Finite Model Theory on Tame Classes of Structures

Anuj Dawar

University of Cambridge Computer Laboratory, William Gates Building,
J.J. Thomson Avenue, Cambridge, CB3 0FD, UK
Anuj.Dawar@cl.cam.ac.uk

Abstract. The early days of finite model theory saw a variety of results establishing that the model theory of the class of finite structures is not well-behaved. Recent work has shown that considering subclasses of the class of finite structures allows us to recover some good model-theoretic behaviour. This appears to be especially true of some classes that are known to be algorithmically well-behaved. We review some results in this area and explore the connection between logic and algorithms.

1 Introduction

Finite model theory is the study of the expressive power of various logics—such as first-order logic, second-order logic, various intermediate logics and extensions and restrictions of these—on the class of finite structures. Just as model theory is the branch of classical mathematical logic that deals with questions of the expressive power of languages, so one can see finite model theory as the same study but carried out on finite interpretations. However, finite model theory is not simply that as it has evolved its own specific methods and techniques, its own significant questions and a core of results specific to the subject that all make it quite distinct from model theory. These methods, questions and results began to coalesce into a coherent research community in the 1980s, when the term *finite model theory* came into common use. The core of the subject is now well established and can be found in books such as [17,33,23]. Much of the motivation for the development of finite model theory came from questions in computer science and in particular questions from complexity theory and database theory. It turns out that many important questions arising in these fields can be naturally phrased as questions about the expressive power of suitable logics (see [1,29]). Moreover, the requirement that the structures considered are available to algorithmic processing leads to the study of such logics on specifically finite structures. Such considerations have provided a steady stream of problems for study in finite model theory.

In his tutorial on finite model theory delivered at LICS in 1993, Phokion Kolaitis [31] classified the research directions in finite model theory into three categories that he called *negative*, *conservative* and *positive*. In the first category are those results showing that theorems and methods of classical model theory

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fail when only finite structures are considered. These include the compactness theorem, the completeness theorem and various interpolation and preservation theorems. In the second category are results showing that certain classical theorems and methods do survive when we restrict ourselves to finite structures. One worth mentioning is the result of Gaifman [22] showing that any first-order sentence is equivalent to a Boolean combination of local sentences. This has proved to be an extremely useful tool in the study of finite model theory. A more recent example in the vein of conservative finite model theory is Rossman's result [38] that the homomorphism preservation theorem holds in the finite (a topic we will return to later). Finally, the third category identified by Kolaitis is of results exploring concepts that are meaningful only in the context of finite structures. Among these are work in descriptive complexity theory as well as 0-1 laws.

Much early work in finite model theory focussed on the negative results, as researchers attempted to show how the model theory of finite structures differed from that of infinite structures. The failure of compactness and its various consequences led to the conclusion that the class of finite structures is not model-theoretically well behaved. Indeed, Jon Barwise once stated that the class of finite structures is not a natural class, in the sense that it is difficult to define (in a formal logic) and does not contain limit points of sequences of its structures. However, recent work in finite model theory has begun to investigate whether there are subclasses of the class of finite structures that may be better behaved. We call such classes *tame*. It is impossible to recover compactness in any reasonable sense in that any class that contains arbitrarily large finite structures but excludes all infinite ones will not have reasonable compactness properties. Thus, interesting subclasses of the class of finite structures will not be natural in the sense of Barwise, but as we shall see, they may still show interesting model-theoretic behaviour. The subclasses we are interested in are motivated by the applications in computer science. It is often the case in a computational application where we are interested in the expressive power of a logic that the structures on which we interpret the logic are not only finite but satisfy other structural restrictions. Our aim is to understand how such restrictions may affect the model-theoretic tools available.

