uses the limit values of a class to find a representative test case. It allows us to select the relevant values of the impact parameters of the concrete scenarios for validation and evaluation. However, the combinations of remaining parameter values still generate a large number of possible test cases. Suppose *pair-wise* combinations are applied to six parameters ($N = 6$) with certain range. If the parameter is discretized with S counts in the range, the total number of concrete scenarios is calculated as follows (Tatar, 2018):

$$S^N = \text{dist}_{\text{EOLC}} \times a_{\text{cut-in}} \times \Delta t_{\text{cut-in}} \times \text{offset} \times v_{\text{Ego}} \times v_{\text{TSV\#1}}$$

where $\text{dist}_{\text{EOLC}}$ is longitudinal distance of ego and target vehicle when the cut-in is initiated, $a_{\text{cut-in}}$ is longitudinal acceleration of target vehicle, $\Delta t_{\text{cut-in}}$ is the completion time of the cut-in, the offset is the lateral distance between ego and target vehicle, and v_{Ego} and $v_{\text{TSV\#1}}$ are velocity of ego and target vehicle, respectively. If it is assumed that $S = 10$, the number of simulations for the concrete scenarios is 10^6 . Moreover, if 10 s per simulation are needed, the simulation time required for single logical scenario will reach up to 2,777 h. This is a reason why cloud-based simulation platforms are introduced recently to accelerate 10^3 – 10^4 times faster by running the simulations simultaneously.

Besides the computational acceleration methods, the combinatorial test case generation, which is the last step in the systematic scenario generation above, aims

to reduce the number of possible concrete scenarios (i.e., the *pair-wise* combination). Some generic algorithms are introduced in the literature: orthogonal arrays or covering arrays (Grindal et al., 2005), In Parameter Order General (IPOG) to satisfy *t-wise* combinations (or coverage) (Lei et al., 2007), machine learning (Lee et al., 2020a, 2020b). It is however interesting to remark that since all the test (logical) scenarios based on crash databases may not reflect the safety-critical scenarios for AVs (Peng & Leblanc, 2012).

For the second category in the evaluation methods, the worst-case scenario evaluation was proposed to identify the truly challenging scenarios for any vehicle with or without active control systems based on a dynamic game theory and optimization techniques (Peng & Leblanc, 2012). Similarly, with consideration of sensor and actuator faults, the worst-case scenario was evaluated on upper bound of performance degradation of AV in a term of a quadratic function calculated by a convex optimization technique (Song & Hedrick, 2011). It is noted that the worst-case scenario approach is limited in applications when the test algorithms are not available in analytical or numerical form (e.g., black-box model) or complex (e.g., with millions of lines of code) (Zhao, 2016).

While the worst-case scenario approach can identify the weakness of a vehicle and vehicle control systems, it did not consider the probability of the occurrence of the worst-case scenarios. The Monte Carlo simulations, based on stochastic models with the probability of the occurrence, had been used to evaluate collision warning and collision mitigation technologies (Lee, 2004, Yang & Peng, 2010). Since both scenarios and models are in general extracted from naturalistic driving database, the non-safety-critical scenarios will be dominant thus this approach cannot accelerate the evaluation. (Bucklew, 2004). In the Kalra and Paddock (2016), a question such as “how many miles would autonomous vehicles have to be driven to demonstrate their failure rate to a particular degree of precision?” is given, and a statistical method is applied with the given specifications (e.g., a failure rate (λ_*) is based on a true fatality rate of 1.09 per 100 million miles, a desired degree of precision is given as $\delta = 20\%$, and $100(1 - \alpha)\%$ confidence interval (CI) is 95%) to calculate the required millage as follows (Mathews, 2010):

$$\chi = \frac{(z_{\alpha/2}/\delta)^2}{\lambda_*} = \frac{(1.96/0.2)^2}{1.09 \times 10^{-8}} = 8,811,009,174$$

where $z_{\alpha/2}$ is the $100(1 - \alpha/2)$ th quantile of standard normal distribution. This is approximately 8.8 billion miles and is one of reasons why the accelerated evaluation is needed in the virtual environments. Recently a series of papers for the accelerated evaluation are published with respect to different scenarios and stochastic models (Zhao, 2016, Zhao et al., 2017, Zhao et al., 2018). Basically, based on the naturalistic-field operational test (N-FOT), which is tested under naturalistic driving conditions, the stochastic model for a specific driving scenario is built for Monte Carlo simulations. Second, the disturbance statistics are skewed to emphasize the safety-critical scenarios which are rare in the crude Monte Carlo method. Then,

importance sampling is used to skew back the results to understand crash rate in the naturalistic driving conditions. While more technical details may be referred to the series of papers above, it is also remarked that a logical scenario (e.g., car-following, lane-change) is given for the accelerated evaluation.