Preservation Theorems. Consider classical preservation theorems, which relate syntactic restrictions on first-order formulas with semantic counterparts. A key example is the extension preservation theorem of Łoś and Tarski which asserts that a first-order formula is preserved under extensions on all structures if, and only if, it is logically equivalent to an existential formula (see [27]). One direction of this result is easy, namely that any formula that is purely existential is preserved under extensions, and this holds on any class of structures. The other direction, going from the semantic restriction to the syntactic restriction makes key use of the compactness of first-order logic and hence of infinite structures. Indeed, this direction is known to fail in the case of finite structures as it was shown by Tait [41] that there is a first-order sentence whose finite models are closed under extensions but that is not equivalent *on finite structures* to an existential sentence. Thus, we can consider the extension preservation question relativised to a class of structures $\mathcal{C}$ as: if a first-order sentence φ is preserved under extensions on $\mathcal{C}$, is it equivalent on $\mathcal{C}$ to an existential sentence? If we replace $\mathcal{C}$ by a class $\mathcal{C}'$ that is contained in $\mathcal{C}$, we are weakening both the hypothesis and the consequent of the question. Thus, one cannot deduce the truth or otherwise of the preservation theorem on $\mathcal{C}'$ from that on $\mathcal{C}$. The question arises anew for every class $\mathcal{C}$. The extension preservation theorem for various classes of finite structures $\mathcal{C}$ is explored in [3].

A related preservation result of classical model theory is the *homomorphism preservation theorem* which states that a first-order formula is preserved under homomorphisms on all structures if, and only if, it is logically equivalent to an existential positive formula. For many years it was an open question whether this preservation theorem was true in restriction to the class of finite structures. The question was finally settled by Rossman [38] who showed that it is indeed true in this case. This provides a rare example of a preservation theorem that sits in the *conservative* rather than the *negative* category in Kolaitis' classification of results. Once again, for every class $\mathcal{C}$ of finite structures, the question of whether the homomorphism preservation theorem holds on $\mathcal{C}$ is a new question. The preservation property is established for a large variety of classes in [4].

Descriptive Complexity. In the *positive* research direction, the most prominent results are those of descriptive complexity theory. The paradigmatic result in this vein is the theorem of Fagin [19] which states that a class of finite structures is definable in existential second-order logic if, and only if, it is decidable in NP. Similar, descriptive, characterisations were subsequently obtained for a large number of complexity classes (see [29]). In particular, Immerman [28] and Vardi [42] showed that LFP—the extension of first-order logic with a least fixed point operator—expresses exactly those classes of finite *ordered* structures that are decidable in P (a similar result is shown by Livchak [35]). Whether or not there is a logic that expresses exactly the polynomial time properties of finite structures, without the assumption of order, remains the most important open question in descriptive complexity. It was shown by Cai, Fürer and Immerman [8] that $\mathsf{LFP}+\mathsf{C}$, the extension of LFP with a counting mechanism, does not suffice. However, it turns out that on certain restricted classes of structures, $\mathsf{LFP}+\mathsf{C}$ is sufficient to express all properties in P. We will see examples of this below.

2 Tame Classes of Structures

We consider classes of finite structures defined in terms of restrictions on their underlying adjacency (or Gaifman) graphs. The adjacency graph of a structure $\mathbb{A}$ is the graph $G\mathbb{A}$ whose vertices are the elements of $\mathbb{A}$ and where there is an edge between vertices a and b if, and only if, a and b appear together in some tuple of some relation in $\mathbb{A}$. The restrictions we consider on these graphs are obtained from graph structure theory and algorithmic graph theory. They are restrictions which have, in general, yielded interesting classes from the point of view of algorithms. Our aim is to explore to what extent the classes are also well-behaved in terms of their model-theoretic properties. From now on, when we say that a class of structures $\mathcal{C}$ satisfies some restriction, we mean that the collection of graphs $G\mathbb{A}$ for $\mathbb{A} \in \mathcal{C}$ satisfy the restriction.

The restrictions we consider and their interrelationships are depicted in Figure 1.

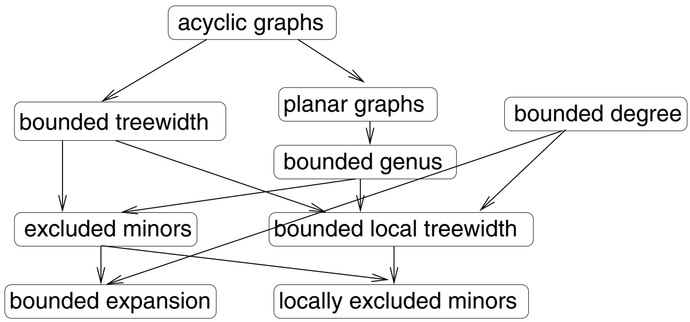

Fig. 1. Relationships between tame classes

Among the restrictions given in Figure 1, that of acyclicity and planarity are of a different character to the others in that they apply to single graphs. We can say of graph G that it is acyclic or planar. When we apply this restriction to a class $\mathcal{C}$, we mean that all structures in the class satisfy it. The other conditions in the figure only make sense in relation to classes of graphs. Thus, it makes little sense to say of a single finite graph that it is of bounded degree (it is necessarily so). When we say of a class $\mathcal{C}$ that it is of bounded degree, we mean that there is a uniform bound on the degree of all structures in $\mathcal{C}$.

The arrows in Figure 1 should be read as implications. Thus, any graph that is acyclic is necessarily planar. Similarly, any class of acyclic graphs has bounded treewidth. The arrows given in the figure are *complete* in the sense that when two restrictions are not connected by an arrow (or sequence of arrows) then the first does not imply the second and separating examples are known in all such cases.

The restrictions of acyclicity, planarity and bounded degree are self-explanatory. We say that a class of graphs $\mathcal{C}$ has bounded genus if there is a fixed orientable surface S such that all graphs in $\mathcal{C}$ can be embedded in S (see [37]). In particular, as planar graphs are embeddable in a sphere, any class of planar graphs has bounded genus. The treewidth of a graph is a measure of how tree-like it is (see [16]). In particular, trees have treewidth 1, and so any class of acyclic graphs has treewidth bounded by 1. The measure plays a crucial role in the graph structure theory developed by Robertson and Seymour in their proof of the graph minor theorem. We say that a graph G is a minor of H (written $G \prec H$) if G can be obtained from a subgraph of H by a series of edge contractions (see [16] for details). We say that a class of graphs $\mathcal{C}$ excludes a minor if there is some G such that for all $H \in \mathcal{C}$ we have $G \not\prec H$. In particular, this includes all classes $\mathcal{C}$ which are closed under taking minors and which do not include all graphs. If G is embeddable in a surface S then so are all its minors. Since, for any fixed integer k, there are graphs that are not of genus k, it follows that any class of bounded genus excludes some minor.

The notion of bounded local treewidth was introduced as a common generalisation of classes of bounded treewidth and bounded genus. A variant, called the diameter width property was introduced in [18] while bounded local treewidth is from [21]. Recall that the r-neighbourhood of an element a in a structure $\mathbb{A}$, denoted $N^r_{\mathbb{A}}(a)$, is the substructure of $\mathbb{A}$ induced by the set of elements at distance at most r from a in the graph $G\mathbb{A}$. We say that a class of structures $\mathcal{C}$ has bounded local treewidth if there is a nondecreasing function $f : \mathbb{N} \to \mathbb{N}$ such that for any structure $\mathbb{A} \in \mathcal{C}$, any a in $\mathbb{A}$ and any r, the treewidth of $N^r_{\mathbb{A}}(a)$ is at most $f(r)$. It is clear that any class of graphs of bounded treewidth has bounded local treewidth (indeed, bounded by a constant function f). Similarly, any class of graphs of degree bounded by d has local treewidth bounded by the function d^r, since the number of elements in $N^r_{\mathbb{A}}(a)$ is at most d^r. The fact that classes of bounded genus also have bounded local treewidth follows from a result of Eppstein [18].

We say that a class of structures $\mathcal{C}$ locally excludes minors if there is a nondecreasing function $f : \mathbb{N} \to \mathbb{N}$ such that for any structure $\mathbb{A} \in \mathcal{C}$, any a in $\mathbb{A}$ and any r, the clique $K_{f(r)}$ is not a minor of the graph $GN^r_{\mathbb{A}}(a)$. This notion is introduced in [11] as a natural common generalisation of bounded local treewidth and classes with excluded minors. Classes of graphs with bounded expansion were introduced by Nešetřil and Ossona de Mendez [40] as a common generalisation of classes of bounded degree and proper minor-closed classes. A class of graphs $\mathcal{C}$ has bounded expansion if there is a function $f : \mathbb{N} \to \mathbb{N}$ such that for any graph $G \in \mathcal{C}$, any subgraph H of G and any minor H' of H obtained from H by contracting neighbourhoods of radius at most r, the average degree in H' is bounded by $f(r)$. In particular, classes that exclude a minor have bounded expansion witnessed by a constant function f.