6.3 The Case Study: Automatic Emergency Braking System

This case study is aimed to show how to follow the systematic process for critical scenario generation in Sects. 6.1 and 6.2. Suppose an automatic emergency braking system (AEB) is implemented and the goal is to generate critical test scenarios for the AEB. First, to define a logical scenario, the scenario classification based on iGLAD codebook is used. Second, to choose the critical scenarios as described in Table 6, the fatal accidents due to vehicle-to-vehicle collision on Korean road in 2012–2014 were analyzed as shown in Table 7. 1,370 accident cases out of 1,726 are classified into four collision types (from C1 to C4 in the table) and classified in detail as 15 pre-crash scenarios based on iGLAD (2019). For more clear description of the case study, it is assumed that the operation design domain (ODD) is defined as follows:

- ODD 1: AEB is not activated to avoid collision of oncoming vehicles.
- ODD 2: AEB is not activated during lane change of ego vehicle.

In addition, if low frequency scenarios (e.g., less than 1% frequency in Table 7) are not considered, 4 logical scenarios (in C3 and C4 collision type) out of 15 ones become candidates for the concrete scenario generation. It is interesting to remark that test scenarios in the C1 and C2 type, which are the collision with oncoming vehicle and lane-change in Table 7, may be used for extended or advanced functions of AEB or collision avoidance. For example, an ODD monitoring, which determines

Table 7 Statistics of vehicle-to-vehicle collision scenarios (Lee et al., 2020a, 2020b)

Collision type		iGLAD	Freq. (%)	
C1	Collision with oncoming vehicle	681/682 543	16.1/12.7 <1.0	 
C2	Collision with vehicle which turns into or crosses a road	635/646 722/721/561/571/645	2.1/1.2 <1.0	 
C3	Collision with vehicle moving ahead or waiting	601	16.9	
C4	Collision with vehicle which starts, stops or is stationary	621/501/741 701	8.7/5.8/4.4 <1.0	  

whether the AV stays within the given ODD, is a prerequisite and can be tested via a set of C1 and C2 type scenarios as false positive or true negative ones. It is also noted that additional scenarios may be added based on other traffic accident analysis, e.g., a test case catalogue based on SHRP 2 NDS (Zhou & Re, 2017). While the frontal collision accidents classified as the scenario 601 are the most frequent accident scenarios as listed in Table 7, more detailed class of the so-called *test scenario suite* can be derived for verification and validation.

For example, a fatal accident is described in Table 8a and it is known that there was the cut-in maneuver just before the frontal collision. Furthermore, based on the GPS information, the road and traffic infrastructure, i.e., the level 1 and 2 in the scenario model of Fig. 95, can be identified by map matching (see also the pictures in Table 8b). While the scenario 601 is called a base scenario, which is used as a test scenario for NCAP of AEB (EuroNCAP, 2020), the test scenario suite can be defined to describe its variances from 601-b to 601-g as shown in Table 8c. It is noted that the scenarios from 601-b to 601-e come from the Korean accident database in traffic accident analysis system (TAAS) and both 601-f and 601-g are added based on the fatal accident reports of ADAS (e.g., ones of Autopilot) (Thorn et al., 2018). If the fatal accident in Table 8 is finally classified as 601-b, its concrete scenarios can be described in the framework of OpenSCENARIO in the last column of Table 6.

Next, the virtual testing environment is implemented using IPG CarMaker and MATLAB. The concrete test scenarios are generated by MATLAB and synchronized with CarMaker through CM4SL (IPG automotive, 2014). A vehicle model and an AEB system in the CarMaker are chosen respectively for simulations. The virtual testing environment corresponding to the fatal accident in Table 8a is shown in Table 8d. For the simulation of the concrete scenarios, suppose the parameter space is given as Table 9 and the five samples are selected in the range of six parameters. Then, the number of the generated concrete scenarios becomes $S^N = 5^6 = 15,625$ and various cut-in trajectories are thus considered as shown in Fig. 98. However, that the concrete scenarios increase exponentially, i.e., combinatorial explosion, when the number of parameters and samples increase linearly.