3 Logic and Algorithms on Tame Classes

The interest in tame classes of structures from the point of view of algorithms is that it is often the case that problems that are intractable in general become tractable when a suitable restriction on the structures is imposed. For instance, for any class of graphs of bounded treewidth, there are linear time algorithms for deciding Hamiltonicity and 3-colourability and on planar graphs there is a polynomial time algorithm for the Max-Cut problem. On the other hand, many problems remain hard as, for instance, 3-colourability is NP-complete even on planar graphs.

What is of interest to us here is that in many cases the good algorithmic behaviour of a class of structures can be explained or is linked to the expressive power of logics. This is especially the case with so-called *meta-theorems* that link definability in logic with tractability. Examples of such meta-theorems are Courcelle's theorem [10] which shows that any property definable in monadic second-order logic is decidable in linear time on classes of bounded tree-width and the result of Dawar *et al.* [13] that first-order definable optimization problems admit polynomial-time approximation schemes on classes of structures that exclude a minor. Also among results that tie together logical expressiveness and algorithmic complexity on restricted classes, one can mention the theorem of Grohe and Mariño [26] to the effect that $\mathsf{LFP} + \mathsf{C}$ captures exactly the polynomial-time decidable properties of classes of structures of bounded treewidth. In this section, we take a brief tour of some highlights of such results.

Acyclic Structures. To say that the adjacency graph $G\mathbb{A}$ of a structure $\mathbb{A}$ is acyclic is to say that all relations in $\mathbb{A}$ are essentially unary or binary and the union of the symmetric closures of the binary relations is a forest. One interesting recent result on such classes of structures is that of Benedikt and Segoufin [6] that any first-order sentence that is order-invariant on trees is equivalent to one without order. This contrasts with a construction of Gurevich (see [1, Exercise 17.27]) that shows that there is a first-order sentence that is order-invariant on the class of finite structures but is not equivalent to any first-order sentence without order. The theorem of Benedikt and Segoufin can be seen as a special case of interpolation. The general version of Craig's interpolation theorem (see [27]) is known to fail on the class of finite structures and even on the class of finite acyclic structures.

Another important respect in which acyclic structures are well-behaved is that while the validities of first-order logic on finite structures are not recursively enumerable, the validities on acyclic structures are decidable. Indeed, it is well-known that even monadic second-order logic (MSO) is decidable on trees (see [7] for a treatment). Moreover, by Courcelle's theorem mentioned above, we know that the problem of deciding, given a formula φ of MSO and an acyclic structure $\mathbb{A}$, whether or not $\mathbb{A} \models \varphi$ is decidable by an algorithm running in time $O(f(|\varphi|)|\mathbb{A}|)$ for some computable function f. We express this by saying that the *satisfiability* problem for the logic (also often called the *model-checking* problem) is *fixed-parameter tractable.* It has also been known, since results of Immerman and Lander and Lindell that $\mathsf{LFP} + \mathsf{C}$ captures polynomial time on trees [30,34].

Finally, it has been proved that the homomorphism and extension preservation theorems hold on the class of acyclic structures (see [4] and [3] respectively). Indeed these preservation properties hold of any class of finite acyclic structures which is closed under substructures and disjoint unions, but may fail for other subclasses.

Bounded Treewidth. Let $\mathcal{T}_k$ denote the class of all structures of treewidth at most k. It is known that many of the properties of acyclic structures that make it a well-behaved class also extend to $\mathcal{T}_k$ for values of k larger than 1. However, it is not known if the order-invariance result of Benedikt and Segoufin is one of these properties. This remains an open question. Monadic second-order logic is as tame on $\mathcal{T}_k$ as it is on $\mathcal{T}_1$ since it is known that the satisfiability problem is decidable [9] and the satisfaction problem is fixed-parameter tractable [10].

It has been shown that $\mathcal{T}_k$ has the homomorphism preservation property [4] as well as the extension preservation property [3]. The former holds, in fact, for all subclasses of $\mathcal{T}_k$ that are closed under substructures and disjoint unions, but this is not true of extension preservation. Indeed, it is shown in [3] that extension preservation fails for the class of all planar graphs of treewidth at most 4, which is a subclass of $\mathcal{T}_4$.

We have mentioned above that Grohe and Mariño [26] proved that $\mathsf{LFP}+\mathsf{C}$ captures polynomial time computation on $\mathcal{T}_k$ for any k. Recently, this has been shown to be optimal, in the following sense. For any nondecreasing function $f : \mathbb{N} \to \mathbb{N}$, let $\mathcal{T}_f$ denote the class of structures where any structure $\mathbb{A}$ of at most n elements has treewidth at most $f(n)$. Then, we can show [15] that as long as f is not bounded by a constant, there are polynomial time properties in $\mathcal{T}_f$ that are not expressible in $\mathsf{LFP}+\mathsf{C}$. Note, this does not preclude the possibility that $\mathsf{LFP}+\mathsf{C}$ capture P on subclasses of $\mathcal{T}_f$ of unbounded treewidth. Indeed, just such a possibility is realised by the result of Grohe that $\mathsf{LFP}+\mathsf{C}$ captures P on planar graphs [24] and more generally on graphs of bounded genus [25].

Bounded Degree Structures. Bounding the maximum degree of a structure is a restriction quite orthogonal to bounding its treewidth and yields quite different behaviour. While graphs of maximum degree bounded by 2 are very simple, consisting of disjoint unions of paths and cycles, structures of maximum degree 3 already form a rather rich class. That is, if $\mathcal{D}_k$ is the class of structures with maximum degree k, then the MSO theory of $\mathcal{D}_3$ is undecidable as is its first-order theory. Indeed, the first-order theory of planar graphs of degree at most 3 is also undecidable [12]. Furthermore, the satisfaction problem for MSO is intractable as one can construct sentences of MSO which express NP-hard problems on planar grids. However, it is the case that the satisfaction problem for first-order logic is fixed-parameter tractable on $\mathcal{D}_k$ for all k. This was shown by Seese [39].

The question of devising a logic in which one can express all and only the polynomial-time properties of bounded degree structures is an interesting one. The graph isomorphism problem is known to be solvable in polynomial time on graphs of bounded degree [36], and indeed, there is a polynomial-time algorithm for canonical labelling of such graphs [5]. It follows from general considerations about canonical labelling functions (see [17, Chapter 11]) that there is some logic that captures exactly P on $\mathcal{D}_k$, for each k. However, we also know, by the construction of Cai, Fürer and Immerman [8] that $\mathsf{LFP}+\mathsf{C}$ is too weak a logic for this purpose. It remains an open question to find a "natural" logic that captures P on bounded degree classes.

On the question of preservation properties, both the homomorphism and extension preservation theorems have been shown to hold, not only on $\mathcal{D}_k$, but also on subclasses closed under substructures and disjoint unions [4,3].

Excluded Minor Classes. Classes with excluded minors are too general a case for good algorithmic behaviour of MSO. This logic is already undecidable, and its satisfaction problem intractable, on planar graphs. Indeed, first-order logic is also undecidable on planar graphs. However, it has been shown that the satisfaction problem for first-order logic is fixed-parameter tractable on any class of structures that excludes a minor [20]. While the extension preservation theorem fails in general on such classes, and was even shown to fail on planar graphs [3], the homomorphism preservation property holds of all classes which exclude a minor and are closed under taking substructures and disjoint unions [4]. It remains an open question whether one can construct a logic that captures P on excluded minor classes. Grohe conjectured [25] that $\mathsf{LFP}+\mathsf{C}$ is actually sufficient for this purpose. Indeed, he proved that $\mathsf{LFP}+\mathsf{C}$ captures P on all classes of bounded genus.

Further Extensions. Frick and Grohe showed that the satisfaction problem for first-order logic is fixed-parameter tractable, even on classes of structures of bounded treewidth [21]. This result was recently extended to classes of graphs that locally exclude a minor [11] by an algorithmic analysis of the graph structure theorem of Robertson and Seymour. It is an open question whether or not it can also be extended to classes of graphs of bounded expansion. The model-theoretic and algorithmic properties of classes of graphs of bounded expansion and that locally exclude minors are yet to be studied in detail and a number of open questions remain.

4 Preservation Theorems

Among the results in the last section, we looked at classes of structures where the homomorphism and the extension preservation theorems are known to hold. Indeed, the homomorphism preservation theorem survives all the restrictions we considered, while the extension preservation is available in some. We now take a brief look at the methods used to establish the homomorphism and extension preservation theorems in the tame classes where they have been shown.