Table 8 Scenario categories with respect to collision with vehicle moving ahead

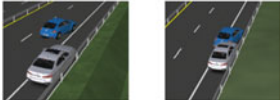
	(a) Fatal accident description	(b) Road scene and scenario
C3	#1 vehicle (A) drives on the shoulder of 2 lane road. #2 vehicle (B) changed the lane from 2nd lane to the shoulder. The #1 vehicle (A) is braked immediately and crashed into the left rear corner of the #2 vehicle (B)	
	(c) Critical/edge case	(d) Virtual environment
		

Table 9 Parameter space for AEB in cut-in (601-b) scenario

	Param	Range (unit)	Param	Range (unit)
SC	v_{Ego}	[30 110] (km/h)	$v_{TSV\#1}$	[30 110] (km/h)
TR	$dist_{EOLC}$	[10 110] (m)	$offset$	[$w/2$ $3w/2$] (m)
	a_{cut-in}	[−9.81 3.92] (m/s^2)	Δt_{cut-in}	[1 5] (s)

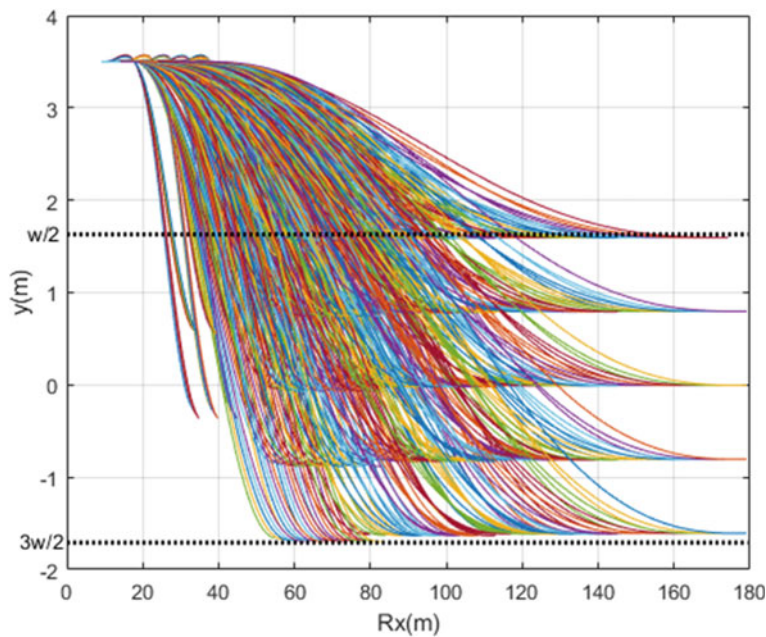
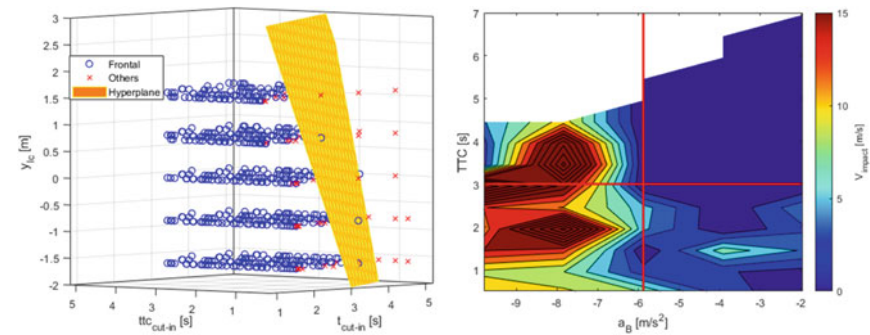


Fig. 98 Cut-in trajectories of vehicle B in AEB in cut-in (601-b) scenarios

To accelerate the validation process, scenario classification is applied to reduce the number of concrete scenarios for testing. In this example, the support vector machine (SVM), which is one of well-known machine learning algorithms, is used to classify between frontal collision and others for finding unnecessary combinations in the lateral parameter space (see also in Fig. 99) and between safe and unsafe (or crash) scenarios in the longitudinal parameter space (Lee et al., 2020a, 2020b). Finally, the initial number of the concrete scenarios is reduced to 10,886 after omitting the unnecessary combinations in the lateral parameter space, and 3,307 after reducing ones in the longitudinal and lateral parameter space. It is summarized in Table 10 that about 80% of safe scenarios and other collision scenarios are omitted for simulations while the critical concrete scenarios (i.e., unsafe frontal collision scenarios) are almost preserved. It is known that other machine learning or deep learning algorithms may be applied to preserve the critical concrete scenarios better. It is concluded in the example that the proposed validation process from scenario



(a) Parameter search of lateral parameter space via SVM (b) Parameter search of longitudinal parameter space via SVM

Fig. 99 Parameter search of parameter space via SVM

Table 10 KPI for AEB in cut-in (601-b) scenarios

	Concrete scenarios	Safe		Unsafe	
		AEB on	AEB off	Frontal collision	Others
Original	15,625	2,252	12,219	1,037	11
Lateral parameter space search	10,886	2,250	7,538	1,023	75
Test scenario suite	3,037	1027	923	1,012	75

generation to virtual testing via scenario search may be effective and efficient to find critical ones for the given AEB, i.e., frontal collision ones in Table 10.

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