The key idea in these proofs is to establish an upper bound on the size of minimal models of a first-order sentence that has the relevant preservation property. For instance, suppose φ is a sentence that is preserved under extensions on a class of structures $\mathcal{C}$. Then, we say that a structure $\mathbb{A}$ is a minimal model of φ in $\mathcal{C}$ if $\mathbb{A} \models \varphi$ and no proper induced substructure of $\mathbb{A}$ is a model of φ. It is then immediate that the models of φ in $\mathcal{C}$ are exactly the extensions of minimal models. It is not difficult to show that φ is equivalent to an existential sentence on $\mathcal{C}$ if, and only if, it has finitely many minimal models. The same holds true for sentences preserved under homomorphisms if we take minimal models, not with respect to induced substructures, but allowing substructures that are not induced (see [4] for details). The preservation properties for tame classes mentioned above are then proved by showing that from every sentence φ we can extract a bound N such that all minimal models of φ have at most N elements. This bound is obtained by considering structural properties that a minimal model must satisfy.

It can be shown that if φ is preserved under homomorphisms on a class $\mathcal{C}$ (closed under disjoint unions and substructures) then there are positive integers d and m such that no minimal model of φ in $\mathcal{C}$ contains a set of m elements that are pairwise distance d or greater from each other. This result is essentially obtained from a construction of Ajtai and Gurevich [2] and is a consequence of Gaifman's locality theorem for first-order logic. A more involved construction, again based on Gaifman's theorem establishes this density property also for formulas preserved under extensions. An immediate consequence is the preservation theorem for certain classes we call wide. A class of structures $\mathcal{C}$ is *wide* if for all d and m, there is an N such that every structure in $\mathcal{C}$ with at least N elements contains a set of m elements that are pairwise distance at least d from each other. For instance, any class of bounded degree is easily seen to be wide.

The construction of Ajtai and Gurevich shows further that for any sentence φ preserved under homomorphisms on $\mathcal{C}$, and for every positive integer s, there are d and m such that no minimal model of φ in $\mathcal{C}$ contains a set of m elements that are pairwise distance d or greater from each other, even after s elements are removed from it. This leads to a definition of classes that are almost wide: $\mathcal{C}$ is *almost wide* if there is an s such that for all d and m there is an N such that in every structure $\mathbb{A}$ in $\mathcal{C}$ with at least N elements, one needs to remove at most s elements to obtain a set of m elements that are pairwise distance at least d from each other. A combinatorial construction is needed to prove that classes of graphs that exclude a minor are almost wide (see [4] and also [32]). Almost wideness is not sufficient in itself to establish the extension preservation property (as is witnessed by the class of planar graphs). However, we can strengthen the requirement of closure under disjoint unions to closure under unions over "bottlenecks" (see [3]) and obtain a sufficient condition. This leads, in particular, to the proof that the extension preservation theorem holds for the classes $\mathcal{T}_k$.

It is not clear if classes of structures of bounded expansion or with locally excluded minors are almost wide. However, they can be shown to satisfy a weaker condition. Say a class of structures $\mathcal{C}$ is *quasi-wide* if for all d there is an s such that for all m, there is an N such that if $\mathbb{A} \in \mathcal{C}$ has N or more elements, then there is a set B of at most s elements in $\mathbb{A}$ such that $\mathbb{A} \setminus B$ contains a set of m elements that are pairwise at least distance d from each other. It can be shown that classes of structures of bounded expansion and that locally exclude minors are quasi-wide. Furthermore, it seems that a strengthening of the Ajtai-Gurevich lemma can establish the homomorphism preservation theorem for quasi-wide classes that are closed under disjoint unions and minors [14].

5 Conclusion

The class of all finite structures is not a model-theoretically well-behaved class. Recent work has investigated to what extent considering further restricted classes may enable us to discover interesting model-theoretic properties. The restrictions that have been found that yield tame classes are also those that yield good algorithmic behaviour. The interaction between logical and algorithmic properties of these classes remains an active area of investigation. Besides preservation theorems, many model-theoretic properties of these classes remain to be explored. In the absence of the Compactness Theorem, which is the bedrock of the model theory of infinite structures, the methods used on tame classes of finite structures are varied and often combinatorial in nature. However, methods based on locality appear to play a central role.

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